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The Derivation of the Gradient of the Acoustic Pressure on a Moving Surface for Application to the Fast Scattering Code (FSC)

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Abstract- In this paper we develop an analytic expression for calculation of the gradient of the acoustic pressure from a rotating blade on a moving surface for application to the Fast Scattering Code (FSC). The analytic result is intended to be used in the helicopter noise prediction code PSU-WOPWOP. One of the goals of the derivation is obtaining a result that will not use any more information than are needed for the prediction of the thickness and loading noise. The result derived here achieves this goal and its incorporation in PSU-WOPWOP is straight forward and attainable.

1. Introduction

The Fast Scattering Code (FSC) developed by Mark H. Dunn [1] requires the satisfaction of impenetrability condition on the scattering code. This condition requires the knowledge of the acoustic velocity produced by the incidence acoustic wave on the scattering surface. To find this velocity, one needs to find the gradient of the acoustic pressure on this surface. The particular incidence field that we are interested is generated by a helicopter rotor and is calculated from the rotor noise prediction code PSU-WOPWOP [2- 4]. Any other code for noise prediction of rotating blades based on the acoustic analogy can also be used to generate the incident wave.

Ideally, one would like to derive an analytic result for the gradient of the acoustic pressure which would require no more input data than those needed to calculate the thickness and loading noise. If we could get such a result, then the gradient of the acoustic pressure could be computed concurrently with the acoustic pressure using the geometric and kinematic data prepared for the acoustic calculations. This scheme will result in an efficient algorithm for calculating the gradient of the acoustic pressure on a computer. We will show in this paper that it is possible to obtain an analytic expression for the gradient of the acoustic pressure which requires the same data as the thickness and loading noise for its evaluation. In the next section, we will present our derivation of this result.

2. Derivation of The Gradient of the Acoustic pressure

2.1. The Governing Wave Equations

The governing wave equations that we will work with are those generating the thickness and loading noise in the Ffowcs Williams-Hawkings (FW-H) equation [5]. These are

$$\square^2\, p_T' = \frac{\partial}{\partial t}[\rho_o\, v_n\, \delta(f)] \tag{1}$$

$$\square^2\, p_L' = -\frac{\partial}{\partial x_i}[p\, n_i\, \delta(f)] \tag{2}$$

$$p' = p_T' + p_L' \tag{3}$$

The blade surface is described by the equation $f(\boldsymbol{x},\, t) = 0$, where $f(\boldsymbol{x},\, t)$ is defined in such a way that $\nabla f = \boldsymbol{n}$, $\boldsymbol{n}$ being the unit outward normal to the blade with components n_i. Here p' is the acoustic pressure, and p_T' and p_L' are the thickness and loading noise, respectively. The density of the undisturbed medium is ρ_o and the local normal velocity of the blade surface is v_n. The blade surface pressure is denoted p and $\delta(f)$ is the Dirac delta function with the support on the blade surface $f = 0$. The solution of the above wave equations was given by Farassat and Succi [6] and Brentner [7]. The resulting solution, known as *Formulation 1A of Farassat*, is used in PSU-WOPWOP. The readers need some familiarity with the derivation of this Formulation to follow our mathematical work below. We recommend reference [7] for this purpose.

2.2. The Gradient of the Acoustic Pressure

We have

$$\nabla p' = \nabla p_T' + \nabla p_L' \tag{4}$$

We will now work with the formal solutions of eqs. (1) and (2) and manipulate them without integrating the delta functions in the integrands for several steps. We first work on finding the gradient of the thickness noise.

Let $(\boldsymbol{x},\, t)$ and $(\boldsymbol{y},\, \tau)$ be the observer and the source space-time variables, respectively. The Green's function of the wave equation in the unbounded domain is:

$$\begin{aligned} G(\boldsymbol{y},\, \tau;\, \boldsymbol{x},\, t) &= \tfrac{\delta(g)}{4\pi r} \qquad \tau \le t \\ &= 0 \qquad\quad\; \tau > t \end{aligned} \tag{5}$$

$$g = \tau - t + \frac{r}{c} \tag{6}$$

where $r = |\boldsymbol{x} - \boldsymbol{y}|$ and c is the speed of sound in the undisturbed medium.

2.2.1. The Gradient of the Thickness Noise

We introduce a new symbol for the thickness source strength as follows: $Q = \rho_o\, v_n$. The formal solution of eq. (1) is:

$$4\pi\, p_T'\left(\boldsymbol{x},\, t\right) = \frac{\partial}{\partial t}\int \frac{Q}{r}\, \delta(f)\, \delta(g)\, d\,\boldsymbol{y}\, \mathrm{d}\tau \tag{7}$$

where the volume integration is over the entire 3-dimensional space and the source time integration is over $(-\infty,\, t]$.

From eq. (7) we find that

$$4\pi\, \nabla p_T'\left(\boldsymbol{x},\, t\right) = \nabla \frac{\partial}{\partial t}\int \frac{Q}{r}\, \delta(f)\, \delta(g)\, d\,\boldsymbol{y}\, \mathrm{d}\tau = \frac{\partial}{\partial t}\int Q\, \delta(f)\, \nabla_x\left(\frac{\delta(g)}{r}\right) d\,\boldsymbol{y}\, \mathrm{d}\tau \tag{8}$$

where the symbol ∇_x stands for gradient operator with respect to the observer variable $\boldsymbol{x}$. We now use the following result in the integrand of the last integral on the right of eq. (8):

$$\nabla_x\left(\frac{\delta(g)}{r}\right) = -\frac{1}{c}\,\frac{\partial}{\partial t}\left(\frac{\hat{\boldsymbol{r}}\, \delta(g)}{r}\right) - \frac{\hat{\boldsymbol{r}}\, \delta(g)}{r^2} \tag{9}$$

where $\hat{\boldsymbol{r}} = (\boldsymbol{x} - \boldsymbol{y})/r$ is the unit radiation vector from the source to the observer. Using this relation in eq. (8) and bringing the observer time derivative out of the integral, we obtain

$$4\pi \nabla p_T^{'}(\boldsymbol{x}, t) = -\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial}{\partial t}\int \frac{\hat{\boldsymbol{r}}\, Q}{r}\,\delta(f)\,\delta(g)\, d\boldsymbol{y}\, \mathrm{d}\tau + \int \frac{\hat{\boldsymbol{r}}\, Q}{r^2}\,\delta(f)\,\delta(g)\, d\boldsymbol{y}\, d\tau\right) \tag{10}$$

The interpretation of the integrals in this equation using generalized function theory is fairly straight forward [5- 8]. We will just write down the final result here:

$$4\pi \nabla p_T^{'}(\boldsymbol{x}, t) = -\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r(1-M_r)}\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS\right) \equiv -\frac{\partial \boldsymbol{E}_1}{\partial t} \tag{11}$$

where M_r is the local Mach number in radiation direction and

$$\boldsymbol{E}_1 = \frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r(1-M_r)}\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS = \frac{1}{c}\int_{f=0}\frac{\partial}{\partial t}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r(1-M_r)}\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS \tag{12}$$

We now use the following result in the first integral on the right side of eq. (12):

$$\frac{\partial}{\partial t}[\,....\,]_{\mathrm{ret}} = \left[\frac{1}{1-M_r}\frac{\partial}{\partial \tau}\,....\right]_{\mathrm{ret}} \tag{13}$$

We obtain

$$\begin{aligned}\boldsymbol{E}_1 &= \frac{1}{c}\int_{f=0}\frac{\partial}{\partial t}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r(1-M_r)}\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS = \\ &\frac{1}{c}\int_{f=0}\left[\frac{1}{1-M_r}\frac{\partial}{\partial \tau}\left(\frac{\hat{\boldsymbol{r}}\, Q}{r(1-M_r)}\right)\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS = \\ &\frac{1}{c}\int_{f=0}\left[\frac{\hat{\boldsymbol{r}}}{1-M_r}\frac{\partial}{\partial \tau}\left(\frac{Q}{r(1-M_r)}\right)\right]_{\mathrm{ret}} dS + \frac{1}{c}\int_{f=0}\left[\frac{Q}{r(1-M_r)^2}\frac{\partial \hat{\boldsymbol{r}}}{\partial \tau}\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{\hat{\boldsymbol{r}}\, Q}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS\end{aligned} \tag{14}$$

Examining the integrand of the first integral on the right of the second equality sign, we note that

$$E_T \equiv \left[\frac{1}{1-M_r}\frac{\partial}{\partial \tau}\left(\frac{Q}{r(1-M_r)}\right)\right]_{\mathrm{ret}} \tag{15}$$

is the integrand of the thickness noise [6, 7]. This we will write down shortly in detail. Furthermore, we can show that [7]

$$\frac{\partial \hat{\boldsymbol{r}}}{\partial \tau} = \frac{c(M_r\,\hat{\boldsymbol{r}} - \boldsymbol{M})}{r} = -\frac{c(1-M_r)\,\hat{\boldsymbol{r}}}{r} + \frac{c(\hat{\boldsymbol{r}} - \boldsymbol{M})}{r} \tag{16}$$

where $\boldsymbol{M}$ is the local Mach number of the blade surface based on the sound speed c. We remind the readers that the blade surface velocity used in $\boldsymbol{M}$ is described in the frame fixed to the undisturbed medium.

Using eqs. (15) and (16) in eq. (14), we get

$$\boldsymbol{E}_1 = \frac{1}{c}\int_{f=0}[\hat{\boldsymbol{r}}]_{\mathrm{ret}}\, E_T\, dS + \int_{f=0}\left[\frac{(\hat{\boldsymbol{r}} - \boldsymbol{M})\, Q}{r^2(1-M_r)^2}\right]_{\mathrm{ret}} dS \tag{17}$$

From references [6, 7], we have

$$E_T = \left[\frac{\dot{Q}}{r(1-M_r)^2}\right]_{\mathrm{ret}} + \left[\frac{\dot{\boldsymbol{M}}\bullet\hat{\boldsymbol{r}}\; Q}{r(1-M_r)^3}\right]_{\mathrm{ret}} + \left[\frac{c(M_r - M^2)\, Q}{r^2(1-M_r)^3}\right]_{\mathrm{ret}} \tag{18}$$

where the dot on a symbol denotes the rate of change in the source time variable. We note that, by eq. (11), we have

$$4\pi \nabla p_T^{'}(\boldsymbol{x}, t) = -\frac{\partial}{\partial t}\left\{\frac{1}{c}\int_{f=0}[\hat{\boldsymbol{r}}]_{\mathrm{ret}}\, E_T\, dS + \int_{f=0}\left[\frac{(\hat{\boldsymbol{r}}-\boldsymbol{M})\,Q}{r^2(1-M_r)^2}\right]_{\mathrm{ret}} dS\right\} \tag{19}$$

where $\boldsymbol{E}_1$ is given by eq. (17). <u>We propose that the observer time derivative in eq. (19) be taken numerically in PSU-WOP-WOP.</u> Then we see that for calculating the gradient of the thickness noise, we do not need any more data than is needed to calculate the thickness noise. This is the result we were looking for.

2.2.2. The Gradient of the Loading Noise

We will now find the gradient of the loading noise. We write eq. (2) in vector form as follows:

$$\Box^2\, p_L^{'} = -\nabla\bullet[p\,\boldsymbol{n}\,\delta(f)] \tag{20}$$

The Green's function solution of this equation is

$$4\pi\, p_L^{'}\left(\boldsymbol{x}, t\right) = -\nabla\bullet\int\frac{p\,\boldsymbol{n}}{r}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,\mathrm{d}\tau = -\int p\,\delta(f)\,\boldsymbol{n}\bullet\nabla_x\left(\frac{\delta(g)}{r}\right)d\boldsymbol{y}\,d\tau \tag{21}$$

We now use eq. (9) in the integrand of the last integral and then take the observer time derivative outside the integral. The result is

$$4\pi\, p_L^{'}\left(\boldsymbol{x}, t\right) = \frac{1}{c}\,\frac{\partial}{\partial t}\int\frac{p\,\boldsymbol{n}\bullet\hat{\boldsymbol{r}}}{r}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,\mathrm{d}\tau + \int\frac{p\,\boldsymbol{n}\bullet\hat{\boldsymbol{r}}}{r^2}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,d\tau \tag{22}$$

From this result, we get

$$4\pi\nabla\, p_L^{'}\left(\boldsymbol{x}, t\right) = \frac{1}{c}\,\frac{\partial}{\partial t}\int p\,\delta(f)\,\boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}\,\delta(g)}{r}\right)d\boldsymbol{y}\,\mathrm{d}\tau + \int p\,\delta(f)\,\boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}\,\delta(g)}{r^2}\right)d\boldsymbol{y}\,d\tau \tag{23}$$

We remind the readers that at this stage of algebraic manipulations, the observer and the source space-time variables are independent because none of Dirac delta functions have been integrated. This approach makes it easy to interpret the differentiation operators. Had we been working with the integrated results, we would be dealing with heavy algebraic manipulations and the differentiation operators would require careful interpretation.

To carry the analysis in eq. (23) further, we use the following two relations which can be derived easily

$$\boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}\,\delta(g)}{r}\right) = \boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}}{r}\right)\delta(g) + \frac{\boldsymbol{n}\bullet\hat{\boldsymbol{r}}\,\hat{\boldsymbol{r}}}{c\,r}\,\delta^{'}(g) = \frac{\mathbf{n}-2\cos\theta\,\hat{\boldsymbol{r}}}{r^2}\,\delta(g) - \frac{\cos\theta\,\hat{\boldsymbol{r}}}{c\,r}\,\frac{\partial}{\partial t}\,\delta(g) \tag{24}$$

$$\boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}\,\delta(g)}{r^2}\right) = \boldsymbol{n}\bullet\nabla_x\left(\frac{\hat{\boldsymbol{r}}}{r^2}\right)\delta(g) + \frac{\boldsymbol{n}\bullet\hat{\boldsymbol{r}}\,\hat{\boldsymbol{r}}}{c\,r^2}\,\delta^{'}(g) = \frac{\mathbf{n}-3\cos\theta\,\hat{\boldsymbol{r}}}{r^3}\,\delta(g) - \frac{\cos\theta\,\hat{\boldsymbol{r}}}{c\,r^2}\,\frac{\partial}{\partial t}\,\delta(g) \tag{25}$$

where $\cos\theta = \boldsymbol{n}\bullet\hat{\boldsymbol{r}}$, i.e., θ is the angle between local outward normal to the blade surface and the radiation direction. Substituting eqs. (24) and (25) in eq. (23) and gathering terms, we obtain

$$\begin{aligned}4\pi\nabla\, p_L^{'}\left(\boldsymbol{x}, t\right) &= \frac{1}{c}\,\frac{\partial}{\partial t}\left\{-\frac{1}{c}\,\frac{\partial}{\partial t}\int\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,d\tau + \int\frac{p\left(\boldsymbol{n}-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^2}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,d\tau\right\} + \\ &\int\frac{p\left(\boldsymbol{n}-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^3}\,\delta(f)\,\delta(g)\,d\boldsymbol{y}\,d\tau = \\ &\frac{1}{c}\,\frac{\partial}{\partial t}\left\{-\frac{1}{c}\int_{f=0}\left[\frac{1}{1-M_r}\,\frac{\partial}{\partial\tau}\left(\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r(1-M_r)}\right)\right]_{\mathrm{ret}} dS + \int_{f=0}\left[\frac{p\left(\boldsymbol{n}-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^2(1-M_r)}\right]_{\mathrm{ret}} dS\right\} + \\ &\int_{f=0}\left[\frac{p\left(\boldsymbol{n}-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^3(1-M_r)}\right]_{\mathrm{ret}} dS\end{aligned} \tag{26}$$

We now manipulate the first integral in the curly bracket on the right side of this equation to pull out the integrand of the loading noise. To help the readers to follow the algebraic steps below, we mention that the integrand of the loading noise is [6, 7]:

$$\begin{gathered} E_L = \frac{1}{c}\left[\frac{1}{1-M_r}\,\frac{\partial}{\partial \tau}\left(\frac{p\cos\theta}{r\,(1-M_r)}\right)\right]_{\text{ret}} + \left[\frac{p\cos\theta}{r^2\,(1-M_r)}\right]_{\text{ret}} = \\ \frac{1}{c}\left[\frac{\dot{p}\cos\theta}{r\,(1-M_r)^2}\right]_{\text{ret}} + \left[\frac{p\,(\cos\theta - M_n\,)}{r^2\,(1-M_r)^2}\right]_{\text{ret}} + \frac{1}{c}\left[\frac{\dot{\boldsymbol{M}}\bullet\hat{\boldsymbol{r}}\;p\cos\theta}{r\,(1-M_r)^3}\right]_{\text{ret}} + \left[\frac{p\cos\theta\,(\,M_r - M^2\,)}{r^2\,(1-M_r)^3}\right]_{\text{ret}} \end{gathered} \tag{27}$$

With this knowledge, we manipulate the following integral as follows

$$\begin{gathered} \frac{1}{c}\int_{f=0}\left[\frac{1}{1-M_r}\,\frac{\partial}{\partial \tau}\left(\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r\,(1-M_r)}\right)\right]_{\text{ret}} dS = \frac{1}{c}\int_{f=0}\left[\frac{p\cos\theta}{r\,(1-M_r)^2}\,\frac{\partial\hat{\boldsymbol{r}}}{\partial\tau}\right]_{\text{ret}} dS + \\ \int_{f=0}\left\{\frac{\hat{\boldsymbol{r}}}{c}\left[\frac{1}{1-M_r}\,\frac{\partial}{\partial \tau}\left(\frac{p\cos\theta}{r\,(1-M_r)}\right)\right]_{\text{ret}} + \left[\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r^2\,(1-M_r)}\right]_{\text{ret}} - \left[\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r^2\,(1-M_r)}\right]_{\text{ret}}\right\} dS = \\ \frac{1}{c}\int_{f=0}\left[\frac{p\cos\theta}{r\,(1-M_r)^2}\left(-\frac{c(1-M_r)\,\hat{\boldsymbol{r}}}{r} + \frac{c(\hat{\boldsymbol{r}}-\boldsymbol{M})}{r}\right)\right]_{\text{ret}} dS = \\ \int_{f=0}[\hat{\boldsymbol{r}}]_{\text{ret}}\,E_L\,dS - 2\int_{f=0}\left[\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r^2\,(1-M_r)}\right]_{\text{ret}} dS + \int_{f=0}\left[\frac{p\cos\theta\,(\,\hat{\boldsymbol{r}}-\boldsymbol{M})}{r^2\,(1-M_r)^2}\right]_{\text{ret}} dS \end{gathered} \tag{28}$$

Here, we have used eq. (16) on the right of the first equality sign. We now use this result in eq. (26) which looks without simplification as:

$$\begin{gathered} 4\pi\nabla\,p_L^{'}\left(\boldsymbol{x},\,t\right) = \frac{1}{c}\,\frac{\partial}{\partial t}\left\{-\left\{\int_{f=0}[\hat{\boldsymbol{r}}]_{\text{ret}}\,E_L\,dS - 2\int_{f=0}\left[\frac{p\cos\theta\,\hat{\boldsymbol{r}}}{r^2\,(1-M_r)}\right]_{\text{ret}} dS + \int_{f=0}\left[\frac{p\cos\theta\,(\hat{\boldsymbol{r}}-\boldsymbol{M})}{r^2\,(1-M_r)^2}\right]_{\text{ret}} dS\right\} + \right. \\ \left. \int_{f=0}\left[\frac{p\left(\boldsymbol{n}\,-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^2(1-M_r)}\right]_{\text{ret}} dS\right\} + \int_{f=0}\left[\frac{p\left(\boldsymbol{n}\,-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^3(1-M_r)}\right]_{\text{ret}} dS \end{gathered} \tag{29}$$

We next simplify this result to get the result we are seeking:

$$\begin{gathered} 4\pi\nabla\,p_L^{'}\left(\boldsymbol{x},\,t\right) = \frac{1}{c}\,\frac{\partial}{\partial t}\left\{-\int_{f=0}[\hat{\boldsymbol{r}}]_{\text{ret}}\,E_L\,dS + \int_{f=0}\left[\frac{p\left(\boldsymbol{n}\,-\,\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^2(1-M_r)}\right]_{\text{ret}} dS - \int_{f=0}\left[\frac{p\cos\theta\,(\,\hat{\boldsymbol{r}}-\boldsymbol{M})}{r^2\,(1-M_r)^2}\right]_{\text{ret}} dS\right\} + \\ \int_{f=0}\left[\frac{p\left(\boldsymbol{n}\,-3\cos\theta\,\hat{\boldsymbol{r}}\right)}{r^3(1-M_r)}\right]_{\text{ret}} dS \end{gathered} \tag{30}$$

We propose again that the observer time derivative in eq. (30) be taken numerically in PSU-WOPWOP. We see that we do not need any more data than those needed to calculate the loading noise to find the gradient of the loading noise.

3. Concluding Remarks

This analysis was performed to find the gradient of the acoustic pressure in PSU-WOPWOP which will be used in the Fast Scattering Code of Dunn [1]. Our main results, eqs. (19) and (30), use only the geometric and kinematic data available for thickness and loading noise calculation. This means that the gradient of the acoustic pressure can be calculated concurrently with the thickness and loading calculation in PSU-WOPWOP. The inclusion of our results in PSU-WOPWOP is quite easy and minimal programming will be involved. One does not expect noticeable increase in the execution time on a computer as compared to that of acoustic calculation only.

We mention here that the choice of our main results was dictated by the use of Formulation 1A in PSU-WOPWOP. Another option available to us is to take the time derivatives in eqs. (19) and (30) analytically using the same method we have followed above. Such a method will give a new analytic expression for the gradient of the acoustic pressure without the need for numerical differentiation with respect to the observer time. We have, however, rejected this procedure for the following reasons:

1. The new analytic expression will have many more terms than those in eqs. (19) and (30). Checking the accuracy of the algebraic manipulations used in the derivation will be much harder. Programming and debugging of the new expression will be time consuming.

2. There will be some new quantities needed in the evaluation of the new analytic expression, e.g., the second time derivative of the blade surface pressure, that will have to be prepared in the preprocessor. These quantities are not needed in Formulation 1A and the preparation of these quantities for the input file will need new programming effort and debugging.

3. Although numerical differentiation is avoided and this will reduce the execution time on a computer, more time will be spent on the evaluation of quantities that are not needed in Formulation 1A which will increase the execution time. Therefore, there may not be any net reduction of execution time by getting rid of the numerical differentiation.

The extreme simplicity of implementing eqs. (19) and (30) in PSU-WOPWOP makes them ideal for the calculation of the gradient of the acoustic pressure needed in the Fast Scattering Code (FSC).

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14. ABSTRACT

In this paper we develop an analytic expression for calculation of the gradientof the acoustic pressure from a rotating blade on a moving surface for application to theFast Scattering Code (FSC). The analytic result is intended to be used in the helicopternoise prediction code PSU-WOPWOP. One of the goals of the derivation is obtaining aresult that will not use any more information than are needed for the prediction of thethickness and loading noise. The result derived here achieves this goal and its incorpora-tion in PSU-WOPWOP is straight forward and attainable.

15. SUBJECT TERMS
Aeroacoustics; Fast scattering code (FSC); Helicopter noise; Noise Prediction; PSU WopWop; Fuselage scattering

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