Science Goals of the Primary Atomic Reference Clock in Space (PARCS) Experiment.

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The PARCS (Primary Atomic Reference Clock in Space) experiment will use a laser-cooled Cesium atomic clock operating in the microgravity environment aboard the International Space Station (ISS) to provide both advanced tests of gravitational theory and to demonstrate a new cold-atom clock technology for space. PARCS is a joint project of the National Institute of Standards and Technology (NIST), NASA's Jet Propulsion Laboratory (JPL), and the University of Colorado (CU). This paper concentrates on the scientific goals of the PARCS mission. The microgravity space environment allows laser-cooled Cs atoms to have Ramsey times in excess of those feasible on Earth, resulting in improved clock performance. Clock stabilities of 5×10^{-14} at one second, and accuracies better than 10^{-16} are projected. The relativistic frequency shift should be measureable at least 35 times better than the previous best, Gravity Probe A.[1] PARCS is scheduled for launch in 2007 and will probably fly with the Stanford Superconducting Microwave Oscillator (SUMO), which will allow a Kennedy-Thorndike type experiment with an improvement of about three orders of magnitude compared to previous best results. PARCS will also provide a much-improved realization of the second, and a stable time reference in space. Significant improvements in testing fundamental assumptions of relativity theory, and in testing non-metric theories of gravity, are expected.

1 Introduction

The PARCS laser-cooled atomic clock takes advantage of the microgravity environment of space to achieve improvements in clock performance. We describe here the scientific and technical measurements to be performed with PARCS.

1.1 Gravitational Measurements

Relativity predicts how clocks behave while moving or in varying gravitational fields. The PARCS clock will be used to test such predictions. Improvements in relativistic frequency shift measurements by nearly two orders of magnitude, and improvements in Kennedy-Thorndike type tests at an even higher level are expected. Earth-based tests of Local Position Invariance (LPI) have recently improved significantly.[2] Space-based tests of LPI therefore no longer offer important improvements, but can complement such earth-based tests with clocks of different structures, in a different environment. These measurements provide a basis for possible future gravitational experiments in highly elliptical earth orbit or for a solar probe.

1.2 Other Technical Measurements

The PARCS clock will realize the second's definition approximately ten times more accurately than that now done on earth. It can provide accurate time interval and frequency signals to laboratories worldwide, thus contributing to the coordination of clocks maintained by standards laboratories around the world. Since the ISS will be above the troposphere and part of the ionosphere, propagation-delay variations for signals traveling between GPS satellites and the ISS will be smaller than those observed on earth. PARCS provides an opportunity to study the clocks, ephemerides, and propagation delay mechanisms in GPS with high precision.

1.3 Atomic Clocks in Microgravity

Earth-based clocks using neutral atoms are limited in accuracy by strong gravitational forces which pull atoms downward out of the apparatus and limit the interaction time over which their resonance frequency can be measured. On earth this time limit, attained in laser-cooled cesium-fountain clocks, is about one second.[3] In the ISS microgravity environment, atom-observation time will be increased by an order of magnitude or more. The resonance linewidth decreases as observation time increases, which simplifies locating the center of the resonance. Many systematic frequency shifts scale as the observation time, so accuracy is improved. We project a fractional frequency uncertainty of 5×10^{-17} (for an averaging time of the order of 10 days) for PARCS. The best earth-based atomic clocks have an uncertainty of order 1×10^{-15} [3,4,5].

The best configuration for a very slow-atom clock in space is the same as that of the traditional atomic-beam clock, but the space clock will involve balls of laser-cooled atoms rather than a continuous beam of thermal atoms. At the projected stability $\sigma_y(\tau) = 5 \times 10^{-14}\tau^{-1/2}$, the projected accuracy of the PARCS clock cannot be achieved without the ISS microgravity environment.

1.4 Concurrent Flight with Other Experiments

Concurrent flight with other clock experiments would provide opportunities for useful comparisons among clocks of different structures. Stanford's superconducting microwave oscillator (SUMO) is scheduled to fly with PARCS. Other possibilities include RACE, a rubidium atomic-clock experiment; ACES,

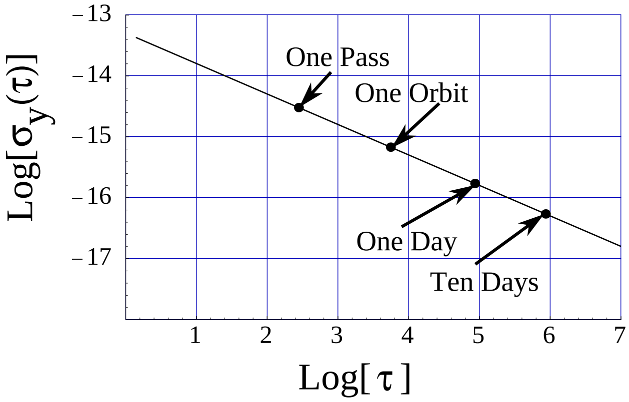

Figure 1: Allan deviation of the PARCS cesium clock showing averaging times τ (in seconds) needed to achieve various measurement uncertainties.

Europe's Atomic Clock Ensemble in Space; the cooled-sapphire oscillators developed at JPL and the University of Western Australia; and the linear-ion clock developed at JPL. In particular, concurrent flight of PARCS and SUMO will allow a Kennedy-Thorndike experiment with a projected performance 770 times greater than previously achieved on earth. A slight improvement in the Michelson-Morley experiment can also be achieved.

2 Experimental Objectives

Relativistic effects on clocks in low-altitude earth orbits can be characterized by the orders of magnitude of the fractional frequency shift they cause. For example, first-order Doppler shifts are of order $v/c \approx 10^{-5}$ for an orbiting clock, where v is the spacecraft speed in a local, freely-falling, earth-centered inertial frame and c is the defined speed of light. If Φ represents the Newtonian gravitational potential at the location of the clock, then gravitational frequency shifts and second-order Doppler shifts are of order $\Phi/c^2 \approx (v/c)^2 \approx 10^{-10}$. The data analysis technique that is planned for PARCS is discussed in the Appendix. It uses only even-order terms; fortunately fourth-order terms are negligible. Fig. 1 shows the projected Allan Deviation of PARCS as a function of averaging time τ. The absolute uncertainty is projected to be 5×10^{-17}, after about 12 days of averaging. We assume a time-transfer uncertainty (to the earth) with a stability of 220 ps over at least 12 days. We assume the following orbital parameters: altitude 400 km, inclination 51.6°, and eccentricity 0.001. Then one pass over a fixed ground station takes about 400 s. The orbital period is about 5500 s, and during 1 day the satellite will be in position to exchange direct transmission with a single ground-based reference during four or five passes. During a single pass, the time available for direct frequency comparison with a ground-based reference clock is not sufficient to realize the full capability of the PARCS clock. Frequency comparisons are instead expected to use GPS satellites as intermediaries, which do not require such line-of-sight exchange.

2.1 Measurement of the Gravitational Frequency Shift

Here the space-borne clock's frequency is compared with the frequency of a clock on the earth employing a measurement of the accumulated phase of the orbiting clock (see Appendix). Accumulated phase measurements make best use of the long-term stability of the space-borne clock.

The ISS altitude is only about 400 km, so the second-order Doppler shift is the dominant contribution to the net frequency shift. The fractional frequency shift due to second-order Doppler (time dilation) is approximately 3×10^{-10}, while that due to gravitation is about 4×10^{-11}. Significant contributions come from the monopole potential of the earth, the quadrupole moment ($\approx 3 \times 10^{-14}$) and a few higher moments. The Stokes coefficients are known sufficiently well known[6] that uncertainties in a frequency-shift test, arising from uncertainties in the Stokes coefficients, are negligible.

One pass is too brief to yield scientifically significant new results with direct frequency comparison to an earth-bound standard. However, with the accumulated-phase measurement method, the long-term stability of the clock can be used to advantage, since many passes over the ground station, lasting for days, are available for the measurement. Fig. 2 shows the results of a covariance analysis for this experiment in which time transfer errors, clock stability, tracking errors, and inaccuracy of the ground clock used for comparison have been accounted for. If the experiment lasts only a few hundred seconds, the uncertainty in determining the fractional frequency shift is dominated by the time-transfer uncertainties. Eventually these become small compared to clock instabilities. At long times, the uncertainty of less than 2 parts per million is dominated by the inaccuracy of the ground clock (frequency uncertainty of 5×10^{-16}). The level at which the corresponding test of GR was achieved in Gravity Probe A was 70 parts per million,[1] so the proposed experiment should result in improvement of measurement of the total gravitational frequency shift by between one and two orders of magnitude. To obtain this result, satellite position uncertainties of less than 50 cm must be achieved.

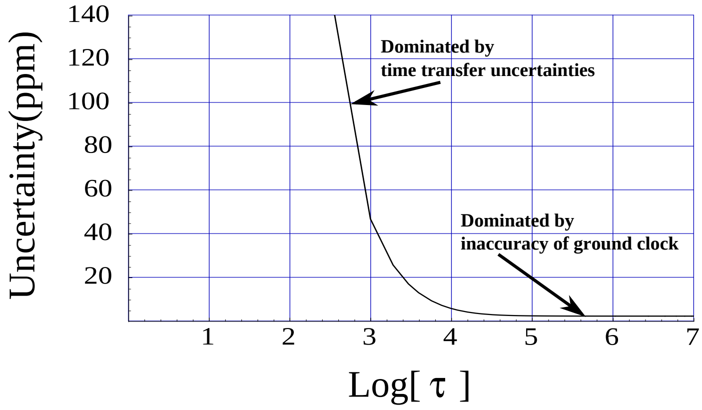

Figure 2: The total measurement uncertainty as a function of averaging time τ (in seconds) for the total frequency shift using the accumulated-phase measurement method. Beyond about 10^5 seconds, the uncertainty in the measurement drops below a value of 2 ppm.

2.2 Second-Order Doppler Shift

Measurement of the second-order Doppler shift would test Local Lorentz Invariance (LLI), at an uncertainty comparable to that of the best previous test. In the Mansouri-Sexl test theory of special relativity,[7] the time dilation terms are multiplied by a coefficient α, where it is currently known that $\alpha = -1/2 \pm (1 \times 10^{-6})$, a result obtained using fast ^{20}Ne atoms.[8] This experiment will probe the effect in a different parameter range.

2.3 Test of Local Position Invariance (LPI)

LPI implies that two clocks of different structure but equal frequencies should suffer identical frequency shifts when moved together through a gravitational field. For such tests the longer-term stability of the clocks is relevant, rather than accuracy. The same control of systematic effects that yields high accuracy also leads to high stability. LPI can still be tested with stable, (but possibly inaccurate) clocks by studying variations in frequency differences as the orbit radius and orbital speed vary. A highly eccentric orbit is most desirable. If LPI is violated, then for nearby clocks A and B $\Delta f/f = c_{AB}\Delta\Phi/c^2$, where $\Delta\Phi$ is the change in the gravitational potential of the clocks, $\delta f = f_A - f_B$, and $f = f_A \approx f_B$. In general relativity (GR) the coefficient c_{AB} is exactly zero. A recent experiment, lasting many months, made use of variations in the sun's potential arising from earth's orbital eccentricity.[2] An upper limit $c_{AB} \leq$

2.1×10^{-5} was obtained, a significant improvement over the previous best[9]. For a clock on the ISS, one may expect a variation in earth's gravitational potential of $\approx \delta\Phi/c^2 \approx 1.3 \times 10^{-12}$. If the comparison between clocks can be performed at the full stabilities of PARCS and SUMO, then the value of c_{AB} can be tested in 30 days at a level such that the error in c_{AB} is

$$\Delta c_{AB} < 9 \times 10^{-6}\,. \tag{1}$$

This is only a small improvement, but complements the earth-based experiments by using clocks of markedly different internal structure. A larger orbital eccentricity for the ISS would benefit this comparison even more.

2.4 Kennedy-Thorndike Experiment

For this experiment, the laser-cooled cesium clock is compared to a clock (such as SUMO) with a resonance based on the length of the oscillator cavity. This oscillator is analogous to an arm of an optical interferometer. As the spacecraft turns, the oscillator cavity turns, and the frequency of the resonance could be influenced by any spatial anisotropy in the speed of light. In contrast, the cesium frequency is not expected to change since any cavity pulling associated with changes in the microwave cavity of this clock is negligible. Comparison of the cesium frequency with that of SUMO thus tests for spatial anisotropy.

Mansouri and Sexl's [7] theory provides a basis for analysis of interferometer experiments testing local Lorentz invariance. Stability, not accuracy, of the laser-cooled clock for an orbital period is crucial in performing such tests. Also, one can hope to reach a precision better than the absolute uncertainty of the clock (5×10^{-17}) because the signals have a characteristic signature due to orbital motion that can be used to average down the noise over many orbits.

Assuming the existence of a preferred frame (e.g., one at rest with respect to the cosmic microwave background radiation), in which the speed of light is isotropic, then in a laboratory moving with velocity $\mathbf{v}$ relative to this frame, the two-way speed of light propagating at angle θ from $\mathbf{v}$ is given by

$$c(\theta)/c = 1 + (1/2 - \beta + \delta)v^2/c^2 \sin^2\theta + (\beta - \alpha - 1)v^2/c^2\,, \tag{2}$$

where α, β, and δ are parameters (to be studied experimentally) introduced in the Mansouri and Sexl test theory. In special relativity the time dilation parameter $\alpha = -1/2$; the Lorentz contraction parameter $\beta = 1/2$; and $\delta = 0$. (δ describes contraction normal to $\mathbf{v}$.) The light speed $c(\theta)$ in Eq. (2) determines the frequencies of a local cavity oscillator of fixed length L.

The parameter $(\beta - \alpha - 1)$ is measured in a Kennedy-Thorndike experiment. The square of the clock's velocity relative to the preferred frame should be

$$\mathbf{v}^2 = (\mathbf{v}_{sun} + \mathbf{v}_{earth} + \mathbf{v}_{sat})^2 \tag{3}$$

where the terms on the right side of Eq. (3) are respectively, the velocity of the sun relative to the preferred frame (which could be taken to be 377 km/s derived from the anisotropy of the cosmic background blackbody radiation), plus the velocity of the earth relative to the sun, plus the velocity of the satellite relative to earth. The coefficient of the last term in Eq. (2) can be quite significant. One contribution to the fractional frequency shift of the cavity oscillator is a cross term in the expansion of Eq. (3), giving rise to

$$\frac{\Delta f}{f} = (\beta - \alpha - 1)\,\frac{2\mathbf{v}_{sun} \cdot \mathbf{v}_{sat}}{c^2}\,. \tag{4}$$

The time signature of such a term is highly correlated with that of the change of potential which is of interest in testing LPI. There it is the change in the earth's (or the sun's) gravitational potential that drives the effect. Here, it is the orientation of $\mathbf{v}_{sat}$ relative to $\mathbf{v}_{sun}$ that drives the effect. These two relativistic effects should be separable since they differ in phase.

Currently, the combination of parameters $(\beta - \alpha - 1)$ is only known experimentally to be $< 6.6 \times 10^{-5}$.[10] If an upper limit of 5×10^{-16} (assumed stability of the cavity oscillator at 5500 s) can be put on the frequency change of Eq. (4), and we assume that $\mathbf{v}_{sun} \approx 377$ km/s, then a limit of order $(\beta - \alpha - 1) < 9 \times 10^{-10}$ results, an improvement by almost three orders of magnitude. A limit on α provides independent confirmation of the special relativity predictions of time dilation. Smaller upper limits on the parameters α, β, and δ will help in eliminating some preferred frame theories.

2.5 Michelson-Morley (MM) Experiment

In a MM type experiment, the θ-dependent term in Eq. (2) is measured. This can be done using the slow rotation of the spacecraft in its orbit, which naturally changes while the laser-cooled cesium clock provides frequency memory. The fractional frequency shift for a 90° rotation starting from θ_0 is $(1/2 - \beta + \delta)v^2/c^2 \cos(2\theta_0)$. One cross-term in the expansion of v^2 varies with the orbital period. Placing an upper limit of 5×10^{-17} on such a term would lead to $(1/2 - \beta + \delta) < 1.5 \times 10^{-9}$, slightly better than the previous best result.[11] This experiment has the advantage of a much larger orbital velocity than the velocity due to earth rotation in an earth-bound experiment.

Table 1: Summary of science objectives for the PARCS mission

Measurement/Test	Expected Uncertainty	Previous Best Uncertainty	Improvement (Ratio)
Net Frequency Shift, $\Delta f/f$	1.7×10^{-6}	70×10^{-6}	35
Gravitational Frequency Shift, $\Delta f/f$	12×10^{-6}	140×10^{-6}	12
Kennedy-Thorndike, $\beta - \alpha - 1$	9×10^{-10}	6.9×10^{-7}	770
Local Position Invariance, c_{AB}	9×10^{-6}	2.1×10^{-5}	2.3
Michelson-Morley, $\frac{1}{2} - \beta + \delta$	5×10^{-10}	5×10^{-9}	10
Atom Drift Time, s	10	1	10
Most Accurate Space Clock, $\Delta f/f$	5×10^{-17}	1×10^{-12}	20,000
Realization of the Second, $\Delta f/f$	5×10^{-17}	1.2×10^{-15}	24

2.6 Realization of the Second

In an earth-based cesium clock, gravity simply pulls the atoms out of the apparatus. The linewidth of the observed transition is then broader, limiting the determination of the resonance center. Also, the atoms in an earth-based clock must move at higher velocities relative to the clock enclosure where the Doppler shift and several other velocity-dependent systematic shifts are larger and more difficult to evaluate. In microgravity, atoms can be launched much more slowly, increasing the observation time by an order of magnitude and reducing the uncertainty in realization of the second by a comparable amount.

2.7 The $TH - \epsilon\mu$ Theory of Lightman and Lee

In the $TH - \epsilon\mu$ theory, charged massive particles in a spherically symmetric gravitational field couple to "gravitationally modified" electromagnetic field equations. To leading order, predictions of gravitational frequency shifts and violations of LPI are expressed through two parameters Γ_0 and Λ_0.[12,13] For example, when comparing the frequencies of a superconducting cavity-stabilized clock and a Hydrogen maser moving together through a varying gravitational potential, $c_{AB} = 3(\Gamma_0 - \Lambda_0)/2$. For clocks at different locations, $\Delta f/f = (1 - 3\Gamma_0 + \Lambda_0)\Delta\Phi/c^2$. The PARCS measurements should give significantly improved upper limits on these two linear combinations of parameters.

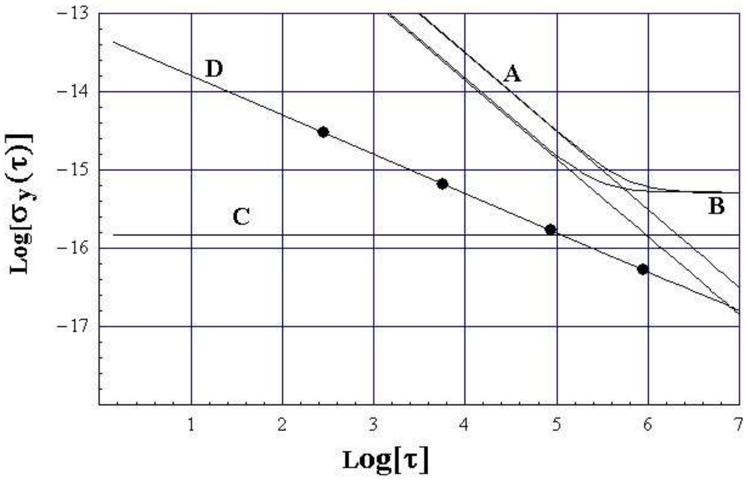

Figure 3: Allan variance plot of the stability limits for PARCS. See the text for a description of the different curves. The long-term limit of curve B is determined by the inaccuracy of the ground clock.

2.8 Realization of the Second

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It is difficult to transfer frequency between laboratories at the accuracy of the best earth-bound standards, so real-time access to the highest accuracy frequency references is limited. PARCS should outperform its earth-based counterparts by an order of magnitude. The best current realization of the second on earth has an uncertainty of 1.4×10^{-15} [3]. The projected uncertainty for the proposed space clock is 5×10^{-17}. Transfer of the second at this accuracy assumes that GR is correct, in order to correct for clock frequency shifts. At this level, uncertainties in our knowledge of the gravitational potential will contribute a few parts in 10^{17} to the overall uncertainty; spacecraft position and velocity will have to be known to 10 cm and 0.12 mm/s, respectively.

Fig. 3 shows stabilities of the critical components of PARCS. The projected clock stability is curve D. The straight curve A shows the frequency-transfer limitation at short integration times, consistent with time transfer at an uncertainty of 220 ps. (The unlabelled curve shows the effect of time transfer at an uncertainty level of 100 ps.) Curve C shows the uncertainty in calculated net frequency shift contributed by position uncertainty alone, from spacecraft tracking at the 1 m level for position and 0.0013 mm/sec level for velocity. The composite uncertainty is given by curve B, where the limit of 5×10^{-16} is due to estimated inaccuracy of the ground clock. The measurement objective for $\Delta f/f$ is achieved in 12 days. Curve D shows the full uncertainty of the space clock being achieved at about 30 days.

2.9 Analyses of GPS Satellite Signals

The ISS is above the troposphere and most of the ionosphere, so the PARCS mission affords the opportunity of viewing GPS satellite signals from a different vantage point. Observations will be limited primarily by the high speed of the ISS and multipath effects associated with signal reflections off ISS structures. Analyses of GPS signals could add to our understanding of the system. When more than four GPS satellites are observed from a receiver with a very stable time base, the navigation equations are highly constrained; this can be turned around to study a particular satellite. Issues of possible interest in GPS include temperature and attitude dependencies of transmitter phase centers.

3 SUMMARY–SCIENCE OBJECTIVES

Table 1 summarizes the scientific objectives for PARCS . These involve the SUMO oscillator that can support a clock stability of $5 \times 10^{-14}\tau^{-1/2}$ and can be used for the on-board, two-clock (LPI) experiments. These results could be enhanced by the concurrent flight of one or more of the clocks being developed elsewhere. If PARCS flies concurrently with SUMO as currently planned, bo the Kennedy-Thorndike (improvement factor 770) and Michelson-Morley experiments (improvement factor 10) can be performed. The objectives and the science requirements for the proposed flight have been dictated primarily by the time-transfer stability considerations shown in Fig. 3.

Appendix

Transformation of second-order Doppler shift of a space-borne atomic clock

This Appendix describes an alternative treatment of the second-order Doppler shift contribution to orbiting clock frequency shift. Over long integration times, systematic errors in the determination of orbiting clock velocity are a major source of uncertainties in comparing proper time predictions with observation. Such errors can be greatly reduced by transforming this frequency shift contribution into an alternative form appropriate for a satellite in nearly free fall, in which the terms can be evaluated with less uncertainty. This approach has been adopted as the principal method of data analysis for PARCS. Accumulated phase involves integration of the second-order Doppler shift contribution over coordinate time. Velocity is a coordinate time derivative, so this term may be evaluated by integration by parts. The metric of GR, valid to order c^{-2} in the neighborhood of the earth is:[14]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left[1 + \alpha_G \frac{2(\Phi - \Phi_0)}{c^2}\right] (cdt)^2 - \alpha_D \left[1 + \frac{2\Phi}{c^2}\right] (dx^2 + dy^2 + dz^2) \tag{5}$$

where dt is the increment of coordinate time, Φ is the Newtonian gravitational potential, and we have inserted coefficients α_G and α_D (which are exactly equal to unity in GR) to identify the sources of various contributions. Φ_0 is the effective gravitational potential on the earth's rotating geoid. For an orbiting clock in free fall, the equations of motion are

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = \left[\frac{d^2 x^\alpha}{ds^2}\right]_{NG} . \tag{6}$$

where $\Gamma^\alpha_{\mu\nu}$ is the Christoffel symbol of the second kind. The subscript "NG" means the non-gravitational part of the acceleration. If the orbiting satellite is in free fall the right-hand side of the above equation is zero. These equations reduce approximately, in the Newtonian or classical limit, to

$$\alpha_D \frac{d\mathbf{v}}{dt} + \alpha_G \nabla\Phi = \left.\frac{d\mathbf{v}}{dt}\right|_{NG} . \tag{7}$$

This shows that the gravitational part of acceleration is related to the gradient of the potential through the coefficient ratio α_G/α_D.

Let τ_B and τ_B be proper times elapsed on the orbiting clock and the earth-fixed reference clock, respectively, between coordinate times t_1 and t_2. The fractional time difference observable is

$$\frac{\Delta\tau}{\tau} = \frac{\tau_B - \tau_A}{\tau_A} = \frac{\alpha_G}{\tau_A} \int_{t_1}^{t_2} dt \left[\frac{\Phi_B - \Phi_A}{c^2}\right] - \frac{\alpha_D}{\tau_A} \int_{t_1}^{t_2} dt \left[\frac{v_B^2 - v_A^2}{2c^2}\right] . \tag{8}$$

The largest contribution to measurement uncertainty of this observable comes from the second-order Doppler shift term $\int v_B^2 dt$. Transforming this term by integration by parts, using Eq. (7), gives

$$\begin{aligned}\frac{\tau_B-\tau_A}{\tau_A} &= \frac{\alpha_G}{\tau_A}\int_{t_1}^{t_2} dt \left[\frac{\Phi_B-\frac{1}{2}\mathbf{r}_B\cdot\nabla\Phi_B-\Phi_A}{c^2}\right] + \frac{\alpha_D}{\tau_A}\int_{t_1}^{t_2} dt \left[\frac{v_A^2}{2c^2}\right] \\ &\quad - \frac{\alpha_D}{2c^2\tau_A}\mathbf{r}_B\cdot\mathbf{v}_B\Big|_{t_1}^{t_2} + \frac{\alpha_D}{2c^2\tau_A}\int_{t_1}^{t_2} dt \left[\mathbf{r}_B\cdot\mathbf{a}_B\right]\Bigg|_{NG}.\end{aligned} \tag{9}$$

Mathematically, this transformation is exact. The boundary terms involve the dot products of the position and velocity evaluated at times t_1 and t_2. These contributions can be made small. The dot product $\mathbf{r}\cdot\mathbf{v}$ is proportional to the orbital eccentricity and for the planned space station orbit the eccentricity will be in the neighborhood of 0.001. Also, it is possible to select starting and ending points for the experiment for which either the dot product vanishes, such as at apogee, or for which the upper boundary term cancels the lower boundary term. Finally, the boundary term is divided by τ_A and decreases with integration time. For long integration times, this term does not contribute significantly to uncertainties in the prediction of the observable. For integration times as short as one orbital period, if velocity can be measured to better than 1 mm/s and position to better than 1 m, the contribution to the fractional error from one such boundary term is less than 5×10^{-17}.

Second, the non-gravitational acceleration aboard the ISS is projected to be less than about 3×10^{-6} g or 3×10^{-5} m/s^2. The contribution of the uncertainty in this term to the net fractional uncertainty can be reduced to negligible levels by monitoring the non-gravitational acceleration to an accuracy of 400×10^{-9} g. This is well within the expected capabilities of the MAMS (Microgravity Acceleration Measurement System) accelerometers, which is already operating on the ISS. A third contribution is proportional to the time integral of $\mathbf{r}_B \cdot \nabla\Phi_B$. Here it is position errors rather than velocity errors that give rise to uncertainties. This term involves the gravitational coefficient α_G and demonstrates that testing the total accumulated phase shift tests the term proportional to α_G in the metric, which is responsible for the gravitational part of the frequency shift.

Thus errors in predicting the accumulated phase arise from clock instabilities, inaccuracy of the ground clock, time transfer (errors in t_1 and t_2), errors in position and velocity determination of B and A. A similar transformation of the second-order Doppler term $\int dt v_A^2/(2c^2)$ is not necessary. Fig. 3 illustrates application of the integration-by-parts method.

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