HUMBLE PROBLEMS

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I have worked on innumerable problems that you would call humble, but which I enjoyed and felt very good about because I sometimes could partially succeed ... Richard P. Feynman, letter to Koichi Mano

INTRODUCTION

Harold Morton introduced a talk by saying that when you wind up an old professor, he tends to talk for a microcentury. I will attempt to keep my comments to that canonical time span. Having failed to find some unifying theme for this talk, I decided to just ramble through my career with a focus on the algorithms, spacecraft, and people I've had the privilege and pleasure to work with. The algorithms, and certainly the spacecraft, are not all mine. The people are some of those whose ideas that have most influenced and inspired my career. The organization of the paper is largely chronological, but I do not hesitate to jump forward or backward in time when the material demands it. The coverage is broad but necessarily shallow; the interested reader can find more detail in the references.

PREHISTORY

My father was an engineer, and engineering was a natural career path for my older brother and me. I much preferred arithmetic to other courses in school, because arithmetic problems have right and wrong answers. It was not until late in my undergraduate education that the realization dawned on me that engineering, and research in general, required dealing with uncertainty and ambiguity. However, the instinct to search out exactly solvable problems has never left me.

When I was an adolescent, I found an article on Calculus in an encyclopedia. It contained the well-known problem illustrated at the right. A tinsmith is to cut corners out of a square of metal and fold up the edges to make an open box of maximum volume. The optimum cut is given by the solution of

$$0 = d[x(a-2x)^2]/dx = (a-6x)(a-2x) . \tag{1}$$

This is a humble problem indeed, but I found it incredibly exciting that a "practical" problem could be solved by simple algebra. One could say that this marks the beginning of my interest in optimization problems, and I've never really lost my sense of wonder at the power of mathematics.

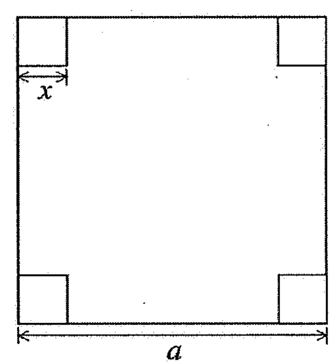

Figure 1. First Humble Problem

I followed my brother to Cornell University, but chose to study Engineering Physics rather than Chemical Engineering as he did, mostly because it was reputed to be the most challenging major. The Cornell Mathematics Department offered a five-semester course in applied mathematics for Engineering Physics majors, taken after the usual three-semester calculus sequence. This was an outstanding sequence taught in small sections not by teaching assistants but by faculty members, including the brilliant applied mathematician Mark Kac, who taught differential equations to sophomores. In my ninth semester of the five-year Engineering Physics program, I elected to take a course in probability theory taught by Jacob

Wolfowitz, whose son currently heads the World Bank. An undergraduate course in quantum mechanics seduced me away from Engineering Physics to pure physics, and I pursued graduate studies in theoretical physics at the University of California at Berkeley. I spent a wonderful five years there, with future Nobel laureates as teachers and one colleague. In retrospect, though, the only practical things I took away from Berkeley were a familiarity with group theory, the only useful mathematics I hadn't learned at Cornell, and the experience of writing a short FORTRAN program for my dissertation.

After Berkeley, I was awarded a postdoctoral fellowship at the University of Maryland. My principal achievements there were to meet Malcolm Shuster, who was a graduate student in physics at the time, and to reinforce my conviction that theoretical physics was not the field in which I would make my mark. Believing that undergraduate teaching would be more to my taste, I declined a second year of my fellowship to accept a position as Assistant Professor of Physics at Williams College.

It's been said many times that you never really learn a subject until you teach it, and my years at Williams certainly bore this out. Aside from deepening my knowledge of physics, teaching at an excellent small liberal arts college also fostered interactions with mathematicians, other scientists, and humanities faculty. I greatly enjoyed this life, perhaps too much, for I did not earn tenure. The spring of 1974 found me at the Washington meeting of the American Physical Society, searching with many others for a congenial teaching position. As luck would have it, the Computer Sciences Corporation (CSC) was at the meeting to recruit scientists with experience in computer programming to write software for NASA. My resume included a course on *Use of the Computer in the Physical Sciences* that I had taught on an IBM 1130 at Williams. This helped earn me an interview with CSC, and I was offered a position one month before I would have become unemployed.

COMPUTER SCIENCES CORPORATION (CSC)

Quaternions and Kalman Filters

CSC offered me a position in either the Attitude Systems Operation or the Orbit Systems Operation. I chose the former, mostly because I felt a rapport with the two physicists in that group who had interviewed me: Jerry Lerner, who had been Malcolm Shuster's roommate in graduate school, and Dale Headrick. Dale was the first to arouse my interest in quaternions and Kalman filters. I had never heard of quaternions, since they did not appear in any of the five classical mechanics texts I had learned or taught from. Goldstein did not introduce the quaternion components as the real and imaginary parts of the Cayley-Klein parameters until the second edition of his classic text [1]. I had heard of Kalman filters, though. The math department at Williams offered annual informal discussion groups of selected topics of interdisciplinary interest. Ironically, the topic chosen for my last year at Williams was Kalman Filtering; but I was too busy looking for a job to take advantage of that opportunity.

My first supervisor at CSC was Mike Plett, who said that he didn't want to see any clever programming from me. I thought that was very peculiar advice until I wrote a clever program that inadvertently assigned a new value to the constant 1. After that, I concentrated on good, solid programming, following the example of Myron Shear, who was described by Dale Headrick as the best FORTRAN programmer he'd ever known.

Attitude/Orbit Analysis

My first task at CSC was under the direction of Mel (Tom) Velez at Goddard Space Flight Center (GSFC). He was pursuing simultaneous orbit/attitude estimation of the spin-stabilized Synchronous Meteorological Satellite (SMS), since he realized that Earth horizon sensor data contains orbit information. This led to some interesting work, but the vastly different time scales of the orbit and attitude dynamics impeded progress. The orbital equations of motion could be integrated with time steps of tens of minutes, while time steps of fractions of a spin period were required by the rigid-body attitude dynamics. This led Velez to pursue variation-of-parameter (VOP) approaches to rigid-body dynamics. Harold Morton, John Junkins, and their students had done some interesting work in this area, but I tried a different approach [2, 3].

The usual approach was to integrate the equation obeyed by the inertial-to-body attitude quaternion

$$\frac{dq}{dt} = \frac{1}{2}\begin{bmatrix} \boldsymbol{\omega} \\ 0 \end{bmatrix} \otimes q \tag{2}$$

and the equation of motion for either the inertial components of the angular momentum

$$d\mathbf{L}_I / dt = \mathbf{N}_I \tag{3a}$$

or for the body-frame components

$$d\mathbf{L}_B / dt = \mathbf{N}_B - \boldsymbol{\omega} \times \mathbf{L}_B , \tag{3b}$$

where $\boldsymbol{\omega}$ denotes the angular velocity and $\mathbf{N}$ the external torque. This approach has one redundant equation, which is constrained by the unity norm of the attitude quaternion. The problem with this formulation is that all the quaternion components are "fast" variables unsuitable for a VOP approach. My idea was to use both of Eqs. (3a) and (3b) and not Eq. (2). There again is one constraint, since the norm L of the angular momentum is the same in both frames. One more variable is needed to complete the parameterization, which I chose to be an angle variable χ, obeying the "fast" equation

$$d\chi / dt = L(L^2 + \mathbf{L}_B \cdot \mathbf{L}_I)^{-1}[(\mathbf{L}_B + \mathbf{L}_I) \cdot \boldsymbol{\omega} + L^{-2}(\mathbf{L}_B \times \mathbf{L}_I) \cdot (\mathbf{N}_B + \mathbf{N}_I)] . \tag{4}$$

Unfortunately, the VOP approaches to attitude dynamics never bore fruit, and these equations (in a much cruder form) were put away in a drawer. Many years later, they found a home in the special issue on Attitude Representations of *The Journal of the Astronautical Sciences* edited by John Junkins and Malcolm Shuster [4]. These variables have never found widespread use, but they have recently been employed for attitude estimation [5].

Gene Lefferts

After about a year of this attitude/orbit analysis, Velez moved on to other matters, and my task fell under the supervision of Eugene J. Lefferts. This was the most felicitous transition of my career. Gene Lefferts became my most important mentor, and I owe my success from that point onward to his influence. He was also directly responsible for two of my future career moves. When I first began working with Gene, one of my CSC associates advised me to talk to him about fishing, because that was his interest outside of work. I quickly found that my complete lack of interest in fishing was no drawback, because Gene was interested in everything: politics, music, art, and culture in general, as well as fishing. He advised an Explorer post that explored fishing, caving, avant garde theater, and gamelan music, among many other things. My daughter joined Gene's post as an adolescent, and he was an important influence on her as well.

Gene actively promoted the use of quaternions, Kalman filters, and dynamic time-domain simulation; and it is owing to his influence that these have been major themes in my career. One of Gene's insights was that optimal estimation, in the form of the Kalman filter, had been very important, but that optimal control had never demonstrated any superiority to classical control techniques. This insight will sound very strange to the astrodynamics community, where optimal maneuver planning has been invaluable in space mission design. I believe that Gene was correct in the context of attitude control, where the control effort can be generated with little or no expenditure of fuel.

Gene also helped me develop my intuitive feel by means of humble toy problems. One example is the simple problem of finding the steady-state covariance of a one-dimensional state with constant dynamics and measurements processed at a time interval Δt. The pre- and post-update covariances are given by

$$P(-) = P(+) + Q\Delta t \tag{5a}$$

$$P(+) = [I - KH]P(-) \quad \text{with} \quad K = P(-)H^T[HP(-)H^T + R]^{-1} \tag{5b}$$

We can assume $H = 1$ and $R = \sigma_n^2$, and the multiplication of 1×1 matrices is commutative, so these equations are easily solved to give

$$P(-) = \frac{1}{2}\left[\sqrt{(Q\Delta t)^2 + 4\sigma_n^2 Q\Delta t} + Q\Delta t\right] \quad \text{and} \quad P(+) = \frac{1}{2}\left[\sqrt{(Q\Delta t)^2 + 4\sigma_n^2 Q\Delta t} - Q\Delta t\right]. \tag{6}$$

Filtering is only really useful when $Q\Delta t \ll R$, in which case

$$P(-) \approx P(+) \approx \sigma_n \sqrt{Q\Delta t}. \tag{7}$$

These equations are obviously useful in giving the expected estimation accuracies given specified levels of measurement noise R and process noise Q. Gene pointed out to me the less obvious use of these equations to determine the level of dynamic measurement fidelity, as represented by Q, needed to reduce estimation errors to a required level given specified measurements. It's interesting to note that Eq. (7) is more easily derived as the solution of the differential equation arising from the continuous-measurement limit of the Kalman filter [6], which gives the steady-state covariance as the solution of

$$dP/dt = Q - PH^T(R\Delta t)^{-1}HP = 0. \tag{8}$$

I have gained many very useful insights by looking at the steady-state behavior of simple Kalman filters.

Working with Gene, I developed a general quaternion-based simulation that accommodated a variety of sensor models and control strategies. We used this in an attempt to use dynamics modeling to improve the attitude estimation of the Nimbus spacecraft [7]. This was of interest to Harry Stallings, an engineer in Henry Hoffman's Guidance and Control Branch at Goddard who had been a colleague of Gene's at the Martin Aircraft Corporation in Baltimore. The Nimbus analysis was inconclusive, but Gene and I had established a connection that was to prove important when Goddard began development of the Solar Maximum Mission (SMM) spacecraft.

Solar Maximum Mission (SMM)

SMM was the first of the Multimission Modular Spacecraft (MMS) series, and was later to be the first spacecraft to be serviced in-orbit by astronauts. The MMS had three modules: a power module, a communication and data handling module, and the Modular Attitude Control System (MACS). The first analysis we performed for SMM was a dynamics simulation of the despin of the spacecraft after release from its launch vehicle, using magnetic torquers. We showed that despin from worst-case conditions could require ten hours, although the initial analysis had given a time of one hour. It turned out that the earlier analysis had misplaced a decimal in one of the conversions of magnetic moment from Ampere-ft^2 to Ampere-m^2, magnetic field strength from gammas to nT, and the resulting torque from oz-in to Nm. This was an early lesson in the advantage of using SI units consistently, as we did in our dynamic simulator, with a penalty much smaller than that paid in 1999 by the Mars Climate Orbiter. Our analysis also led to providing SMM with an additional magnetic torquer bar external to the MACS. I was delighted that my contribution had actually led to a change in hardware launched into space.

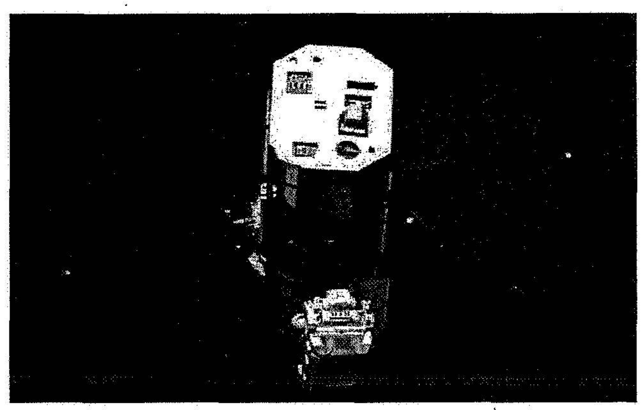

Figure 2. SMM with Attached Astronaut

Jim Murrell, of General Electric Space had developed a multimission quaternion Kalman filter for the MACS being built by GE [8]. This Kalman filter was later used on Landsats D and E and on the Upper Atmosphere Research Satellite (UARS); but Arun Guha, Henry Hoffman's Lead ACS Engineer for SMM, chose not to use it for SMM. Instead, he developed a decoupled-axis Kalman filter based on the filter employed on GSFC's International Ultraviolet Explorer (IUE) spacecraft. With our credibility established, Gene and I were given the task of developing a FORTRAN prototype of the ACS software for SMM and validating it in our dynamic simulator. This was the most exciting project I had engaged in thus far, and I attacked it with great enthusiasm. After the prototype had been developed and testing was underway, Henry Hoffman decided that his branch should have a peer review of our work. The review panel included Jim Donohue, an outstanding classical controls engineer, also from the Martin Aircraft Corporation, who mentored an entire generation of engineers in Hoffman's branch. He had also one time played on a semipro baseball team with the future Hall of Fame baseball player Al Kaline, which impressed me very much. During my presentation, Donohue suggested that we apply a lead-lag filter to the digital sun sensor data. I had no idea what he was talking about but said that we'd look into it. When I later confessed my ignorance to Guha, he advised me to read the Classical Controls book in the Schaum's Outline series so I could speak the language [9]. I took Arun's advice, and later told him that I was taking a course in control theory. When he asked where, I said that it was the course that he was teaching me. I presented my SMM analysis at the same GN&C conference where Murrell presented his MACS Kalman filter [10].

SMM was launched in 1980, after I had left CSC, and within a few months the fuses on the reaction wheels in the MACS began to blow, causing a loss of attitude control. Henry Hoffman and Tom Flatley, an extraordinary attitude dynamicist in Henry's branch, used SMM's magnetic torquers to spin-stabilize the spacecraft until astronauts could replace the entire MACS in 1984 (Figure 2). Henry liked nothing better than rescuing a satellite in distress, which led to his being called a satellite savior [11].

Spacecraft Attitude Determination and Control

During my stay at CSC, their contract for support of GSFC neared its expiration date. Roger Werking, who was responsible for the greater share of CSC's support, tasked Jim Wertz to document the algorithms and software that CSC had developed under their contract. He intended this to be a Contractor Report that would be available for the use of another contractor that might outbid CSC for the continued support. Jim somehow talked Roger into producing a book rather than a Contractor Report, convinced CSC to contribute to its creation, and found a publisher. The resulting book has been an indispensable reference in spacecraft attitude determination and control and a monument to Jim's initiative and perseverance [12].

Jim developed an overall outline for the book, and solicited the staff of the Attitude Systems Operation for proposals to write various sections. I offered to write the sections on attitude kinematics and dynamics, assuming that I only needed to paraphrase the corresponding sections of Goldstein [1]. You can imagine my chagrin when Mike Plett said, in reviewing my proposed outline of these sections, "I hope you can make this clearer than Goldstein." I apparently succeeded, and found my small first taste of fame. Malcolm Shuster joined the CSC Attitude systems operation in 1977, too late to contribute significantly to *Spacecraft Attitude Determination and Control.* During his first year at CSC, he developed the QUEST algorithm, to which I had made some small contributions. This algorithm was announced to the world at the same GN&C Conference where Murrell's and my papers appeared [13]. The history of the development of QUEST was the topic of Shuster's Brouwer Lecture [14].

After the completion of *Spacecraft Attitude Determination and Control,* many of the authors began to leave CSC, owing partly to their employment opportunities increasing along with their visibility, and partly to their desire to avoid the uncertainties of employment by a government contractor. Ironically, CSC was to retain its supporting role at GSFC into the next millennium. I also became impatient to leave, preferably to enter the Civil Service. To my delight, an ad appeared in the April 23, 1978 Washington Post, seeking an Aerospace Engineer/Physicist/Mathematician to conduct "research in satellite orbit and attitude dynamics and the mathematical modeling of dynamical systems" at the Naval Research Laboratory. I applied with a cover letter stating, "I believe that my qualifications, as presented in the enclosed resume, are perfectly suited to this position." My wife found this breathtakingly arrogant, but I thought it was a simple statement of the fact that the ad seemed to be written specifically for me. It turned out that Bernie Kaufman, who was seeking to fill this position, had worked with Gene Lefferts at Goddard and was checking out all the Goddard-related applicants with him. I later found that when Bernie told Gene that I had applied, Gene told him that he could stop looking further. This was the first of the two career moves I owe to Gene.

NAVAL RESEARCH LABORATORY (NRL)

Optimal Control

Bernie Kaufman's section comprised himself, Bob Dasenbrock, Bill Harr, and me. Bernie and Bob were the orbit dynamics experts, and Bill was an excellent programmer who showed me how to make my FORTRAN readable. Bernie is the best supervisor I ever had, supportive, encouraging, successful in bringing interesting work into our section, and insulating us from outside interference.

Ironically, Terry Alfriend, who headed a different branch at NRL, posed the first problem that really engaged me. It was the rest-to-rest attitude maneuver about the symmetry axis of an axially symmetric spacecraft with one flexible mode. The equations of motion are

$$I_b\ddot{\theta} = N + k\psi \quad \text{and} \quad I_a(\ddot{\theta}+\ddot{\psi}) = -k\psi\,, \tag{9a}$$

with the boundary conditions

$$\theta(\pm T/2) = \pm\theta/2, \quad \dot{\theta}(\pm T/2) = \psi(\pm T/2) = \dot{\psi}(\pm T/2) = 0\,. \tag{9b}$$

where θ is the angular position of the main body with moment-of-inertia I_b, ψ is the relative angular displacement of the appendage with moment-of-inertia I_a, N is the torque applied to the main body, and k is the torsional coupling constant. I set up an optimal control problem with quadratic penalties on the control and the flexible mode. The problem has closed-form solutions with θ, ψ and N all odd functions of time. When the penalty on the control was less than some critical value, the solutions involve hyperbolic trig functions. Replacing the quadratic penalty on the control with an absolute limit on its magnitude gives a more complicated solution with a singular control arc and only a slight reduction in maneuver time. Additionally removing the penalty on the amplitude rate of the flexible mode led to chattering arcs that defied analysis. At this point, I realized that, although the control effort became a more complicated function as I tried to improve the performance, θ and ψ remained smooth functions of time. This led me to consider a simple non-optimal maneuver, choosing θ, ψ and N to be the lowest order polynomials that could satisfy the equations of motion and the boundary conditions. The maneuver time using this polynomial profile was only about 10% longer than the maneuver time of the optimal profile. This reinforced my agreement with Gene Lefferts' belief that optimal control was less useful than optimal estimation. I presented this paper at the 1979 AAS Astrodynamics Conference, the first AAS conference I attended [15].

The Lefferts, Markley, Shuster (LMS) Paper

Shortly after I moved from CSC to NRL, the analysis of steady-state Kalman filters regained my attention. The state equation for single-axis attitude estimation using a gyro in "model-replacement mode" is

$$\frac{d}{dt}\begin{bmatrix}\theta\\ b\end{bmatrix} = \begin{bmatrix}\omega_g\\ 0\end{bmatrix} + \begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ b\end{bmatrix} + \begin{bmatrix}-n_v\\ n_u\end{bmatrix} = \begin{bmatrix}\omega_g\\ 0\end{bmatrix} + F\begin{bmatrix}\theta\\ b\end{bmatrix} + \begin{bmatrix}-n_v\\ n_u\end{bmatrix}, \tag{10}$$

where θ is the rotation angle, ω_g is the gyro output, b is the gyro bias, an n_v and n_u are uncorrelated white noise processes. Assuming effectively continuous angle measurements, the steady-state covariance obeys

$$dP/dt = FP + PF^T + Q - PH^T(R\Delta t)^{-1}HP = 0\,, \tag{11}$$

with $H = [1\ 0]$, $R = \sigma_n^2$, and $Q = \mathrm{diag}([\sigma_v^2, \sigma_u^2])$. The solution, found without too much effort, is

$$P_{\theta b} = -\sigma_u \sigma_n (\Delta t)^{1/2},\ P_{\theta\theta} = \sigma_n [(\sigma_v^2 - 2P_{\theta b})\Delta t]^{1/2} \approx \sigma_v \sigma_n (\Delta t)^{1/2},\ \text{and}\ P_{bb} = \sigma_u (\sigma_v^2 - 2P_{\theta b})^{1/2} \approx \sigma_v \sigma_u . \quad (12)$$

The approximate form of $P_{\theta\theta}$ is basically the same as Eq. (7).

The corresponding equations for discrete measurements lead to a quartic equation that is not so easy to analyze. Amazingly enough, Robert L. Farrenkopf found a factorization of this quartic into two quadratic equations, which led to an incredibly useful estimate of the accuracy of attitude estimation systems. His result was published in a note in the July-August 1978 issue of the *Journal of Guidance and Control*, right at the time I was moving from CSC to NRL [16].

I decided to try to generalize Farrenkopf's result to the three-axis case, starting with the quaternion equivalent of Eq. (10). It soon became apparent that no closed-form steady-state solution would be forthcoming, but I persisted in developing the general equations. After much analysis, I arrived at a formulation that looked familiar; it was identical to the MACS filter developed by Murrell. Greatly disappointed, I sent my analysis off to Gene Lefferts at GSFC, asking if he thought it had any value at all. He said that Shuster, who had taken over my task after I left CSC, had developed a parallel analysis, and that he thought that our combined work was worth publishing. The resulting paper is the most cited paper that Malcolm Shuster or I ever wrote [17]. The final version was written on weekends at Gene's house. He provided the vector measurement model, the only real innovation in the paper, and also served as a buffer to prevent Malcolm and me from coming to blows over disagreements in presentation.

Autonomous Navigation

My principal research activity at NRL was a covariance analysis of autonomous navigation using landmarks. This coupled attitude/orbit problem used some of the expertise I had developed with Velez, which had led to my being hired by NRL. The attitude and orbit errors turn out to be highly correlated, and it is not surprising that they tend to cancel when geolocating unknown objects on the Earth. My first paper on this subject was presented at the AAS Astrodynamics Conference at Lake Tahoe [18]. Connie Carrington, then a graduate student of John Junkins, presented a very nice paper on magnetic momentum unloading at the same conference. When I introduced myself and complimented her on her paper, she said that a paper Peter Camillo and I had written on our momentum unloading analysis for SMM had been very useful to her, which greatly pleased me [19].

Further work on autonomous navigation augmented the landmark measurements with measurements of the line-of-sight (LOS) between two spacecraft in a formation. Remarkably, I found that the orbits of both spacecraft could be obtained using only measurements of the magnitude and inertial direction of the LOS between the spacecraft, except in a few highly symmetric situations [20]. This result was rediscovered independently by Mark Psiaki at Cornell [21].

As part of this covariance analysis, I developed a convenient form for the orbit state transition matrix expressed in Cartesian coordinates [22]. Bob Dasenbrock later told me that this algorithm was used in the onboard software of at least one NRL spacecraft.

Although the research atmosphere at NRL was very congenial, I missed the connection with real spacecraft that I had experienced in my work on SMM. In 1985, Gene Lefferts announced his retirement. Jerry Teles, his supervisor in the Flight Dynamics Analysis Branch, asked if there was a replacement who could supply the capabilities that he had provided to GSFC. He said that I could, and the position was offered to me. It was impossible for me to refuse, as hard as it was to leave Bernie Kaufman and NRL. Once again, I owed a career move to Gene Lefferts.

GODDARD SPACE FLIGHT CENTER

Attitude Determination Error Analysis System (ADEAS)

My first really interesting project after moving to GSFC was developing the Attitude Determination Error Analysis System (ADEAS). This was an attitude error covariance analysis program facilitating the analysis of the accuracies attainable with different sensor complements, different observation schedules, and different estimation methods, including mistuned estimators. It was developed for a mainframe computer, but has since been ported to the desktop computer environment and is still widely used at GSFC. One of my colleagues in this effort was Ed Seidewitz, a brilliant recent MIT graduate. Ed and I worked through the covariance analysis equations for both batch estimators and Kalman filters with solve-for and consider parameters. One of the innovations of ADEAS was its ability to assess the influence of process noise in a batch estimator. Mark Nicholson, the best FORTRAN programmer I have ever known, did most of the programming. He also uncovered errors in the equations Ed and I gave him to code. I believe that much has been lost in the increasing separation of the functions of computer scientist and control engineer. The results of Ed's, Mark's, and my seamless collaboration were documented in the best conference paper that I never published in an archival journal [23]. One interesting discovery of our ADEAS work was that the attitude of Goddard's Cosmic Background Explorer (COBE) spacecraft could be much more simply represented by 3-1-3 Euler angles than by any other representation of the attitude, weakening my aversion to this parameterization.

Wahba's Problem

In 1965, Grace Wahba had posed the problem of finding the proper orthogonal matrix that minimized a certain loss function [24]. This problem was quickly solved [25], but Paul Davenport provided the first really useful solution [12]. He rewrote Wahba's original loss function in quaternion form as

$$L \equiv \tfrac{1}{2}\Sigma_i a_i \left|\mathbf{b}_i - A\mathbf{r}_i\right|^2 = \lambda_0 - \mathrm{trace}(AB^T) = \lambda_0 - q^T K q, \tag{13a}$$

where $\mathbf{r}_i$ is a vector to an observed object coordinatized in a reference frame, $\mathbf{b}_i$ is the corresponding vector coordinatized in the spacecraft body frame , and a_i is the non-negative weight assigned to this observation; and where

$$\lambda_0 \equiv \Sigma_i a_i, \quad B \equiv \Sigma_i a_i \mathbf{b}_i \mathbf{r}_i^T, \quad \text{and} \quad K \equiv \begin{bmatrix} B + B^T - I_{3\times3}\,\mathrm{trace}B & \Sigma_i a_i \mathbf{b}_i \times \mathbf{r}_i \\ \left(\Sigma_i a_i \mathbf{b}_i \times \mathbf{r}_i\right)^T & \mathrm{trace}B \end{bmatrix} \tag{13b}$$

It follows that the optimal quaternion is the eigenvector of K with the maximum eigenvalue:

$$K q_{\mathrm{opt}} = \lambda_{\max} q_{\mathrm{opt}}. \tag{14}$$

This is Davenport's q method, which is greatly appealing to physicists who love to work with eigenvectors and eigenvalues. Although very robust, Davenport's q method is also very slow, and Shuster's QUEST has been much more widely applied.

I was aware of the Singular Value Decomposition (SVD) algorithm, but it was not until Malcolm Shuster applied it to a sensor calibration problem that it occurred to me to use this technique to solve Wahba's Problem [26]. The SVD of the matrix B in Eq. (13b) is given as

$$B = USV^T, \tag{15a}$$

where S is diagonal and U and V are orthogonal. It was easy to show that the optimal attitude matrix is

$$A_{\mathrm{opt}} = U\,\mathrm{diag}([1 \quad 1 \quad \det U \det V])\,V^T. \tag{15b}$$

The SVD algorithm is as robust as Davenport's q method, but even slower. It has provided useful analytical insights into the solution of Wahba's problem, however.

One drawback of all these solutions to Wahba's Problem is that they estimate the attitude only, unlike more general estimators that can simultaneously solve for other parameters such as gyro drifts. I attempted to remedy this by considering the body and reference frame components of the observations to be functions of some parameters, and using a perturbation expansion in these parameters. The application of this theory did not live up to my hopes, but the papers contain some interesting algebraic identities that Ed Seidewitz and I discovered along the way [27, 28]. It was a great loss for the aerospace community when Ed left Goddard to pursue his first love of software engineering.

Although the parameter estimation papers were not successful, they led to the discovery of a fast optimal attitude matrix (FOAM) solution to Wahba's problem [29]:

$$A_{\text{opt}} = \zeta^{-1}[(\kappa + \|B\|_F^2)B + \lambda_{\max}\,\text{adj}B^T - BB^TB], \tag{16a}$$

where $\|\cdot\|_F$ denotes the Frobenius norm, $\text{adj}(\cdot)$ denotes the classical adjoint,

$$\kappa \equiv \tfrac{1}{2}(\lambda_{\max}^2 - \|B\|_F^2), \quad \text{and} \quad \zeta \equiv \kappa\lambda_{\max} - \det B. \tag{16b}$$

As 1990 approached, Malcom Shuster proposed that we write a survey paper entitled "25 Years of Wahba's Problem." We could never find the time for this, but in 1995 I gave a lunchtime talk called, unsurprisingly, "30 Years of Wahba's Problem." This talk came to the attention of Daniele Mortari, who had invented several new solutions to the problem. He proposed that we write a survey presenting the different methods and comparing them for speed and accuracy. In preparing the paper, I developed first-order perturbation forms of Daniele's ESOQ and ESOQ2 algorithms. We found that ESOQ and ESOQ2 are the fastest of currently known methods, although not significantly faster than the better-known QUEST algorithm [30].

Orbit Analysis

About the time I went from NRL to Goddard, the radical idea was proposed that every engineer should have a computer on his or her own desk. Among the applications proposed to sell this idea to management was a simple mission analysis program to plot ground tracks of near-Earth satellites. This used a simple analytic orbit propagator incorporating the secular effects of the Earth's oblateness. Comparison with a more precise propagator showed an in-track error increasing linearly with time. Remembering something Bob Dasenbrock had told me about semianalytic propagators, I gave him a call. He said that the errors arose from the difference between the mean and osculating semimajor axis, leading to an incorrect value for the mean motion. I added a linear mean-to-osculating transformation and its exact nonlinear inverse to the propagator, giving a zeroth-order extended-time algorithm. This fixed the problem, and the propagator was incorporated by Jim Jeletic into the Mission Planning Graphics Tool (MPGT) [31], and was also used to compute orbits for a wall display at the Smithsonian Air and Space Museum.

An opportunity arose in 1989 to transfer to Henry Hoffman's Guidance and Control Branch. Remembering my work on SMM as a high point of my career, I seized this opportunity to work with Hoffman, Donohue, and Flatley, and to get closer to the actual design of attitude control systems.

Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX)

The first spacecraft I worked on I after my transfer was SAMPEX, the first of the Small Explorer (SMEX) series of spacecraft. Its attitude control system had been largely defined by Tom Flatley before I entered the picture [32]. It had a single reaction wheel providing an angular momentum bias along the spacecraft's Sun-pointing y axis. Reaction wheel torques controlled the pointing of the z axis science instruments. Magnetic torquers were used to damp nutation and keep the y axis pointed at the Sun. The attitude was sensed by a three-axis magnetometer and a two-axis digital Sun sensor with a $\pm$ 64° field of view centered on the y axis and with 0.5° resolution. The TRIAD algorithm, or vector method [12], was used to find the attitude matrix A. This required the spacecraft to carry onboard spacecraft and Sun ephemerides and a magnetic field model. The spacecraft ephemeris was a simple integration of $\mathbf{F} = m\mathbf{a}$, with a geopotential incorporating J_2, J_3, J_4, S_{22}, and C_{22} corrections and a drag correction using a Jacchia-Roberts density model, reinitialized periodically with uplinked Cartesian orbit vectors. My innovation was adding S_{22}, C_{22} and the Jacchia-Roberts model. The propagator exhibited in-track errors that were negligible for SAMPEX, but became problematic for more precise applications. Their cause and cure were not discovered until much later by Mark Beckman [33]. The cure is related to the solution of the errors in the MPGT propagator pointed out to me earlier by Bob Dasenbrock; the orbit must be put on the correct energy shell by initializing the propagation at a point where the approximate onboard gravitational potential is numerically close to the more exact potential used to compute the initial Cartesian orbit elements.

The spacecraft angular velocity was found by differentiating the attitude matrix

$$[\boldsymbol{\omega}\times] = -\dot{A}A^T, \tag{17}$$

where $[\cdot\times]$ denotes the cross-product matrix. Then the total system angular momentum was given by

$$\mathbf{H} = I\boldsymbol{\omega} + H_{\text{wheel}}\mathbf{j}, \tag{18}$$

The magnetic control maintained the angular momentum with a magnitude of H_0 pointed simultaneously along the spacecraft y axis and at the Sun. An angular momentum error was computed as

$$\Delta\mathbf{H} = (\mathbf{H} - H_0\mathbf{j}) + (\mathbf{H} - H_0\mathbf{s}), \tag{19}$$

where $\mathbf{s}$ is the Sun unit vector in the body frame. This was driven toward zero by a magnetic dipole

$$\mathbf{m} = k_{\text{mag}}(\Delta\mathbf{H} \times \mathbf{B}), \tag{20}$$

where $\mathbf{B}$ is the magnetic field in the body frame.

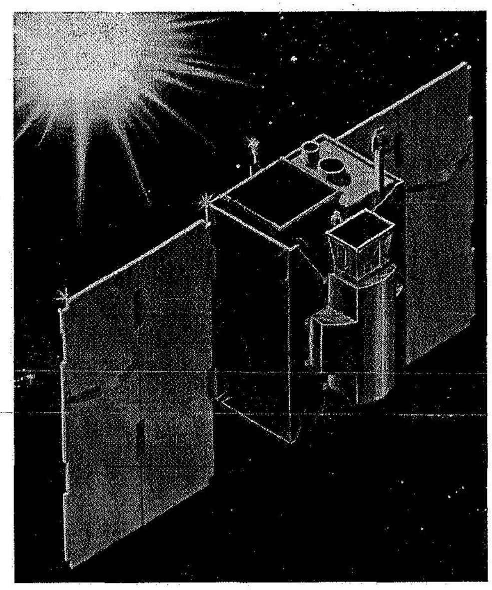

Figure 3. SAMPEX

This simple control law worked well in simulations until the finite resolution of the digital Sun sensor was modeled. When this resolution was included, the TRIAD attitude solutions had discrete jumps, and the extremely noisy angular rates computed by Eq. (17) destabilized the attitude control. My solution was to add a simple constant-gain Kalman filter [34]

$$\mathbf{H} = (1-K)\mathbf{H}_{\text{predicted}} + K\mathbf{H}_{\text{derived}}, \tag{21a}$$

where K is the filter gain, $\mathbf{H}_{\text{derived}}$ is given by Eq. (18) and

$$\mathbf{H}_{\text{predicted}}(t) = A(t)A^T(t-\Delta t)\mathbf{H}(t-\Delta t) + (\mathbf{m}\times\mathbf{B})\Delta t. \tag{21b}$$

SAMPEX was launched on July 3, 1992 with a lifetime requirement of two years and a goal of three years. It is still producing valuable scientific data, and its z axis pointing law has been reprogrammed several times to meet changing observational goals [35].

Geostationary Operational Environmental Satellite (GOES)

In 1990 NASA and NOAA, the National Oceanic and Atmospheric Administration, began a study of an improved version of the Geostationary Operational Environmental satellite (GOES), which was itself an improvement on the SMS satellite that I had studied for Mel Velez many years earlier. Frank Bauer headed up the ACS portion of this GOES-Next study. My contribution was to develop an attitude determination concept to provide the required arcsecond pointing accuracy. The GOES-I/M spacecraft had pushed the accuracy attainable with Earth horizon sensors about as far as it seemed possible to go. The obvious (to me) solution was a stellar-inertial system similar to that developed by Jim Murrell for MMS. The major effort was to find orientations for the star trackers that would protect them from Sun impingement and to use both Farrenkopf's analytic equations and numerical simulations to show that the required pointing accuracy could be attained. The idea of a Landsat ACS in geostationary orbit seemed obvious to me, but one reviewer for the *Journal of Guidance, Control, and Dynamics*, where the concept was published, found it very innovative [36]. It formed the basis for the winning proposal for the GOES-N/P spacecraft.

Hubble Space Telescope (HST)

HST was released by STS-31 in April 1990, less than a year after I joined Henry Hoffman's branch. Before my transfer, I had worked with Paul Davenport on some of the optical calibration algorithms being developed for HST. This was the only time I worked with Paul, whose reluctance to publish prevented him from achieving the wider recognition he deserved.

HST exhibited two problems almost immediately after launch. The most famous is the aberration of primary mirror, which was not a control problem. The second problem was the appearance of attitude disturbances that were most severe at entry and exit from the Earth's shadow, but that persisted throughout the daylight portion of the orbit. Henry immediately attributed these disturbances to the bistem booms used in the solar arrays, which had a tendency to twist as shown in Figure 5. Henry, Tom Flatley, and Jim Donohue immediately set to work to develop a disturbance-attenuating control algorithm for HST. Engineers at Marshall Space Flight Center and at Lockheed-Martin, the HST prime contractor, also attacked the problem. This work led to the Solar Array Gain Augmentation (SAGA) algorithm that HST employed until the solar arrays were replaced with rigid arrays in 2002. I was largely an admiring spectator of this development, but I did play a role in the development of a second-generation SAGA-II algorithm.

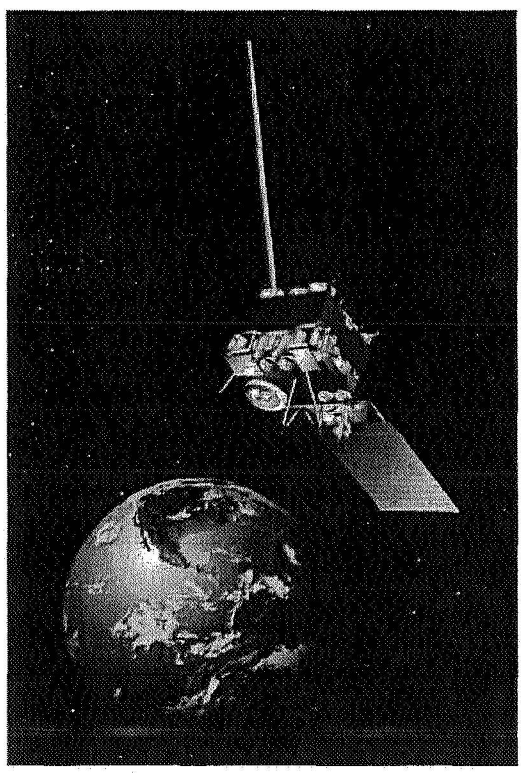

Figure 4. GOES-N

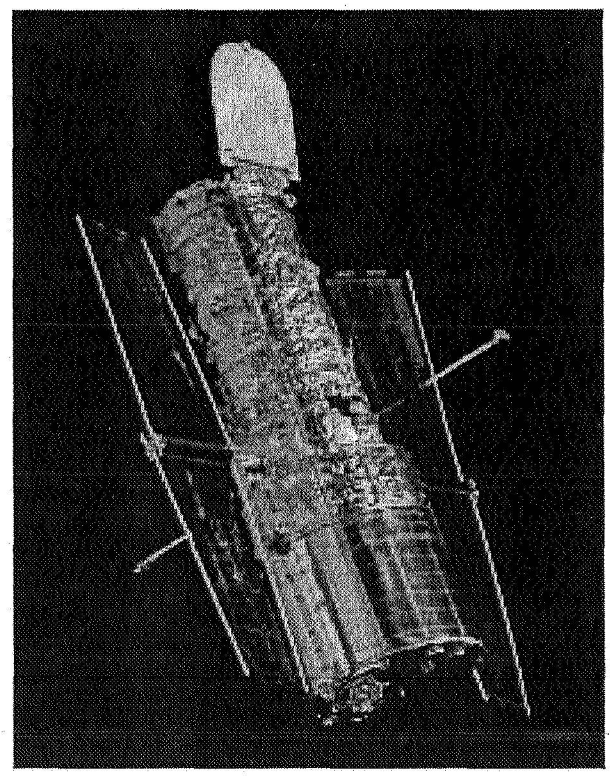

Figure 5. Hubble Space Telescope

The next problem to surface on HST was the failure of two of its six precise air-bearing gyros. Since HST needs three gyros for control, failure of two more gyros would have actuated an undesirable firmware controller using limited-life ball-bearing gyros, so the HST Project decided in 1991 to develop a software safe pointing mode requiring fewer than three gyros. As Goddard's lead engineer for this effort, I worked with a contractor team headed by John Nelson of Lockheed-Martin. Their preliminary study had shown that a zero-gyro safe pointing mode could be implemented, and I quickly agreed that this was preferable to a two-gyro or one-gyro mode. We developed the Zero-Gyro Sunpoint (ZGSP) mode from initial concept to flight readiness in four months [37]. This mode points a preferred axis toward the Sun during the daylight part of the orbit and uses a momentum bias in the reaction wheels along this axis to hold attitude during eclipse. This is similar to the science mode of SAMPEX, but ZGSP does not rely on an onboard ephemeris or magnetic field model, so TRIAD is out of the question. Instead, the ZGSP mode controls rotation around the Sun line by locking it to the Earth's magnetic field. This has the unfortunate property that for some orientations of the orbit plane, magnetic equator, and Sun line, the apparent rotation of the inertial magnetic field as HST orbits can cause the body angular momentum to cancel a significant fraction of the bias momentum. The ZGSP mode has exhibited some large attitude excursions arising from this effect during eclipse, but it maintained HST in a power and thermally safe state for 38 consecutive days in November and December of 1999 after the failure of four gyros and until the replacement of all six gyros.

It warmed my physics-trained heart that Henry Hoffman always emphasized the importance of the conservation of angular momentum. He was quick to discredit any analysis that reached erroneous conclusions by ignoring this principle, something that I once experienced personally to my great embarrassment. I was delighted to find an application of angular momentum conservation to onboard detection of a spacecraft failure. The HST computer calculates the total system angular momentum as the vector sum of reaction wheel momentum sensed by the wheel tachometers, and spacecraft body momentum sensed by the gyros. The high torque capability of the reaction wheels can cause both of these components to change rapidly, but the total system momentum only changes slowly as a result of environmental disturbance torques. Propulsion torques would violate this assumption, but HST has no propulsion system. Thus a rapid change in system momentum would indicate a failure of either a reaction wheel tachometer or of a gyro. A reaction wheel tachometer failure can be identified on HST by an independent test, a comparison of the reaction wheel momentum change to applied torque, so the system momentum test was implemented to identify gyro failures [38]. It has not identified any failures to date, however.

Research in Attitude Estimation

In his last year at Goddard, Gene Lefferts had a National Research Council Resident Research Associate (NRC RRA), S. Vathsal, whom I inherited after I replaced Gene. Vathsal developed a second-order Kalman filter based on the methods presented in the Lefferts, Markley, and Shuster paper [39]. I have had the privilege and the pleasure of working with several other NRC RRAs, including Itzhack Bar-Itzhack from 1987 to 1989, Nadav Berman from 1992 to 1994, and Yaakov Oshman from 1996 to 1998. Although I was nominally their advisor, I learned more from them than they from me, and our collaborations produced some interesting research that I certainly could not have accomplished without them [40–42].

Working with these NRC RRAs, most especially with Yaakov Oshman, increased my understanding of spacecraft attitude estimation — deepening my knowledge of estimation theory, opening my eyes to a variety of approaches, and challenging my assumptions and approximations. When I finally thought I understood the issues well enough, I wrote an update the Lefferts, Markley, and Shuster paper [43]. Extensive discussions with Russell Carpenter at Goddard sharpened my arguments and significantly improved the clarity of the final paper. Russell also guided me into the mysteries of Daum's approach to estimation [44], which contributed greatly to my most recent work on attitude filtering [45].

I have saved my most rewarding NRC RRA experience until last. John Junkins' student Joe Mook had presented some of their work on Minimum Model Error Estimation at a Goddard Flight Mechanics/Estimation Theory Symposium in 1988 [46]. All of John Junkins' many graduate students that I ran into did interesting work, so I began to regard this as a theorem. I saw Joe periodically at conferences, and we became friends. His reward to me was to send his best graduate student, John Crassidis, to be my NRC RRA for 1994–96. I quickly discovered how creative and prolific John is; during his stay at Goddard he presented more than a dozen conference papers and submitted a half-dozen for publication, with me as a co-author. An example of my contribution was to sketch a method for approximate GPS attitude estimation that John worked out in a week while I was away, along with an alternative method, a complete covariance analysis, and a comparison with the covariance of the optimal estimate [47]. Our collaboration did not end with the termination of John's tenure at Goddard, but continues to this day, much to my delight.

I also returned to Farrenkopf's analysis of the steady-state single-axis attitude estimation errors using gyros and angle measurements. Spacecraft generally use rate-integrating gyros, which have random error on the angle output with standard deviation σ_e that he did not include [12]. I discovered that including this term led to a quartic equation that could be solved exactly like Farrenkopf's. Using the same notation as in Eqs. (10)–(12) and defining

$$S_e \equiv \sigma_e/\sigma_n, \quad S_u \equiv (\Delta t)^{3/2}\sigma_u/\sigma_n, \quad \text{and} \quad S_v \equiv (\Delta t)^{1/2}\sigma_v/\sigma_n, \tag{22}$$

the steady-state errors are

$$P_{\theta b}(-) = -\zeta\sigma_u\sigma_n(\Delta t)^{1/2}, \qquad P_{\theta\theta}(-) = (\zeta^2-1)\sigma_n^2, \qquad P_{\theta\theta}(+) = (1-\zeta^{-2})\sigma_n^2,$$

and
$$P_{bb}(\mp) = \sigma_u[\sigma_v^2 + 2\gamma\sigma_u\sigma_n(\Delta t)^{1/2} + \tfrac{1}{3}\sigma_u^2(\Delta t)^2]^{1/2} \pm \tfrac{1}{2}\sigma_u^2\Delta t, \tag{23}$$

where
$$\zeta = \gamma + \tfrac{1}{4}S_u + \tfrac{1}{2}(2\gamma S_u + S_v^2 + \tfrac{1}{3}S_u^2)^{1/2} \quad \text{and} \quad \gamma \equiv (1 + S_e^2 + \tfrac{1}{4}S_v^2 + \tfrac{1}{48}S_u^2)^{1/2}. \tag{24}$$

These results are identical to Farrenkopf's if $\sigma_e = 0$. I showed my original messy derivation of these equations to Reid Reynolds, and together we found the much more straightforward path to the results that appears in the published version [48].

Tropical Rainfall Measuring Mission (TRMM)

In 1991, GSFC embarked on the simultaneous development of two spacecraft in-house: the Tropical Rainfall Measuring Mission (TRMM) and the X-Ray Timing Explorer (XTE, later renamed RXTE in honor of Bruno Rossi). The two spacecraft used a common procurement for reaction wheels and gyros, but their attitude control systems were quite different. XTE chose a stellar-inertial reference designed by Mike Femiano based on Murrell's MACS design to satisfy its arcsecond pointing requirements [49], while TRMM chose an Earth-referenced system to satisfy its less stringent 0.3° (3σ) attitude knowledge requirement. TRMM employed a static Earth horizon sensor similar to that used on the NOAA low-Earth-orbiting spacecraft and on the Defense Meteorological Satellite Program (DMSP) spacecraft. In early 1994, after development of the TRMM ACS was well underway, we became aware of a progressive loss of optical transmission through of some of the windows of the horizon sensors on the DMSP spacecraft. We worried that TRMM, with its lower 350 km altitude orbit, might experience a more severe problem, and possibly even complete loss of the attitude reference. To protect against this possibility, we considered adding a star tracker to TRMM, which would have incurred severe monetary and schedule penalties.

Recalling that Joe Hashmall at CSC had found some promising results on batch attitude determination using magnetometers and gyros, I sought more information from Eleanor Ketchum of the Flight Dynamics Analysis Branch, who was sponsoring his work. They and Joe Sedlak responded enthusiastically, and within a month had developed a Kalman filter that gave 0.3° attitude accuracy using real data from the Upper Atmosphere Research Satellite (UARS) [50]. The common procurement of XTE and TRMM gyros permitted this filter to be used on TRMM, since the gyros were much more accurate than required by the Earth-sensor-based TRMM ACS but provided the dynamics memory of approximately one orbit period required by the filter to reduce the effects of magnetic field modeling errors.

John Crassidis, Kong Ha, and I studied alternative Kalman filter implementations using magnetometer, Sun sensor, and gyro data [51]. Although various simpler algorithms were promising, the TRMM ACS group decided to adapt the XTE Kalman filter for this purpose; and this was accomplished under the direction of Steve Andrews. In 2001, it was realized that TRMM had insufficient propellant to continue compensating for atmospheric drag in its 350 km orbit, so the orbit was raised to 402 km. The static Earth sensor that had performed according to specification at the 350 km altitude ceased providing valid data above 380 km, so the Kalman filter was enabled. After some retuning, the filter is providing attitude accuracies of approximately 0.2°, about the same as the horizon sensor had provided [52].

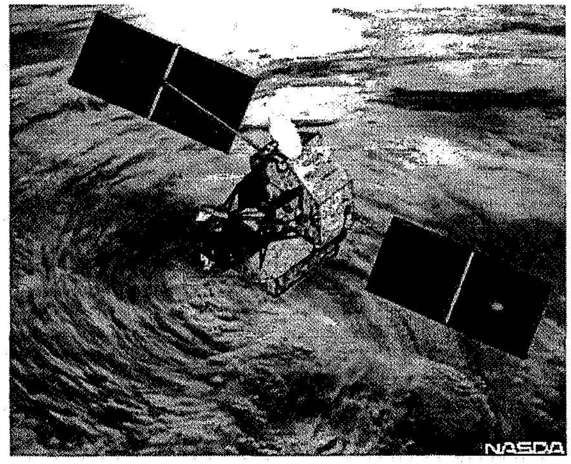

Figure 6. TRMM

Wilkinson Microwave Anisotropy Probe (WMAP)

The MAP proposal effort began in late 1994, anticipating a Mid-sized Explorer (MIDEX) Announcement of Opportunity in early 1995. The Principal Investigator (PI) was Chuck Bennett of Goddard, who had been deputy PI on the Differential Microwave Radiometer (DMR) instrument on COBE. The MAP proposal was a Goddard-Princeton collaboration to repeat the DMR measurement of the cosmic microwave background (CMB) with increased resolution and precision. To that end, MAP was to be launched to the Earth-Sun L2 libration point to avoid the Earth's atmosphere, radiation, and magnetic field. The Princeton team included David Wilkinson, the dean of CMB studies, in whose honor MAP was rechristened WMAP in 2002. It was a great privilege to participate in planning meetings with him. Cliff Jackson, who headed the Goddard engineering team and later became the MAP Instrument Systems Engineer, deserves a lot of credit for the success of the proposal.

The scientists wanted to obtain a highly interconnected set of measurements over an annulus between 45° and 90° from the anti-Sun line, requiring MAP to execute a fast spin at about 0.5 rpm and a slow precession of its spin axis at one revolution per hour at a constant angle of 22.5° from the Sun line. I realized that this scan pattern could be easily represented by 3-1-3 Euler angles and could be accomplished by a zero-momentum ACS controlled by reaction wheels, just as for COBE. John Crassidis took the Euler angle description and used MATLAB to produce the "spirograph" shown in Figure 7. The science team were delighted that we could produce this pattern; and a modified version of Figure 7 was included in the proposal. Tom Flatley tried without success to find a dual-spin dynamical configuration that could provide this scan pattern; and I believe that if he couldn't, nobody could. MIDEXs were to be single-string spacecraft, so the MAP ACS concept had only three reaction wheels to serve the dual function of counterbalancing the body's spin angular momentum to maintain zero system momentum and applying control torques to provide the desired attitude. The wheel axis orientations were chosen to bias all wheel speeds away from zero to avoid undesirable zero-speed crossings.

Attitude sensing was by a star tracker and a digital Sun sensor, despite the preference of another Goddard engineer (not in Hoffman's branch) to use an Earth sensor in place of a star tracker. My GOES and TRMM experience had left me with a prejudice against Earth sensors, and the Earth/Sun separation of less than 10° as viewed from the MAP orbit at L2 would have led to marginal attitude determination accuracy. I was relieved to find two star tracker manufacturers who could provide a tracker to acquire and track stars at 3 deg/sec, as required by MAP. Ironically, the star tracker contract was eventually awarded to a third vendor.

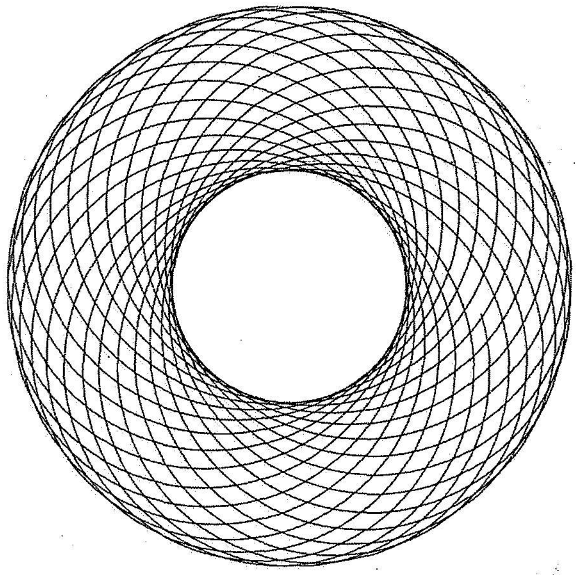

Figure 7. MAP Scan Pattern

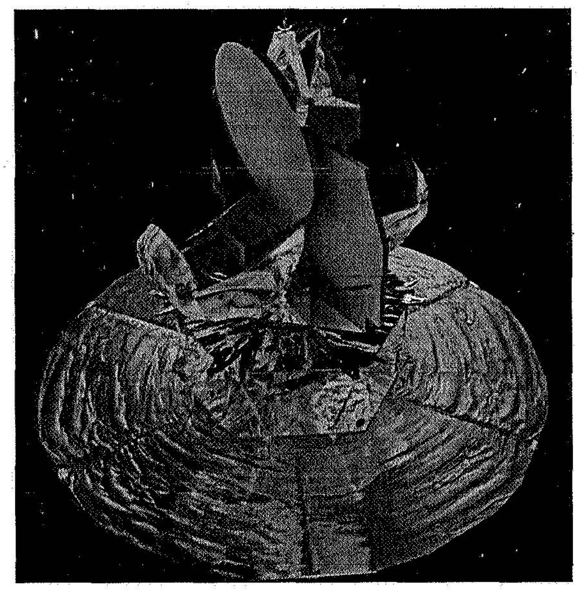

Figure 8. MAP

I had laid out this basic ACS concept before the end of 1994, and John Crassidis developed a simple MATLAB simulation to confirm its viability [53]. MAP was selected in 1996 to be one of the first two MIDEX missions. The ACS was developed by a MIDEX ACS group already in place at Goddard, headed by David Ward. A two-wheel backup mode was later developed to provide reduced performance in the unlikely event of a reaction wheel failure, which has not occurred [54].

MAP was launched in 2001, and its data have enabled scientists to determine the values of key cosmological parameters and to answer questions about the origin of structure in the early universe and the fate of the universe.

SUMMARY

My career did not end with MAP's launch in 2001, although I still see MAP as its high point. I could mention Goddard's Swift spacecraft, a MIDEX mission launched in 2004, that uses six reaction wheels to slew 60° in 60 sec to catch elusive short-lived gamma-ray bursts on the fly. I continue to follow HST, which is currently observing in two-gyro fine-pointing mode, saving the remaining two operable gyros until a new servicing mission can replace all six gyros, and with a one-gyro science mode under development. I am anxiously awaiting the launch of GOES-N, held up since last winter by a variety of difficulties, most recently a labor dispute. I have largely been an interested observer and friendly critic of these developments, however. The torch has been passed to a new generation.

My aerospace career has been extremely satisfying, but I will especially cherish the Dirk Brouwer Award because of the high regard in which I hold you, my colleagues in the AAS. Your opinion means the world to me. I look back on my career as having been largely fortuitous; a matter of finding myself in the right place at the right time. It has of course also required me to make the best use of my opportunities when I found them.

My greatest reward has been the opportunity it has given me to work with incredibly talented colleagues. Many of the people named in this paper are well known to you, including three previous recipients of the Dirk Brouwer Award, while others have achieved only more limited reputations. I hope that my talk will serve to rescue some of these valued colleagues from undeserved obscurity.

ACKNOWLEDGEMENTS

I've tried in this paper to recognize those who have been important to me during my aerospace career. I'd also like to acknowledge some crucial earlier influences: my father, Francis L. Markley, my grandfather, Harry J. Tschan, my eighth grade algebra teacher, Mary Huzzard, my freshman English composition teacher, Joseph A. Mazzeo, who taught me to write, and David Park at Williams College. Many others have been omitted for lack of space; to all of you, I offer my sincere apologies.

REFERENCES

[1] Goldstein, H., *Classical Mechanics*, 2nd ed., Reading, MA, Addison-Wesley, 1980.

[2] Morton, H. S., Jr., Junkins, J. L., and Blanton, J. N., "Analytical Solutions for Euler Parameters," *Celestial Mechanics*, Vol. 10, November 1974, pp. 287–301.

[3] Kraige, L. G., and Junkins, J. L., "Perturbation Formulations for Satellite Attitude Dynamics," *Celestial Mechanics*, Vol. 13, February 1976, pp. 39–64.

[4] Markley, F. L., "New Dynamic Variables for Momentum-Bias Spacecraft," *Journal of the Astronautical Sciences*, Vol. 41, No. 4, 1993, pp. 557–567.

[5] Sedlak, J. E., "Spinning Spacecraft Attitude Estimation Using Markley Variables: Filter Implementation and Results," 2005 Flight Mechanics Symposium, Goddard Space Flight Center, Greenbelt, MD, October 1995, NASA Conference Publication NASA/CP-2005-212789.

[6] Gelb, A., ed., *Applied Optimal Estimation*, Cambridge, MA, the MIT Press, 1974.

[7] Lefferts, E. J., and Markley, F. L., "Dynamic Modeling for Attitude Determination," AIAA Paper 76–1910, AIAA Guidance and Control Conference, San Diego, CA, August 1976.

[8] Murrell, J. W., "Precision Attitude Determination for Multimission Spacecraft," AIAA Paper 78-1248, AIAA Guidance and Control Conference, Palo Alto, CA, August 1978, pp. 70–87.

[9] Di Stefano, J. J., III, Stebberud, A. R., and Williams, I. J., *Schaum's Outline of Theory and Problems of Feedback and Control Systems*, New York, NY, McGraw-Hill, 1967.

[10] Markley, F. L., "Attitude Control Algorithms for the Solar Maximum Mission," AIAA Paper 78-1247, AIAA Guidance and Control Conference, Palo Alto, CA, August 1978, pp. 59–69.

[11] Kuznik, F., "Satellite Saviors," *AIR & SPACE/Smithsonian*, Vol. 6, No. 3, 1991, pp. 66–70.

[12] Wertz, J. R., ed., *Spacecraft Attitude Determination and Control*, D. Reidel, Dordrecht, Holland, 1978.

[13] Shuster, M. D., "Approximate Algorithms for Fast Optimal Attitude Computation," AIAA Paper 78-1249, AIAA Guidance and Control Conference, Palo Alto, CA, August 1978, pp. 88–95.

[14] Shuster, M. D., "In Quest of Better Attitudes: Plenary Lecture," in *Spaceflight Mechanics 2001, Advances in the Astronautical Sciences, Vol.108, Part II*, ed. by Louis A. D'Amario, Lester L. Sackett, Daniel J. Scheeres, and Bobby G. Williams, San Diego, CA: Univelt, Inc. 2001, pp. 2089–2117.

[15] Markley, F. L., "Large Angle Maneuver Strategies for Flexible Spacecraft," in *Astrodynamics 1979, Advances in the Astronautical Sciences, Vol. 40, Part II*, ed. by Paul Penzo, Bernard Kaufman, Louis Friedman, and Richard Battin, San Diego, CA: Univelt, Inc. 1980, pp. 625–647.

[16] Farrenkopf, R.L., "Analytic Steady-State Accuracy Solutions for Two Common Spacecraft Attitude Estimators," *Journal of Guidance and Control*, Vol. 1, No. 4, 1978, pp. 282–284.

[17] Lefferts, E. J., Markley, F. L., and Shuster, M. D., "Kalman Filtering for Spacecraft Attitude Estimation," *Journal of Guidance, Control, and Dynamics*, Vol. 5, No. 5, 1982, pp. 417–429.

[18] Markley, F. L., "Autonomous Satellite Navigation Using Landmarks," in *Astrodynamics 1981, Advances in the Astronautical Sciences, Vol. 46, Part II*, ed. by Alan L. Friedlander, Paul J. Cefola, Bernard Kaufman, Walt Williamson, and G. T. Tsang, San Diego, CA: Univelt, Inc. 1982.

[19] Camillo, P. J., and Markley, F. L., "Orbit-Averaged Behavior of Magnetic Control Laws for Momentum Unloading," *Journal of Guidance and Control*, Vol. 3, No. 6, 1980, pp. 563–568.

[20] Markley, F. L., "Autonomous Navigation Using Landmark and Intersatellite Data," AIAA Paper 84-1987, AIAA/AAS Astrodynamics Conference, Seattle, WA, August 1984.

[21] Psiaki, M. L., "Autonomous Orbit determination for Two Spacecraft from Relative Position Measurements," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 305–312.

[22] Markley, F. L., "Approximate Cartesian State Transition Matrix," *Journal of the Astronautical Sciences*, Vol. 34, No. 2, 1986, pp. 161–169.

[23] Markley, F. L., Seidewitz, E., and Deutschmann, J., "Attitude Determination Error Analysis: General Model and Specific Application," *Mécanique Spatiale: Space Dynamics*, Cepadues-Editions, Toulouse, France, 1989, pp. 251–266.

[24] Wahba, G., "A Least Squares Estimate of Spacecraft Attitude," *SIAM Review*, Vol. 7, No. 3, July 1965, p. 409.

[25] Farrell, J. L. and Stuelpnagel, J. C., Wessner, R. H., Velman, J. R., and Brock, J. E. "A Least Squares Estimate of Spacecraft Attitude," *SIAM Review*, Vol. 8, No. 3, July 1966, pp. 384-386.

[26] Markley, F. L., "Attitude Determination Using Vector Observations and the Singular Value Decomposition," *Journal of the Astronautical Sciences*, Vol. 36, No. 3, 1988, pp. 245–258.

[27] Markley, F. L., "Attitude Determination and Parameter Estimation Using Vector Observations: Theory," *Journal of the Astronautical Sciences*, Vol. 37, No. 2, 1989, pp. 41–58.

[28] Markley, F. L., "Attitude Determination and Parameter Estimation Using Vector Observations: Application," *Journal of the Astronautical Sciences*, Vol. 39, No. 3, 1991, pp. 367–381.

[29] Markley, F. L., "Attitude Determination Using Vector Observations: a Fast Optimal Matrix Algorithm," *Journal of the Astronautical Sciences*, Vol. 41, No. 2, 1993, pp. 261–280.

[30] Markley, F. L., and Mortari, D., "Quaternion Attitude Estimation Using Vector Observations," *Journal of the Astronautical Sciences*, Vol. 48, No. 2/3, 2000, pp. 359–380.

[31] Markley, F. L., and Jeletic, J. F., "A Fast Orbit Propagator for Graphical Display," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 2, 1991, pp. 473–475.

[32] Flatley, T. W., Forden, J. K., Henretty, D. A., Lightsey, E. G., and Markley, F. L., "Onboard Attitude Determination and Control Algorithms for SAMPEX," Flight Mechanics/ Estimation Theory Symposium, Goddard Space Flight Center, Greenbelt, MD, NASA Conference Publication 3102, May 1990, pp. 379–398.

[33] Beckman, M., "Reducing On-Board Computer Propagation Errors Due to Omitted Geopotential Terms by Judicious Selection of Uploaded State Vector," Flight Mechanics/Estimation Theory Symposium, Goddard Space Flight Center, Greenbelt, MD, May 1996, NASA Conference Publication 3333, pp. 375–386.

[34] McCullough, J. D., Flatley, T. W., Henretty, D. A., Markley, F. L., and San, J. K., "Testing of the Onboard Attitude Determination and Control Algorithms for SAMPEX," Flight Mechanics/ Estimation Theory Symposium, Goddard Space Flight Center, Greenbelt, MD, May 1992, NASA Conference Publication 3186, pp. 55–68.

[35] Markley, F. L., Flatley, T. W. and Leoutsakos, T., "SAMPEX Special Pointing Mode," Flight Mechanics/Estimation Theory Symposium, Goddard Space Flight Center, Greenbelt, MD, May 1995, NASA Conference Publication 3299, pp. 201–215.

[36] Markley, F. L., Bauer, F. H., Deily, J. J., and Femiano, M. D., "Attitude Control System Conceptual Design for the GOES-N Spacecraft Series," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 2, 1995, pp. 247–255.

[37] Markley, F. L., and Nelson, J. D., "Zero-Gyro Safemode Controller for the Hubble Space Telescope," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 4, 1994, pp. 815–822.

[38] Markley, F. L., Kennedy, K. R., Nelson, J. D., and Moy, E. W., "Autonomous Spacecraft Gyro Failure Detection Based on Conservation of Angular Momentum," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, 1994, pp. 1385–1387.

[39] Vathsal, S., "Spacecraft Attitude Determination Using a Second-Order Nonlinear Filter," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 6, 1987, pp. 559–566.

[40] Bar-Itzhack, I. Y., and Markley, F. L., "Minimal Parameter Solution of the Orthogonal Matrix Differential Equation," *IEEE Transactions on Automatic Control*, Vol. 35, No. 3, 1990, pp. 314–317.

[41] Markley, F. L., Berman, N., and Shaked, U.,"Deterministic EKF-like Estimator for Spacecraft Attitude Estimation," Paper AIAA-65, *Proceedings of the 1994 American Control Conference*, Baltimore, MD, June-July 1994, pp. 247–251.

[42] Oshman, Y., and Markley, F. L., "Sequential Attitude and Attitude-Rate Estimation Using Integrated-Rate Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 3, 1999, pp. 385–394.

[43] Markley, F. L., "Attitude Error Representations for Kalman Filtering," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 2, 2003, pp. 311–317.

[44] Daum, F. E., "New Exact Nonlinear Filters," Chapter 8 in *Bayesian Analysis of Time Series and Dynamical Models*, ed. By J. C. Spall, Marcel Dekker, New York, 1988.

[45] Markley, F. L., "Attitude Filtering on SO(3)," AAS Paper 05-460, The Malcolm D. Shuster Astronautics Symposium, Grand Island, NY, June 2005.

[46] Mook, D. J., and Junkins, J. L., "Minimum Model Error Estimation for Poorly Modeled Dynamic Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 3, 2003, pp. 256–261.

[47] Crassidis, J. L., and Markley, F. L., "New Algorithm for Attitude Determination Using Global Positioning System Signals," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 5, 1997, pp. 891–896.

[48] Markley, F. L., and Reynolds, R. R., "Analytic Steady-State Accuracy of a Spacecraft Attitude Estimator," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1065–1067.

[49] Bauer, F. H., Femiano, M. D., and Mosier, G. E., "Attitude Control System Conceptual Design for the X-Ray Timing Explorer," AIAA Paper 92-4334, AIAA Guidance, Navigation, and Control Conference, Hilton Head Island, SC, August 1992, pp. 236–250.

[50] Sedlak, J., and Hashmall, J., "Accurate Magnetometer/Gyroscope Attitudes Using a Filter with Correlated Sensor Noise," *Flight Mechanics Symposium 1997*, NASA Conference Publication 3345, Greenbelt, MD, 1976, pp. 83–87.

[51] Crassidis, J. L., Andrews, S. F., Markley, F. L., and Ha, K., "Contingency Designs for Attitude Determination of TRMM," Flight Mechanics/ Estimation Theory Symposium, Goddard Space Flight Center, Greenbelt, MD, May 1995, NASA Conference Publication 3299, pp. 419–433.

[52] Andrews, S. F., and Bilanow, S., "Recent Flight Results of the TRMM Kalman Filter," AIAA Paper 2002-5047, AIAA Guidance, Navigation, and Control Conference, Monterey, CA, August 2002.

[53] Markley, F. L., Andrews, S. F., O'Donnell, J. R., Jr., and Ward, D. K., "The Attitude Control System of the Wilkinson Microwave Anisotropy Probe," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 3, 2005, pp. 385–397.

[54] Starin, S. and O'Donnell, J. R. Jr., "Development of a Two-Wheel Contingency Mode for the MAP Spacecraft," *2002 AIAA Guidance, Navigation and Control Conference*, AIAA Paper 2002-4584, Monterey, CA, 2002.