The Multipole Structure of Earth's STEP Signal

Kenneth Nordtvedt
Northwest Analysis
118 Sourdough Ridge Road, Bozeman MT 59715 USA
kennordtvedt@imt.net

March 25, 2004

Abstract

If there is an interaction in physical law which differentially accelerates the test bodies in a STEP satellite, then the different elements that compose the Earth will most likely have source strengths for this interaction which are not proportional to their mass densities. The rotational flattening of Earth and geographical irregularities of our planet's crust then produces a multipole structure for the Equivalence Principle violating force field which differs from the multipole structure of Earth's ordinary gravity field. Measuring these differences yields key information about the new interaction in physical law which is not attainable by solely measuring differences of test body accelerations.

Introduction

The purpose of a Space Test of the Equivalence Principle (STEP) is to measure with extremely high precision any differences between the acceleration of different materials (elements of the periodic table). Any difference will most likely be the result of a previously undetected, long range force field in physical law, which acting between objects leads to an interaction energy between each pair of source elements of the form

$$V(\vec{r}_{ij}) \;=\; -\frac{G\,M_i M_j}{R_{ij}} \;\pm\; \frac{K_i K_j}{R_{ij}}\, exp(-\mu R_{ij})$$

with the new interaction's coupling strengths K_i being different than bodies' mass-energies; and μ being a possible Yukawa inverse range parameter related to the *mass* of the boson field particle responsible for the new interaction. If the new field is *massless* then its spatial dependence is inverse square just as Newtonian gravity. Letting $\kappa_e(\vec{r})$ be the source density in Earth pertinent to this new interaction, then two experimental test bodies would fall toward Earth with rate differences given by

$$\frac{|\vec{a}_i - \vec{a}_j|}{\vec{g}_e} \;=\; \left(\frac{K_1}{M_1} - \frac{K_2}{M_2}\right) \frac{1}{GM_e} \vec{\nabla} \int \frac{\kappa_e(\vec{r}')}{|\vec{r} - \vec{r}'|}\, d^3 r'$$

Normally a STEP experiment views its role as just measuring the different K_i/M_i ratios of different materials. This note highlights some interesting further possibilities for the experiment which depend on details of the source integral in this fundamental equation. A generalization of Newton's law of action and reaction usually manifests itself in field theory based interactions, particularly at the static limit. So if objects respond to a force field with coupling strength different than their masses, then objects generally are sources for this force field with strengths different than their masses. So a non-spherically symmetric Earth (in shape and composition) can have novel features in its Equivalence Principle violating (EPV) force field [1].

The Earth's ordinary Newtonian gravitational potential and (EPV) potential can both be expanded in spherical harmonics which reflect the deviations from perfect spherical symmetry of the Earth

$$U(\vec{r}) \quad = \quad G \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3r' \;=\; \frac{GM}{r}\left(1 + \sum_{l=2}^{\infty}\sum_{m=-l}^{m=l}\left(\frac{r_e}{r}\right)^l J_{lm}\, Y_{lm}(\theta,\,\phi)\right) \tag{1}$$

$$K(\vec{r}) \quad = \quad \int \frac{\kappa_e(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3r' \;=\; \frac{K}{r}\left(1 + \frac{\vec{d}\cdot\hat{r}}{r} + \sum_{l=2}^{\infty}\sum_{m=-l}^{l}\left(\frac{r_e}{r}\right)^l K_{lm}\, Y_{lm}(\theta,\,\phi)\right) \tag{2}$$

Three things should be observed about the expansion for the EPV potential: 1) Since the center of mass-energy has been chosen as origin of the coordinate frame, the mass dipole term vanishes in expansion for $U(\vec{r})$, but generally there will be a dipole term in the potential $K(\vec{r})$; 2) because of the rotational flattening of Earth there will be substantial quadrupole moments J_{20} and K_{20} for the two potentials. Because the Earth's core material and mantel material may havea different ratio of source strength densities for generating the EPV interaction than their mass density ratio, K_{20} will differ from the well-measured J_{20} moment of the Earth; 3) while Earth's mulitipoles other than J_{20} are very small in spite of irregular distributions of high mountains, plains, and deep ocean basins, probably due to the approximate isostacy of the Earth's crust, this isostacy will not so suppress the multipoles in the EPV potential.

Quadrupole Moments

One third the mass of Earth is in its iron-dominated core while the remaining mantle and crustal materials of Earth are composed of relatively low-Z elements. Because the Earth is rotatonally flattened as shown in Figure (1), the matter distributions will produce significant quadrupole moment parameters J_{20} and K_{20}. And because these quadrupole moment integrals for a body weight each differential of matter by the square of distance from the center, the mantle material of Earth contributes more strongly to these moments than their fractional mass fraction. If these two parts of Earth have different ratios in their strengths for producing ordinary gravity and the EPV force field, then the difference between the two quadrupole moment parameters can be estimated to be

$$\begin{aligned} 1 - \frac{K_{20}}{J_{20}} \quad &\cong \quad \frac{\kappa_{core}/\rho_{core} - \kappa_m/\rho_m}{\kappa_{core}/\rho_{core} + (M_m/M_{core})\,\kappa_m/\rho_m}\left(\frac{I_m - f\,(M_m/M_{core})\,I_{core}}{I_m + f\,I_{core}}\right) \\ &\cong \quad \frac{\kappa_{core}/\rho_{core} - \kappa_m/\rho_m}{\kappa_{core}/\rho_{core} + 2\kappa_m/\rho_m} \end{aligned}$$

with f being the ratio of the core's flattening ratio to that of the mantle. A measurement of K_{20} is seen to involve the sum of κ/ρ ratios as well as the difference. No number of STEP measurements of only differences

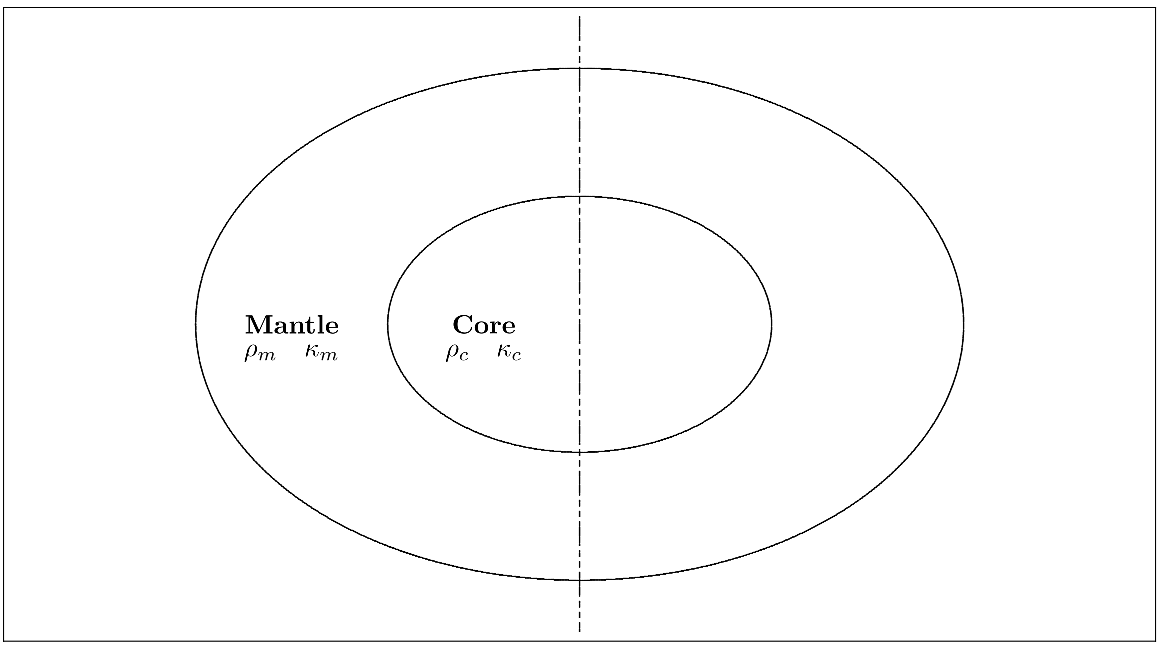

Figure 1: The rotationally flattened Earth is shown with its two main material components — core and mantle/crust. When these two components of Earth have different ratios in the source density strengths for ordinary gravity and the EPV force field, $\kappa_m/\rho_m \neq \kappa_c/\rho_c$, the quadrupole parameter K_{20} will differ from gravity's J_{20}.

in the K/M ratios will yield information on the total strength of the new interaction, so this is vital, new information.

EPV Dipole Moment

The Earth's crust is rather irregular in its composition and thickness, being just a few kilometers thick beneath deep ocean basins and tens of kilometers thick beneath high mountains and plateaus — and it is dynamic, albeit on long geologic time scales, as it tends to reach isostatic equilibrium in which the total weight of the crustal material displaces equal weight of heavier mantle material. The crust approximately floats. This explains the small size of the Earth's gravitational mulitpoles; about just the same amount of mantle mass is displaced at any location as there is crustal mass there. But if the crustal material has ratio of strengths κ_{cr}/ρ_{cr} which differs from that of the displaced mantle material, then the dipole moment parameter that appears in the potential expansion, Equation () [2], will be given by

$$\vec{d} \cong \frac{r_e^3}{K} \int \left(\kappa_{cr} - \rho_{cr} \frac{\kappa_m}{\rho_m} \right) h(\theta,\, \phi)\, \hat{r}\, d\Omega \;\cong\; 3 \frac{\kappa_{cr} - \rho_{cr}\kappa_m/\rho_m}{\kappa_m(1-\xi) + \xi\kappa_c} \int h(\theta,\, \phi)\, \hat{r}\, d\Omega/4\pi$$

with $h(\theta, \phi)$ being the location-dependent thickness of the crust (plus ocean), $\hat{r}$ being unit vector to surface location, $d\Omega$ being solid angle differential, and ξ being volume fraction of the Earth's core. If only these dipole and quadrupole modifications of the EPV force field are then considered, the field that drives differential accelerations of test bodies on the STEP spacecraft will have the form

$$\vec{a} \;=\; -\frac{K}{r^2} \left(\hat{r} \;-\; \frac{\vec{d} - 3\vec{d} \cdot \hat{r}\, \hat{r}}{r} \;+\; \frac{3}{2} K_{20} \frac{r_e^2}{r^2} \left((1 - 5\cos^2\theta)\, \hat{r} \;+\; 2\, \cos\theta\, \hat{z} \right) \right)$$

with the dipole vector $\vec{d}(t)$ being fixed with the rotating Earth and therefore presenting a time-dependence in its equatorial plane components. Measurement of the global EPV parameters, $\vec{d}$ and K_{20} will be facilitated because the signals they produce in differentially accelerating the test bodies in the STEP satellite will generally have different frequencies than the dominant monopolar EPV signal proportional to K.

The Low Altitude STEP Signal Approximation

In order to maximize the strength of the EPV signal originating from Earth's matter, the orbit of a STEP satellite will be as close to Earth as drag considerations permit. So it is appropriate to formulate a low altitude expression for the EPV signal. As illustrated in Figure 2, a satellite is considered which has altitude $\rho << r$ above the Earth's surface. A coordinate system is chosen whose pole, $x = 1 - \cos\theta = 0$, is beneath the instantaneous position of the satellite. Over the entire surface of the Earth there is an inhomogeneous source of the EPV signal proportional to the thickness of the crust, $h(x, \phi)$, and the differences between the crustal materials' EPV source strength and that of the displaced denser mantle material, $\sigma(x, \phi) = \kappa_{cr} - \kappa_m \rho_{cr}/\rho_m$. The vertical and horizontal components of the EPV signal at the satellite can then be expressed as integrals over the Earth's surface.

$$\delta k_v \;=\; \int_0^2 \frac{r^2\, (\rho \,+\, rx)}{\left(2r\,(r \,+\, \rho)\, x \,+\, \rho^2\right)^{3/2}}\, dx \,\int_0^{2\pi} \sigma(x, \phi)\, d\phi$$

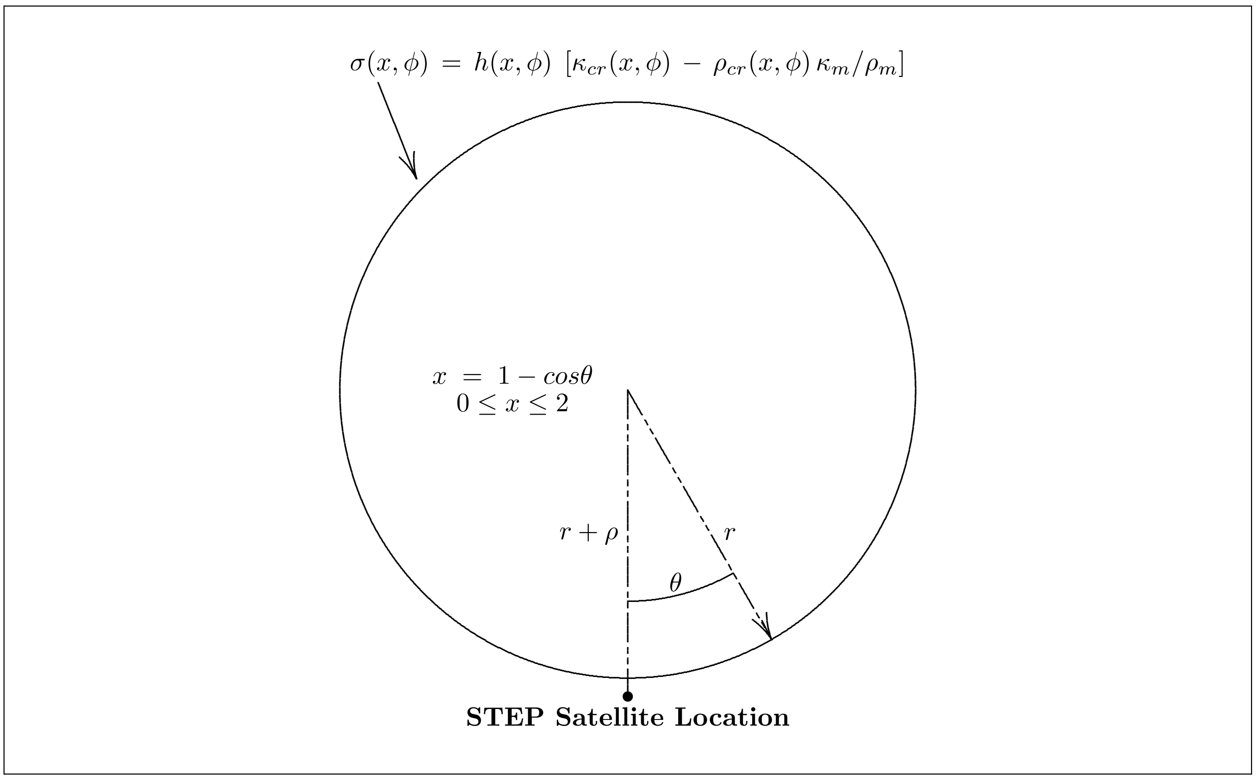

Figure 2: For a low altitude STEP satellite, the vertical component of the EPV signal is found to be in part proportional to the source strength of the new EPV interaction directly below the satellite. There is also, in general, a global contribution from the entire surface of the Earth, and there is a horizontal component of the EPV signal which is also determined by aglobal integral.

$$\delta\vec{k}_h \;=\; \int_0^2 \frac{r^3\,\sqrt{2x\,-\,x^2}}{\left(2r\,(r\,+\,\rho)\,x\,+\,\rho^2\right)^{3/2}}\;dx\;\int_0^{2\pi}\sigma(x,\phi)\,\hat{u}(\phi)\,d\phi$$

which in the limit of a low altitude satellite take the form

$$\delta k_v \;=\; 2\pi\,\sigma(0)\,+\,\int_0^2 \frac{1}{\sqrt{8x}}\;dx\int_0^{2\pi}\sigma(x,\phi)\,d\phi$$

$$\delta\vec{k}_h \;=\; \int_0^2 \frac{\sqrt{1-x/2}}{2x}\;dx\;\int_0^{2\pi}\sigma(x,\phi)\,\hat{u}(\phi)\,d\phi$$

with $\sigma(0)$ is the EPV source strength directly beneath the spacecraft. All components of the EPV signal, however, also receive contributions from global integrals over the entire Earth surface. As the STEP satellite tracks over the entire Earth's surface in the course of its entire mission and many hundreds of orbits, a robust EPV should map out the geographic distribution of the Earth surface's source strength for this new interaction.

This work supported by N.A.S.A. contract NAG3-2911

References

[1] Nordtvedt K (2000) STEP's equivalence principle violating force field due to non-spherical Earth *Class. Quantum Grav.* **17** 2531-2536

[2] Nordtvedt K (2001) Earth's equivalence principle violating multipoles: more science from a robust violation signal in STEP *Class. Quantum Grav.* **18** 2467-2473