# CDF and PDF Comparison Between Humacao, Puerto Rico and Florida

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## INTRODUCTION

The knowledge of the atmospherics phenomenon is an important part in the communication system. The principal factor that contributes to the attenuation in a Ka band communication system is the rain attenuation. We have four years of tropical region observations. The data in the tropical region was taken in Humacao, Puerto Rico. Previous data had been collected at various climate regions such as desserts, template area and sub-tropical regions. Figure 1 shows the ITU-R rain zone map for North America.

Rain rates are important to the rain attenuation prediction models. The models that predict attenuation generally are of two different kinds. The first one is the regression models. By using a data set these models provide an idea of the observed attenuation and rain rates distribution in the present, past and future. The second kinds of models are physical models which use the probability density functions (PDF).

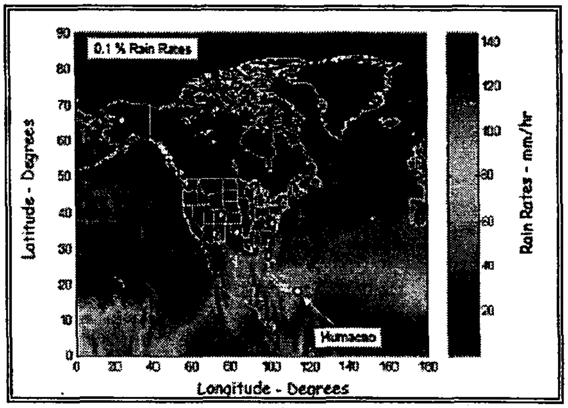


Figure 1.North America ITU-R Rain Zones

This paper presents the analysis of Humacao, Puerto Rico and Boca Ratón, Florida rain data through the development of the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF). The PDF is a function of a continuous variable such that the integral of the function over a specific region yields the probability that its value will fall within the region. The CDF describes a statistical distribution. It has the value, at each possible outcome, of the probability of receiving that outcome or a lower one. The Humacao and Florida data sets are use to obtain the corresponding PDF and CDF. Humacao has 35 months of data, from July 2001 to May 2004. Florida has 46 months of data, from March 1995 to December 1998.

## EXPERIMENT DESCRIPTION

A Ka band Propagation Terminal is deploy in the roof of a building. The terminal is a 1.8 meter offset reflector antenna and weather instruments. These instruments take data for weather statistic like barometric pressure, outside temperature and relative humidity. The rain accumulation is a measure with a tipping bucket rain gauge. A "tip" is a measurement of 0.01 millimeter of rain accumulation. Every time a tip occur the instrument send a signal to the computer data logging system which store it in a file. The amount of rain collected form a gauge located on the roof of a building is usually lower than the amount collected from the gauge located on the ground. The difference in the catch of the gauge is cost by the air flow across the gauge [14]. Humacao rain data logging began on July 2001.

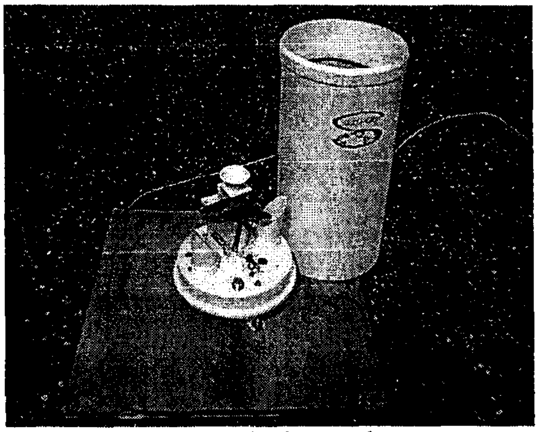

Figure 5.Tipping Bucket

| Latitude Deg. | Longitude Deg. | Azimuth Deg. | Elevation Deg. | Site Altitude |
|---|---|---|---|---|
| 18.1487 | -65.8385 | 108 | 37 | 70 m |

Table 1.Humacao Terminal Description

DATA FILE DESCRIPTION

Humacao rain data file is a text archive that has seven columns and an indeterminate numbers of rows. The number of rows is indeterminate because they depend on the amount of tips recorded. The rain data file contains the following variables:

- Column 1: GMT DAY
- Column 2: Hours
- Column 3: Minutes
- Column 4: Seconds
- Column 5: Fractions of seconds.
- Column 6: Tips
- Column 7: Cumulative rain

An example of Humacao rain data file is:

```
5  22  40  13  40  .01  .0004
5  23  43  58  43  .02  .0092
6  00  02  06  02  .03  -.0008
```

The Florida rain data file is a text archive that has six columns containing the followings variables:

- Column 1: Year
- Column 2: Month
- Column 3: Day
- Column 4: Hours
- Column 5: Minutes
- Column 6: Seconds

An example of a Florida rain data file is shown below:

```
98  01  06  15  42  02.80
98  01  06  15  42  05.03
98  01  06  20  07  40.60
```

## RAIN RATE ANALYSIS

The data files need to be pos-process for example, the time is change from the hours, minutes, seconds format to fraction of days as a define in equation 1 and 2. Equation 1 corresponds to the Humacao data files and equation 2 corresponds to the Florida data files.

$$t_k = A_{k,0} + \frac{A_{k,1} + \frac{A_{k,2}}{60} + \frac{A_{k,3} + A_{k,4}}{3600}}{24} \qquad (1)$$

$$t_k = E_{k,2} + \frac{E_{k,3} + \frac{E_{k,4}}{60} + \frac{E_{k,5}}{3600}}{24} \qquad (2)$$

The terms in equation 1 and 2 are defined as follow. $A_{k,0}$ and $A_{kE,2}$ correspond to the day of the year. $A_{k,1}$ and $A_{k,3}$ correspond to the hours. $A_{k,2}$ and $A_{k,4}$ correspond to the minutes. $A_{k,3}$ and $A_{k,5}$ correspond to the seconds and $A_{k,4}$ is the fraction of a seconds.

To obtain the total rain fall we need to perform the sum of all tips. As previously mention the amount of water per tip is 0.01 inches. Total rainfall is defined in equation 3, where the variable Amp represents the amount of water in one tip.

$$Cum_k = \sum_{n=0}^{k} Amp_n \qquad (3)$$

Equation 3 is redefined as equation 4 to account for the index definition used in the MathCAD software.

$$Rain\_Fall = Cum_{NP-1} \qquad (4)$$

Equation 5 defines the time different of two consecutives tips. Subscript k represents the number of rows in data file.

$$\Delta 0_k = \left(t_{k+1} - t_k\right) \qquad (5)$$

For eliminates two consecutive time values that are not on the same day in the data set we need to do a statement for $\Delta 0_k$. This statement establishes that when $\Delta 0_k$ more than 1 the value that we need to use for $\Delta 0_k$ will be0.000001. In the other case ($\Delta 0_k$ less than 1), $\Delta 0_k$ will be $\Delta 0_k$.

For obtain the total time that the rain rate was between 1 mm/hr to 300 mm/hr we use a sum of all differences between the numbers of point between zero to number of point in the data file.

For calculate the rain rate in millimeters per hours we use the equation 6 that have the conversion of the rain rate in mm/hr units. If the increment in seconds and limit the max rain rate to 300 mm/hr rain rates and the min rain rate to 1 mm/hr is more than three we use that data point and we convert it in mm/hr. In the other hand, if that increment in second is less than 3 we do not use the data point for analysis purpose.

For calculate the Probability Density Function (PDF) we use the following range:

$0<rr<10$

$10<rr<20$

$20<rr<30$

$30<rr<40$

.....

$150<rr<160$

To obtain the percent of the time the indeterminate rain rate can happen (PDF), first we add all samples that fall in a specific rain rate range, then it is divided by the total number of raw data points and the results is then multiply by 100.

$$PP_m = \frac{P_m}{P\_total} \times 100 \qquad (6)$$

In equation 7 we have the add of all samples that fall in a specific rain rate range

$$P_m = \sum_{k=0}^{rows(A)-2} if(m \cdot 10 < RR_k < m \cdot 10 + 10, 1, 0) \qquad (7)$$

In equation 8, we have the total of number of raw data points.

$$P\_total = \sum_{m=0}^{15} P_m \qquad (8)$$

In the other hand, to obtain the Cumulative Distribution Function (CDF), first we add of all samples in a specific rain rate, then it is divided by the total number of points and the results is then multiply by 100.

$$CUM_m = \frac{CUM1_m}{CUM1_0} \times 100 \qquad (9)$$

In equation 10 we have add of all samples in a specific rain rate where m is a number between zero to fifth teen.

$$CUM1_m = \sum_{k=m}^{15} P_k \qquad (10)$$

## RESULTS

The Probability Density Function (PDF) for four years of data in Humacao and Florida is show in figure 2.

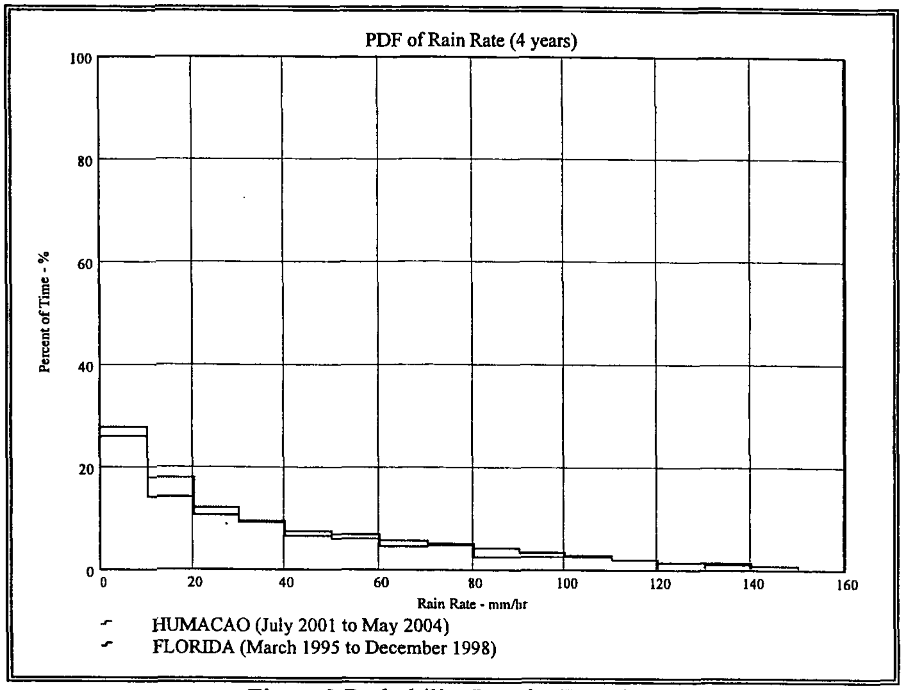


Figure 2.Probability Density Function

The Cumulative Distribution Function (CDF) for four years of data in Humacao and Florida Data is show in figure 3.

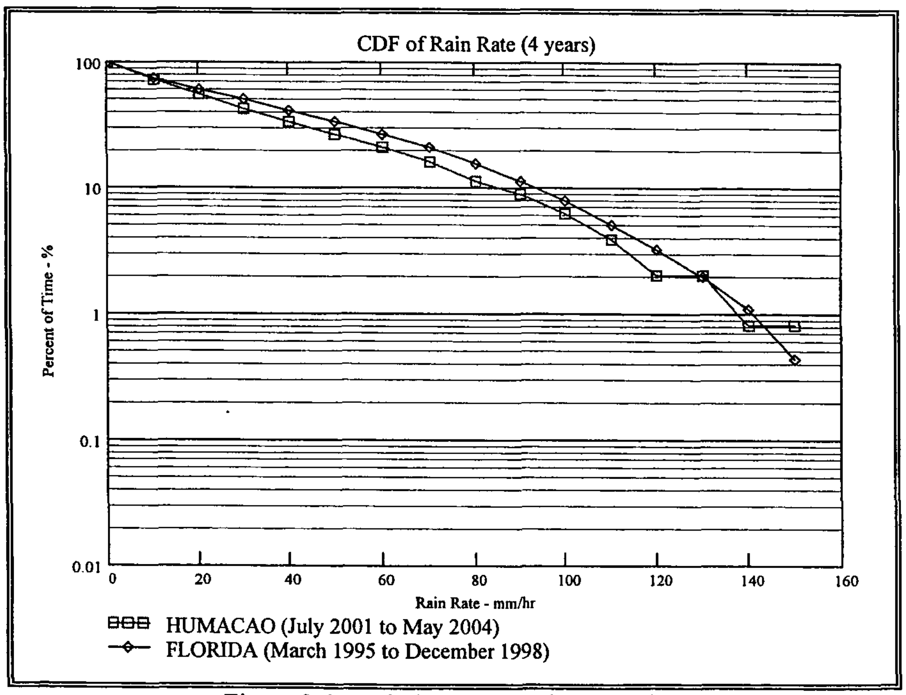


Figure 3.Cumulative Distribution Function

## CONCLUSION

The probability of rain rate between Humacao and Florida are basically the same. Humacao was bigger percent of probability than Florida in the smaller rain rates (0 to 30 mm/hr), but later it gets almost the same. In the larger rain rates (100 to 160 mm/hr) Humacao and Florida has the same percent probability for obtain that quantity of rain. The cumulative distribution function (CDF) shows that Florida has more percent of time to have a determinate rain rate in mm/hr than Humacao. Humacao has more percent of time to have the larger rain rate (between 140 to 160 mm/hr) than Florida. Humacao and Florida has the same percent of time to have a smaller rain rate (0 to 10 mm/hr). Florida has more percent of time to have a rain rate between 20 to 140 mm/hr than Humacao. Florida rain rate data shows larger rain probabilities in the range of 20 to 140 mm/hr than Humacao. The conclusions will be summary in that Florida is a worst case of rain rate than Humacao, Puerto Rico. Now the next step that we need to do is investigate is an atmospheric event occur by analyzed Florida data by month.

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