2004

NASA FACULTY FELLOWSHIP PROGRAM

MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA

CONTROLS FOR REUSABLE LAUNCH VEHICLES DURING TERMINAL AREA ENERGY MANAGEMENT

Prepared By:	Brian J. Driessen
Academic Rank:	Assistant Professor
Institution and Department:	University of Alabama in Huntsville Department of Mechanical and Aerospace Engineering
NASA/MSFC Directorate:	Space Transportation
MSFC Colleague:	Charles E. Hall

Notation

R .... rotation matrix from a frame fixed to the vehicle and a global inertial reference frame. Or, if w is a vector expressed in the vehicle's coordinate frame, then Rw is the same vector expressed in the global frame.

$\vec{i}_G, \vec{j}_G, \vec{k}_G$ .... global x-axis (North), global y-axis (East), global z-axis (Down)

$\vec{i}_A, \vec{j}_A, \vec{k}_A$ .... vehicle x-axis (forward), vehicle y-axis (right), vehicle z-axis (down)

J .... inertia matrix of RLV vehicle, $J \in R^{3\times3}$, $J = J^T > 0$

m .... vehicle mass

g .... scalar, acceleration due to gravity

q .... Euler angles of RLV orientation, $q \in R^3$ (roll, pitch, and yaw), representing the orientation of the vehicle relative to a global inertial reference frame. $q \equiv (q_{roll}, q_{pitch}, q_{yaw})^T$.

ω .... angular velocity of RLV expressed in vehicle coordinates, $\omega \in R^3$

$\vec{a}$ ……………………………………………….. $\in R^3$, acceleration of vehicle mass center, expressed in vehicle coordinates, i.e., $a = R^T \ddot{p}$.

a_n .... projection of a onto the global negative z unit axis $-\vec{k}_A$, i.e., the component of a, also called the "normal acceleration" of the vehicle

υ_m .... vehicle Mach number (at center of mass)

δ .... $\delta \in R^8$ is the vector of control surface positions, for left and right flaps, left and right rudders, two inboard elevons, and two outboard elevons.

$\delta_{\min}$, $\delta_{\max}$ .... $\delta_{\min} \in R^8$, $\delta_{\min} < 0$, $\delta_{\max} \in R^8$, $\delta_{\max} > 0$, denote the lower and upper limits of the control surface positions δ .

$\dot{\delta}_{\max}$ .... $\dot{\delta}_{\max} \in R^n$ denotes an upper limit on $|\dot{\delta}|$, about 30 degrees per second for each control surface, except for the flaps with a limit of $10^\circ s^{-1}$.

τ_a .... $\tau_a \in R^3$, net aerodynamic torque exerted on the vehicle, expressed in the vehicle frame.

F_a .... $F_a \in R^3$, net aerodynamic force exerted on the vehicle, expressed in the vehicle frame.

S_τ .... $\partial\tau_a / \partial\delta$, called the "torque sensitivity matrix."

$A(q)$ .... $A \in R^{3\times3}$. known partial derivative of ω with respect $\dot{q}$. A is nonsingular along any guidance commanded trajectory. A is a (known) function of q. $\omega = A(q)\dot{q}$.

M .... JA

subscript g.. denotes a guidance-commanded quantity. For example, $q_{roll)g}$ is the guidance-commanded value of q_{roll}.

$sk(v)$ .... the 3 by 3 matrix that when post-multiplied by a vector w gives the cross product of v and w. Or, $sk(v)w = v \times w$.

$\dot{p}$ .... the vehicle's linear velocity in global coordinates.

Q .... dynamic pressure, $\frac{1}{2}\rho\|\dot{p}\|_2^2$, where ρ is the air density.

α .... angle of attack

β .... sideslip angle

The Problem

During the terminal energy management phase of flight (last of three phases) for a reusable launch vehicle, it is common for the controller to receive guidance commands specifying desired values for (i) the roll angle q_{roll}, (ii) the acceleration a_n in the body negative z direction, $-\vec{k}_A$, and (iii) ω_3, the projection of ω onto the body-fixed axis $\vec{k}_A$, is always indicated by guidance to be zero. The objective of the controller is to regulate the actual values of these three quantities, i.e make them close to the commanded values, while maintaining system stability.

The equations of motion are given by:

$$\begin{aligned} JA\ddot{q} &= -J\dot{A}\dot{q} - sk(A\dot{q})JA\dot{q} + \tau_a \\ M\ddot{q} &= f + \tau_a \\ \ddot{q} &= F + M^{-1}\tau_a \\ ma &= F_a + mgR^T e_3 \end{aligned} \quad , \quad \text{where} \quad \begin{aligned} f &\equiv -sk(A\dot{q})JA\dot{q} - J\dot{A}\dot{q} \\ M &\equiv JA \\ F &\equiv M^{-1}f \\ e_3 &\equiv (0,0,1)^T \end{aligned} \qquad (1)$$

The following are measured and available to the controller: δ, q, ω, a, α, β, υ_M, and Q. In the next section, we present a controller for this problem, including simulation results.

Proposed Controller

Denote the commanded orientation of the vehicle as q_c, whose value will be designed shortly. The commanded torque, $\tau_{a)c}$, is output by the orientation controller and is chosen so as to make the model-based value of $\ddot{q}$ satisfy:

$$\ddot{q} + [\Lambda_1]\dot{q} + [\Lambda_2](q - q_c) + [\Lambda_3]\int_0^t (q - q_c)dt = 0 \qquad (2)$$

where the $[\Lambda_i]$ are diagonal gain matrices producing stability within the last equation. The values used in our simulation were:

$$\Lambda_1 = diag(2\mu, 3\mu, 3\mu),\ \Lambda_2 = diag(\mu^2, 3\mu^2, 3\mu^2),\ \Lambda_3 = diag(0, \mu^3, \mu^3), \qquad \mu = 4 \qquad (3)$$

giving two-percent settling times of 1 second. First, $q_{c)1}$ is already given as the guidance-command for the roll-angle. Then, the yaw-command $q_{c)3}$ is chosen as $q_{c)3} = \text{atan2}(\dot{p}_2, \dot{p}_1)$, which would be associated with zero side-slip if $\dot{p}_3$ were zero. Finally, the commanded pitch angle, $q_{c)2}$, is chosen as:

$$q_{c)2} \equiv \theta_c , \tag{4}$$

where θ_c is purely model-based and approximates the value of $q_{c)2}$ that would produce a_n as:

$$a_n = a_{n)c} \equiv a_{n)g} - k_i \int_0^t (a_n - a_{n)g}) dt \tag{5}$$

at the current mach, dynamic pressure, zero-sideslip, while $q_1 = q_{c)1}$ and $q_3 = q_{c)3}$. The value of θ_c is constructed as follows. Off-line, before flight, a table is constructed which inputs angle of attack, mach, and dynamic pressure, and outputs the corresponding a_n. At the current time, t, a *simple one-dimensional search within this table* produces the value of θ_c, at virtually no computational cost. The second term in the expression for $a_{n)c}$ in (5) helps to correct for modeling errors. The integral gain k_i was chosen as $k_i = 4/5$, making (5) slower than (2).

The surface deflections δ are driven by actuators with 3rd order (flaps) or 4th order (rudders/elevons) actuator models. These actuators have either a 0.15-second 2% time-constant {flaps} or a 0.05-second 2% time-constant {rudders/elevons}. The commanded deflections, δ_c, are chosen as follows. Since the aerodynamic torque, τ_a, is not a perfectly linear function of δ, using a delayed value of the Jacobean, $S(t-\Delta)$, $\Delta = 0.006s$ as for one controller sample period, we perform a truncated {fixed number} set of damped {half-step} Newton-Rhaphson iterations, with the single *fixed* Jacobean, i.e., fixed for that time instant (or sample period). In particular,

$$\hat{\delta}_{k+1} = sat\left[\left\{ \hat{\delta}_k + \tfrac{1}{2} G^{+} \begin{pmatrix} -\hat{\delta}_k \\ -(\tau_a - \tau_{a)c}) \end{pmatrix} \right\}_{1:8}, 0.75\delta_{\min}, 0.75\delta_{\max} \right], \quad (k=1,\ldots,20) \tag{6}$$

where the subscript $1:8$ denotes the first eight entries of the vector and where we are saturating at 75% of the true actuator bounds to help avoid hitting the true actuator limits. The same percentage is used for rate-limit avoidance. The iteration is initialized at $\delta(t-\Delta)$. The simulations used 20 iterations, thus involving 20 evaluations of the look-up table for the aerodynamic torque (at the current υ_M, Q, α, and $\beta = 0$), 20 back substitutions, and one matrix factorization. The matrix G is the one associated with the stationarity condition for minimizing the sum of the squares of the δ_i subject to achieving the specified torque:

$$G \equiv \begin{bmatrix} I & S(t-\Delta)^T \\ S(t-\Delta) & 0 \end{bmatrix} \tag{7}$$

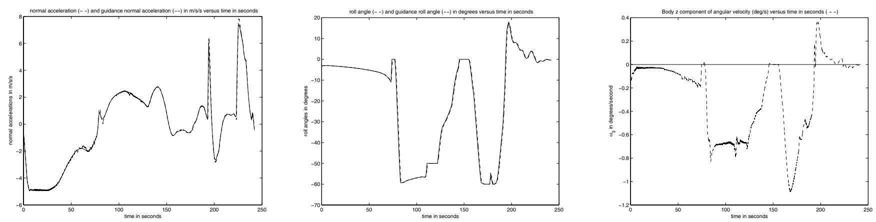
The output of these Newton iterations is fed into a direct rate-limiter, with the rate-limits set at 75% of $\dot{\delta}_{\max}$. This result is in turn fed into a filter $\dfrac{100^2}{s^2 + 200s + 100^2}$, the output of which is declared our commanded actuator deflection, δ_c. In all simulations, this choice of δ_c produced absolutely no saturation of the actuators' *actual* bounds nor any saturation of the actuators' *actual* rate-limits; *thus, the 3rd order (flaps) and 4th order (rudders/elevons) dynamic actuator models were fully satisfied within all simulations, the results of which we now present in the next section*. For possible reference,

$$\begin{aligned} \delta_{\min} &= [-15; -15; -60; -30; -30; -30; -30; -30] \text{ degrees} \\ \delta_{\max} &= [26; 26; 30; 60; 25; 30; 25; 30] \text{ degrees} \end{aligned} \tag{8}$$

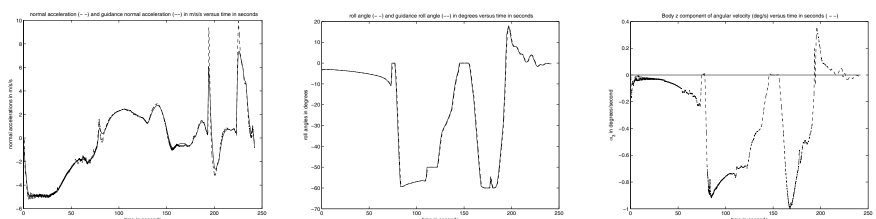
In the next section, we present numerical simulation results for a set of typical guidance commands. In these simulations, the guidance-commands are *open-loop [which is not typical of the actual guidance but gives a practical test of the controller].*

Results

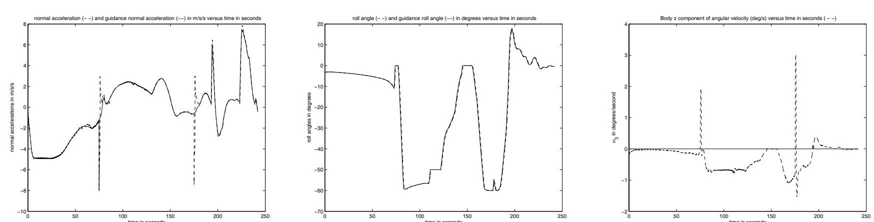
A set of typical guidance commands $(a_{n)d}, q_{roll)d})$ were used to test the proposed controller. The nominal case's result is shown in Figures 1-3 below. In the second case, the actual body-z air force is *two times larger* than that of the model – see Fig's 4-6. In the third case, 1.2g {where 1g is 9.81 m/s/s} *windgusts*, each of duration 1 second, occur at times $t = 75s$ and $t = 175s$ -- Fig's 7-9. In these plots, the axes-ranges are $[0s, 243s]$, $[-70^\circ, 20^\circ]$, $[-6m/s^2, 8m/s^2]$, or $[-1.2^\circ s^{-1}, 0.4^\circ s^{-1}]$, except for Fig's 4, 7 and 9 with ranges $[-6m/s^2, 10m/s^2]$, $[-10m/s^2, 8m/s^2]$ and $[-2^\circ s^{-1}, 4^\circ s^{-1}]$.



Figures 1-3. a_n (m/s/s), roll (deg), ω_3 (deg/s). dashed = actual, solid = guidance



Figures 4-6. a_n (m/s/s), roll (deg), ω_3 (deg/s). dashed = actual, solid = guidance



Figures 7-9. a_n (m/s/s), roll (deg), ω_3 (deg/s). dashed = actual, solid = guidance

Acknowledgements

The author would like to thank the entire team of engineers and scientists in the guidance and control research area of the Marshall Space Flight Center's Transportation Directorate. Special thanks goes to my NASA colleague Charlie Hall and the NFFP program managers Gerry Karr and Jeanelle Bland.