Prospects for an Improved Measurement or Experimental Limit on *G-dot*

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ABSTRACT

The orbital motion of an ultra-drag-free satellite, such as the large test body of the SEE (Satellite Energy Exchange) satellite, known as the "Shepherd," may possibly provide the best test for time variation of the gravitational constant *G* at the level of parts in 10^{14}. Scarcely anything could be more significant scientifically than the incontestable discovery that a fundamental "constant" of Nature is not constant. A finding of non-zero *(G-dot)/G* would clearly mark the boundaries where general relativity is valid, and specify the onset of new physics. The requirements for measuring *G-dot* at the level proposed by SEE will require great care in treating perturbation forces. In the present paper we concentrate on the methods for dealing with the gravitational field due to possible large manufacturing defects in the SEE observatory. We find that, with adequate modeling of the perturbation forces and cancellation methods, the effective time-averaged acceleration on the SEE Shepherd will be $\sim 10^{-18}$ g (10^{-17} m/s^2).

1. Introduction

A thorough understanding of the gravitational force—especially a satisfactory quantum theory of gravity—is the missing link in efforts to achieve a satisfactory unification theory. The question of whether the gravitational constant *G* is truly constant or whether it might be time-varying is of particular importance to modern theories of gravitation and, hence, to efforts to achieve a satisfactory unification theory. A striking feature of recent theories of quantum gravity and string theory is that they cannot retain a constant *G*, but rather require various secular rates of change. Their predictions of *(G-dot)/G* are typically $\sim 10^{-13}$/yr to $\sim 10^{-11}$/yr. Moreover, a test of *(G-dot)/G* is one of the very few ways of discriminating among various modern theories [see, for example, Marciano, 1984; Bronnikov, Ivashchuk & Melnikov, 1988; Melnikov, 1994; Drinkwater *et al*. 1999; and Ivashchuk & Melnikov, 2000].

It was of course Dirac's original conjecture about variation of the fundamental constants, summarized in his "Large Numbers Hypothesis," that opened the door to initial speculations in this area, and his original concept of two metrics (one for "mechanical," *i.e*., orbital processes and another for "atomic" processes) still echoes today in the theories discussing extra dimensions. By roughly mid-century, the scalar-tensor theories of gravity were essentially the first to make quantitative predictions of a non-zero *(G-dot)*/*G*. The essential feature was that the gravitational field would have a secular change . Such theories, particularly that of Brans and Dicke, were hotly debated within the context of general relativity, and eventually gave way to increasingly precise experimental confirmations of general relativistic predictions. However, perhaps the most striking thing about the debate at that point in time was that a non-zero *(G-dot)*/*G* lay outside of the predictions of general relativity; hence, if some evidence for it could be discovered, then Einsteinian geometrodynamics would be incomplete, at best.

Although a number of different theoretical models have subsequently been proposed, experimental/observational evidence is still not sufficiently precise to discriminate among the predictions of different theories with respect to *G-dot* and other variables and, hence, to assess the validity of alternative models.

The question of the variability of *G* has taken on increasing urgency in recent years. One important new motivation for the measurement of *(G-dot)*/*G* has arisen within the context of attempts to reveal the presence of a dynamical background energy in the universe, the so-called "dark energy" or quintessence." Chiba (1999), for instance, has pointed out that a dynamical coupling of the quintessence field to the gravitational field can give rise to a *(G-dot)*/*G* effect, and he has used the existing experimental values to constrain the size of such a coupling. For a review, see Uzan (2003).

The best tests of *G-dot* at present are observational tests from Lunar Laser Ranging (LLR). The basic approach of LLR analysis is to disentangle a number of different effects—Newtonian, Einsteinian, and putative post-Einsteinian—relating to the motion of the Moon in search of putative post-Einsteinian effects, such as the Nordtvedt effect, other Universal Free Fall violations, and non-zero G-dot [Nordtvedt, 1996, 2002, & 2003]. To date no violations have been found. The present limit on *G-dot* is ~10^{-12}/yr [Pitjeva, 1997; Williams *et al*., 2001; Williams *et al*., in press].

There is now reason to believe that the orbital motion of a near-Earth satellite can be made so nearly drag free by design that it may effectively play the role of the Moon and provide a test for time variation of the gravitational constant *G* at the level of parts in 10^{14}—about two orders of magnitude beyond both the current observational results and the predictions of most current theories. To wit, we believe that the large test body of the SEE (Satellite Energy Exchange) satellite [Sanders & Deeds, 1992 and 1993, and Sanders *et al*., 1993, and Sanders & Gillies 1998a], known as the "Shepherd," could play this role. The methods for a test of *G-dot* by SEE are closely analogous to those of LLR, with the notable exception that LLR is entirely *observational*, while the use of a drag-free artificial satellite essentially comprises a *controlled experiment* with very fine accuracy.

There has never been a credible laboratory measurement of *(G-dot)*/*G* (using test masses in a controlled situation) at cosmologically interesting levels of precision. Although there have been perhaps a dozen laboratory experiments proposed to measure *(G-dot)*/*G*, none of them has been successfully carried out at cosmologically significant levels of interest. The existing data through 1997 are reviewed by Gillies (1997).

In short, it would be very significant scientifically to discover that a fundamental "constant" of Nature is not constant. Nothing could do more to invigorate interest in new theories, most of which do allow for time variation of *G* and other fundamental "constants." A finding of non-zero *(G-dot)*/*G* would of course require modification of general relativity, since it assumes a constant value of *G*. More broadly, this would clearly mark the boundaries where general relativity is valid, and specify the onset of new physics. The very precise *experimental* data to be provided by a SEE mission augurs for significant advances in gravitation theory, with concordant implications for unification theory.

3. Experimental method for determining *(G-dot)/G* on the SEE mission

The experimental approach to measuring *(G-dot)/G* by a SEE mission is described in a previous article [Sanders *et al.*, 2000]. The main idea is to use the orbital period as a clock running in comparison with atomic clocks and to infer a possible change in *G* from the dependence of the period on *G*. Thus, unless the orbital period is constant except for various known and/or characterizeable perturbations, *G* will be shown to be changing (strictly speaking, we can look only for changes in the product $M_E G$). We have shown that the accuracy with which *(G-dot)/G* may be measured is limited by (1) available position resolution if the observation time is less than one year and (2) accuracy in accounting for perturbations if the observation time is greater than one year [Sanders *et al.*, 2000]. The capability for measuring time will not be a limiting factor in measuring *(G-dot)/G*, assuming the next generation of atomic clocks is available.

4. Expected Error Budget for *G-dot* Determination by SEE

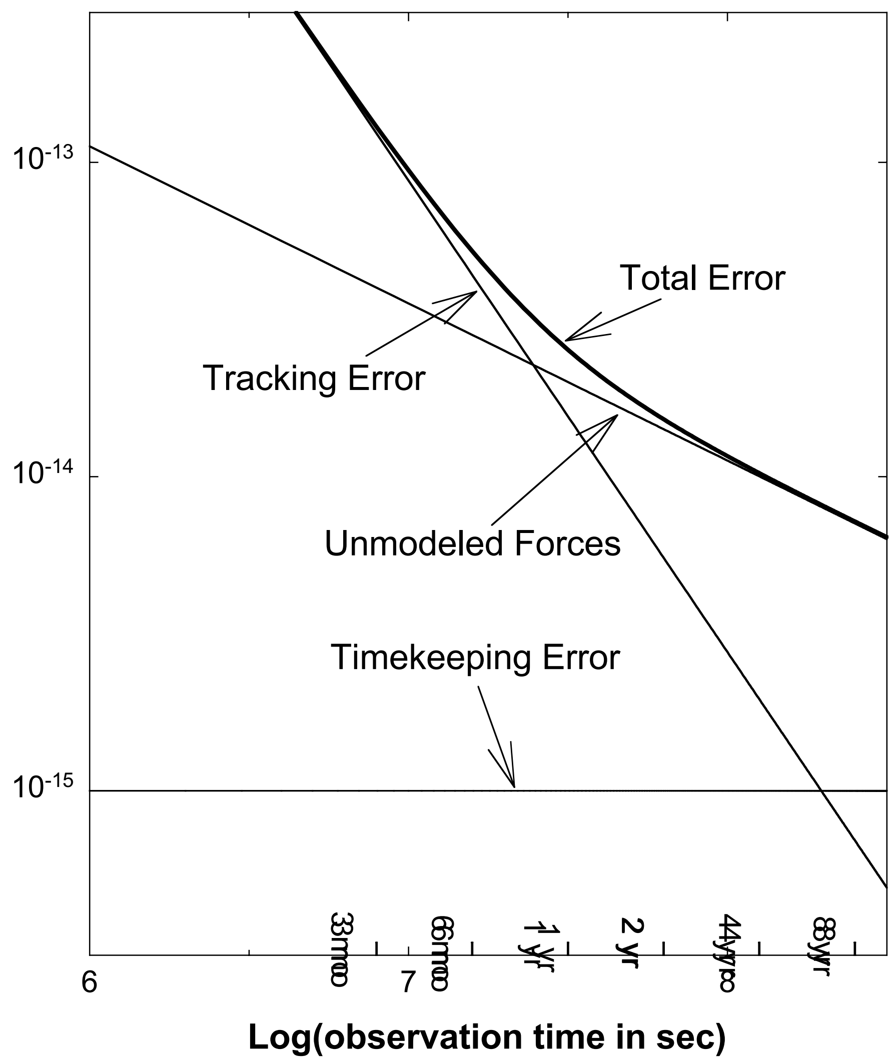
Great caution is required in satellite design to make the Shepherd as nearly drag-free as possible. The various perturbing effects that are thought to have the potential to contribute to error in the measurement of *G*–dot on a SEE mission are being evaluated. The status of this evaluation is shown in Table 1 [after Sanders *et al.*, 2000].

Table 3. Error Budget for *G*-dot (One-Year and Four-Year Observation Periods)

Error Source	Average Force ($\times10^{-17}$ N)	$\delta(G\text{-dot}/G)$ ($\times10^{-15}$)		Brief Comments (details below)
		1 yr	4 yrs	
Tracking error	NA	15.6	2.0	GPS/SLR accuracy = 1 cm
Timekeeping error	NA	~1	~1	Next generation clocks
Blackbody radiation	10.0	8.6	4.3	$\Delta\Theta < 0.1$ mK
Electrostatic forces	<15	<10	<5	Surface potential < 6.4 mV
Lorentz forces	small	zero	zero	Perpendicular to velocity
Earth's field	<1.4	<0.9	<0.5	With GRACE or equivalent
Capsule mass defects	22.2	15	7.4	Many defects ~10 mg
Gravity of particle	<0.22	<0.15	<0.08	Newton's 3rd law
Shepherd's moments	small	small?	small?	Not evaluated yet
Outgassing jets	small	small	small	Obviate by baking
Total	**NA**	**25**	**10**	

The *(G-dot)/G* error budget is summarized by Figure 1 below. The top row from Table 1 (tracking error) appears in Figure 1 as a line that decreases as $t^{-3/2}$. The second row (timekeeping) appears as the horizontal line at 10^{-15}/yr. The collective effect of all other items in Table 1 appears in Figure 1as the line that decreases as $t^{-1/2}$. The total error in Figure 1 is shown as the hyperbola-like curve, which exceeds 10^{-13}/yr if the observation time is a few months, and which falls below 10^{-14}/yr when the observation time is more than 4 years.

Figure 1
Error Budget for $\dot{G}/G$
by Class of Source



5. Perturbations--Internal Gravitational Field of SEE Observatory

In this section we focus on the effects due to the gravitational field of the SEE observatory itself. The original article on SEE pointed out that it is possible, in principle, to distribute the mass of any closed container such that the gravitational field on the interior is zero [Sanders & Deeds, 1992; U.S. Patent No. 5,427,335, June 27, 1995]. In practice, the desired distribution cannot of course be realized exactly, and it is therefore necessary to develop strategies to cope with the “mass defects” which will inevitably exist [Sanders & Gillies, 1996a; Sanders & Gillies, 1996a; Sanders & Gillies, 1996b; Sanders & Gillies, 1997; Sanders & Gillies, 1998b]. We have previously investigated the perturbations due to a large number of small, randomly-located point-like mass defects in the side walls of the SEE experimental chamber [Corcovilos & Gadfort, 1998; Sanders *et al*., 1999 and Sanders *et al*., 2000]. In the present paper we consider the perturbation due to a single large mass defect in the end wall of the SEE experimental chamber.

Detailed simulations are of course required to validate data-reduction methods for disentangling the effects of various combinations of mass defects. The original SEE paper pointed out that a Fourier-Bessel expansion of the potential was suitable for this purpose [Sanders & Deeds, 1992], and we subsequently presented the explicit form of the off-axis coefficients [Antonov, 1999]. The role of the spatial Fourier spectrum for treating the potential on axis is illustrated in Sanders *et al*. (1999).

Although comprehensive approaches such as these will be required for reducing the data from the actual mission, elementary calculations of special cases are very helpful for providing insight into the meaning of comprehensive treatments. In this section we consider one such special case—extra mass at one end of the experimental chamber—and we carry out elementary calculations to illustrate the impact on the *G*-dot determination.

It is necessary to distinguish among three stages in the treatment of perturbations due to mass defects:

(A) How large the mass-defect perturbations actually are. This is essentially an issue of manufacturing tolerances.

(B) How accurately these perturbations can be *modeled* or mapped, We treat this under the heading of "self calibration" in Sanders & Gillies (1996). The error in this modeling is called the “unmodeled force.”

(C) The extent to which the effects of mass defects can be *canceled*, by varying the orientation of the capsule and the position of the Shepherd, and then averaging over these different configurations. The departure from perfect cancellation is called the “uncanceled force.”

These distinctions are discussed in Sanders *et al*. (2000) in the sections titled “The drag-free satellite concept.” The concept of Almost-Zero Time-Averaged Drag (AZTAD) satellites, introduced here, is central to SEE’s *G*-dot determination.

We now demonstrate that a very large defect will not be deleterious, provided the SEE observatory is *calibrated* (“B” above) and the *unmodeled* force is further suppressed by the *cancellation* methods (“C” above).

For this illustration we suppose that the thickness defect in one endplate of the observatory chamber is manufactured 50 microns too thick (an enormous error!). We trace the implications of this mass defect as if the chamber were otherwise perfectly manufactured (*i.e*., the SEE observatory would have zero internal field if the thickness of the endplate were correct). The distance of the Shepherd from the heavier endplate is typically ~1 m, so it will experience a perturbation force of about

$$F = GMm/r^2 \cong 1.4\text{x}10^{-9}\ \text{N}$$

Here we have taken the radius of the endplate as 50 cm and its density as 2700 kg/m^3, so the mass defect m is 106 g. We take the mass of the Shepherd as M=200 kg.

This perturbative force will of course be in the *forward* direction when the heavy end is in the front end of the observatory, and the force will point *backward* when the heavy end is in the back. In either case, the force can be measured by observing the relative motion of the Shepherd and a Particle. The *relative* acceleration of the Shepherd and Particle will be greater when the Shepherd is near the heavy endplate than when it is near the “good” endplate. From observations of such differences, the potential field of the SEE observatory can be mapped. For simplicity we assume that the Particle is located far from either end, so that the interaction between the Particle and the endplate defect may be neglected.

When the Shepherd is near the "heavy" endplate, it experiences an additional acceleration of $F/M \cong 7.0\times10^{-12}$ m/s^2 that does not occur when it is near the "good" endplate. Therefore the size of the net impact of the perturbation during a four-hour observation, may be roughly estimated as a differential displacement equal to

$$1/2\ a\ t^2 = 7.2\times10^{-4} \text{ m}$$

when the Shepherd is near the heavy endplate.

Such a large difference (0.72 mm) would of course be an obvious effect. The question is, just how accurately can it be measured? This will determine *how accurately* the potential field of the capsule can be mapped (modeled). The SEE experimental design calls for the measurement accuracy of the relative positions of the test bodies to be known to less than a micron. A 1-micron difference in, say, four hours of observation (two orbital revolution) corresponds to an acceleration difference δa equal to 1×10^{-14} m/s^2 (*i.e.*, $1/2\ \delta a\ t^2 = 1\times10^{-6}$ m). In turn, this corresponds to a 70-nm error in the thickness of the end wall. That is, an additional 70 nm (150 mg) in the end wall would cause the test-body separation to increase or decrease by an additional 1 micron, which is large enough to be measurable. Thus, the ability to detect test-body positions to <1 micron is equivalent to being sensitive to a thickness difference between the two end walls with a resolution of <70 nanometers on the basis of observation of the Particle and Shepherd for four hours near the heavy end and four hours near the good end.

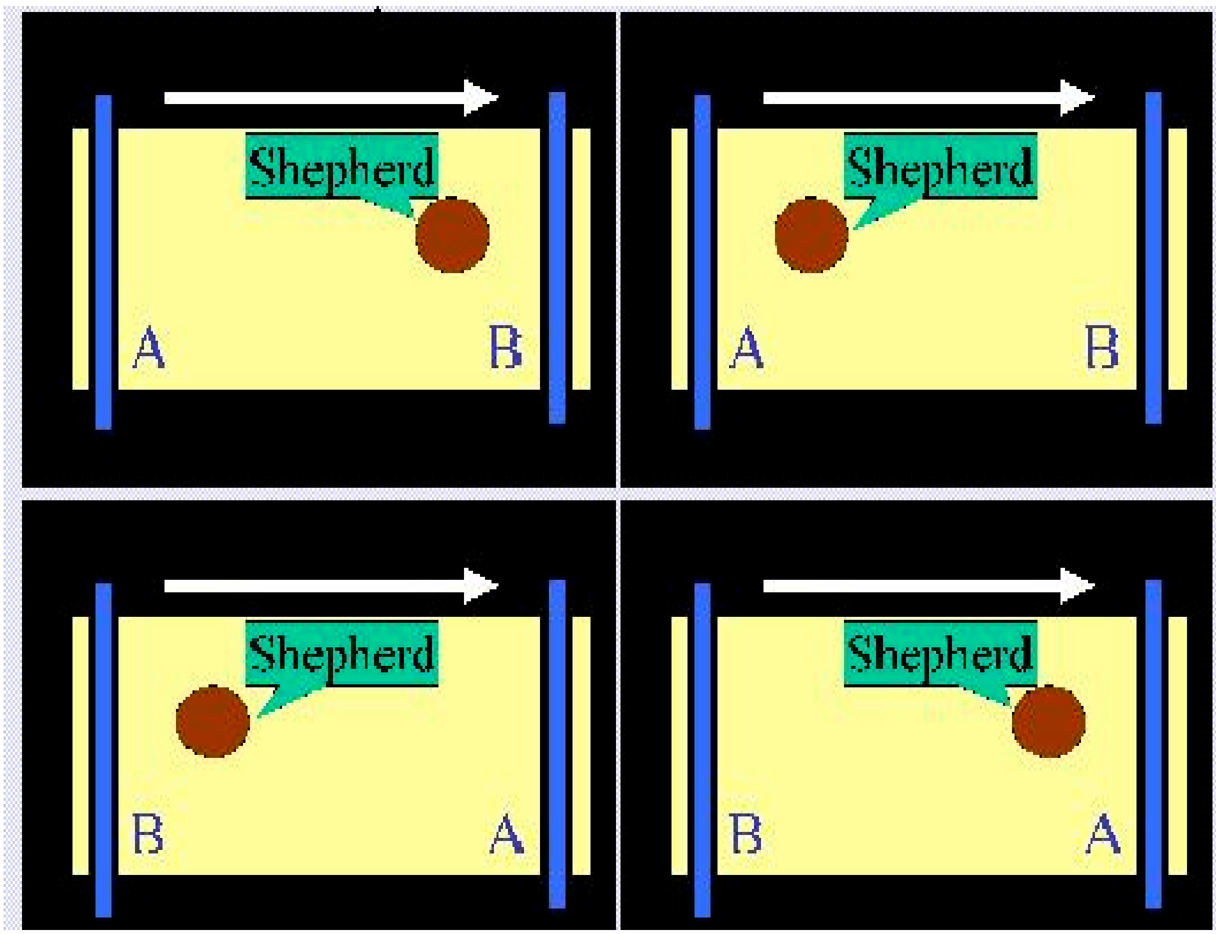
The above calculations demonstrates that—remarkably—although the defect in the end-wall thickness may be very large (50 microns), the resulting difference in the potential may be detected with a resolution equivalent to ~70 nm in endplate thickness.

Moreover, we note that this result is *not a function of the size of the defect*. The reader may easily demonstrate this fact by repeating the above algebra with a different assumption about the manufacturing defect (for example, a 5-micron or 500-micron error in thickness); the result will be that the detection resolution would be unchanged (still ~70 nm).

The discussion to this point entails effects of modeling (mapping, calibrating) of the perturbation forces, *not* cancellation.

Cancellation procedures can result in a further dramatic reduction in the actual time-averaged force on the Shepherd, which is what affects the *G*-dot measurement. Here is the cancellation procedure: Consider the axial component of the un-modeled force along any line parallel to the axis of the SEE observatory. This function may be of course decomposed into the sum of two functions—one even and one odd—and the choice of origin is arbitrary. For any two points located symmetrically with respect to the origin, signs of the odd part of the force are opposite, while those of the even part are the same. To say that the sign of the even part remains the same means that, if the even force on a test body is *forward* when it is in one end of the capsule, then the force will also be forward when the test body is in the opposite end of the capsule. Now, if the capsule is “flipped”—turned 180 degrees, so that the back end becomes the front end—this force will reverse direction: the even force on the test body will now be *backwards* (in both ends of the capsule). The odd part of the force of course still switches sign when the Shepherd moves from one end of the capsule to the other, regardless of the orientation of the observatory. We apply these simple principles to the SEE Shepherd. The two orientations of the SEE observatory and the two positions of the test body yield "Four Flight Configurations" here. The cartoon figure below illustrates the four configurations.

Figure 2
The Four Flight Configurations



Obviously the sum of the even and odd forces over the Four Flight Configurations is exactly zero. In practice it will not be possible to control the position of the Shepherd exactly, so the two positions will not be quite symmetrically located with respect to the origin. We estimate that it will be easy to control the Shepherd to within a centimeter at all times and to control its *average* position to within a millimeter.

A point of secondary importance is that the Shepherd may be intentionally moved about over a "roaming interval" ~1 or 2 meters in length. There are a number of reasons why this procedure would be more desirable than trying to pin it to a single location at each end. One important benefit of the roaming-interval approach is that it avoids the possibility of accidentally choosing the location at a point where the un-modeled force is changing rapidly. That is, the roaming approach averages over such sensitive locations and less-sensitive locations.

We have previously shown that the time-averaged force on the Shepherd will typically be ~3 orders of magnitude smaller than the un-modeled force, assuming the average position of the Shepherd can be controlled to within a few millimeters [Sanders *et al*., 1999 and Sanders *et al*., 2000]. This result was for the case of a large number of small point-mass defects in the walls of the SEE observatory. We now demonstrate that the same cancellation technique is also effective in the case of a large mass defect in the end wall.

The Shepherd will be maneuvered so that it spends equal times in the two Flight Configurations that locate it near the "heavy" endplate, and also equal times in the other two Flight Configurations, in which the Shepherd is near the "good" end of the chamber.

As indicated above, if the mass defect is m=106 g, the resulting perturbation would be about

$$F \cong GMm/r^2 \cong 1.4 \times 10^{-9} \text{ N}$$

This is the approximate **actual perturbation force**.

We would observe this by seeing that the relative acceleration of the two test bodies would be greater when the Shepherd were closer to the "heavy end of the capsule. However, we demonstrated above that, as a consequence of limitations in position-measurement capability, we would make an error in modeling this force that is equivalent to a mass defect of δm ~ 150 mg (thickness ~70 nm thickness). Therefore, the error in the force map would be

$$\delta F = G\,M\,\delta m/r^2 \cong 2\text{x}10^{-12} \text{ N}$$

This is the **un-modeled force** (error in the force calibration).

Now comes the cancellation: The un-modeled force error will be in the *forward* direction in two flight configurations and in the *backward* direction in the other two flight configurations. The time average, taken over all Four Flight Configurations, is exact cancellation in principle. The cancellation will not be exact in practice because the effective distance r of the Shepherd from the mass defect cannot be controlled exactly. We assume that the average positioning error is 1 mm (a large value compared with the accuracy of the measurement system). Thus, if r~1 m, we choose

r_1=1.000 m and r_2=1.001 m. Therefore the *difference* between the resulting forces, as computed from the force map, is two or three orders of magnitude smaller than the un-modeled force, namely:

$$\delta F_1 - \delta F_2 = G M \delta m \times (1/r_1^2 - 1/r_2^2) \cong 4\text{x}10^{-15} \text{ N}$$

This is the **un-cancelled force**.

The corresponding un-cancelled acceleration is obtained by dividing out the Shepherd mass, M (200 kg). The result is

$$a \cong 2\text{x}10^{-17} \text{ m/s}^2 \cong 2\text{x}10^{-18} \text{ g}$$

That is, the time-averaged drag on the Shepherd is ~10^{-18}g, *ceteris paribus*.

To avoid misunderstanding, we must emphasize that the above is only a series of illustrative elementary calculations about a *single* mass defect, *viz*. a defect in the thickness of one end wall. This approach is useful for obtaining order-of-magnitude results, but it is no substitute for a comprehensive analysis of the combined impact of multiple mass defects, including large numbers of randomly distributed mass defects, as described in Sanders *et al*. (1999). Our results for multiple mass defects are summarized in Figure 3 of Sanders *et al*. (1999) and the accompanying text.

6. Summary

These results, plus the calculation above of the impacts of a single large defect, show that time-averaged force on the Shepherd will be remarkably small, given proper treatment of the perturbations, even if the perturbations are relatively large. This is a major accomplishment under our NASA Fundamental Physics in Microgravity grant. We regard it as an important advance in experimental gravitation.

7. Discussion in Q&A Period

Question (Ho-Jung Paik): Since the unmodeled force on the Shepherd due to mass defects is constant—has no time variation—how can it have a random-walk character? How can this contribution to the total unmodeled force continue to decrease with time [as $t^{-1/2}$], as shown in Figure 1?

Answer (during session): I believe you are correct.

Later answer: Although it is true in principle that the force *at any given point in the experimental chamber* due to mass defects is not time-varying, the Shepherd will not be located at exactly the intended positions (if it were, then exact cancellation would result, so the time-averaged force would indeed be constant—in fact, zero). Rather, the actual force on the Shepherd will be time-varying because of positioning errors, and this effect is correctly described by a random walk, so the force will vary as $t^{-1/2}$, as shown in Figure 1.

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