Emission II: Collisional Plasmas

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Introduction

We have covered the basic atomic processes that are important in X-ray emitting plasmas: collisional excitation/ionization, photoexcitation/ionization, radiative decay and so on.

X-ray emitting plasmas are separated into two types:

- Collisional: $k_B T_e \sim$ Ionization energy of plasma ions
- Photoionized: $k_B T_e \ll$ Ionization energy of plasma ions

What about plasmas in local thermodynamic equilibrium (LTE)?

This occurs if $N_e > 1.8 \times 10^{14}\, T_e^{1/2}\, \Delta E_{ij}^{3}$ cm^{-3}.

For $T_e = 10^7$K for H-like Iron, $N_e > 2 \times 10^{27}$ cm^{-3}.

For $T_e = 10^5$K for H-like Oxygen, $N_e > 10^{24}$ cm^{-3}.

Introduction

Astrophysical **collisional** plasmas come in two types:

- Coronal/Nebular: $N_e < 10^{14} - 10^{16}$ cm^{-3}
- Collisional-Radiative: 10^{14} cm^{-3} $< N_e < 10^{27}$ cm^{-3}

In the more common Coronal (or Nebular) plasma, collisions excite ions but rarely de-excite them; the decay is radiative. This is also called the "ground-state" approximation, as all ions are assumed to be in the ground-state when collisions occur.

In a CR plasma, collisions compete with photons in de-exciting levels; a level with a small oscillator strength (transition rate) value may be collisionally de-excited before it can radiate.

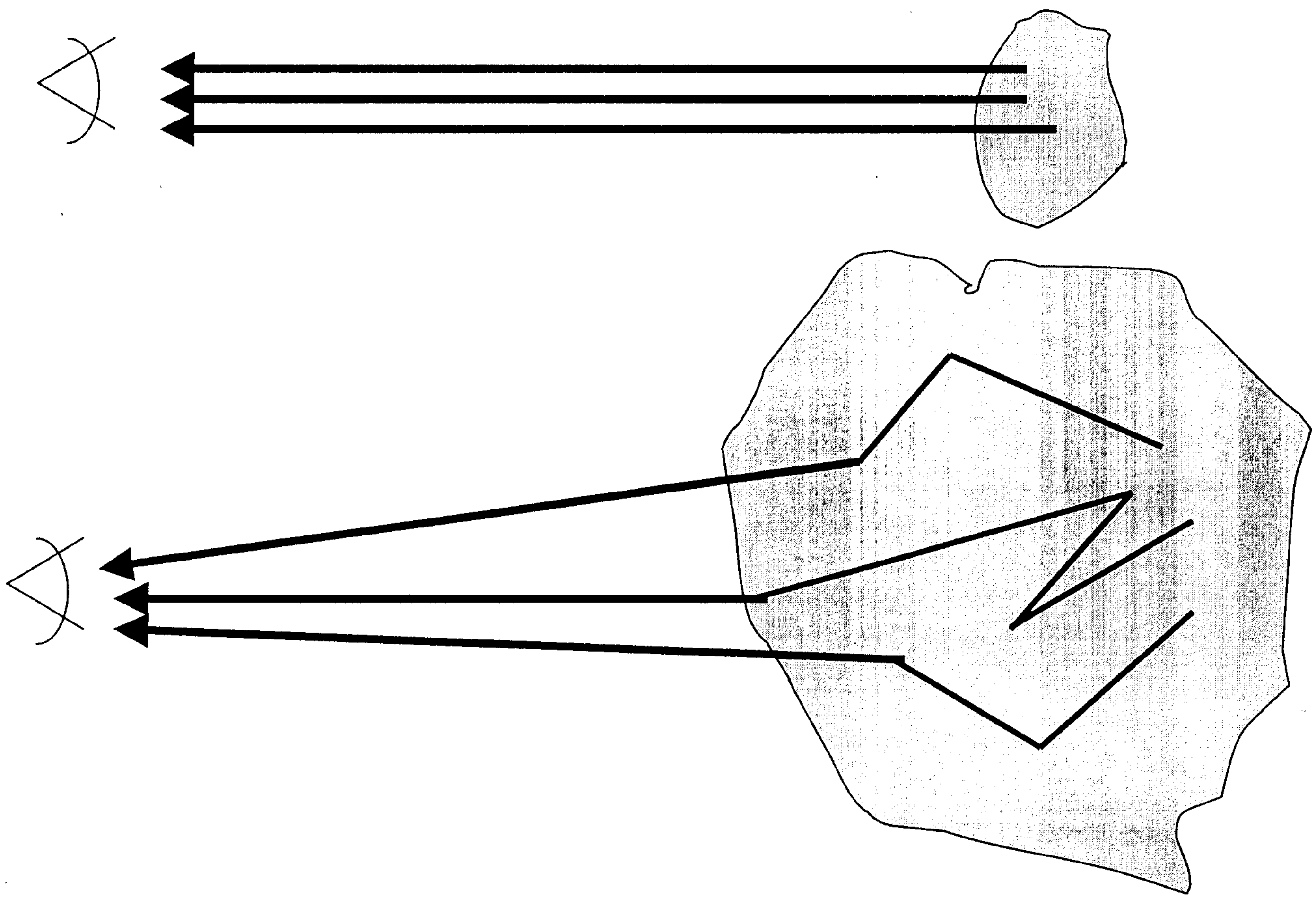
Introduction

We will make some initial assumptions about our "astrophysical plasmas":

- They are dominated by H and He, with trace metals.
- Any magnetic and electric fields do not significantly affect the ion level structure.
- No nuclear transitions are significant.
- The electrons have a thermal (Maxwellian) velocity distribution.

Optical Depth

But what about radiative excitation? Can't photons still interact with ions, even in a collisionally ionized plasma?



Optical Depth

So, is photon scattering an important process?

Yes, but only for allowed transitions; in a collisional plasma, many transitions are forbidden or semi-forbidden.

So couldn't this show up as optical depth in allowed lines, weakening them relative to forbidden lines?

Yes, and this can be calculated after modeling a plasma. Using the ionization balance and the coronal approximation, along with the A value for the transition and the emitting volume, it is easy to calculate the optical depth for a line:

$$\tau = n_l \, \sigma \, l$$

This effect is often not important, and even less often checked!

Equilibrium

Both CR and Coronal plasmas may be in equilibrium or out of it.

- A collisional plasma in ionization equilibrium (usually called a CIE plasma) has the property that

$$I_{rate}(\mathrm{Ion}) + R_{rate}(\mathrm{Ion}) = I_{rate}(\mathrm{Ion}^-) + R_{rate}(\mathrm{Ion}^+)$$

- A non-equilibrium ionization (NEI) plasma may be:
  - Ionizing $[\Sigma I_{rate}(I) > \Sigma R_{rate}(I)]$
  - Recombining $[\Sigma I_{rate}(I) < \Sigma R_{rate}(I)]$
  - Other

Equilibrium

The *best* term to describe the topic of this talk is:

optically-thin collisional (or thermal) plasmas

Frequently, the "optically-thin" portion is forgotten (bad!)

If the plasma is assumed to be in equilibrium, then CIE is often used, as are phrases like:

- Raymond-Smith
- Mekal
- Coronal plasma (even for non-coronal sources...)

Out of equilibrium, either NIE or NEI are used frequently, as are:

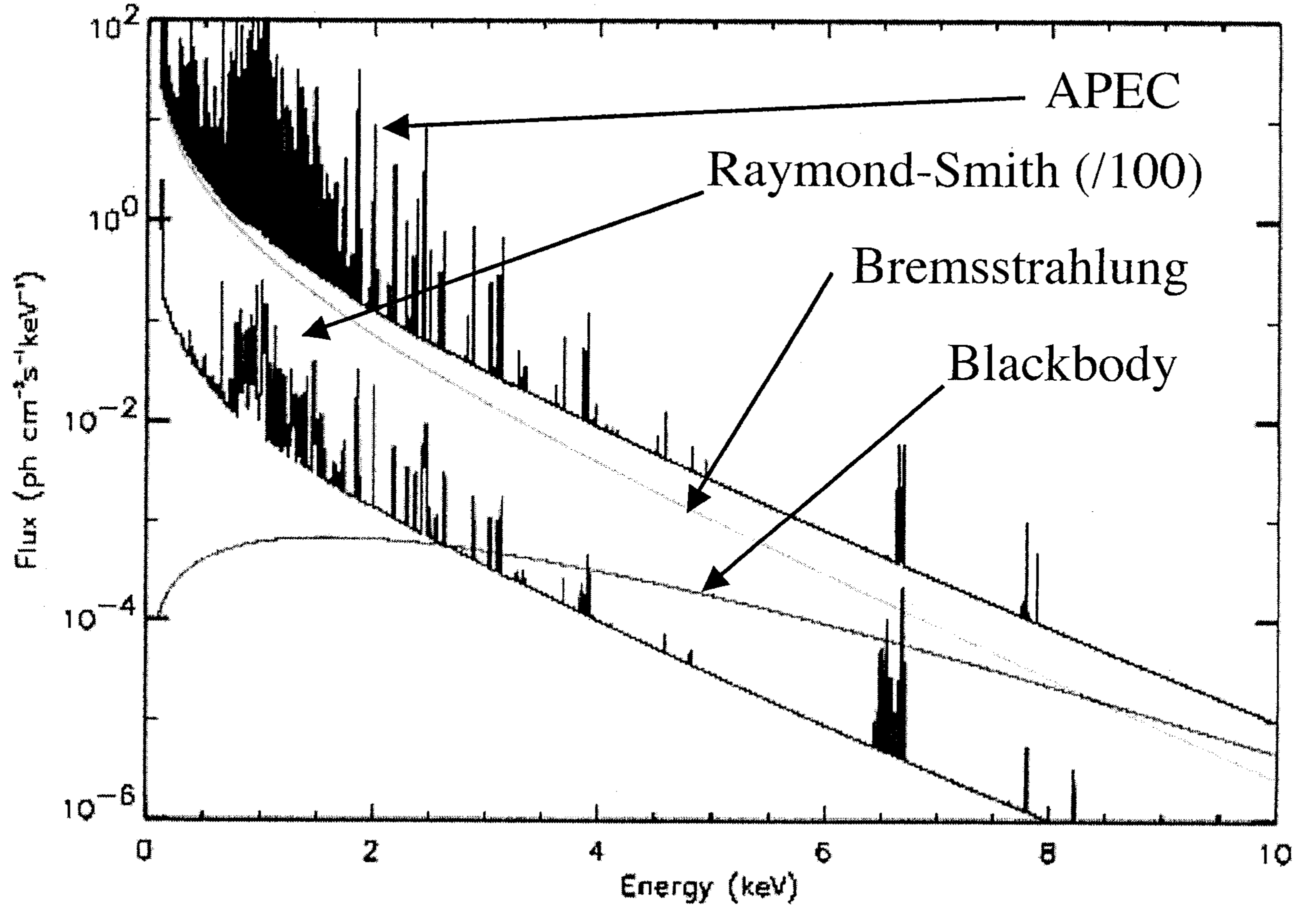
- Ionizing
- Recombining
- Thermal + power-law tail

→ Non-elephant Biology!

Spectral Emission

So what do these plasmas actually look like?

At 1 keV, without absorption:



Bremsstrahlung

Was ist der Bremsstrahlung?

First seen when studying electron/ion interactions. Radiation is emitted because of the acceleration of the electron in the EM field of the nucleus.

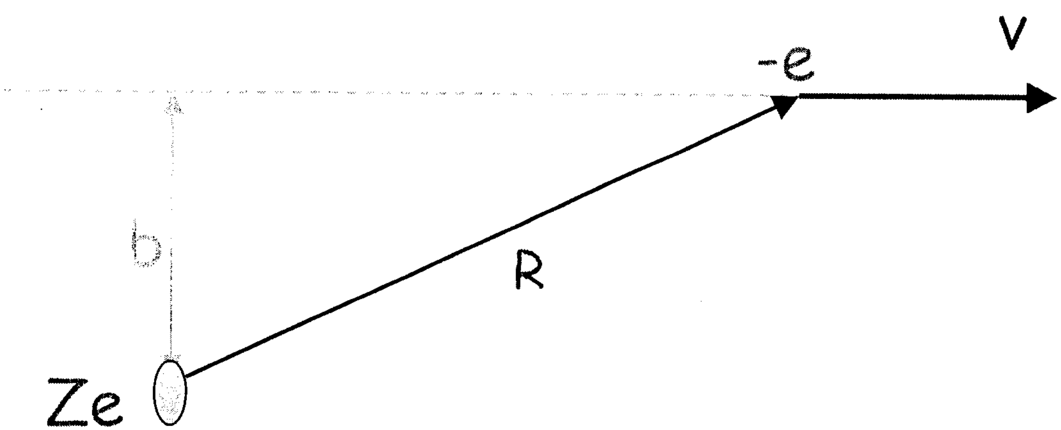
Importance to X-ray astronomy:

- For relativistic particles, may be the dominant coolant.
- Continuum emission shape dependent on the e^- temperature.
- Ubiquitous: hot ionized gas $\Rightarrow$Bremsstrahlung radiation.

The complete treatment should be based on QED, but in every reference book, the computations are made "classically" and modified ("Gaunt" factors) to take into account quantum effects.

Bremsstrahlung

Non-relativistic: Uses the dipole approximation (fine for electron/nucleus bremsstrahlung)



The electron moves mainly in straight line--

$$\Delta v = \frac{Ze^2}{m_e}\int \frac{b}{b^2 + v^2t^2)^{3/2}}dt = \frac{2Ze^2}{mbv}$$

And the electric field is: $E(t) = \dfrac{Ze^3 \sin\theta}{m_e c^2 R(b^2 + v^2t^2)}$

Now use a Fourier transform to get:

$$E(\omega) = \frac{Ze^3 \sin\theta}{m_e c^2 R} \frac{\pi}{bv} \exp(-b\omega/v)$$

And the emitted energy per unit area and frequency is:

$$\frac{dW}{dA d\omega} = c|E(\omega)|^2$$

Integrating over all solid angles, we get:

$$\frac{dW(b)}{d\omega} = \frac{8\pi}{3} \frac{Z^2 e^6}{m_e^2 c^3} \left(\frac{1}{bv}\right)^2 \exp(-2b\omega/v)$$

Bremsstrahlung

Consider a distribution of electrons in a medium with ion density n_i, electron density n_e and constant velocity v. Then the emission per unit time, volume, frequency:

$$\frac{dW}{dVdtd\omega} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega} b\,db$$

Approximate this by considering only contributions up to b_{max} and integrating:

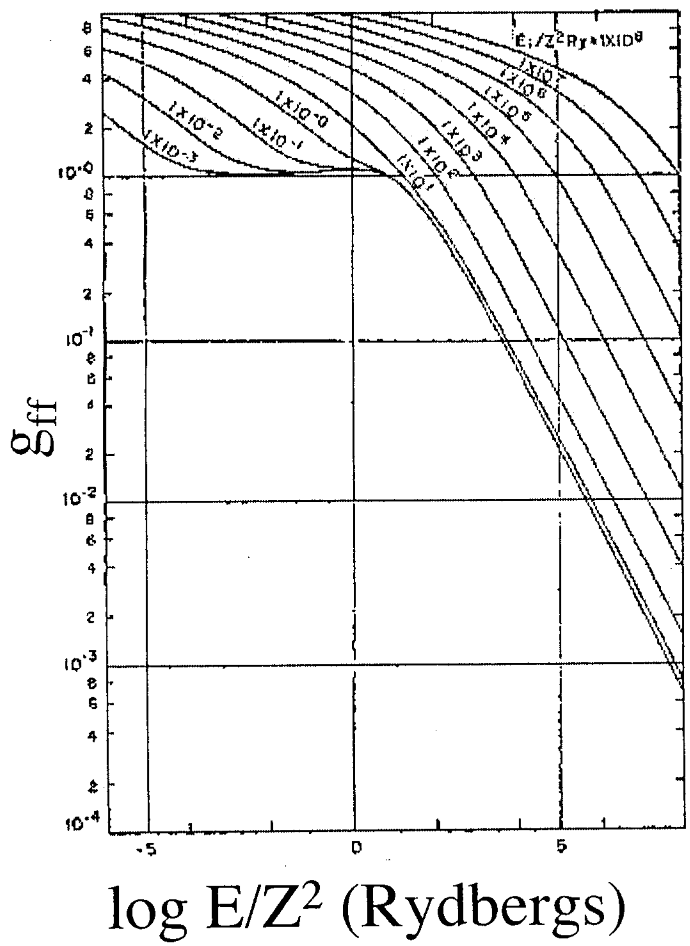
$$\frac{dW}{dVdtd\omega} = \left(\frac{16e^6}{3m_e^2 vc^3}\right) n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

where $b_{\min} \sim \frac{h}{mv}$ and $b_{\max} \sim \frac{v}{\omega}$

Bremsstrahlung

The full QED solution is:

$$\frac{dW}{dV\,dt\,d\omega} = \left(\frac{16\pi e^6}{3^{3/2} m_e^2 v c^3}\right) n_e n_i Z^2 g_{ff}(v,\omega)$$



$g_{ff} = 1$ line

Karzas & Latter, 1961, ApJS, 6, 167

Bremsstrahlung

Now integrate over electrons with a Maxwell-Boltzmann velocity distribution:

$$dP \propto \exp\left(\frac{-E}{kT}\right) d\vec{v} \propto v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

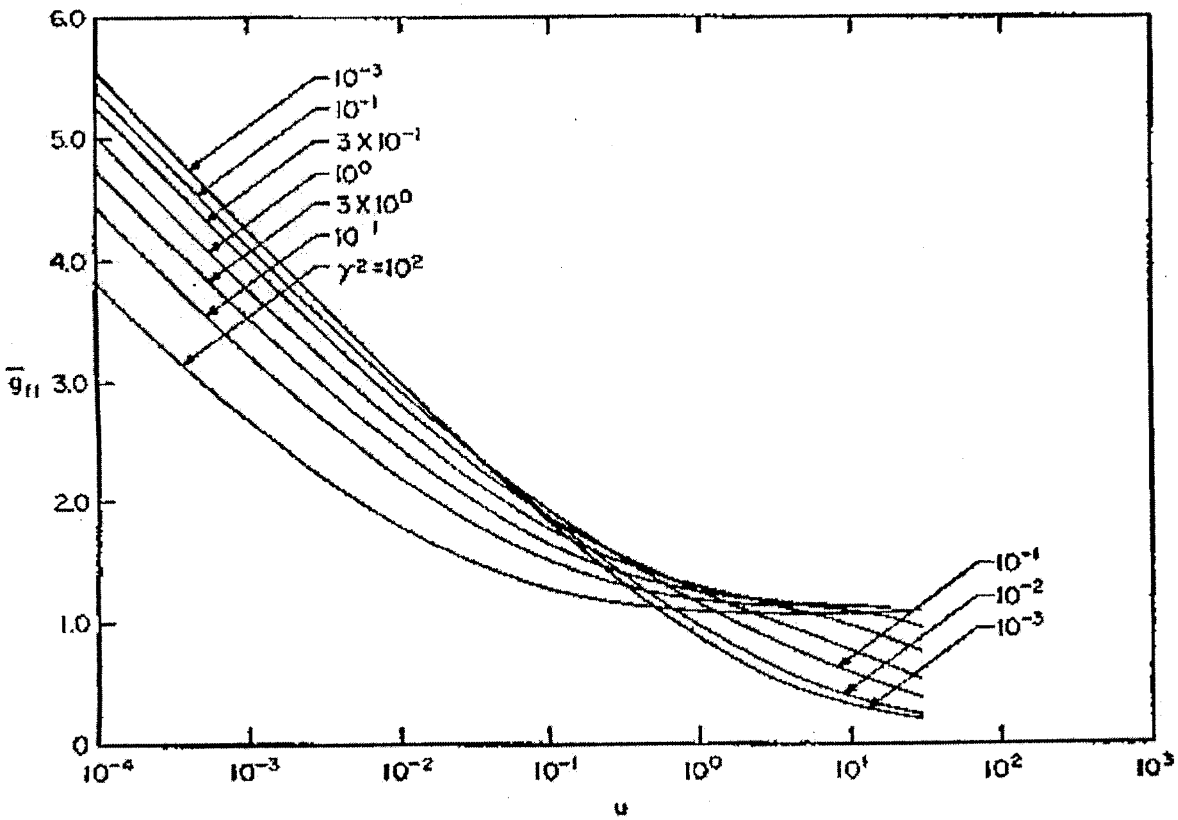
To get:

$$\frac{dW}{dV dt d\nu} = \frac{32\pi e^6}{3 m_e c^3} \sqrt{\frac{2\pi}{3kTm_e}} n_e n_i Z^2 \exp\left(\frac{-h\nu}{kT}\right) \langle g_{ff} \rangle$$

$$= 6.8 \times 10^{-38} \frac{n_e n_i Z^2}{\sqrt{T}} \exp\left(\frac{-h\nu}{kT}\right) \langle g_{ff} \rangle \mathrm{erg\ s^{-1} cm^{-3} Hz^{-1}}$$

where $\langle g_{ff} \rangle$ is the velocity average Gaunt factor

Bremsstrahlung



$u = h\nu/kT$;

$\gamma^2 = \mathrm{Ry}\, Z^2/kT$

$= 1.58 \times 10^5\, Z^2/T$

Numerical values of $\langle g_{ff} \rangle$.

From Rybicki & Lightman
Fig 5.3 -- originally from
Karzas & Latter (1961)

When integrated over frequency (energy):

$$\frac{dW}{dV dt} = \frac{32\pi e^6}{3hm_e c^3}\sqrt{\frac{2\pi kT}{3m_e}}\, n_e n_i Z^2 \langle g_B \rangle$$
$$= 1.4 \times 10^{-27}\sqrt{T}\, n_e n_i Z^2 \langle g_B \rangle \ \mathrm{erg\ s^{-1}\,cm^{-3}}$$

Global Fitting

CCD (or proportional counter) data are regularly fit in a global fashion, using a response matrix. If you believe that the underlying spectrum is from an optically-thin collisional equilibrium plasma, then you can "fit" your choice of collisional plasma model (apec, mekal, raymond, equil are available in XSPEC or sherpa).

By default, the only parameters are temperature and emission measure. If the fit is poor ($\chi^2/N > 1$) you can add more parameters: such as the overall abundance relative to solar, or the redshift.

If the models are still a poor fit, the abundances can be varied independently, or the equilibrium assumption can be relaxed in a few ways.

Are there problems with this method?

Of course there are. However, when your data has spectral data has resolution less than 100, you cannot easily identify and isolate X-ray spectral lines -- but low resolution data is better than no data: the goal is understanding, not perfection.

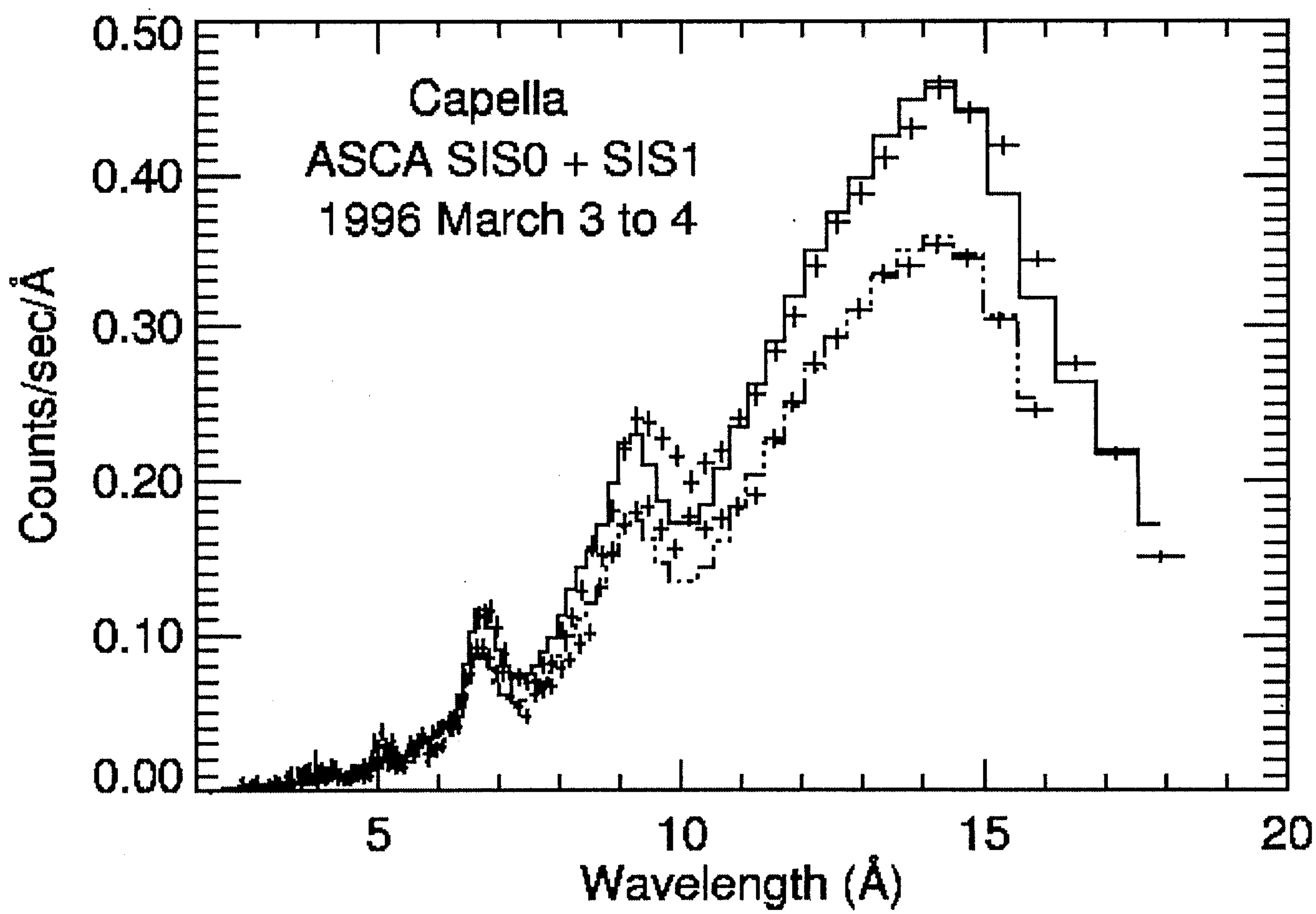
(tm) Keith Arnaud

It is vital to keep in mind:

1. If the underlying model is inadequate, your results may be as well. Beware especially abundances when only one ionization state can be clearly seen.
2. Cross-check your results any way you can. For example, the EM is related to the density and the emitting volume. Are they reasonable?
3. If you can't get a good fit in a particular region, your problem may be the model, not the data.

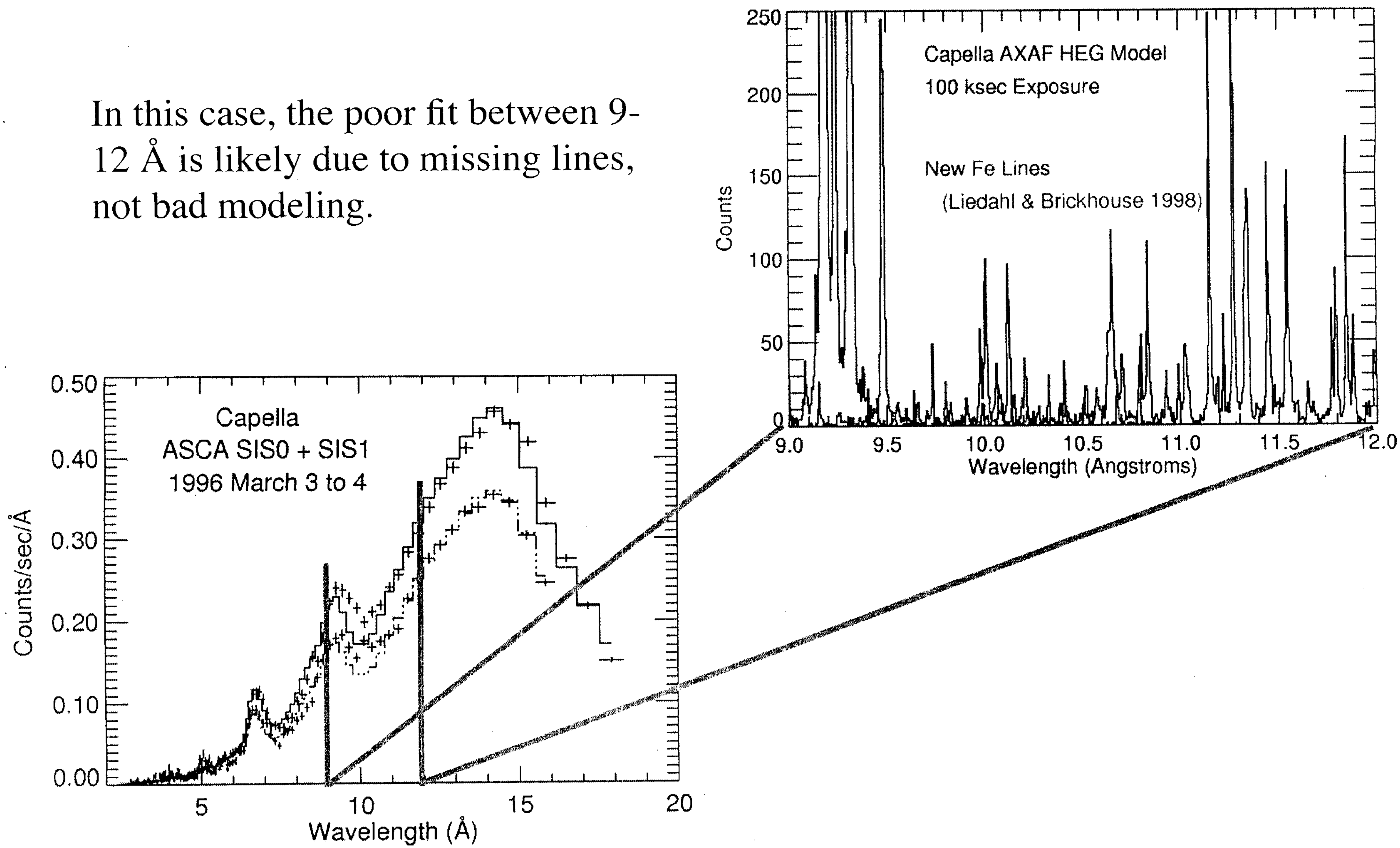
Global Fitting

Consider this ASCA CCD spectrum of Capella, with a collisional plasma model fit:



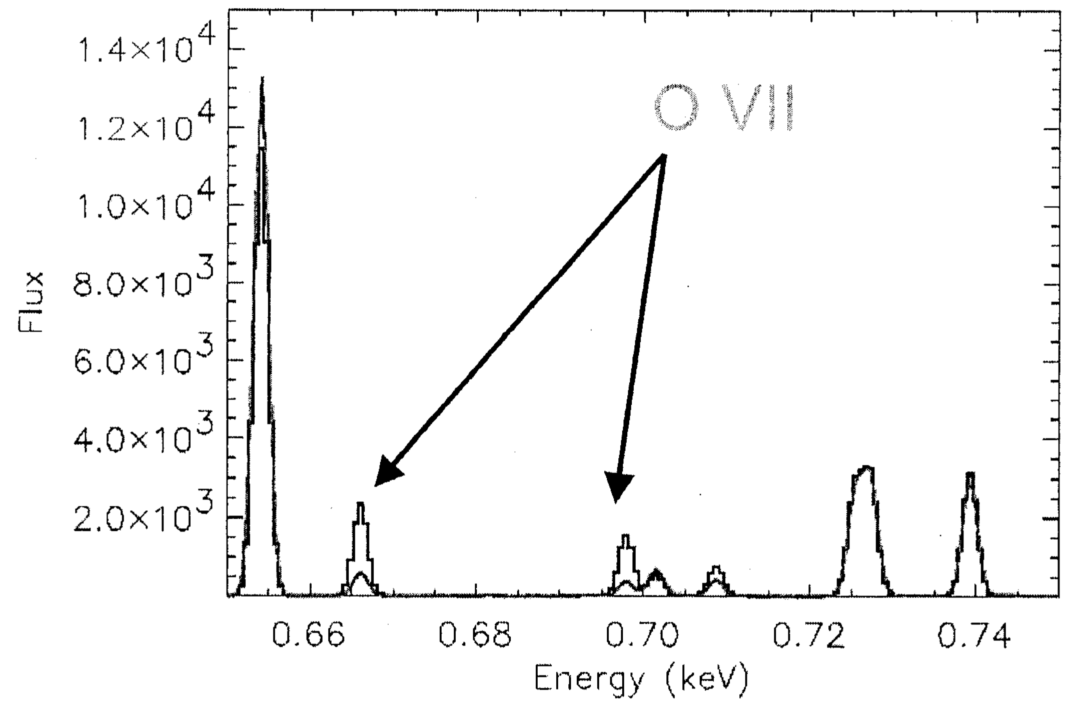
Global Fitting

In this case, the poor fit between 9-12 Å is likely due to missing lines, not bad modeling.



Global Fitting

Here is a parallel shock (pshock, kT=0.7 keV), observed with the ACIS BI:

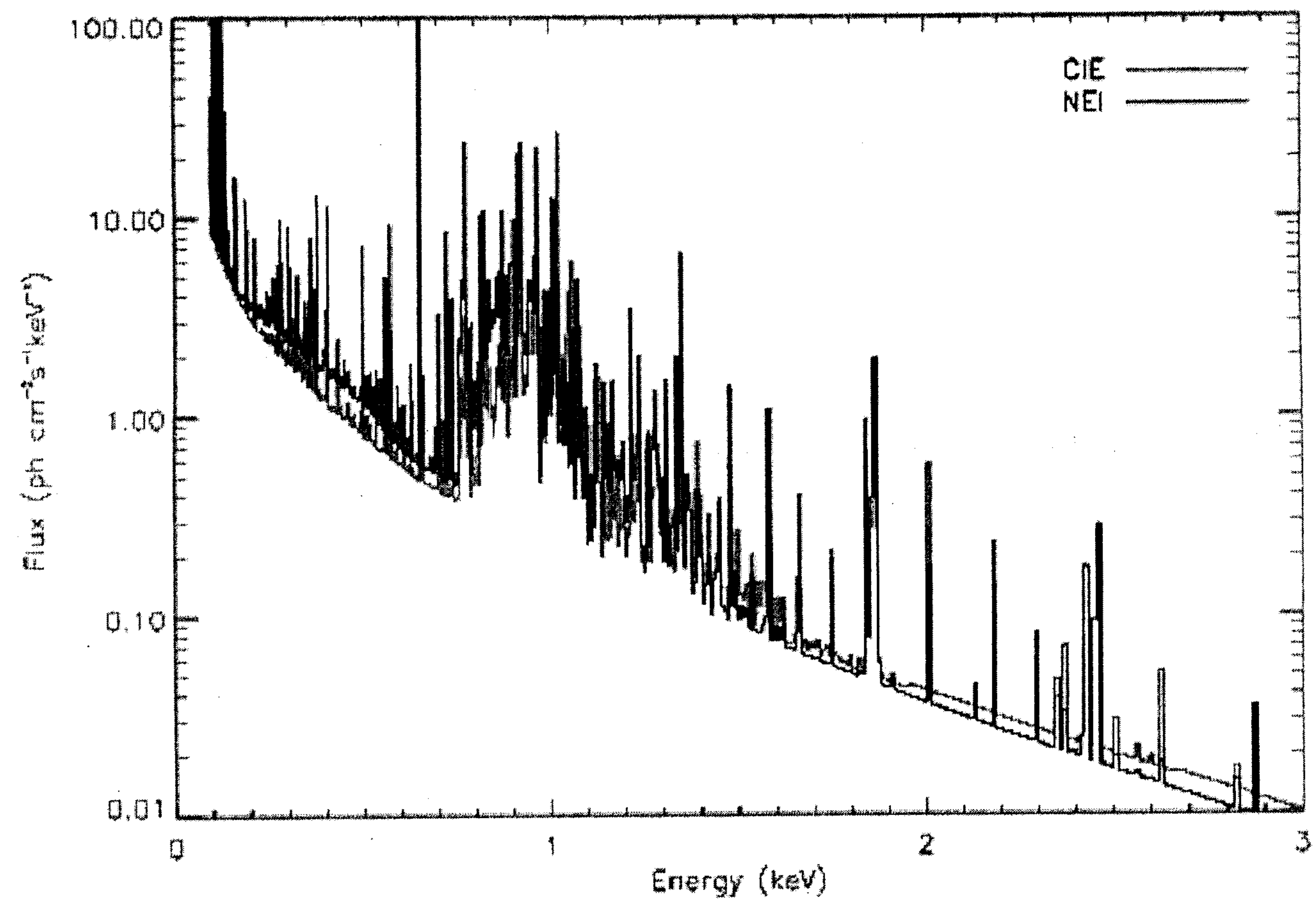


An NEI collisional model fits the data quite well.

But with higher resolution... the NEI model fails, pshock is needed.

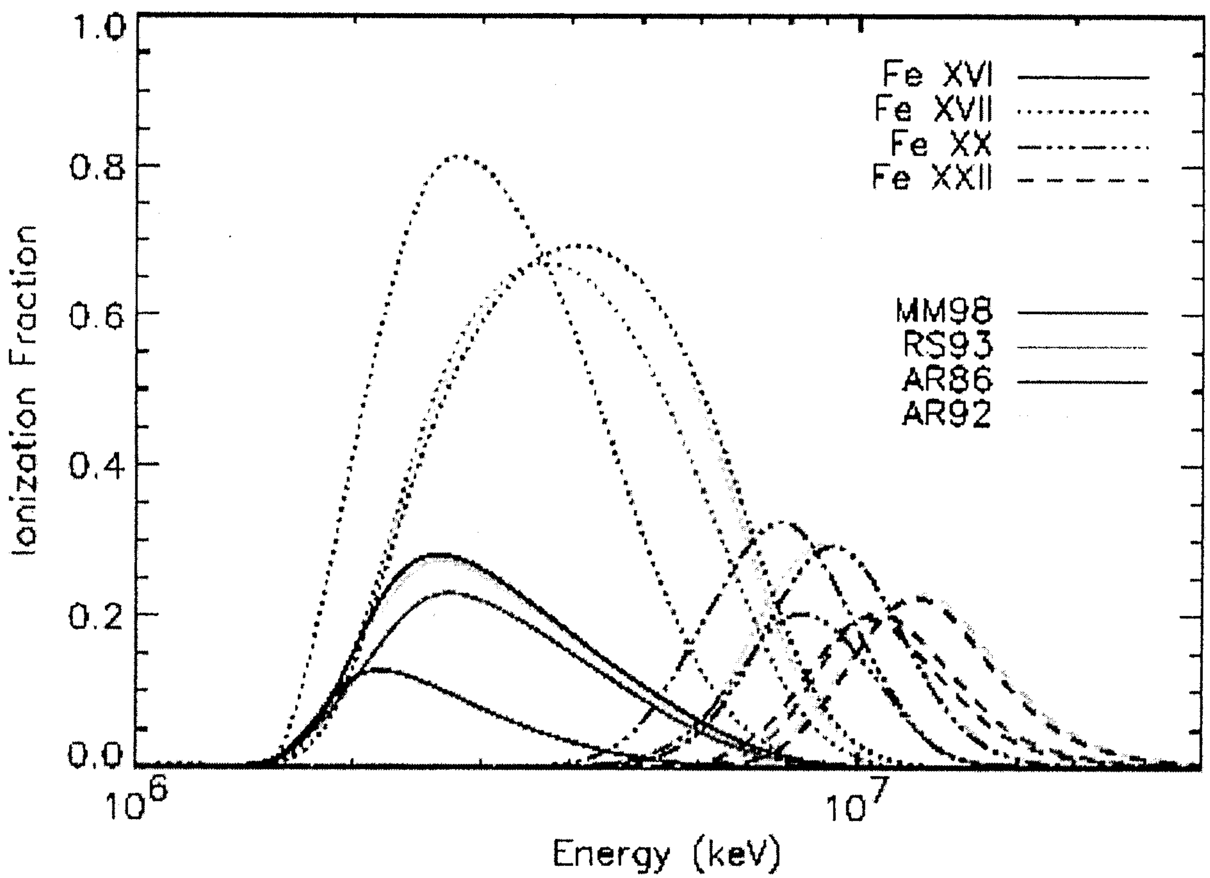
NEI vs CIE Emission

We can compare a CIE plasma against an NEI plasma, in this case an ionizing plasma, also at 1 keV.



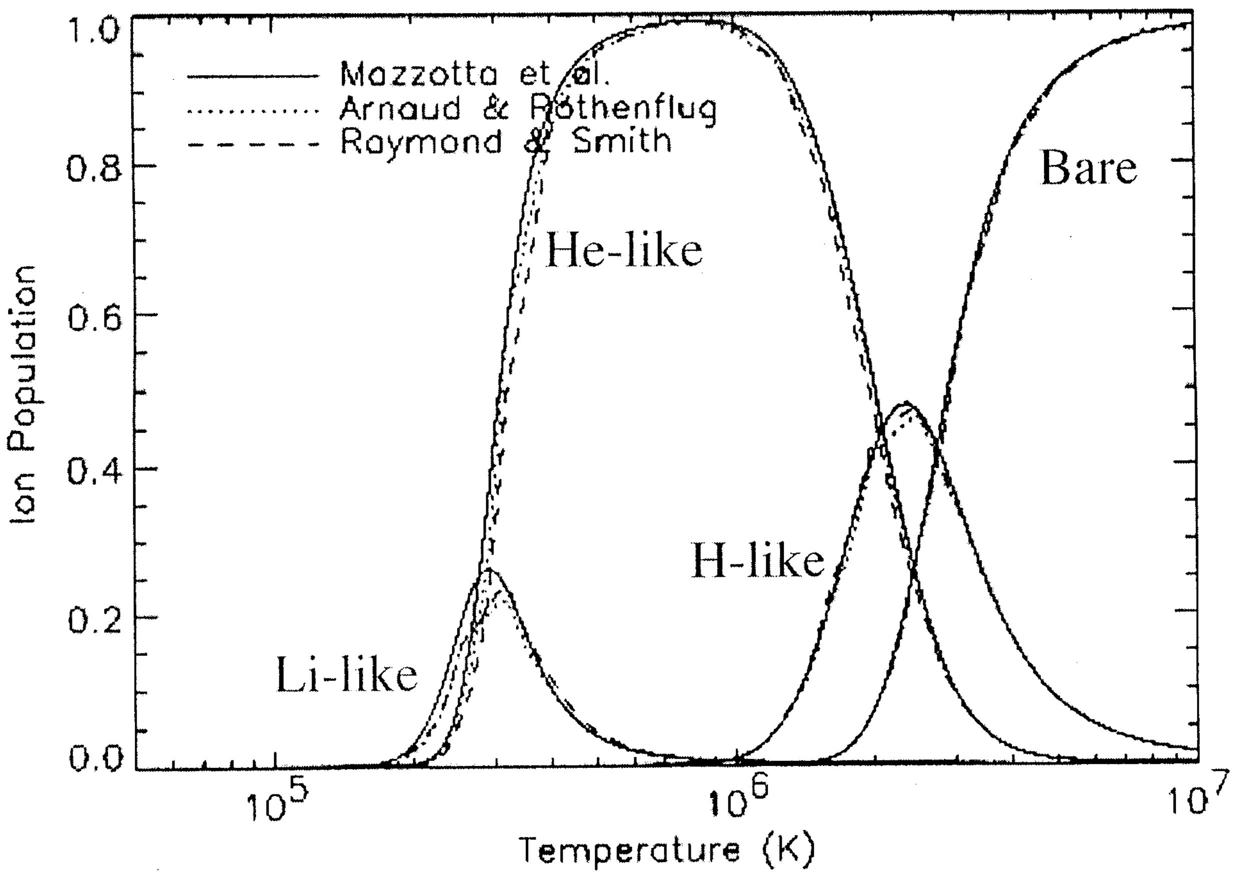
Ionization Balance

In order to calculate an emission spectrum the abundance of each ionization state must be known. Shown here are four equilibrium ionization balance calculations for 4 iron ions:



Ionization Balance

In some cases, the differences are small. Here is a comparison of O VI, VII, VIII, and fully-stripped Oxygen, for three different models:



Ions of Importance

All ions are equal...

...but some are more equal than others.

In collisional plasmas, three ions are of particular note:

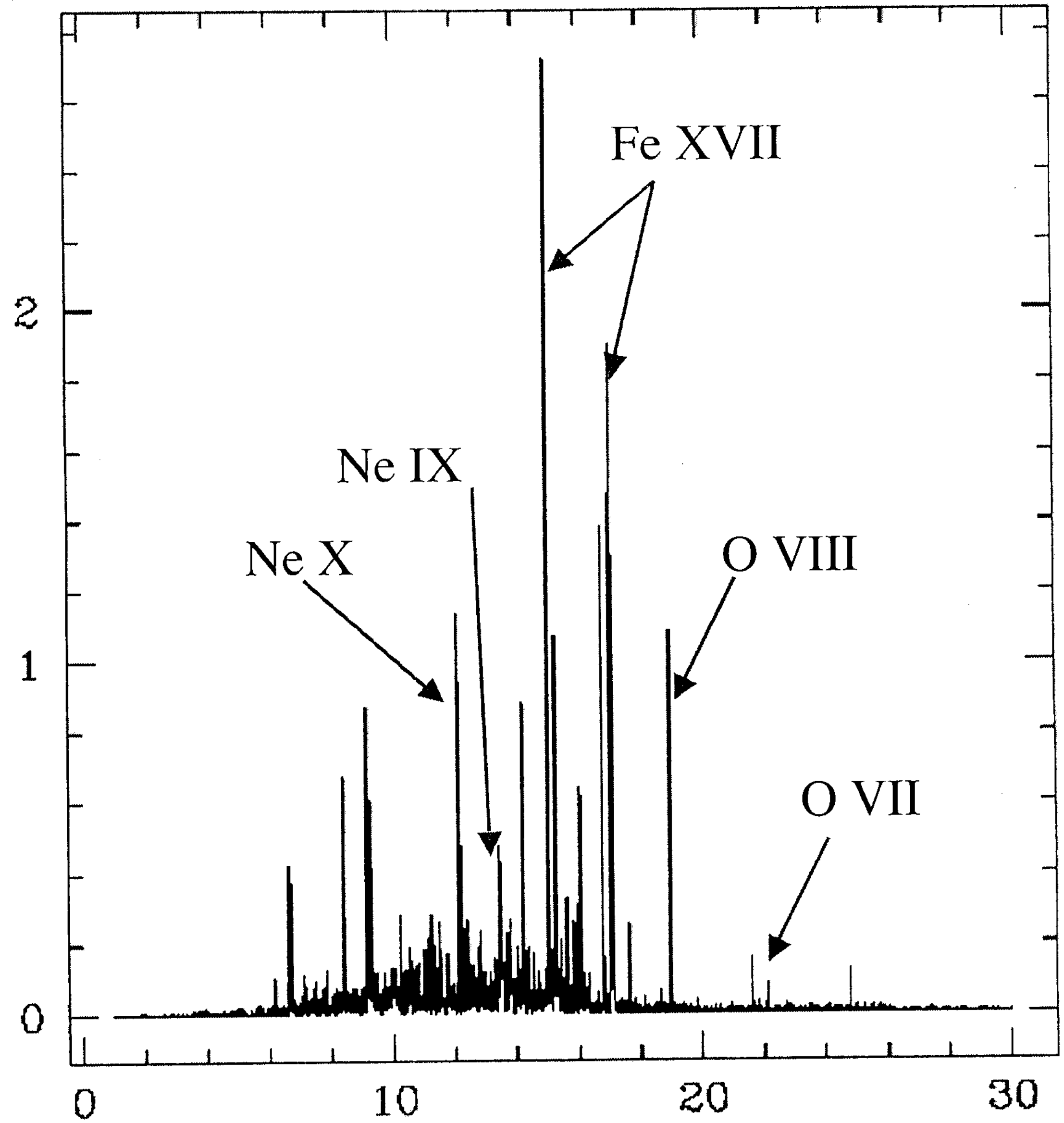
H-like : All transitions of astrophysically abundant metals (C$\rightarrow$Ni) are in the X-ray band. Lyα/Lyβ is a useful temperature diagnostic; Lyα is quite bright.

He-like: $\Delta n \geq 1$ transitions are all bright and in X-ray. The $n=2\rightarrow 1$ transitions have 4 transitions which are useful diagnostics, although R=300 required to separate them.

Ne-like: Primarily Fe XVII; two groups of bright emission lines at 15Å and 17Å; ionization state and density diagnostics, although there are atomic physics problems.

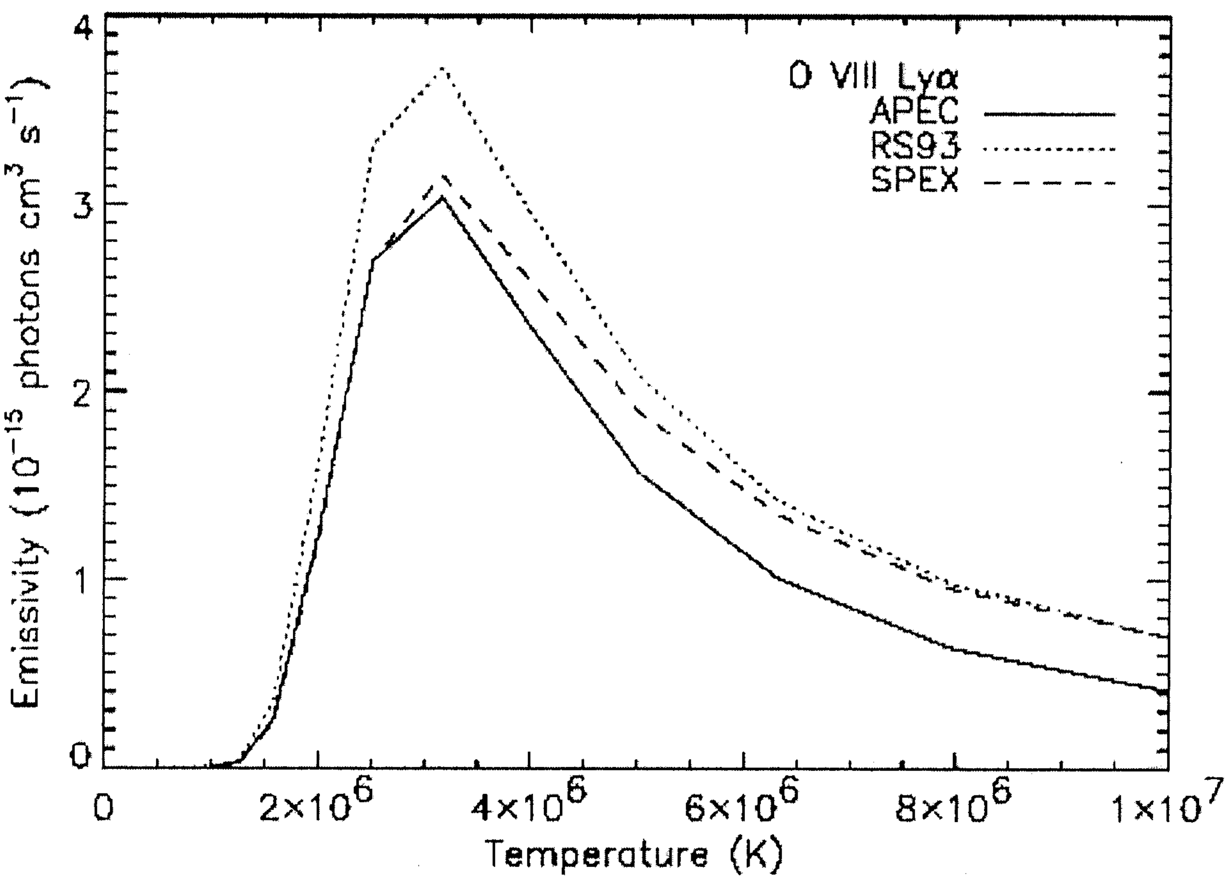
Ions of Importance

Capella observed with the Chandra HETG



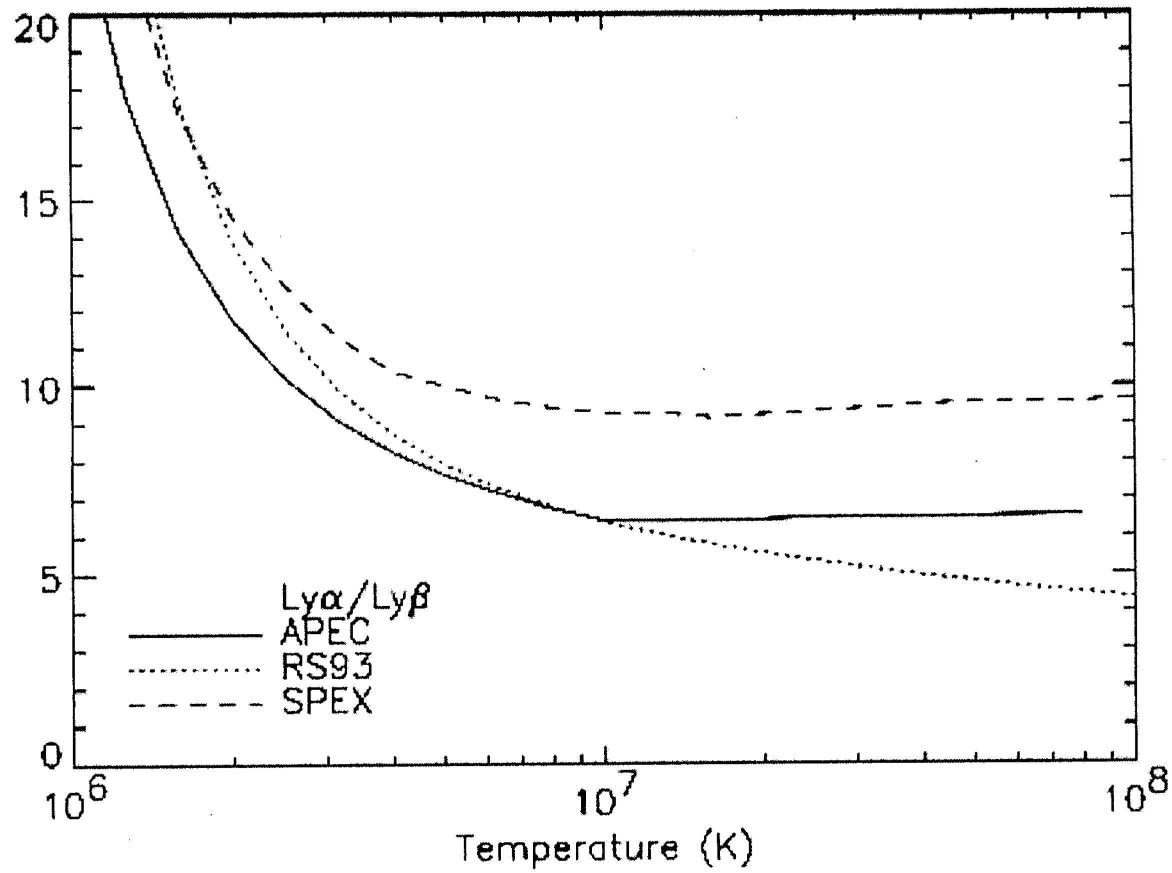
Hydrogenic Lines

Three calculations of the O VIII Lyα line as a function of temperature.



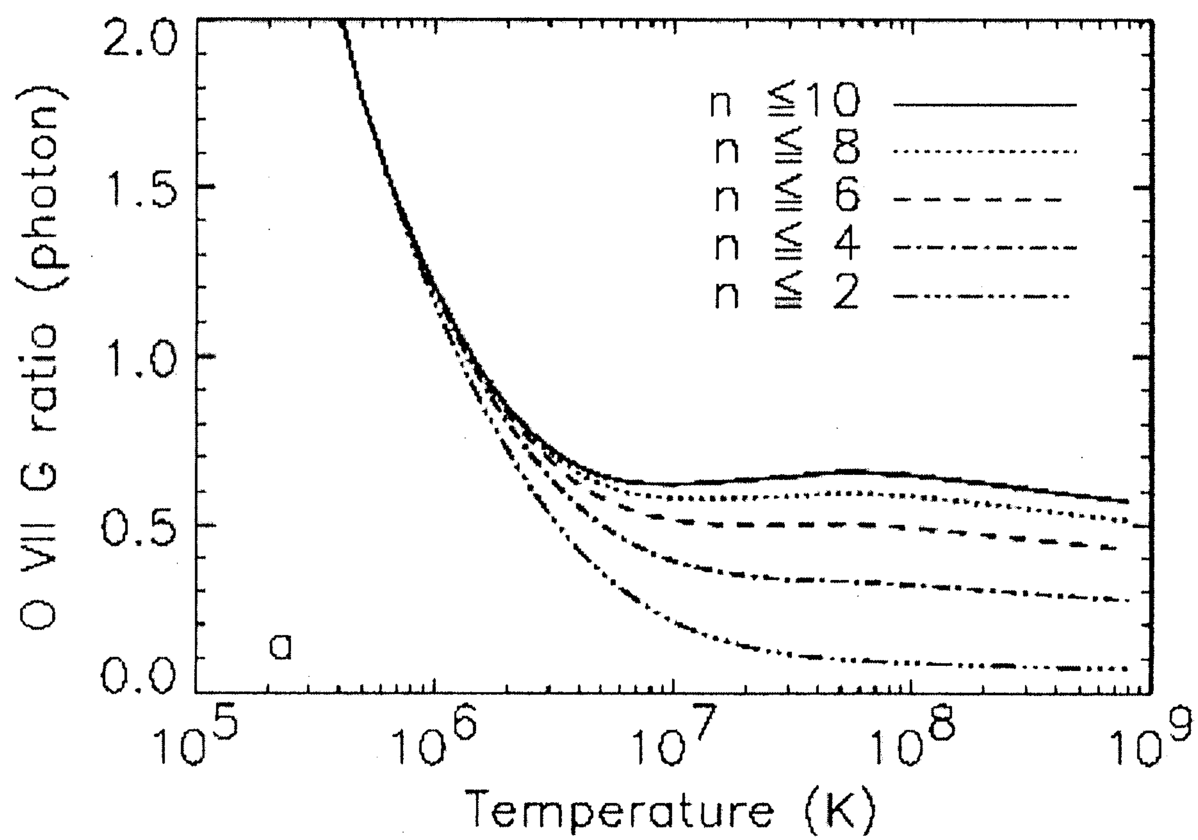
Hydrogenic Lines

Three calculations of the O VIII Lyα/Lyβ line as a function of temperature (APEC agrees with measurements).



Helium-like Lines

One useful He-like diagnostic is the G ratio, defined as (F+I)/R [or, alternatively, (x+y+z)/w]. It is a temperature diagnostic, at least for low temperatures, and it is also measures ionization state.

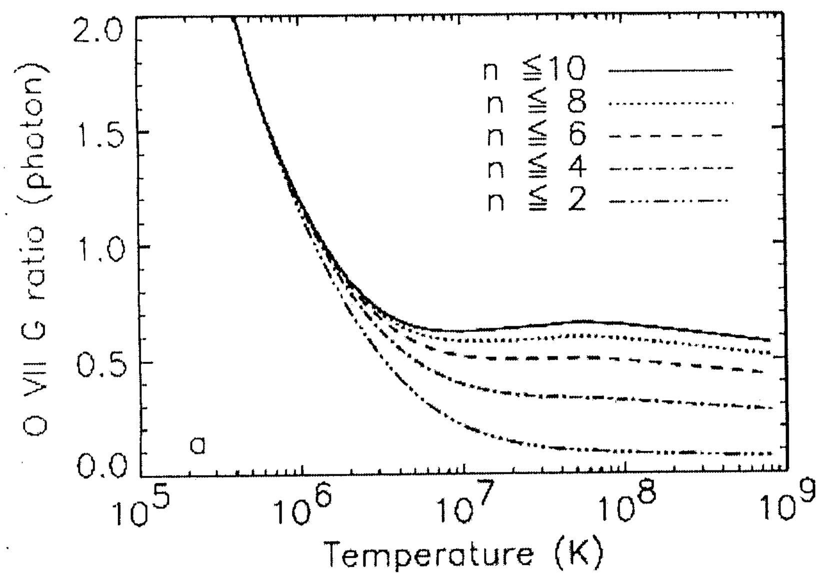


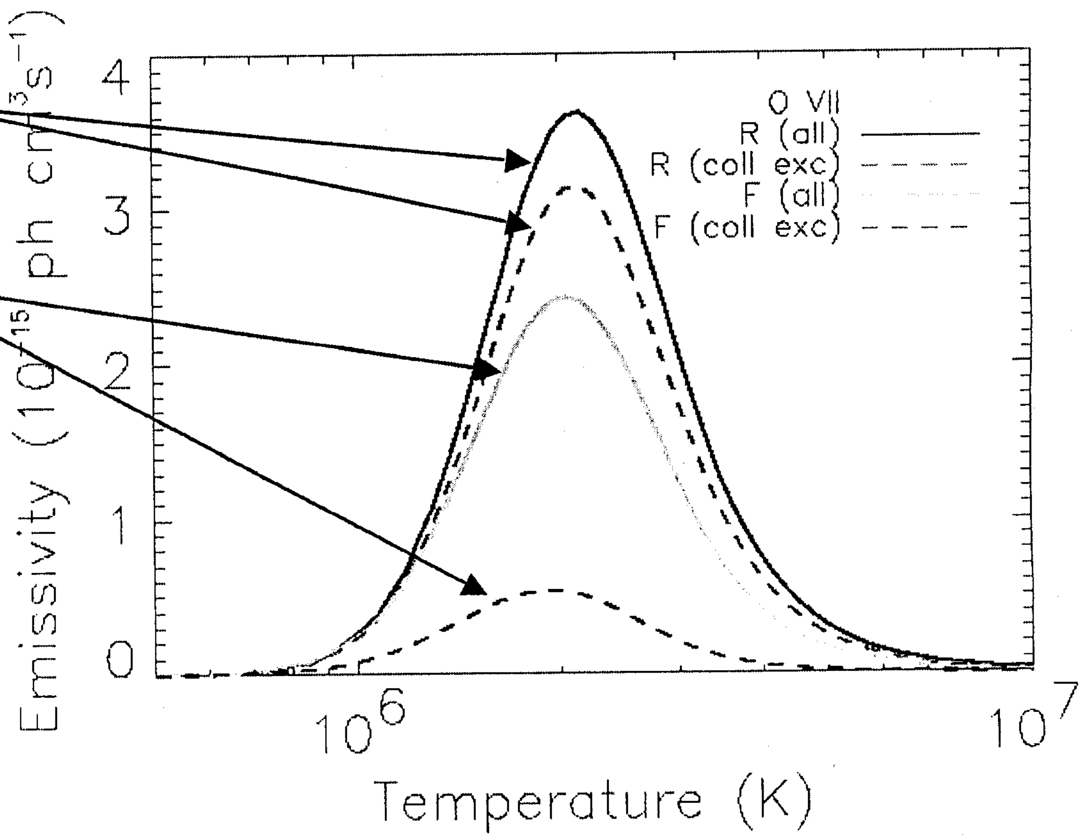
Helium-like Lines

Why does the G ratio measure temperature and ionization state?

Because the resonance line R is excited by collisions, which are temperature dependent, while the F and I lines are excited by recombination and other processes.

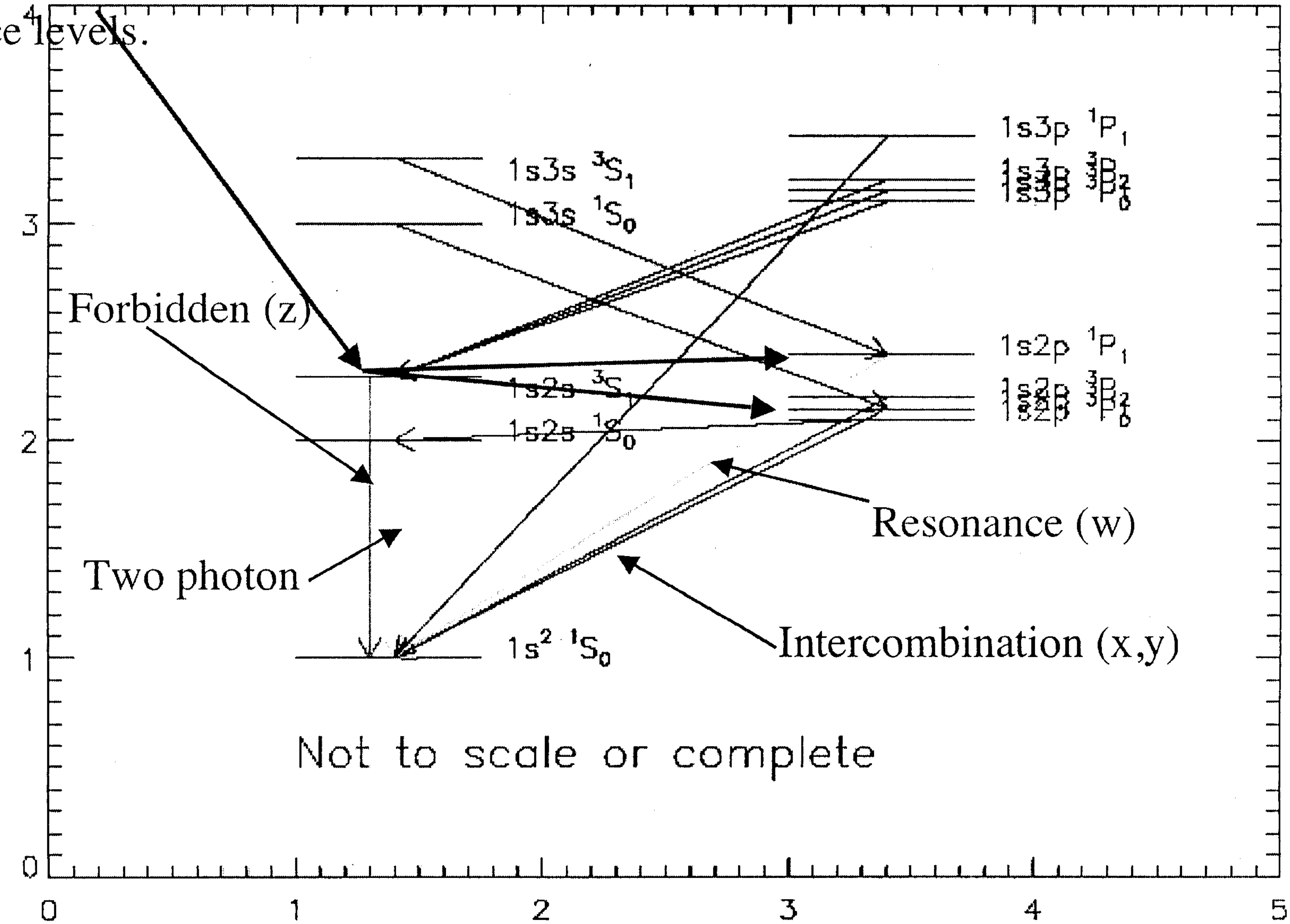
G = (F+I)/R





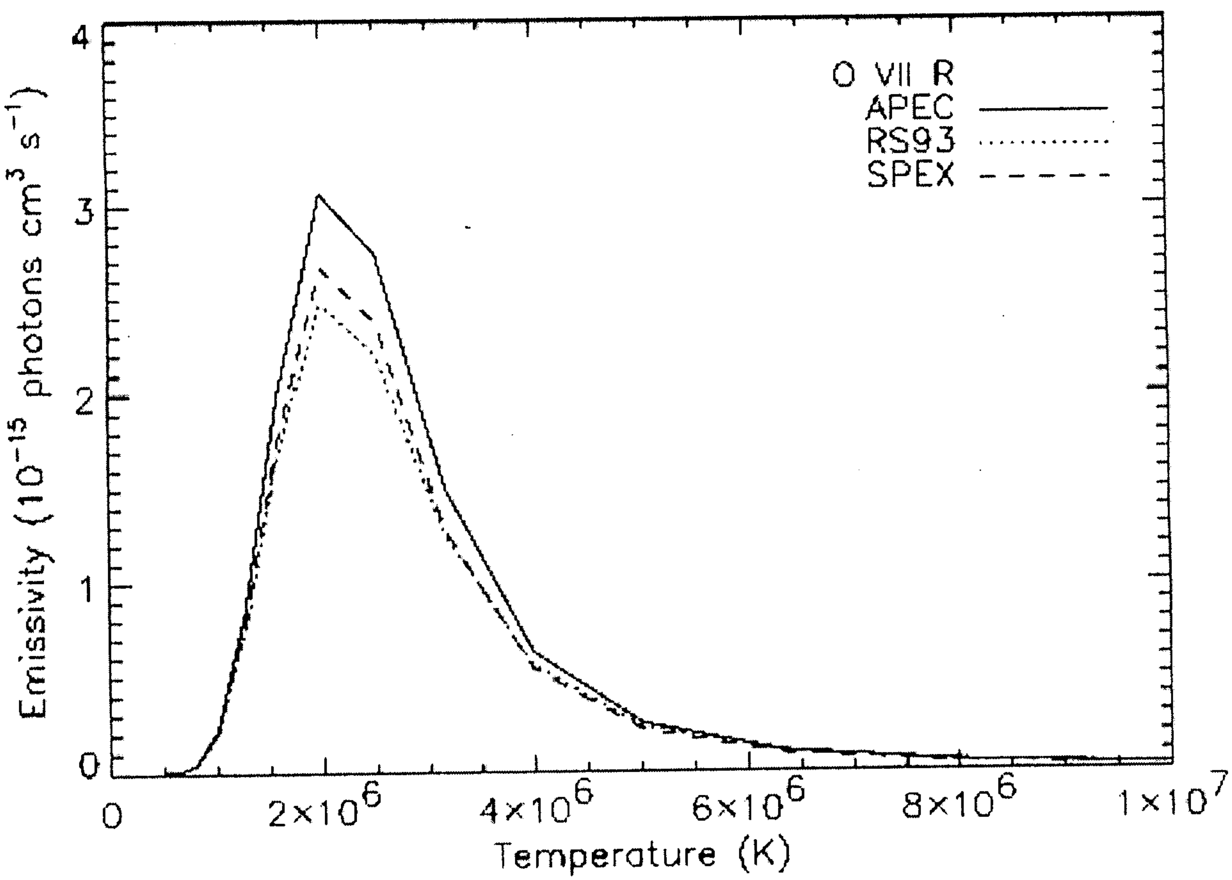
Helium-like Lines

The ratio F/I is normally called the R ratio, and it is a density diagnostic. If n_e is large enough, collisions move electrons from the forbidden to the intercombination and resonance levels.



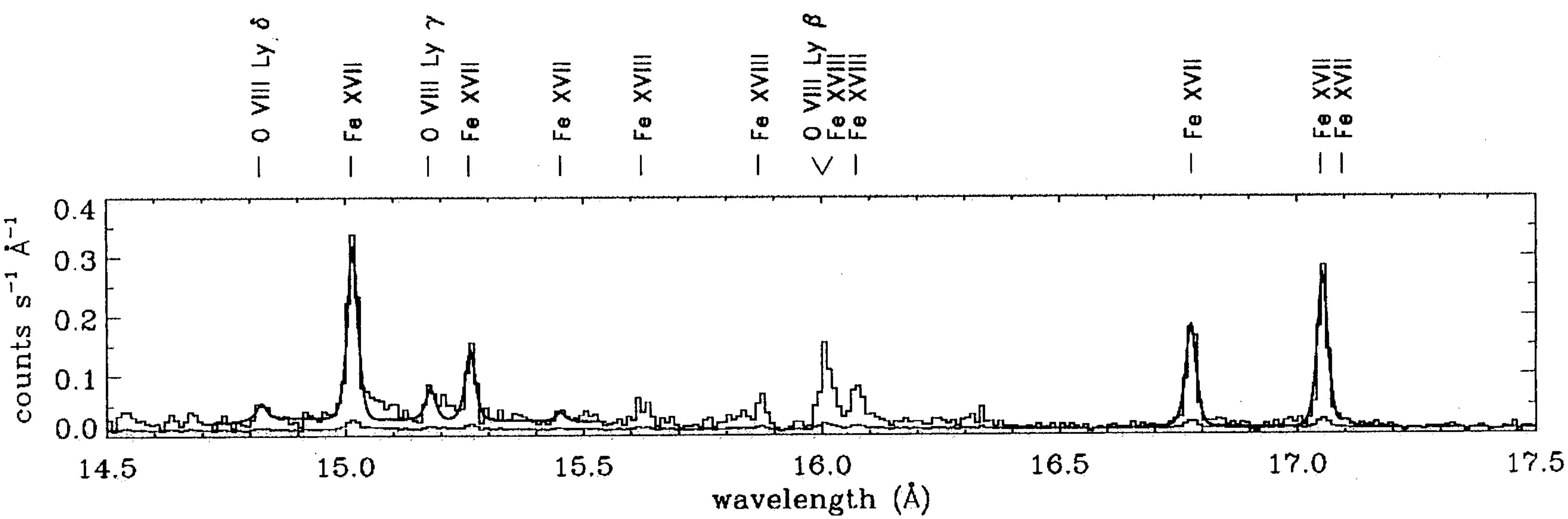
Helium-like Lines

How well are these He-like lines known? Here are three calculations for each of the three lines:

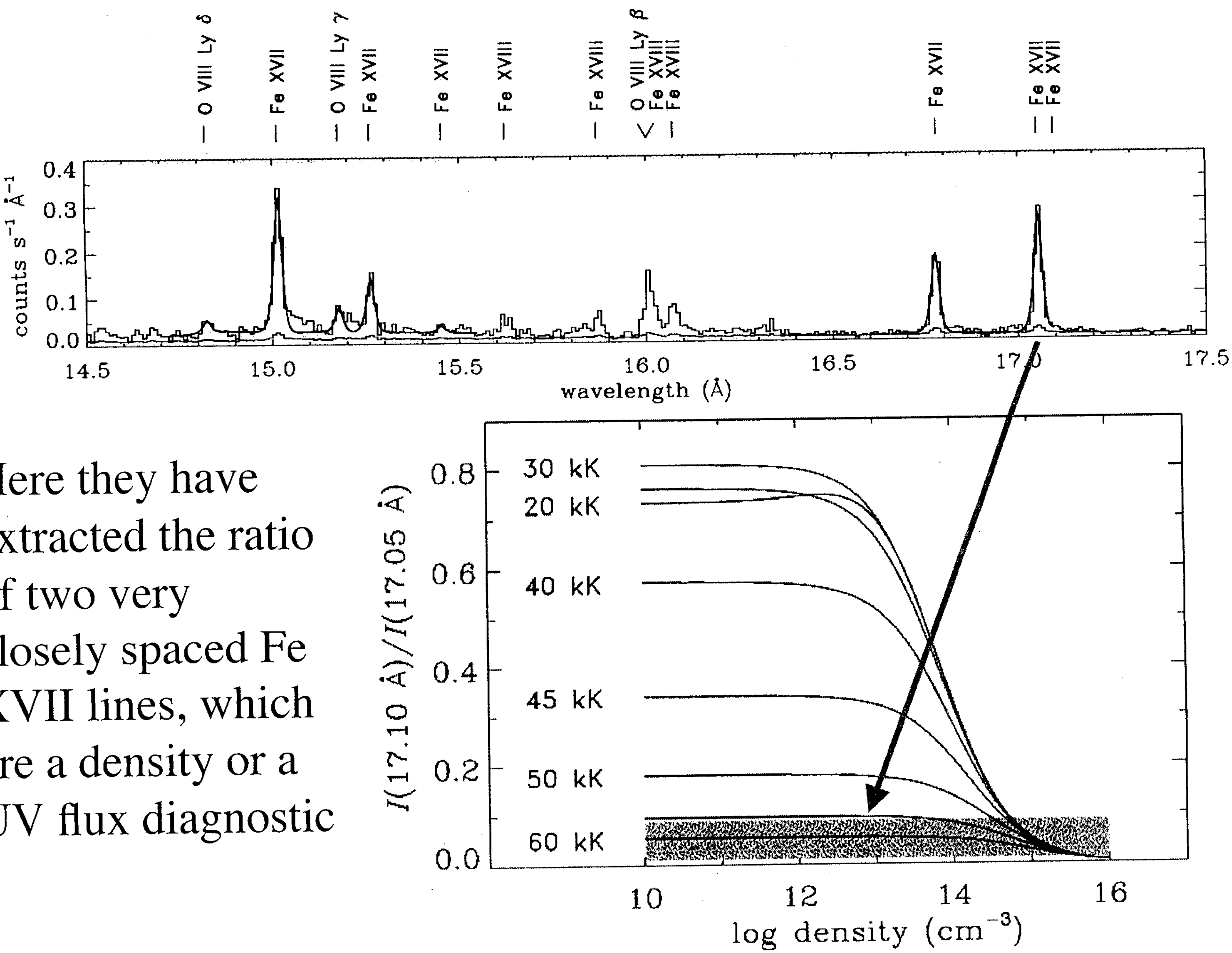


Neon-Like Lines

Fe XVII is the most prominent neon-like ion; Ni XIX is 10x weaker simply due to relative abundances. There are a number of diagnostic features, as can be seen in this grating spectrum of the WD EX Hya (Mauche *et al.* 2001):

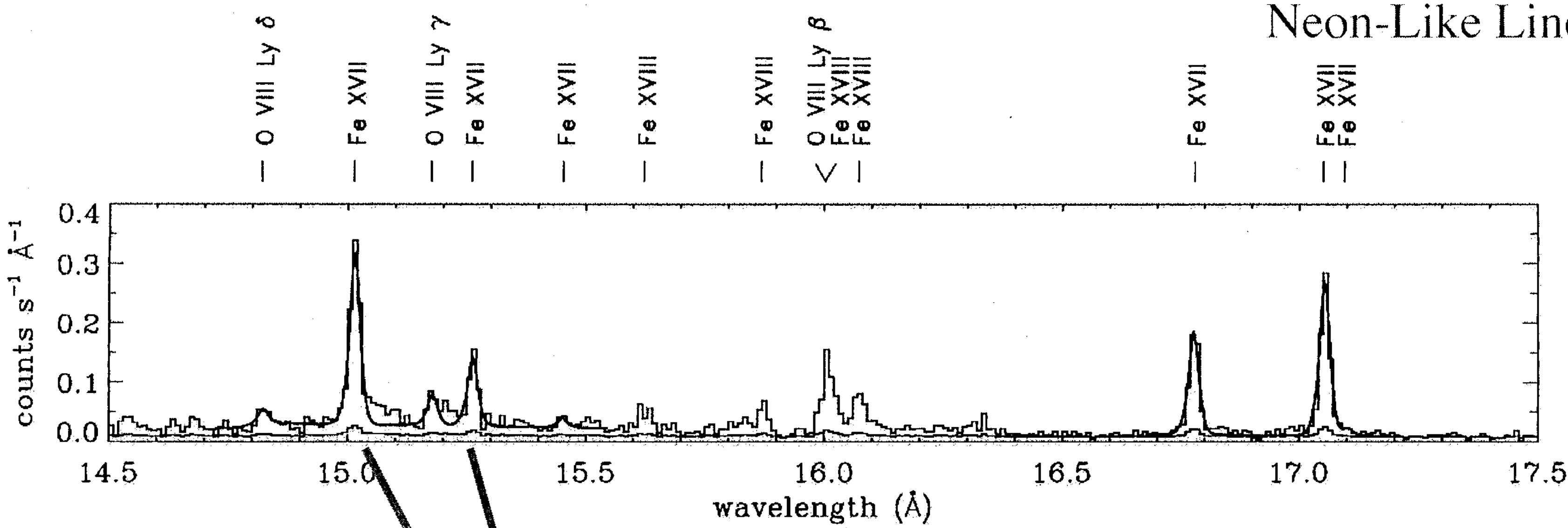


Neon-Like Lines



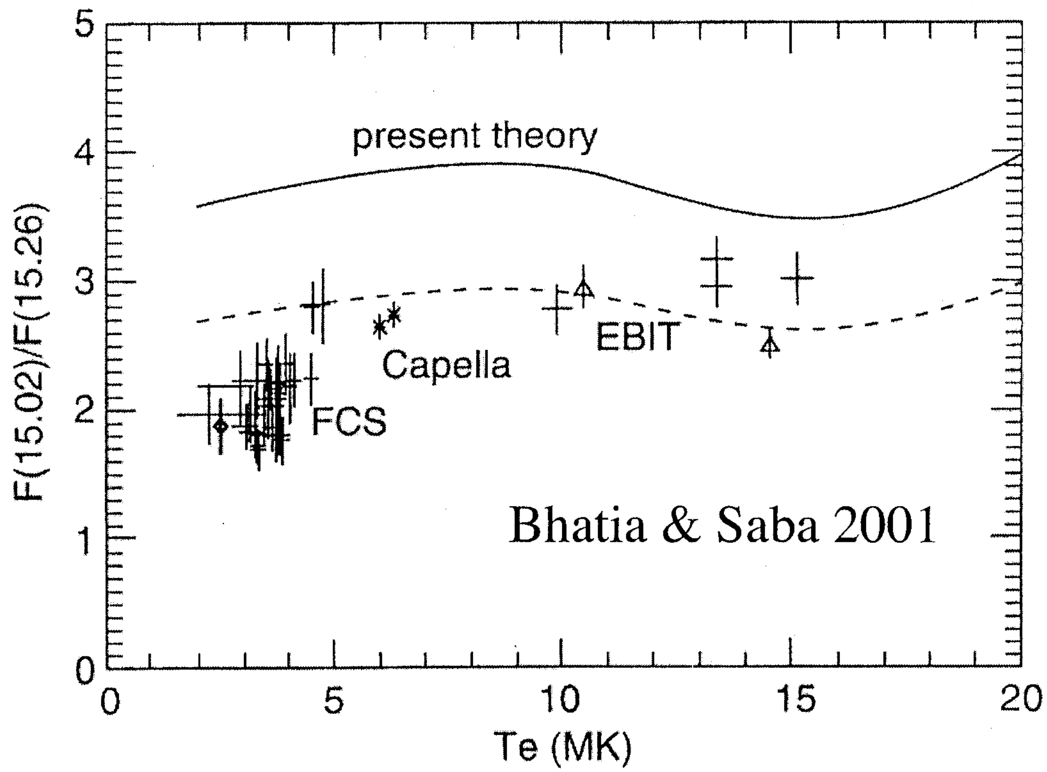
Here they have extracted the ratio of two very closely spaced Fe XVII lines, which are a density or a UV flux diagnostic

Neon-Like Lines



What about the strong 15.02Å and 15.26Å lines?

They should be useful diagnostics, but right now we're still debating their proper ratio...stay tuned



Plasma Codes

Understanding a collisional plasma requires a collisional plasma model. Since even a simple model requires considering hundreds of lines, and modern codes track millions, most people select one of the precalculated codes:

Code	Source
Raymond-Smith	ftp://legacy.gsfc.nasa.gov/software/plasma_codes/raymond
SPEX	http://saturn.sron.nl/general/projects/spex
Chianti	http://wwwsolar.nrl.navy.mil/chianti.html
ATOMDB	http://cxc.harvard.edu/ATOMDB

The calculated spectrum is also known as APEC, and the atomic database is called APED.

Plasma Codes

The collisional plasma models available in XSPEC or Sherpa are:

apec	ATOMDB code; good for high-resolution data
raymond	Updated (1993) Raymond-Smith (1977) code
meka	Original Mewe-Kaastra (Mewe *et al.* 1985) code; outdated
mekal	Mewe-Kaastra-Liedahl code (Kaastra 1992); new Fe L lines
c6mekal	mekal with an polynomial EM distribution
equil	Borkowski update of Hamilton, Sarazin & Chevalier (1983)
nei	Ionizing plasma version of equil
sedov	Sedov (SNR) version of equil
pshock	Plane parallel shock version of equil

Variable abundance versions of all these are available.

Individual line intensities as functions of T, n, etc. are not easily available (yet) in either XSPEC or Sherpa.

Atomic Codes

HULLAC (Hebrew University / Lawrence Livermore Atomic Code) : Fast, used for many APED calculations, not generally available.

R-Matrix : Slow, used for detailed calculations of smaller systems of lines, available on request but requires months to learn.

FAC (Flexible Atomic Code) : Fast, based on HULLAC and written by Ming Feng Gu. Available at

ftp://space.mit.edu/pub/mgfu/fac

So you think you've got a collisional plasma: what do you do?

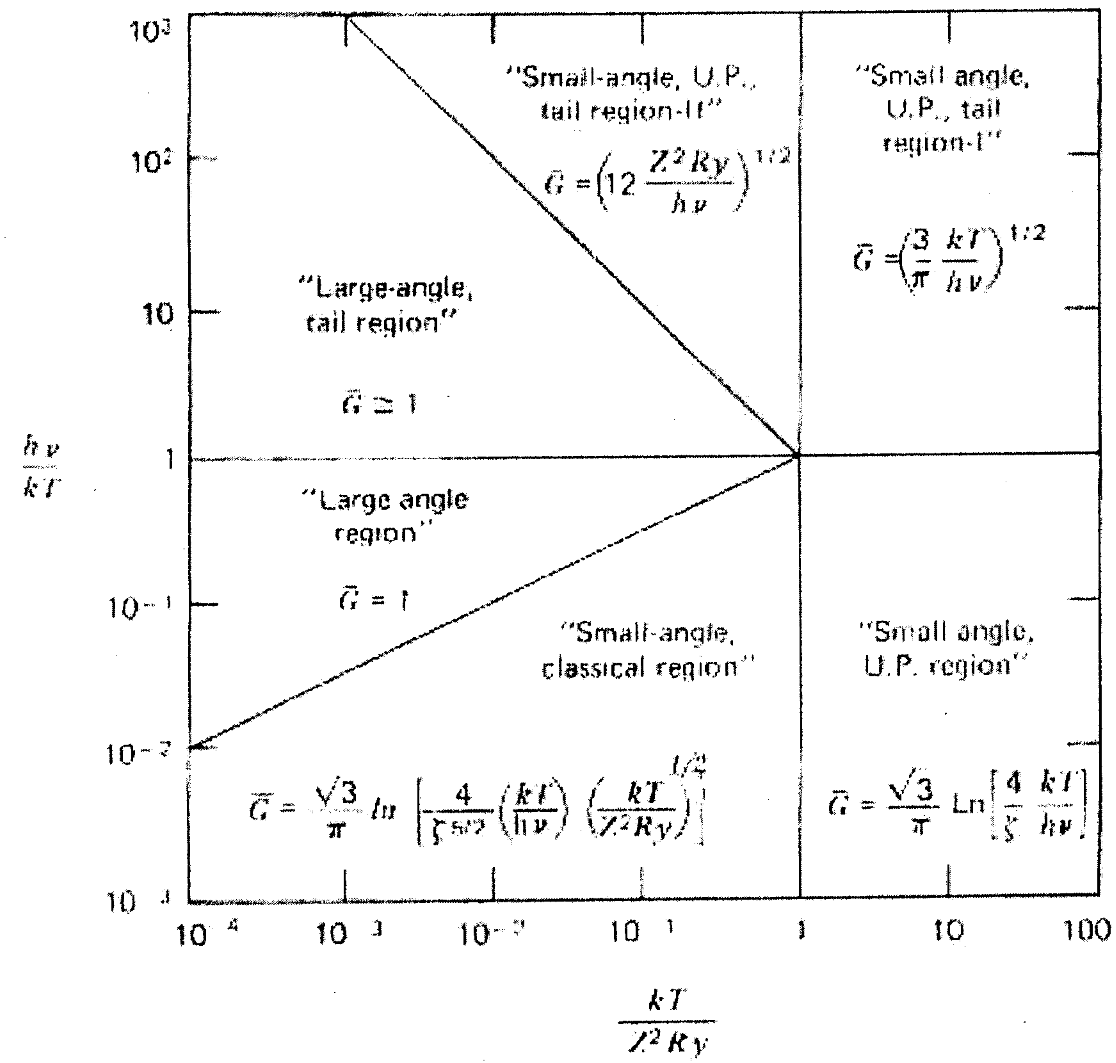
- If high resolution data are available, line-based analysis allows the best control of errors, both atomic and data/calibration.
- If CCD (or worse) is all that you have, remember Clint Eastwood's admonition:

A spectroscopist's gotta know his limitations.

Keep in mind that :

(a) only the strongest lines will be visible,
(b) they could be blended with weaker lines,
(c) plasma codes have at least 10% errors on line strengths,
(d) the data have systematic calibration errors, and finally:
(e) **the goal is understanding, not $\chi^2_n \sim 1$ fits.**

Bremsstrahlung



Approximate analytic formulae for $\langle g_{ff} \rangle$

From Rybicki & Lightman Fig 5.2 (corrected) -- originally from Novikov and Thorne (1973)