Wilkinson Microwave Anisotropy Probe (WMAP) Attitude Estimation Filter Comparison

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Abstract

The Wilkinson Microwave Anisotropy Probe (WMAP) spacecraft was launched in June of 2001. The sensor complement of WMAP consists of two Autonomous Star Trackers (ASTs), two Fine Sun Sensors (FSSs), and a gyro package which contains redundancy about one of the WMAP body axes. The onboard attitude estimation filter consists of an extended Kalman filter (EKF) solving for attitude and gyro bias errors which are then resolved into a spacecraft attitude quaternion and gyro bias. A pseudo-linear Kalman filter has been developed which directly estimates the spacecraft attitude quaternion, rate, and gyro bias. In this paper, the performance of the two filters is compared for the two major control modes of WMAP: inertial mode and observation mode.

INTRODUCTION

The Microwave Anisotropy Probe (MAP) Satellite was launched at 15:46:46 EDT on June 30, 2001 aboard a Delta II-7425-10 launch vehicle. After three phasing loops MAP passed by a swingby point where it was injected by the moon's gravity towards the L2 point. On October 1, 2001, following a three month journey, MAP arrived safely at its permanent observing station near the quasi-stable position, L2 Lagrange Point, 1.5 million km from Earth in the direction opposite the Sun.

MAP used two two-axis gyroscopes to measure its angular rate vector. One input axis of the first gyroscope was aligned along the body x-axis and the other input axis was aligned along the body z-axis. The input axes of the second gyro were aligned along the body y and z-axes. Therefore, if one gyro failed then the rate about two body axes is still measured; namely, about the z-axis and one of the other two. MAP also had two Digital Sun Sensors that measured one vector; namely, the direction to the Sun. In addition, the spacecraft also carried two Autonomous Star Trackers (ASTs) which output quaternions.

The WMAP attitude estimation algorithm consists of a 6 state Extended Kalman Filter (EKF) which estimates attitude and gyro bias errors using primarily at least one AST and during contingencies will also use the digital sun sensors. The attitude and gyro bias errors are resolved into a spacecraft quaternion and gyro bias. This filter has a long heritage over scores of missions.

Some recent work has been on the development of a so-called Pseudo Linear Kalman (PSELIKA) Filter. The PSELIKA filter directly estimates the spacecraft attitude, rate, and gyro bias. This paper will compare the performance of the EKF and PSELIKA filter. In the next section we describe the angular rates at which the problem is examined. This section is followed by a description of the examined filters. In the next section we present comparison results using flight data and then, in the last section, we present our conclusions.

Body Angular Kinematics [ref. 1]
The WMAP control system consisted of a variety of different control modes. In this paper, the inertial and science observation control mode data will be used in the filter comparison. The inertial control mode is straightforward. However, the science mode was much more intricate. The primary WMAP science was to occur at L2. However, the spacecraft was placed in the science mode frequently before L2 was reached. Besides providing some early science data, this mode also minimized the solar radiation pressure force on the spacecraft which assisted the ground based orbit estimation and prediction. Figure 1 demonstrates the spacecraft body rates.
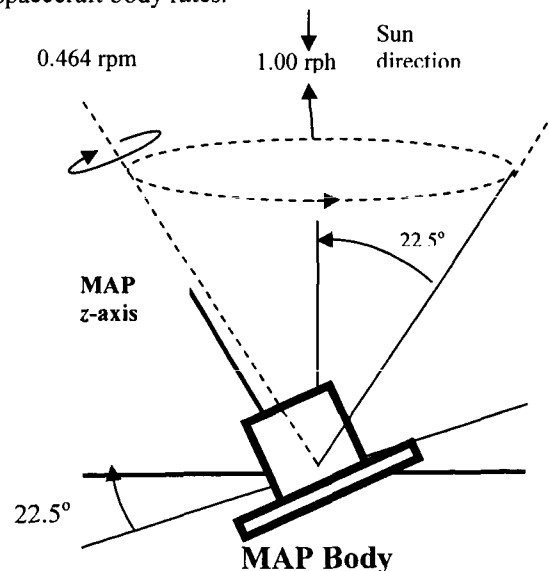

Fig. 1: The Geometry of the Angular Motion Components of the MAP Spacecraft.

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THE FILTERS

Both filters assume the following form for the dynamics model:

$$\dot{x} = f(x(t), t) + w(t) \tag{1}$$

and for the measurement model:

$$z_k = h_k(x(t_k)) + v_k \tag{2}$$

Pseudo Linear Kalman Filter

PSELIKA re-arranges eq. (1) as follows:

$$\dot{x} = F(x(t), t)x(t) + w(t) \tag{3}$$

Correspondingly, eq. (2) is re-arranged as follows:

$$z_k = H_k(x(t_k))x(t_k) + v_k \tag{4}$$

Both eqs. (3) and (4) are now in the form of a standard linear Kalman filter. For a given time period, the best estimate of $x(t)$ is used to form the F and H_k matrices of eqs. (3) and (4) respectively. With this key substitution, the algorithm flow is that of the linear Kalman filter. The dynamics and measurement models will now be described.

Dynamics Model

In this estimator (filter) we use AST measurements which are a direct measurement of the spacecraft quaternion. The dynamics model of Filter I was developed in [ref. 2]. Using

$$Q = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{5}$$

define the system dynamics as

$$\begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega + h)\times] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} I^{-1}(T - \dot{h}) \\ 0 \end{bmatrix} + \begin{bmatrix} w_\omega \\ w_q \end{bmatrix} \tag{6}$$

where $[g\times]$ is the cross product matrix of a general vector g, w_ω accounts for the inaccuracies in the modeling of the spacecraft angular dynamics, and w_q accounts for modeling errors of the quaternion dynamics. w_ω and w_q are defined to be zero mean while noise processes.

Measurement Equations

The measurements are star tracker quaternions and the x,y, and z body rates measured by the two spacecraft gyros. The measured quantities are contaminated by noise so they are not the reference values but rather, q_m and ω_m.

AST Measurement

As mentioned above the AST directly measures the spacecraft quaternion. This relationship can be represented as follows:

$$q_m = q + v_q \tag{7}$$

where v_q is a measurement noise which is assumed to be white.

Gyro Measurement

As the AST directly measures the spacecraft attitude quaternion, the gyros directly measure the spacecraft rate. This relationship can be written as follows:

$$\omega_m = \omega + v_\omega \tag{8}$$

where v_ω is a measurement noise which is assumed to be white.

The Combined measurement

Combining eqs. (7) and (8) we obtain

$$\begin{bmatrix} \omega_m \\ q_m \end{bmatrix} = \begin{bmatrix} I_{3\times3} & 0_{3\times4} \\ 0_{4\times3} & I_{4\times4} \end{bmatrix}\begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} v_\omega \\ v_q \end{bmatrix} \tag{9}$$

This is the combined measurement model for simultaneous measurement of the gyros and AST.

Update Equations

The update equations are the standard linear discrete Kalman filter equations. Namely, the a priori covariance equation is as follows:

$$P_k(-) = \Phi_k P_{k-1}(+)\Phi_k^T + Q_k \tag{10}$$

where $P_k(-)$ is the a priori state covariance, Φ_k is the state transition matrix, $P_{k-1}(+)$ is the updated covariance from the previous time step, and Q_k is the dynamics noise covariance matrix. The Kalman gain is obtained as follows:

$$K_k = P_k(-)H_k^T \left[H_k P_k(-)H_k^T + R_k \right]^{-1} \qquad (11)$$

where H_k is the measurement sensitivity matrix defined in eq. (9) and R_k is the measurement noise covariance matrix. The state update is as follows:

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k(z_k - H_k \hat{x}_k(-)) \qquad (12)$$

where $\hat{x}_k(+)$ is the updated state, $\hat{x}_k(-)$ is the a priori state, and z_k is the measurement. Lastly, the a posteriori state error covariance is:

$$P_k(+) = (I - K_k H_k)P_k(-)(I - K_k H_k)^t + K_k R_k K_k^t \qquad \dots \quad (13)$$

where $P_k(+)$ is the updated state error covariance.

FILTER II

Extended Kalman Filter

As mentioned above, the EKF assumes the dynamics model of eq. (3) and the measurements model in eq. (4), whereas PSELIKA directly estimates the state vector. The EKF estimates attitude and gyro bias errors. Thus, the EKF state vector during the measurement update stage is as follows:

$$\Delta x = \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} \qquad (14)$$

where Δa is the angular attitude error and Δb is the gyro bias error.

The estimate of these errors is used to compensate the a priori quaternion and gyro bias estimates to achieve the a posteriori state estimate. In order to accomplish this task, eqs. (3) and (4) are linearized about the quaternion and bias best estimates. With this description in mind, the dynamics and measurement models will now be described.

Dynamics Model

From [ref. 3], the attitude and gyro bias dynamics model is as follows:

$$\Delta \dot{a} = \left[\omega \times \right] \Delta a - \Delta b + w_a \qquad (15)$$

$$\Delta \dot{b} = w_b \qquad (16)$$

Where w_a is the attitude error process noise and w_b is the gyro bias error process noise. Combining eqs. (15) and (16)

define the EKF dynamics model during the measurement update stage:

$$\begin{bmatrix} \Delta \dot{a} \\ \Delta \dot{b} \end{bmatrix} = \begin{bmatrix} [\omega \times] & -I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} + \begin{bmatrix} w_a \\ w_b \end{bmatrix} \qquad (17)$$

Measurement Equations

The measurement equations of the EKF are straightforward though not as simple as those of PSELIKA. The measurement is of the form expressed in eq. (7). This measurement needs to be resolved into the form of the state represented in eq. (14). The quaternion measurement is assumed to be truth. The resulting attitude error quaternion is formed as follows:

$$\Delta q = q_m^{-1} \hat{q}(-) \qquad (18)$$

where Δq is the error quaternion, q_m^{-1} is the inverse of the quaternion measurement, and $\hat{q}(-)$ is the a priori attitude quaternion estimate. Δq can be decomposed as follows:

$$\Delta q \approx \begin{bmatrix} \delta\psi/2 \\ \delta\theta/2 \\ \delta\varphi/2 \\ 1 \end{bmatrix} \qquad (19)$$

The effective measurement is formed from eq. (19) as follows:

$$\Delta a_m = \begin{bmatrix} \delta\psi \\ \delta\theta \\ \delta\varphi \end{bmatrix} + v_k \qquad (20)$$

where v_k is the measurement noise.

The resulting measurement equation becomes:

$$\Delta a_m = \Delta a + v_k \qquad (21)$$

Using eqs. (14) and (21), the following relationship is obtained:

$$\Delta a_m = \begin{bmatrix} I & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} + v_k \qquad (22)$$

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Update Equations

Even though different states are estimated, the EKF update equations are similar to those described for the PSELIKA filter. The two changes concern how the a priori and a posteriori spacecraft attitude quaternion and gyro bias estimates are computed. The spacecraft quaternion a priori estimate is computed using the quaternion kinematic equation which is shown in eq. (6). Explicitly,

$$\dot{\hat{q}} = \frac{1}{2} \Omega \hat{q} \tag{23.a}$$

where Ω is defined as follows:

$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{23.b}$$

For the components of ω in (23.b), substitute those of $\hat{\omega}_\kappa(-)$ which is defined as follows:

$$\hat{\omega}_\kappa(-) = \omega_{m_k} - \hat{b}_{k-1} \tag{24}$$

where $\hat{b}_{k-1}$ is the latest estimate of the gyro bias and ω_{m_k} is the current gyro measurement at time k. The state update equation is computed as follows:

$$\hat{x}_k(+) = K_k z_k \tag{25}$$

where z_k is the effective measurement specified in eq. (20). As mentioned before, $\hat{x}_k(+)$ from eq. (25) represents estimated attitude and gyro bias errors. The spacecraft attitude quaternion and gyro bias need to be updated using this updated error state. Define the following:

$$\hat{x}_k(+) = \begin{bmatrix} \Delta\hat{a}_k(+) \\ \Delta\hat{b}_k(+) \end{bmatrix} \tag{26}$$

The spacecraft quaternion is updated by converting $\Delta\hat{a}_k(+)$ from eq. (26) into the delta quaternion, $\Delta\hat{q}_k(+)$. This is accomplished by eq. (19) in reverse. The resultant spacecraft quaternion update equation then becomes:

$$\hat{q}_k(+) = \hat{q}_k(-)\Delta\hat{q}_k(+)^{-1} \tag{27}$$

The spacecraft gyro bias is simply updated as follows:

$$\hat{b}_k = \hat{b}_{k-1} + \Delta\hat{b}_k(+) \tag{28}$$

In the next section, the performance of each filter detailed above will be analyzed.

RESULTS

Two spans of WMAP data were used to compare the performance of the two filters. For the first span, the spacecraft was inertial pointing. In the second span, the spacecraft was in the science observation mode. Both modes are described in the introduction. For both spans of data, the WMAP attitude ground system EKF was run with gyro data and AST#1. The filter was set up to solve for the spacecraft attitude and gyro bias. The same span of data was then run through the PSELIKA filter. First, the inertial mode results will be described. A 1200 second span of inertial data was chosen which started before the WMAP spacecraft began gyro calibration maneuvers. Figures 2 and 3 show the difference between the PSELIKA and EKF attitude and gyro bias estimation errors respectively. Figure 4 shows the rate differences between the PSELIKA filter and the gyro measurements.
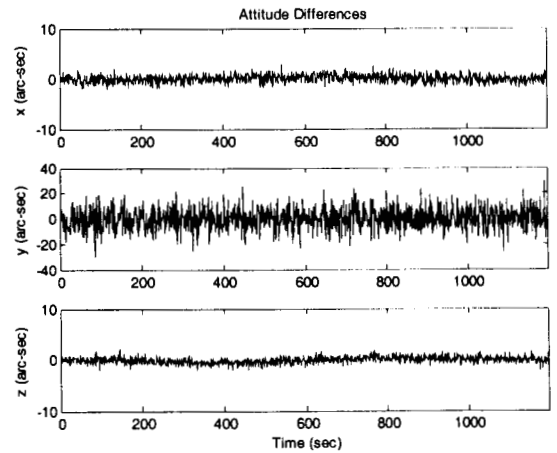

Fig. 2: Attitude Estimation Differences between the PSELIKA and EKF Filters for Inertial mode
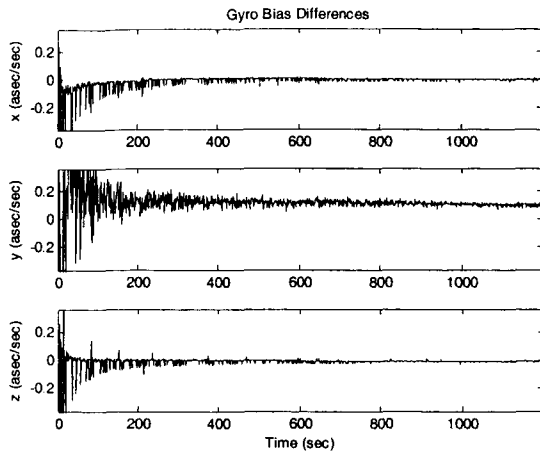
Fig. 3: Gyro Bias Estimation Differences Between the PSELIKA and EKF Filters for Inertial mode
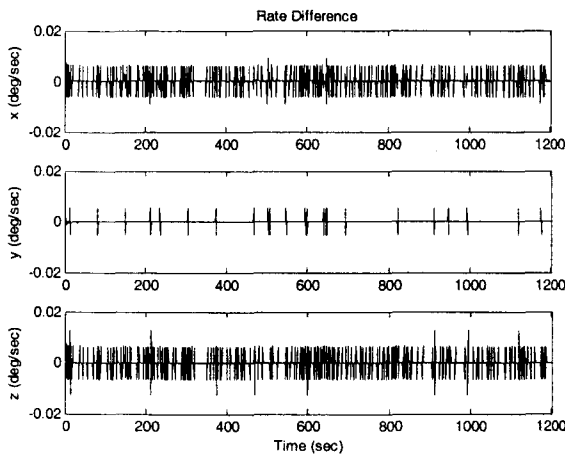

Fig. 4: Difference Between Gyro measurements and PSELIKA Estimated Rate for Inertial Mode

Both the PSELIKA and EKF filters converged within approximately 20 seconds. The RSS of the three components of the mean residuals for the PSELIKA and EKF filters were 0.0007 and 0.087 arc-seconds respectively. The RSS of the three components of the standard deviation for the PSELIKA and EKF filters were 0.077 and 8.52 arc-seconds respectively. Overall the filter performance was comparable though the PSELIKA did have a slightly lower residual mean and standard deviation. A comparison of the residual PSDs was inconclusive.

The second span of data consisted of the spacecraft spinning at approximately 2.7 deg/sec about the z-axis and 1 deg/hour about the other two axes. The rate profile from the WMAP gyros is shown in Figure 5. A 1200 second span of data was selected. The EKF attitude and gyro bias converged within 120 seconds. PSELIKA converged within 240 seconds. The attitude estimation differences between the two filters are shown in Figure 6 with the corresponding gyro bias differences in Figure 7.
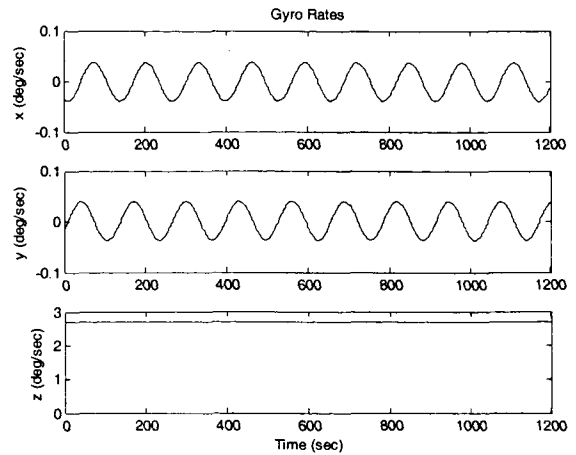

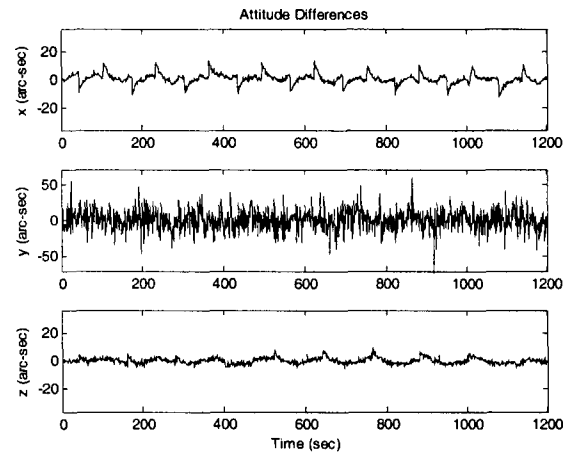**Fig. 5**: WMAP Gyro Rates for Science Observation mode

Fig. 6: Attitude Estimation Differences Between the PSELIKA and EKF Filters for Observation mode
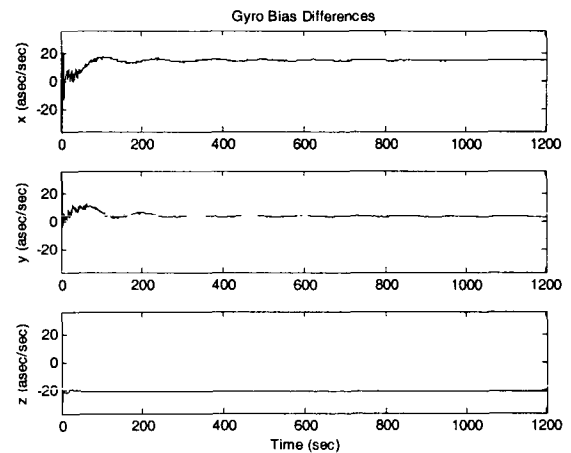

Fig. 7: Gyro Bias Estimation Differences Between the PSELIKA and EKF Filters for Observation mode

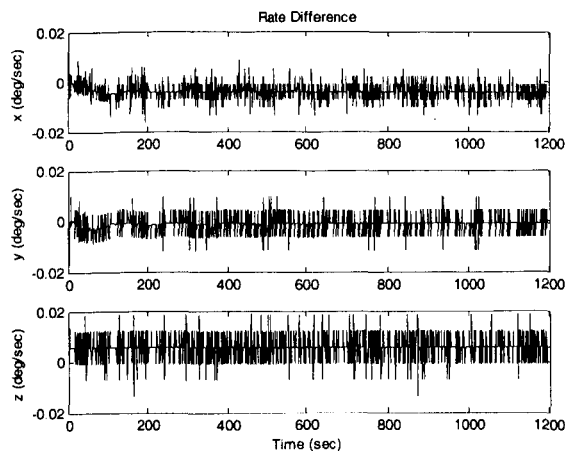The rate estimate was then compared to the gyro rates and the results are shown in Figure 8.

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Fig. 8: Difference Between Gyro measurements and PSELIKA Estimated Rate for Observation Mode

The AST measurement residuals were analyzed for both filters. The RSS of the three components mean value for the PSELIKA was 0.07 arc-seconds and 0.26 arc-seconds for the EKF. The RSS of the three components of the standard deviation of the PSELIKA and EKF filters were 0.12 arc-seconds and 15.34 arc-seconds respectively. The PSD of each set of residuals was generated in an attempt to check which filter produced residuals closer to white noise. However, both PSD results were inconclusive.

As a final test, the PSELIKA filter state was augmented with the spacecraft external torque states as well as the wheel torque states. The added states were modeled assuming a Markov process. The reaction wheel torque states were also added as measurements. The attitude and gyro bias estimation difference between the two filters is shown in Figures 9 and 10 respectively.
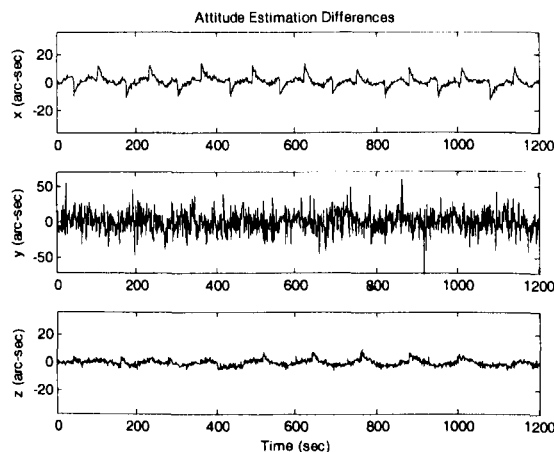

Fig. 9: Attitude Estimation Differences Between the Augmented PSELIKA and EKF Filters for Observation mode
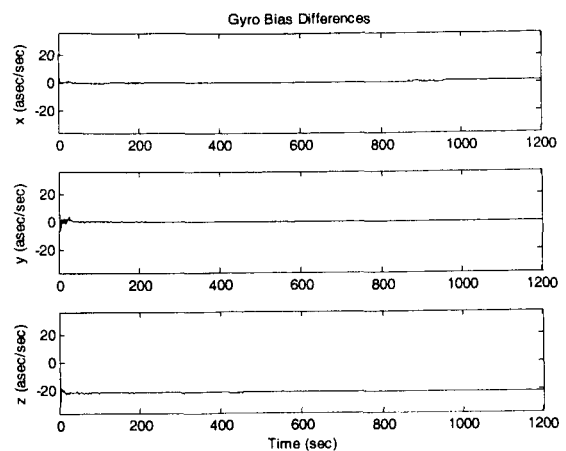

Fig. 10: Gyro Bias Estimation Differences Between the Augmented PSEILIA and EKF Filters for Observation mode.

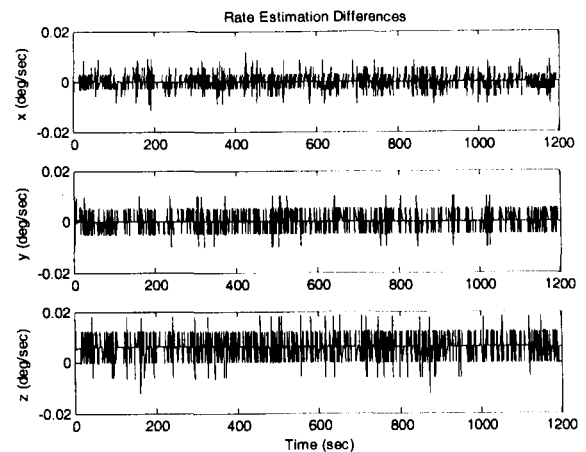The rate estimation differences are shown in Figure 11.

Fig. 11: Difference Between Gyro Rate and Augmented PSELIKA Estimated Rate for Observation mode

The addition of the six dynamic states in the PSELIKA filter reduced the convergence from 240 to 60 seconds. In addition, the augmented states provided information about the total torque on the spacecraft. A plot of that total torque estimate is shown in Figure 12.
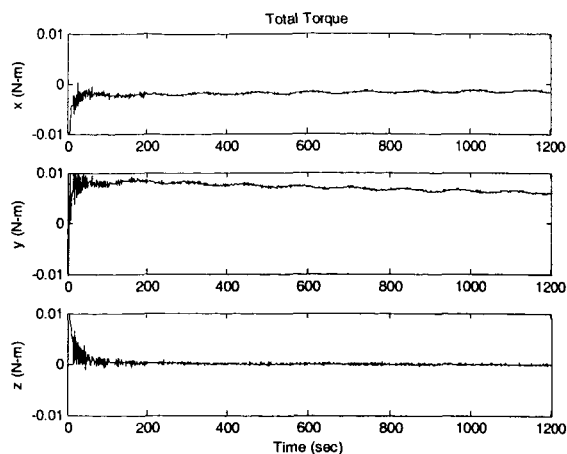
Fig. 12: Total Spacecraft Torque Estimate from Augmented PSELIKA in Observation mode

CONCLUSIONS

The Pseudo-Linear Kalman (PSELIKA) filter and Extended Kalman Filter (EKF) were described and compared in this paper. PSELIKA directly estimated the spacecraft attitude quaternion, rate, and gyro bias. The EKF update stage estimates attitude and gyro bias errors, which are used to update the a priori attitude quaternion and gyro bias. Each filter used AST and gyro measurements. Both filters were shown to have similar performance though the EKF did converge faster for the observation mode span of data and the PSELIKA had smaller mean and standard deviation AST residuals. An augmented PSELIKA was introduced which added the spacecraft external torque and reaction wheel torque states. The addition of those six states dramatically improved the convergence time of the PSELIKA and demonstrated a faster convergence time than the EKF. Future work will look at better quantifying the performance of each filter as well as analyzing mechanisms for tuning each filter.

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[2] Azor, A., I.Y. Bar-Itzhack, J. Deutschmann and R.R. Harman, "Angular-Rate Estimation Using Delayed Quaternion Measurements", *AIAA J. of Guidance, Control, and Dynamics, AIAA J. of Guidance, Control, and Dynamics,* Vol. 24, No. 3, May-June 2001, pp. 436-443.

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