Carrier Modulation Via Waveform Probability Density Function

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Abstract—**Beyond the classic modes of carrier modulation by varying amplitude (AM), phase (PM), or frequency (FM), we extend the modulation domain of an analog carrier signal to include a class of general modulations which are distinguished by their probability density function histogram. Separate waveform states are easily created by varying the pdf of the transmitted waveform. Individual waveform states are assignable as proxies for digital ONEs or ZEROs. At the receiver, these states are easily detected by accumulating sampled waveform statistics and performing periodic pattern matching, correlation, or statistical filtering. No fundamental natural laws are broken in the detection process. We show how a typical modulation scheme would work in the digital domain and suggest how to build an analog version. We propose that clever variations of the modulating waveform (and thus the histogram) can provide simple steganographic encoding.**

Index Terms—**Communication systems, data communication, modulation, and probability.**

I. INTRODUCTION

For many years the definitions of ac-carrier modulation types have remained constant [1], [3], namely forms of amplitude modulation (AM) and angle modulation (PM) and subtypes and variations such as FM and QPSK. We recently have explored waveform types in which the information is impressed onto the carrier by varying the probability density function (pdf) of the wave (histogram), which can be done such that the long-term amplitude- or phase-envelope are of little concern. That is, in a noisy channel where the signal may suffer long-term or short-term variations of amplitude or phase or frequency, it is still possible via statistical analysis of the waveform to recognize patterns in the pdf which imply reception of digital ONE or ZERO. We view pdf-modulation as representing a new modulation domain, which although it might not be described as having a high bit-rate efficiency, it does have the advantage of permitting steganographic masking of information content.

II. MODULATION REQUIREMENTS

In AM and PM and their various subtypes, the resulting modulated waveforms generally appear as ac- or rf-carriers that resemble quasi-sine waves. A well-behaved modulated carrier signal must be stationary and ergodic [2]. We note here that the modulating waveform, to which information has been

added, does not have to be subsequently transmitted over ac-coupled or wireless transmission channels. For example, modulation can be impressed onto a dc carrier, i.e., a baseband carrier. Our discussion does not rule out subsequent modulation of the baseband signal onto a secondary ac-coupled or rf carrier, so long as at the receiving end of the communications channel, the original baseband signal can be detected and extracted from the secondary ac-carrier without gross distortion. Comments on noise distortion of a non-noisy desired signal will follow later.

A. Modulation Model

We introduce here a basis for the new model, pdf-modulation. We begin by rejecting the premise that modulation of amplitude or angle is all that can be done. We generalize to say that if any controlled distortion of a carrier's waveform shape can be recognized by a receiver at a later time or at a different location, then we meet all the needs for modulation of information onto a carrier. The probability density function of a waveform distribution function $F(x)$ can be defined as [2]:

$$f(x) = \frac{dF(x)}{dx} \geq 0 \qquad (1)$$

We presume the ability to distort the carrier signal by implementing (1). Therefore in principle we have gone beyond amplitude or angle modulation as defined long ago [1].

In our model, the information to be modulated onto the carrier conceptually employs a Digital-to-Analog Converter (DAC) to realize a pre-defined controlled distortion to the quasi-sinewave carrier signal as in Fig. 2. Our DAC operates at a sample rate much higher than the information data rate, so that the final modulated carrier signal is comprised of a smoothed but modulated carrier signal comprised of a large number of small step-voltage changes in the DAC output, to become one complete cycle of the modulating signal. The exact nature of the smoothing technique is not important for this model and will not be further discussed.

In our model it is sometimes useful to have from 1-to-n whole half-cycles of carrier signal ("modulation packet") to complete the modulation of a single "unit of information". Thus the zero-crossings of the modulated signal might also be detected in order to keep track of the progress. However, implementation of dynamic LIFO (Last-In-First-Out) techniques and dynamic histogramming can bypass the concern about having to detect waveform zero crossings.

In the simplest version of the modulator, the DAC generates

the modulating signal by being stepped through a large "source table" of numbers representing all the samples necessary to synthesize the packet of carrier signal necessary to carry one unit (e.g., bit) of information. To encode binary digital data, there would be two or more source tables, with each table used to generate one modulation packet for each unique signal state. Although our examples will discuss binary data transmission, but there is no requirement to limit the number of information states to only two if a multilevel digital system is being defined.

Secondly, our model assumes the existence of an Analog-to-Digital Converter (ADC) which transfers a continuous stream of samples of the received signal to a device which generates a data table representing a dynamic histogram of the modulation packets. Histograms [2], and the statistics for pdf-modulation decoding, are thus extracted from the data.

In our model, the resulting successive histograms of the received modulated signal are processed by some algorithmic "black-box," such as by computer-driven statistical analyses, pattern-matching, correlation techniques, or by means such as neural networks, to produce a list of "units of information" that should match the original list that was encoded by the DAC and source-tables in the transmitter. We assume that there may be a "training time" involved in the algorithm for decoding the units of information. In the end, the algorithm must distinguish by some means when the probability density function of the extracted modulation has changed from one information state to another, as part of the novelty of our method.

B. Additional Subtleties

We have purposely not specified nor claimed that there exist only one set of tables used to perform optimum (in the mathematical sense) modulation of the carrier waveforms based on the information to be modulated. We have merely stated that the model requires creating modulating waveforms that have identifiable mathematical statistics after reception and demodulation, where the statistics are determined strictly by use of waveform probability density function (pdf) and dynamic histogramming. Setting aside the problem (outside the versions of this model) where zero-crossings in the received carrier signal might be useful in a design implementation, our model includes the use of any means, analog or digital, for recognition of differences in the modulation statistics based on the received probability density functions.

C. Example Forms of pdf-Modulation

Figs. 1 to 4, show two different sets of pdf-modulation waveforms, which we will also subsequently show have differing advantages and disadvantages. We do not claim either of these two sets to be the only possible forms, nor optimized forms. They merely serve as trial examples for some instructive simulations. In Figs. 3 and 4, a rather obvious distinction in waveform pdf is created by use of a sawtooth waveform and a square-wave waveform for representing the two states of a binary-based state system, state A and state B.

In Figs. 1 and 2 the two binary states are represented by quasi-sinewave waveform having two symmetric but variable levels where kinks occur. If the kinks are all at the same relative sampling level, one state is implied. If one half of the kinks are raised in level, the other binary state is implied. Note that the shape of the distortions, and hence pdf, of the waveforms is important, not the peak-to-peak values.

The reader should at this point discern that a large number of possible pdf-modulation carrier waveforms exist even though we show only two example sets.

D. Signal to Noise Ratio Issues

Because communications channels typically add undesired noise to the signal [4], we test sensitivity of the above pdf-modulation processes to errors by simulating statistical noise in the received signal. We have tested our example modulation types by creating spreadsheet simulations, in which random quasi-white noise is added (with the RAND function), and by using the statistical skewness function from statistics on a distribution, we have empirically derived some indications of process signal-to-noise (S/N) ratio robustness. ((2) is one representative definition of skewness [7].)

$$\text{Skewness} = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_j - \bar{x}}{\sigma} \right)^3 \tag{2}$$

Table I shows a comparison of noise sensitivity of the quasi-sinewave small scale distortion process example of Fig. 5 when compared to the large-scale process of Figs. 6. However, merely listing some values tends to be inconclusive. Therefore we assume a binary "slicing level" exists between the skewness of state A and the skewness of state B, such that if the ratio of levels calculated over reference slicing level is greater than unity, then state A "wins" and if it is less than unity, then state B "wins". Since we already know which waveform represents state B and which waveform represents state A, a false "win" is actually an error condition where the noise level has overridden the signal level and therefore S/N is too low. Note that the following data is based on many random trials, while looking for the minimum values. Anyone performing similar simulations will see slightly different data depending on the number of "runs" attempted.

Viewing the ratios in Table II, we see that set I sensitivity to noise is obviously worse than in Set II. In Set I it was possible to measure SKEWNESS clear across the entire half-waveform in spite of noise level. In Set I, the small-scale steps needed to be resolved by measuring only the SKEWNESS of the histograms in the areas where the steps were presumed to exist. This fact, coupled with the results in Table II, shows that the large-scale example is far less sensitive to noise than the small scale example. *But we do not claim this as proof that the large scale process is mathematically optimum.*

TABLE I
INITIAL SIMULATION RESULTS

Detection Windows (See Set I)	Small-Scale Process		Detection Windows (See Set II)	Large-Scale Process	
Noise Seed for "Rand" Function	Typical State A Skewness	Typical State B Skewness	Noise Seed for "Rand" Function	Typical State A Skewness	Typical State B Skewness
0	0	0	0	0	0
1	0.43	1.09	50	0.97	2.73
1	−0.55	1.52	50	1.48	3.10
2	0.056	3.22	75	1.58	2.66
2	0.54	1.43	75	1.58	2.22
3	0.77	2.15	100	1.58	3.05
3	0.54	0.88	100	1.04	2.02
4	−0.98	0.29	125	1.09	2.09
4	0.55	0.35	125	1.75	1.70

TABLE II
LONG-TERM SKEWNESS RATIOS

Detection Windows (See Set I)	Small-Scale		Detection Windows (See Set II)	Large-Scale	
Noise Seed for "Rand" Function	Lowest Skewness Ratio After Many Trials	Level of Largest Ignorable Noise Level (Ratio > 1.0)	Noise Seed for "Rand" Function	Lowest Skewness Ratio After Many Trials	Level of Largest Ignorable Noise Level (Ratio > 1.0)
0	NA		0		
1	1.04	TBD	50		
2	0.17		75	1.05	
			78	>1.0	TBD
			82	<1.0	
			88	0.97	
			100		
			125		

Set I: measure SKEWNESS @ array (104:119) in first cycle for STATE A
measure SKEWNESS @ array (104:119) in second cycle for STATE B
Set II: measure SKEWNESS @ array (0:255) in sawtooth wave for STATE A
measure SKEWNESS @ array (0:255) in square wave for STATE B
TBD: To be determined (is a statistical value that is best determined by averaging a large number of tests).

E. Analog Detection Model

As another variation of our model, we propose a conceptual analog means of waveform decoding and histogram statistical analysis. As in Fig. 7, we first propose using a window comparator with integrated output [5] to capture each slice of waveform level as a kind of analog histogram recorder. As in Fig. 8, we propose use of a tree of window comparators, where outputs are dynamically and continuously evaluated as analog levels by use of a generic industrial summing amplifier/comparator circuit [6]. Neither of these industry examples, the window comparator/ integrator, nor the summing amplifier/comparator, uses any exotic technology. But whenever any generic analog circuitry is specifically connected to perform an analog equivalent of digital histogram detection of pdf-modulation, the combined model is of novel interest.

F. Steganographic Issues

We have discussed robustness and system noise issues when using pdf-modulation. But the implications of the model are that over a long period of time the system must have some means to accommodate signal fading and/or other signal variations which would lead to long-term changes in the statistics of the histogram. Therefore we assume that receive-side "training" continues as long as the receiving process continues. That is, each new histogram updates the state-of-information in the whole receive correlation process.

As a result, the aggregate statistics of the transmitted signal are variable over a range and in a manner such that the receiver can, through continuous training, adapt to changes in the signal. We therefore propose that added quasi-random noise, bias shifts, changes from square wave to triangle wave, and so forth, can be used intentionally to complicate the appearance of the transmitted waveform, so long as the interference does not confuse receiver training. (Or, in the transmitter, a process would just make a dramatic shift in the waveform statistics and then allow a built-in delay time for the receiver to retrain before starting to send real information). Note that we are neither modeling nor including use of spread-spectrum techniques in our model.

On a telecommunications circuit, where the channel could be "wire-tapped", the intentional use of exotic signal patterns would tend to make signals look more like noise, thereby intentionally attempting to confuse detection by an unwelcome observer. Therefore, it is postulated, but not proven here, that use of pdf-modulation would permit noise-like modulation to carry information, when by its nature the noise-like modulation would resemble real channel noise. Although complicated by the issue of designing a system process for extraction of intentional noise-like modulation from natural real noise, the net effect of successful transmission and detection of information would be that of making surreptitious monitoring more difficult.

III. CONCLUSIONS

We have shown via simulation of pdf-modulation that the model successfully transfers information through a communications channel provided that the signal-to-noise ratio is manageable. We have shown that successful operation of a pdf-modulation process in the presence of noise requires an increasingly higher signal-to-noise ratio when there is a lower degree of distortion or "cleverness" of the modulation waveform. Simulated signals with noise have shown effects that are entirely expected by classical communications theory [4], but with effects that are difficult to define in closed-form equations. The model does not violate any rules covering signal-to-noise ratio versus channel bit rate. Pdf-modulation, being fundamentally unlike the "older" existing industry-practice forms such as AM and PM, merits inclusion in the modulation domain as pdf- or histogram-modulation. We have proposed both analog and digital models for implementing the process.

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[7] *HELP* for SKEW worksheet function, Microsoft Corporation EXCEL 2002™, which is defined differently than the standard SKEWNESS in other sources.
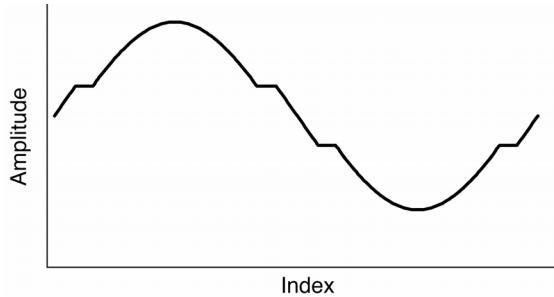
Fig. 1. Modified sinewave modulation with steps even, for state A.
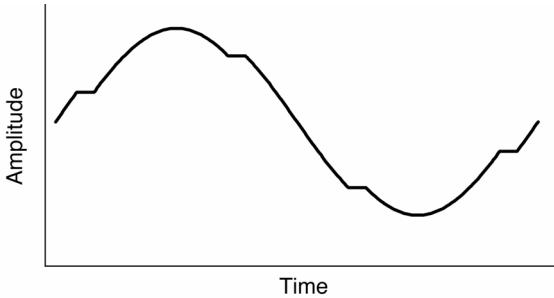
Fig. 2. Modified sinewave modulation with steps offset, for state B.
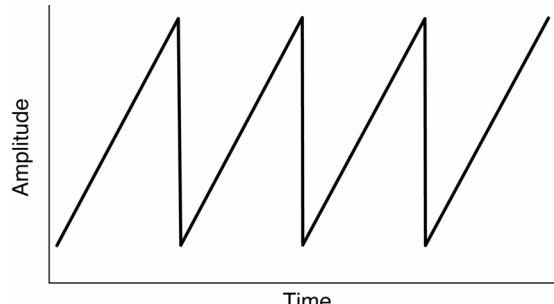

Fig. 3. Triangle modulation for state A.
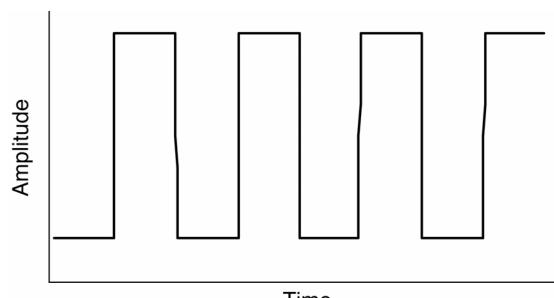

Fig. 4. Square modulation for state B.
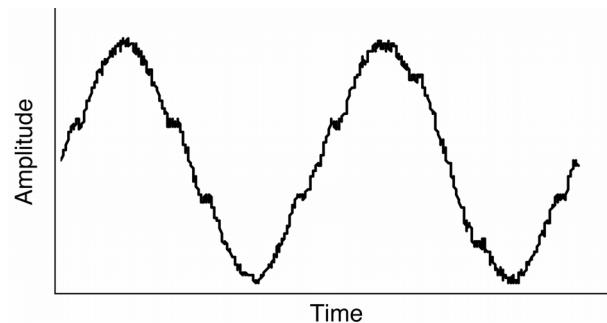
Fig. 5. One cycle of state A is concatenated with one cycle of state B, using modified sinusoidal modulation of Figs. 1 and 2, and with added noise.
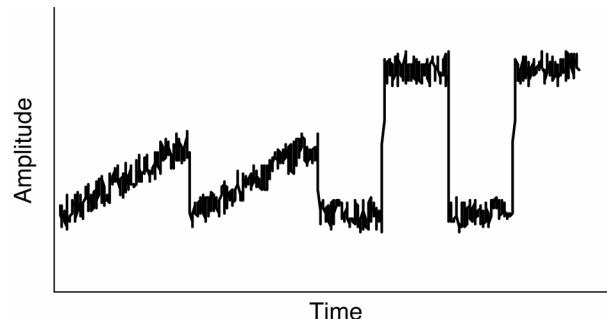

Fig. 6. One cycle of state A is concatenated with one cycle of state B, using triangle and square modulation of Figs. 3 and 4, with considerable added noise.
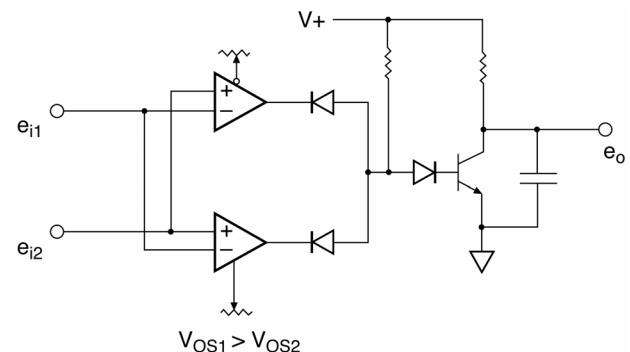

Fig. 7. Representative analog window comparator/integrator.
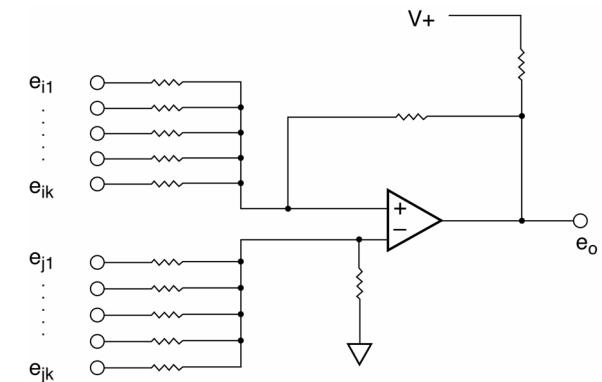
Fig. 8. Representative analog summing amplifier/comparator.