UWB Tracking Algorithms – AOA and TDOA

Abstract

Ultra-Wideband (UWB) tracking prototype systems are currently under development at NASA Johnson Space Center for various applications on space exploration. For long range applications, a two-cluster Angle of Arrival (AOA) tracking method is employed for implementation of the tracking system; for close-in applications, a Time Difference of Arrival (TDOA) positioning methodology is exploited. Both AOA and TDOA are chosen to utilize the achievable fine time resolution of UWB signals.

This talk presents a brief introduction to AOA and TDOA methodologies. The theoretical analysis of these two algorthms reveal the affecting parameters' impact on the tracking resolution. For the AOA algorithm, simulations show that a tracking resolution less than 0.5% of the range can be achieved with the current achievable time resolution of UWB signals. For the TDOA algorithm used in close-in applications, simulations show that the (sub-inch) high traking resolution is achieved with a chosen tracking baseline configuration. The analytical and simulated results provide insightful guidance for the UWB tracking system design.

UWB Tracking Algorithms
AOA and TDOA

Jianjun (David) Ni*, Melinda Refford

Dickey Arndt, Phong Ngo, Chau Phan, Julia Gross, John Dusl

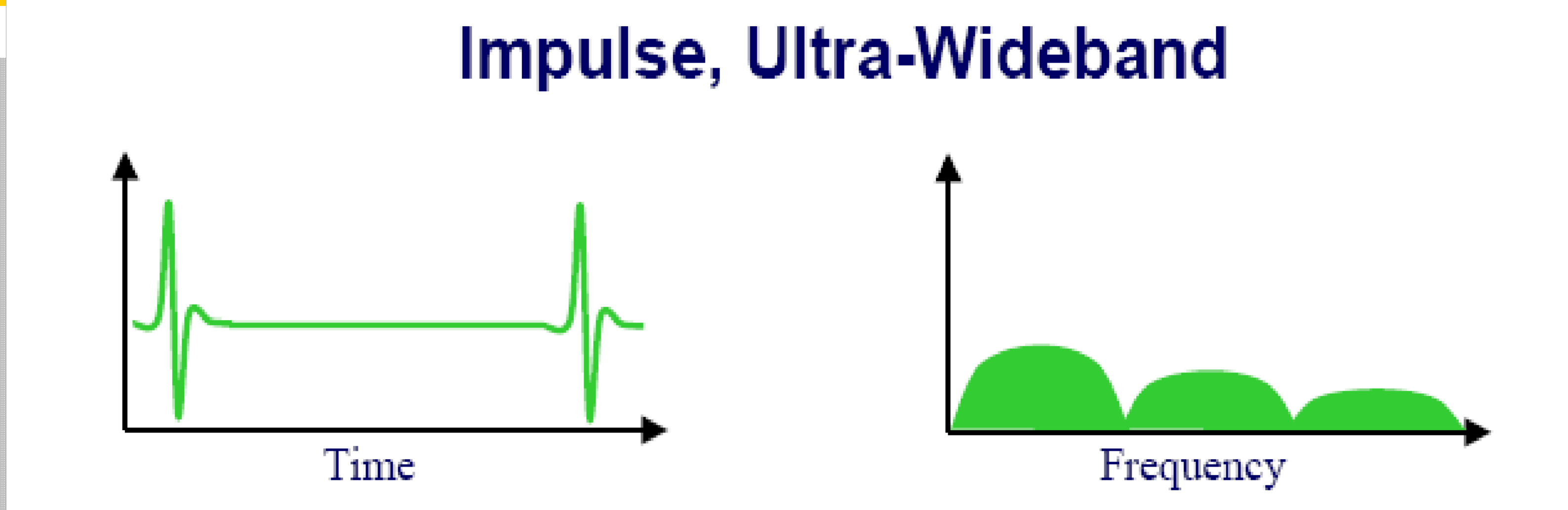
UWB Systems Group / EV4
Avionic Systems Division / EV
NASA Johnson Space Center

This research was performed while the PI held a National Research Council Research Associateship Award at NASA Johnson Space Center.

Outline

- UWB Tracking Algorithm Introduction
- AOA Tracking Methodology
- AOA Resolution Analysis and Simulations
- TDOA Tracking Methodology
- TDOA Resolution Analysis and Simulations
- Conclusion and Future Work

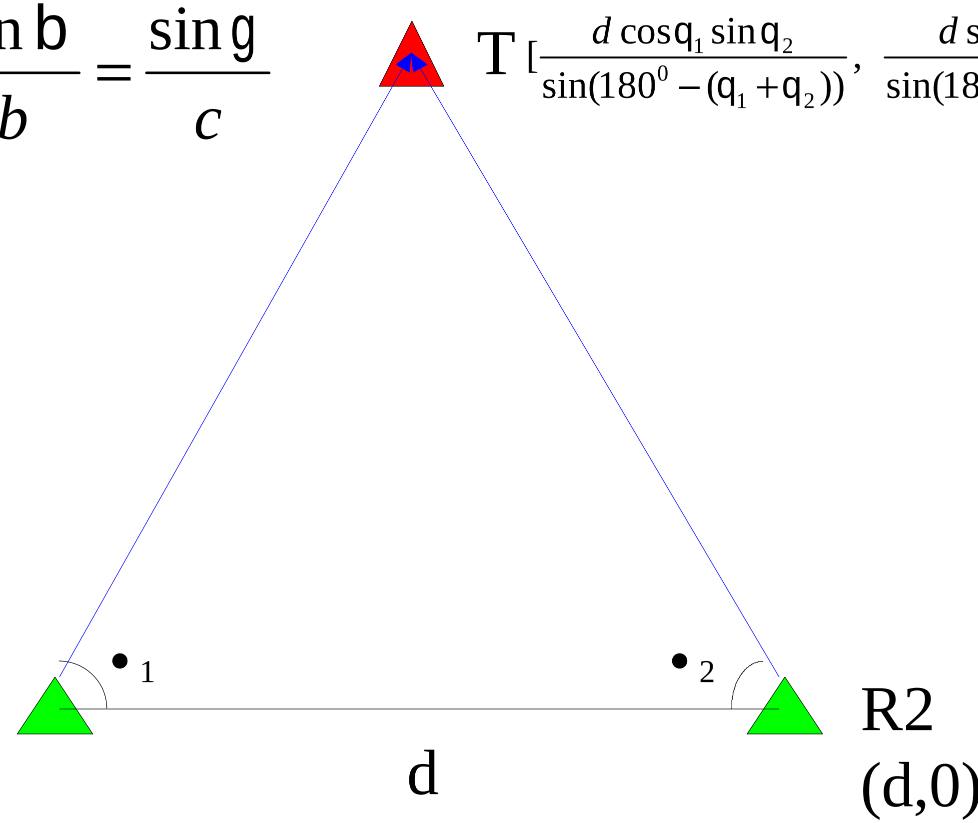
Motivation
(UWB Fine Time Resolution to Precise Tracking)



- 1 ns → 1 foot, 3 ps → 1 mm (ranging, linear)
- Tracking (positioning in 2D/3D) resolution (nonlinear) ?
- Long Range application: AOA (Angle of Arrival)
- Proximity Application: TDOA (Time Difference of Arrival)

Angle of Arrival (AOA)

$$\frac{\sin a}{a} = \frac{\sin b}{b} = \frac{\sin g}{c}$$



$$T\left[\frac{d\cos q_1 \sin q_2}{\sin(180^0 - (q_1 + q_2))}, \frac{d\sin q_1 \sin q_2}{\sin(180^0 - (q_1 + q_2))}\right]$$

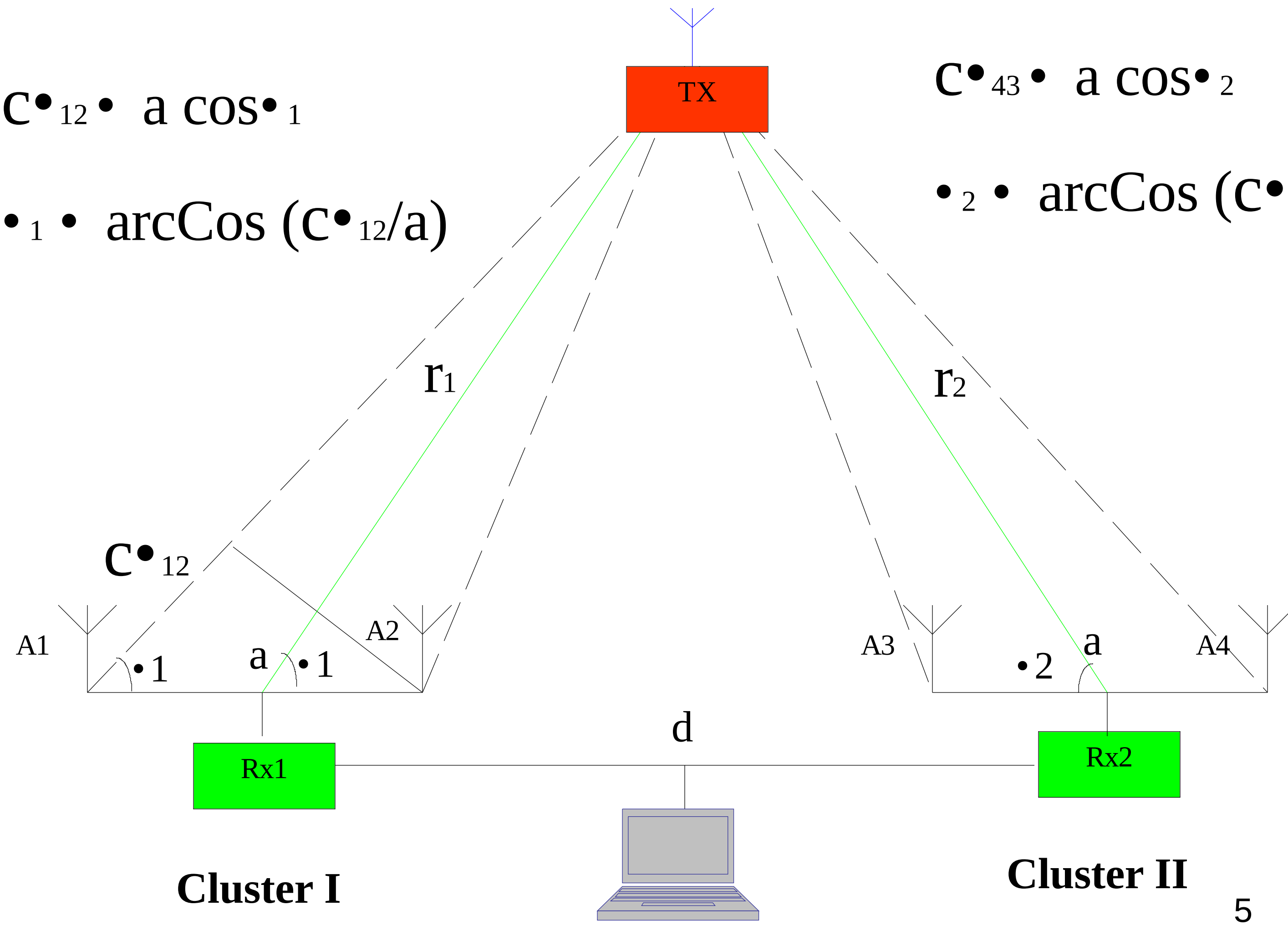
Two Cluster Design

$c\bullet_{12} \bullet$ a $\cos\bullet_{1}$

$\bullet_{1} \bullet$ arcCos $(c\bullet_{12}/a)$

$c\bullet_{43} \bullet$ a $\cos\bullet_{2}$

$\bullet_{2} \bullet$ arcCos $(c\bullet_{43}/a)$



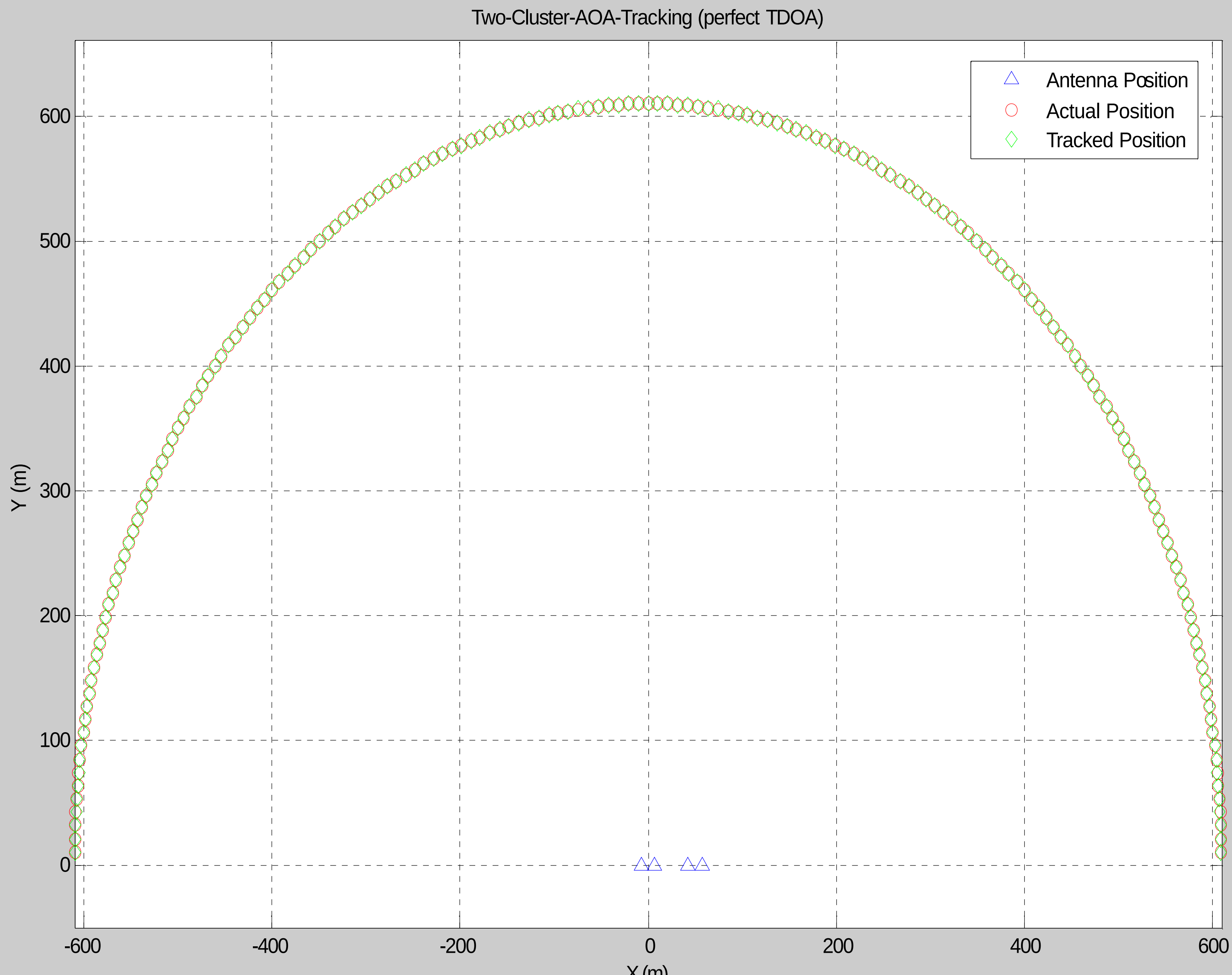
Long Range Assumption: $r_1, r_2 >> a$

Tracking Simulation

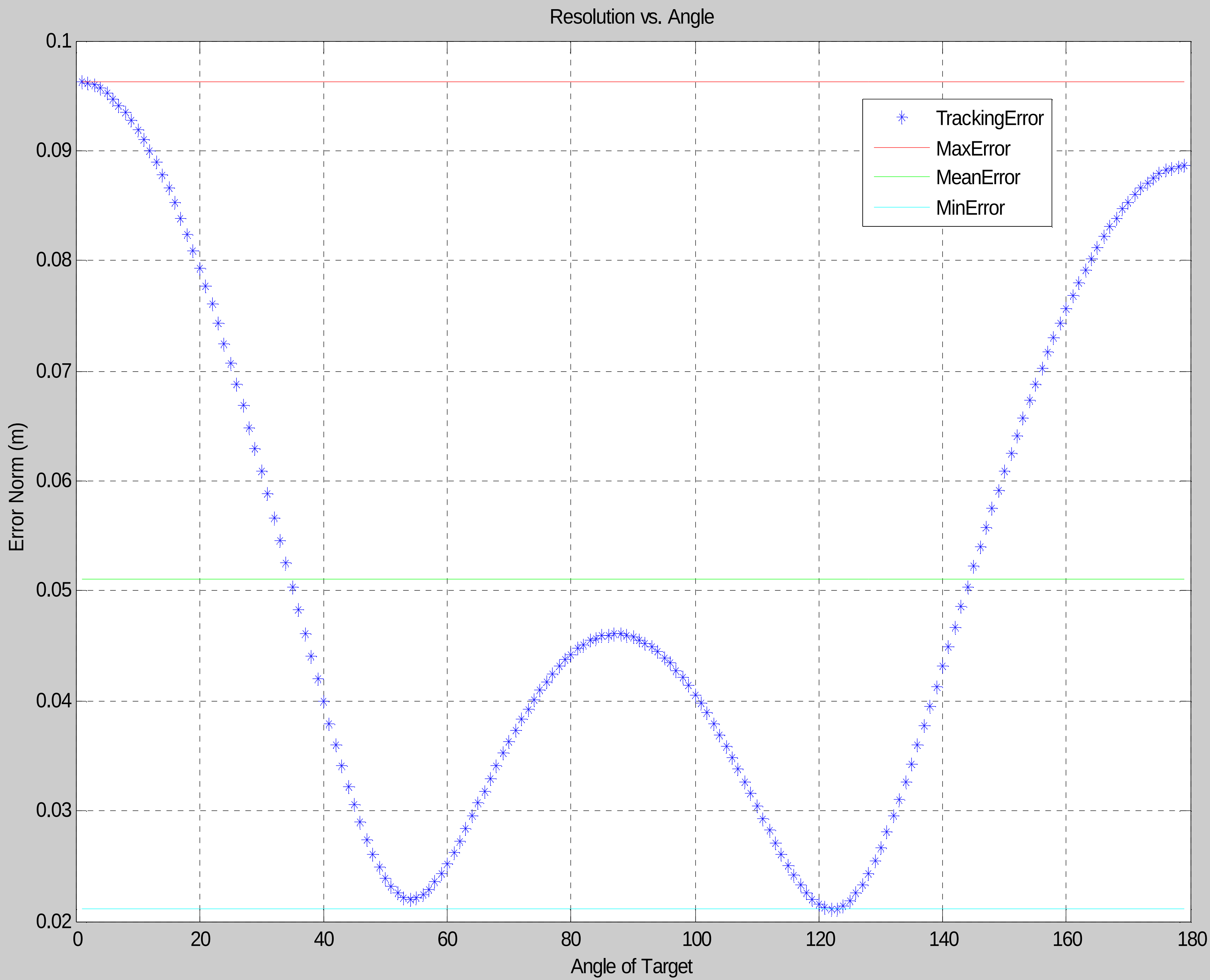
Perfect TDOA Information

Default Setting:

- Cluster Size a=15 meters
- Baseline Size d=50 meters
- Tracking Range r=610 meters (2000 feet)
- Tracking Angle • =0~180 degree



Two-Cluster-AOA-Tracking (perfect TDOA)
Antenna Position
Actual Position
Tracked Position
Y (m)
X (m)
600
500
400
300
200
100
0
-600
-400
-200
0
200
400
600

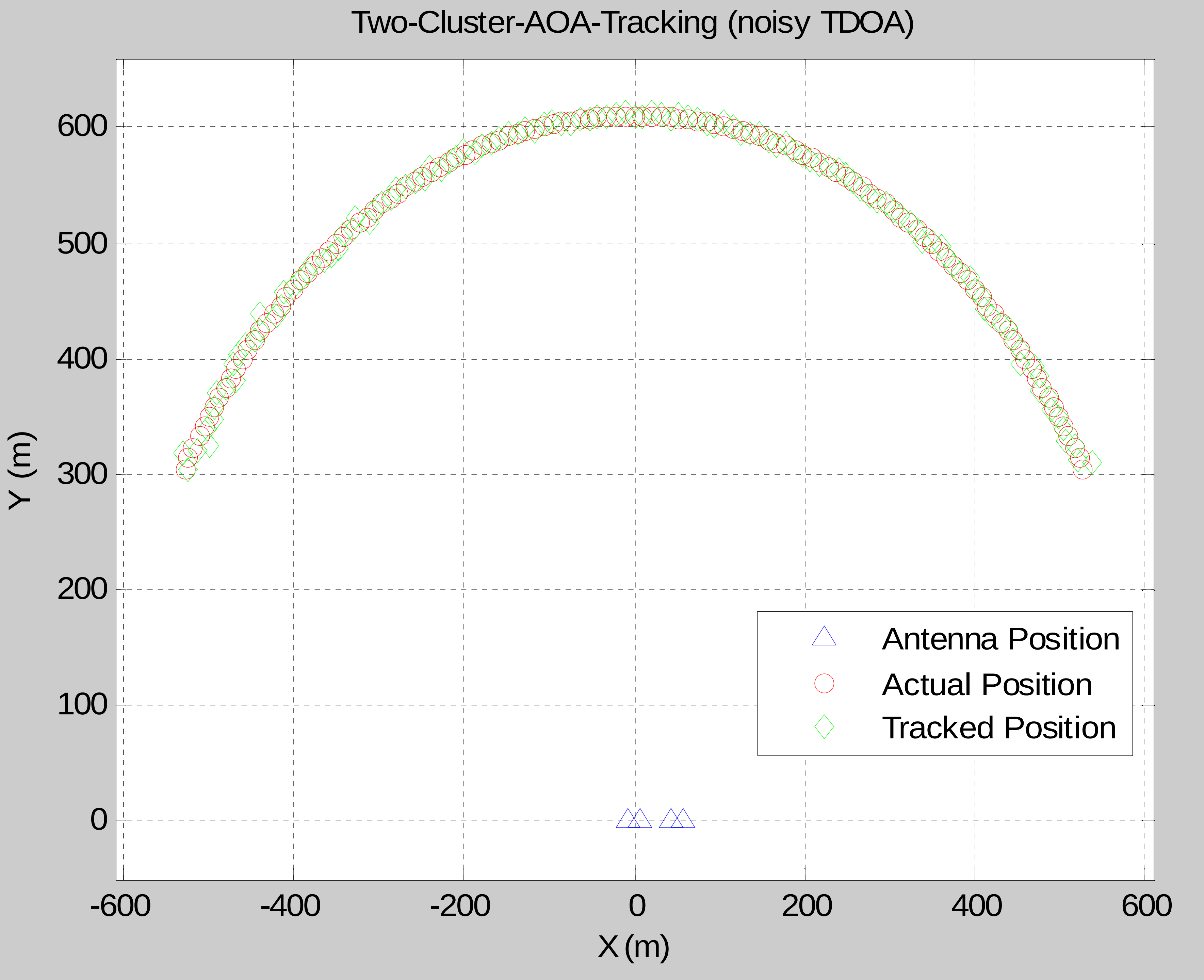

Resolution vs. Angle
TrackingError
MaxError
MeanError
MinError
Error Norm (m)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0
20
40
60
80
100
120
140
160
180
Angle of Target

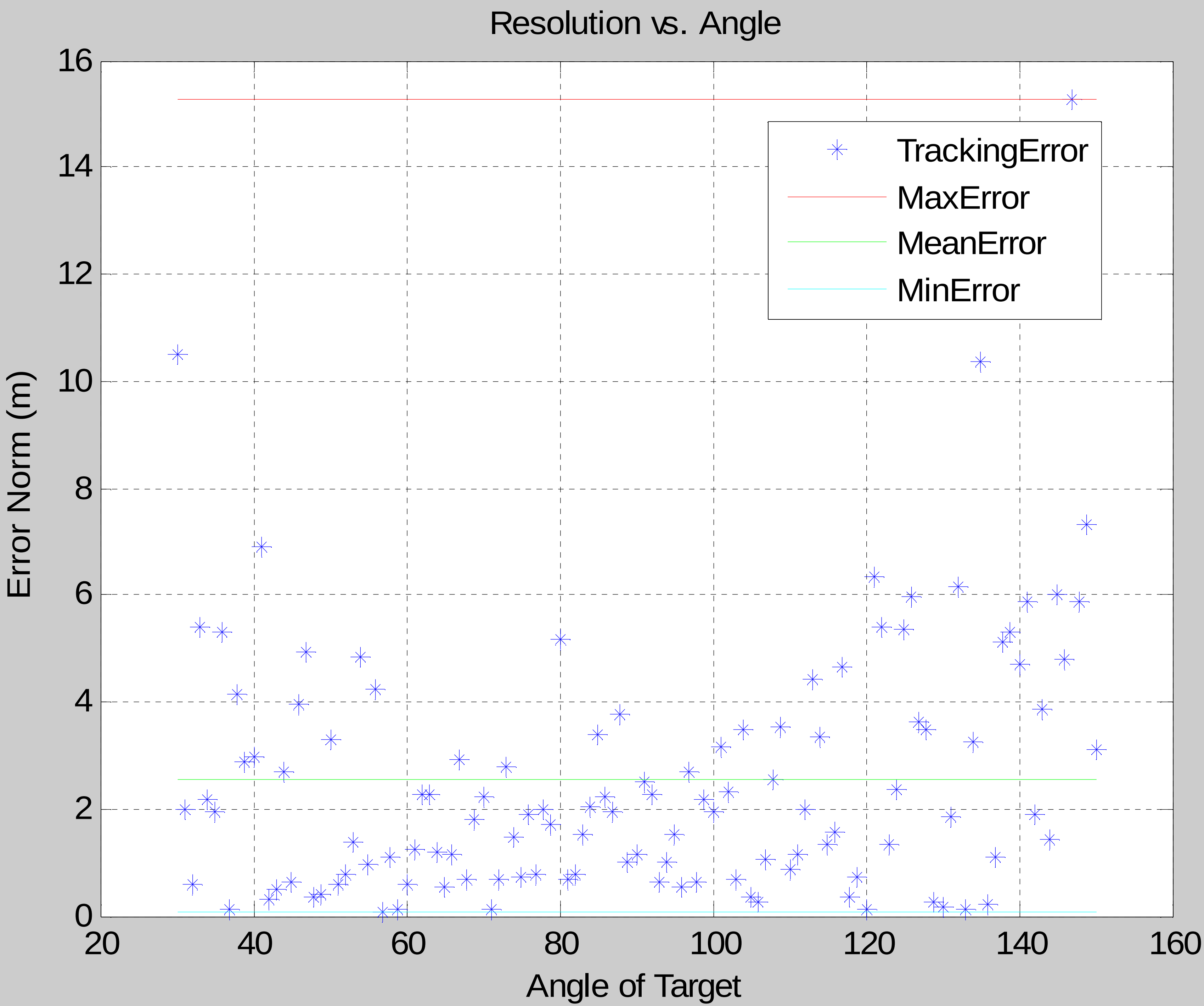
Tracking Simulation

Noisy TDOA Information

Default Setting:

- Cluster Size a=15 meters
- Baseline Size d=50 meters
- Tracking Range r=610 meters (2000 feet)
- Tracking Angle • ≅30~150 degree
- TDOA Noise Level =10 picoseconds


Two-Cluster-AOA-Tracking (noisy TDOA)
Y (m)
X (m)
600
500
400
300
200
100
0
-600
-400
-200
0
200
400
600
Antenna Position
Actual Position
Tracked Position


Resolution vs. Angle
Error Norm (m)
0
2
4
6
8
10
12
14
16
20
40
60
80
100
120
140
160
Angle of Target
TrackingError
MaxError
MeanError
MinError

Resolution vs. Affecting Parameters

$$\mathrm{MSE} = f(a, d, \mathbf{r}, \mathrm{s})$$

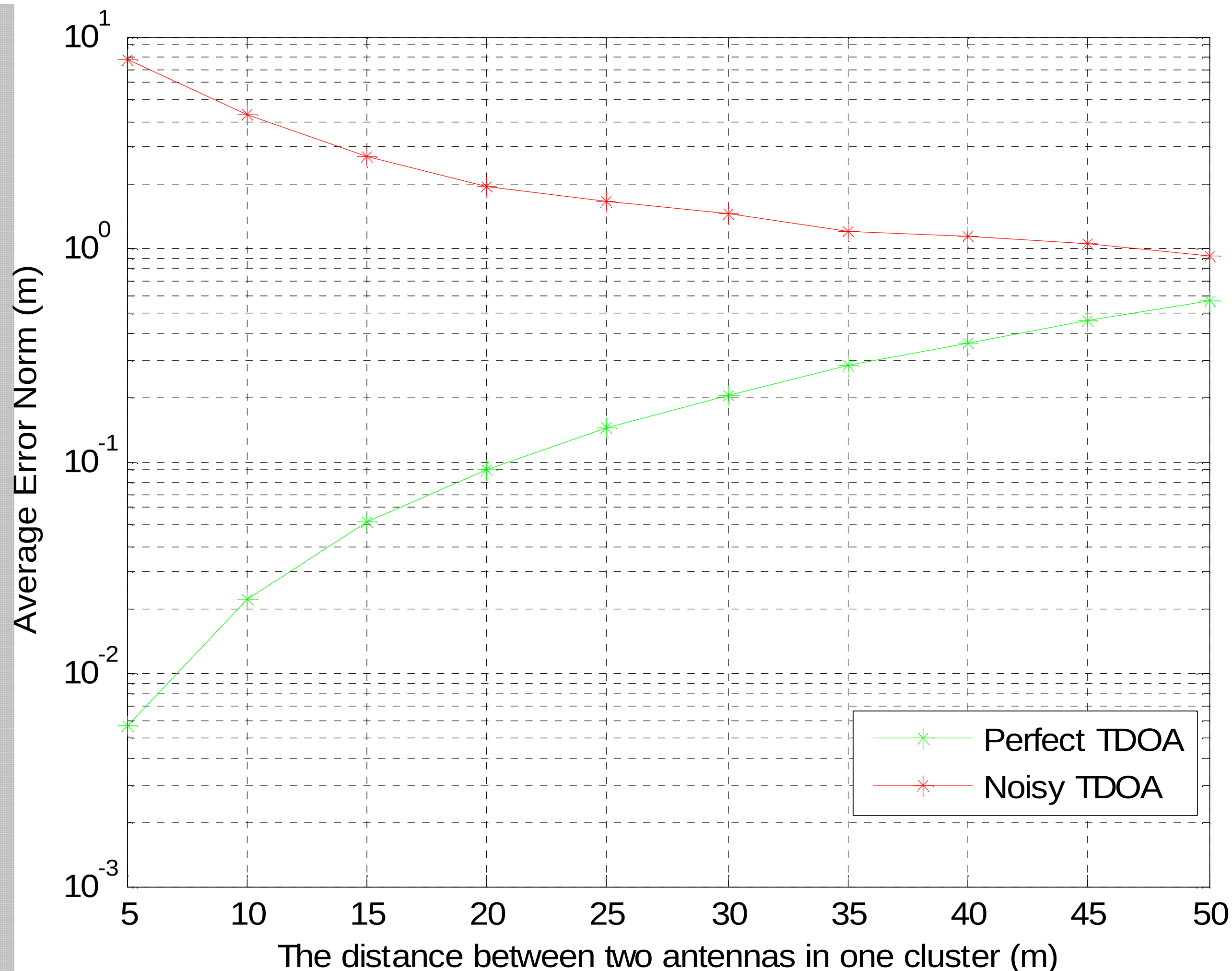
a – cluster size (distance between two antennas)

d – baseline size (distance between two receivers)

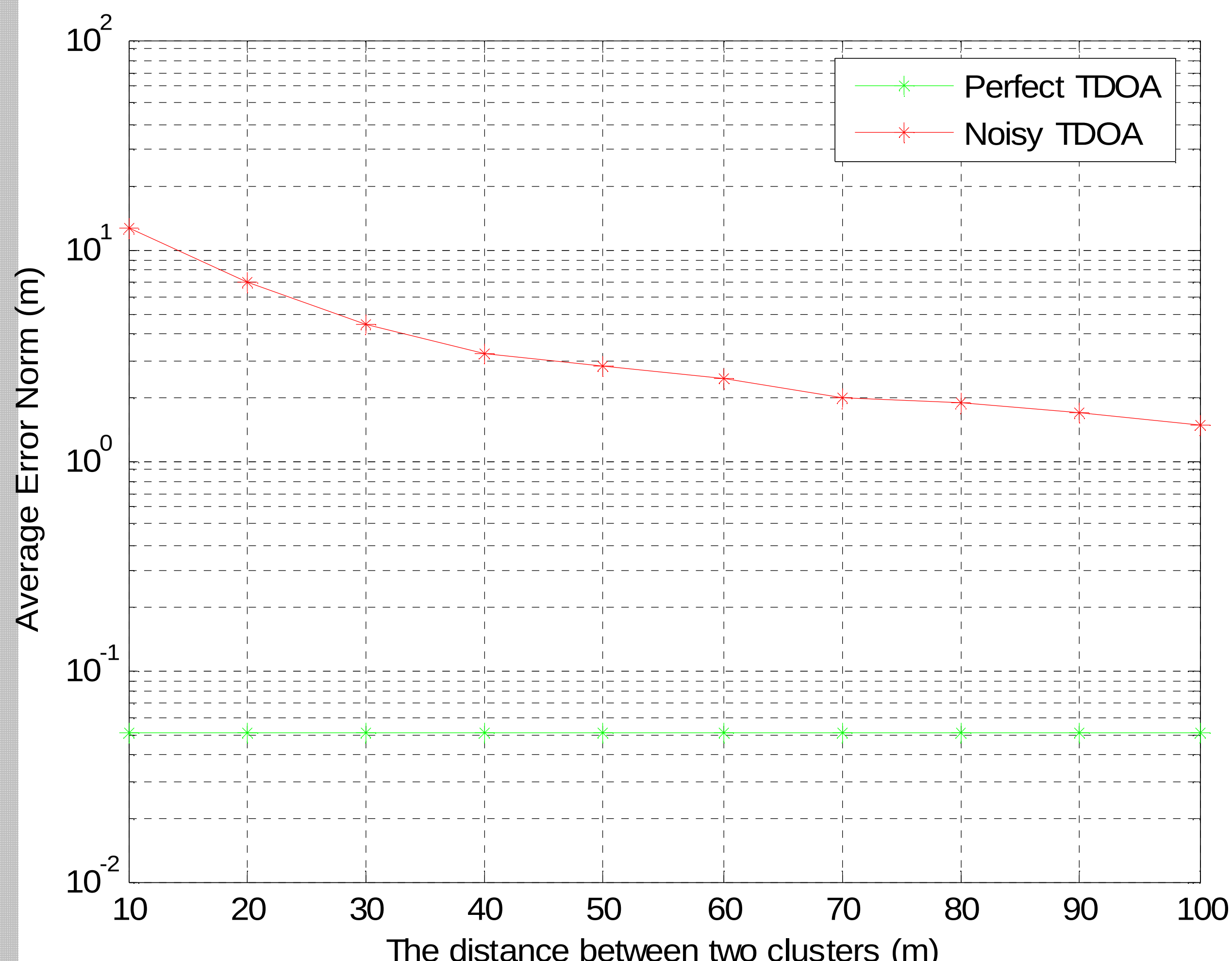
$\mathbf{r}$ – tracking range (distance from target to origin, angle)

s – TDOA noise level (standard derivation of TDOA)

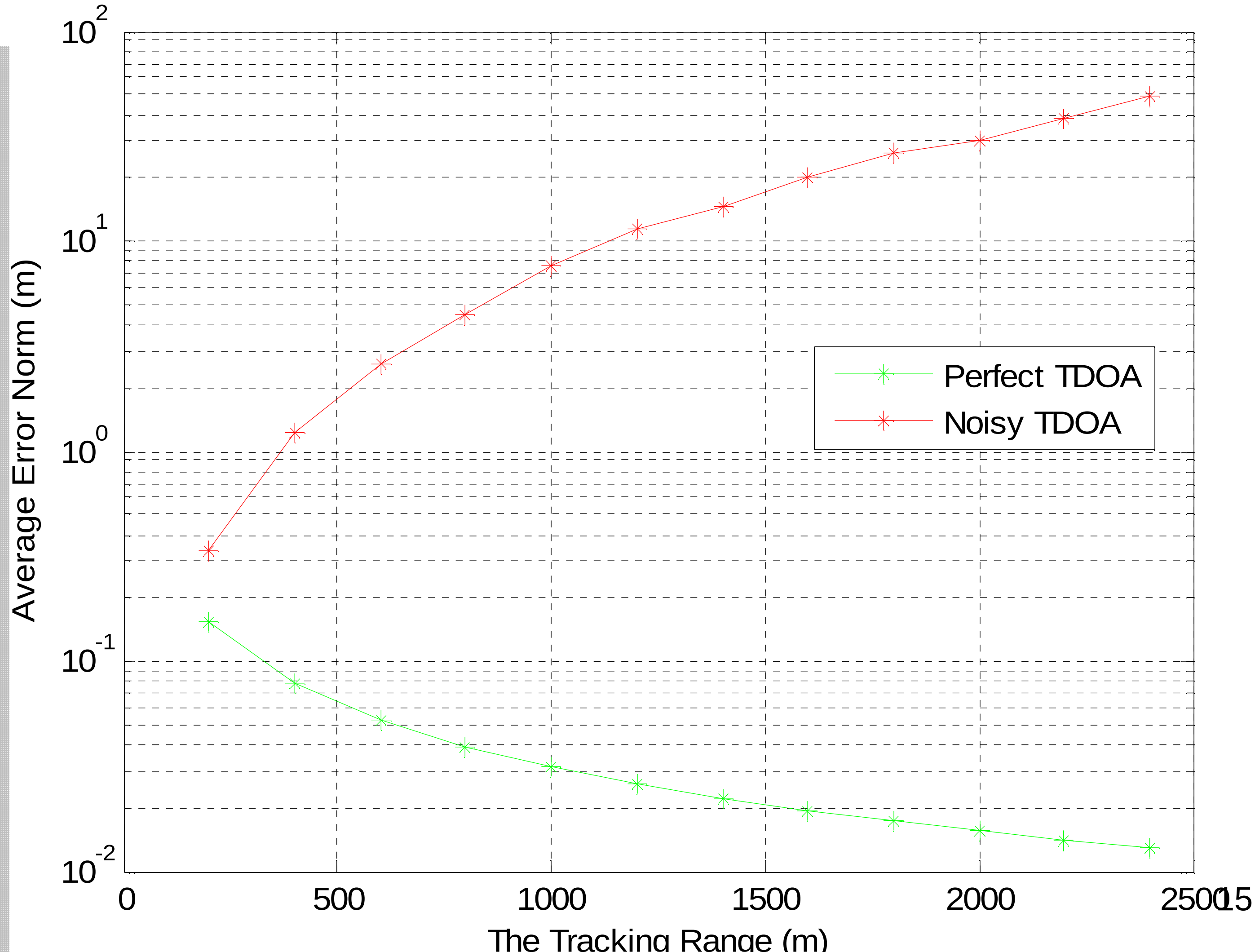
Resolution vs. Cluster Size



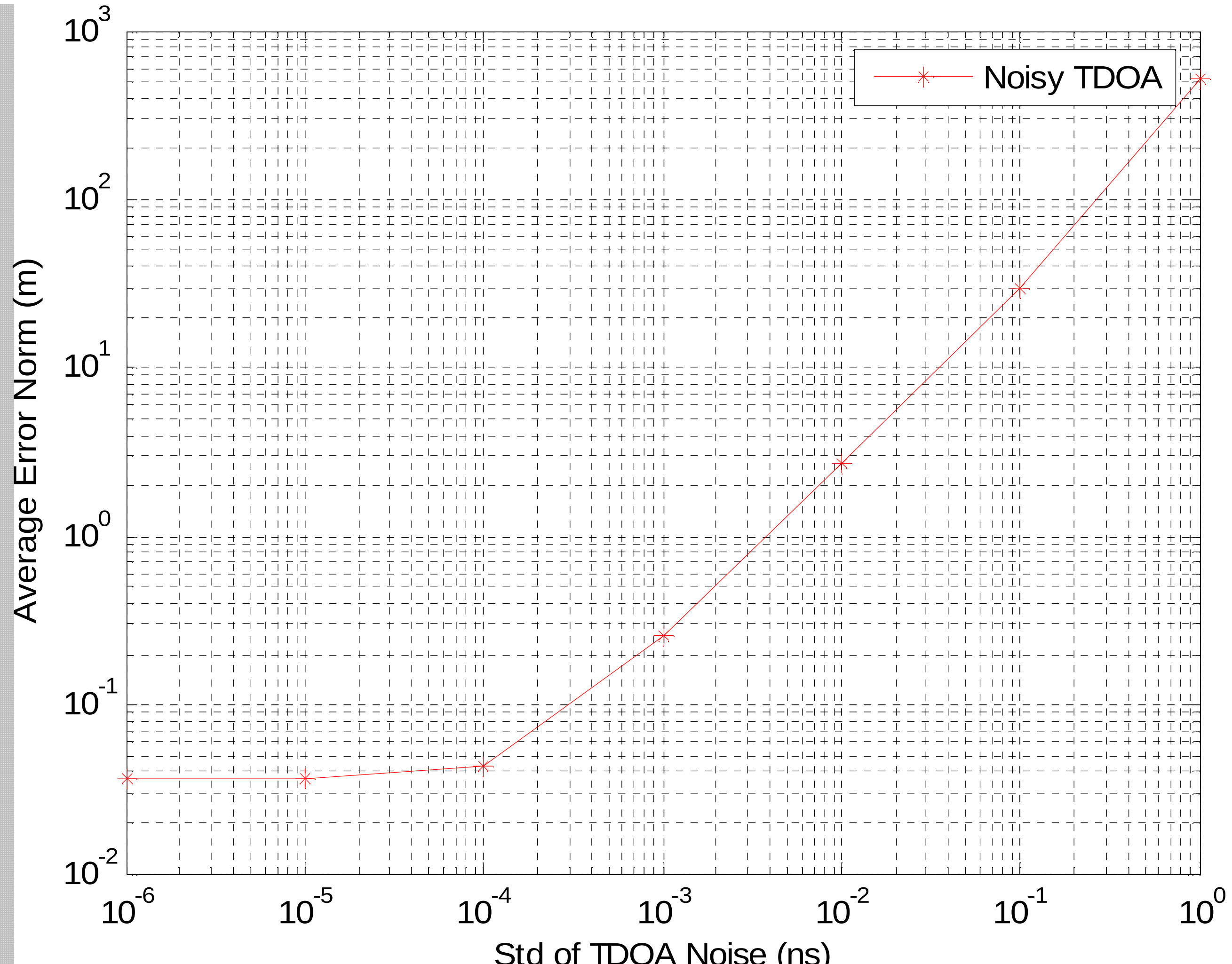
Resolution vs. Baseline Size



Resolution vs. Range



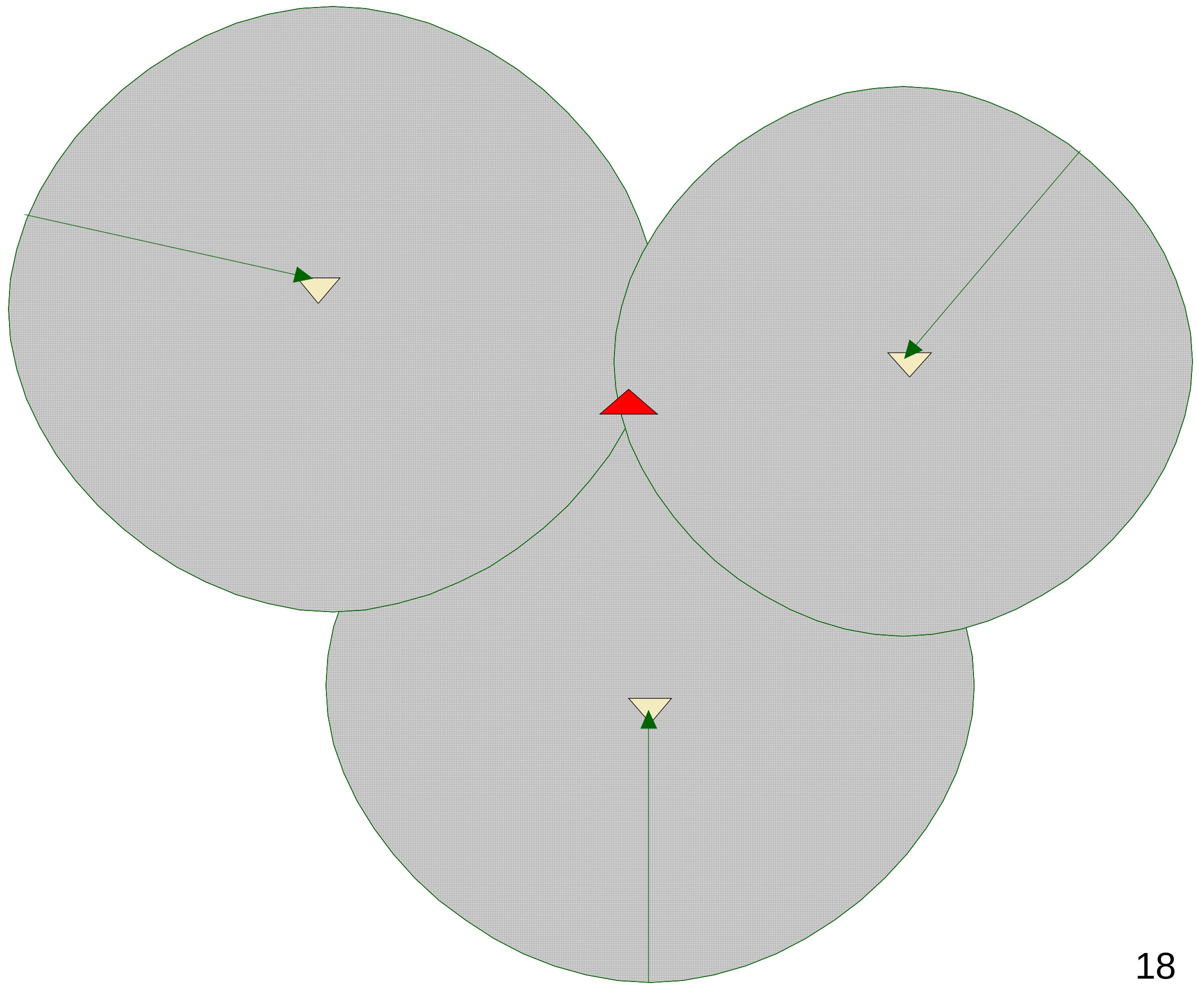
Resolution vs. TDOA Noise



AOA Summary

- Analysis shows that AOA algorithm can achieve fine tracking resolution using TDOA estimates with low noise level
- Analysis provides guidance for system design to improve the tracking resolution
- To increase the cluster size
- To decrease the noise level of TDOA estimates (hardware/DSP techniques)

Time of Arrival (TOA)

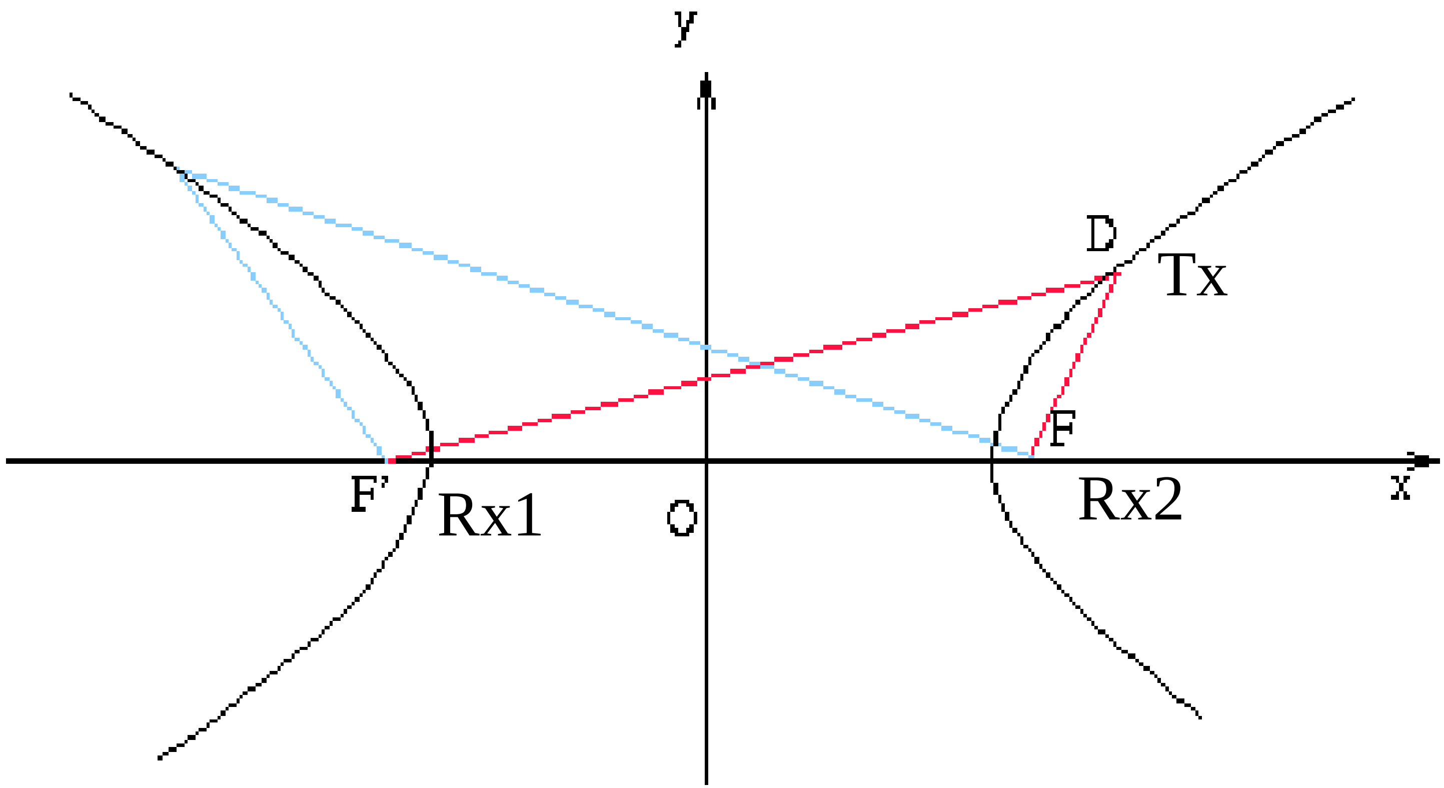


Drawbacks of TOA

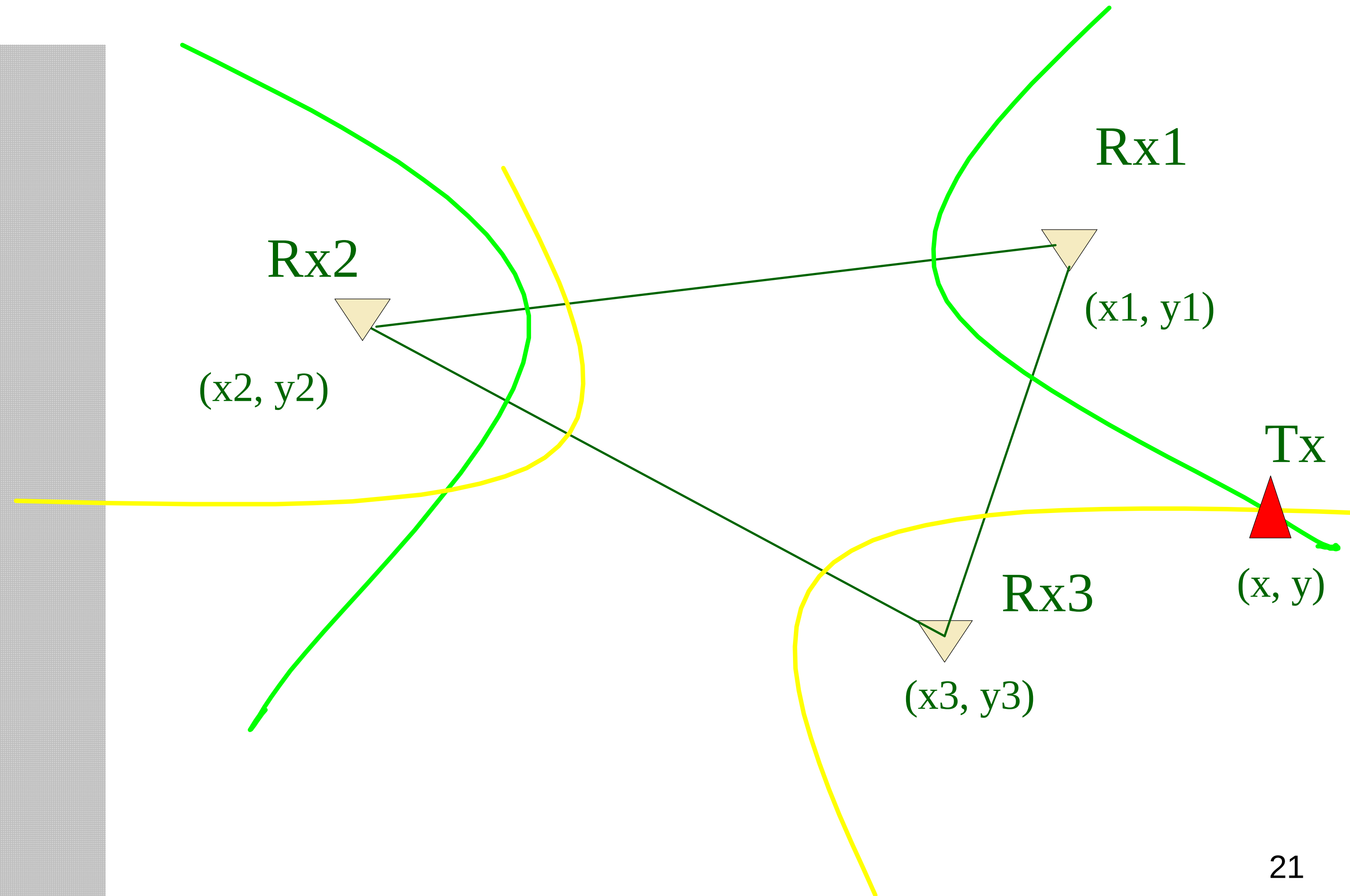
- Ranging: requires duplex transmission and incurs overhead (Asynchronization)
- Synchronization: hard to achieve the synchronization precision between the transmitter and receiver ; 1microsecond synchronization error can easily translate into 300 meters of range error

Time Difference of Arrival (TDOA)

Hyperbola: $b^2 x^2 - a^2 y^2 = a^2 b^2$



Time Difference of Arrival (TDOA)



Advantages of TDOA

- No synchronization between Tx and Rx
- Simplex (one-way) data estimation
- Cross-correlation works well to obtain TDOA

TDOA Equations (2D)

$$D_{12} = \sqrt{(x_1 - x)^2 + (y_1 - y)^2} - \sqrt{(x_2 - x)^2 + (y_2 - y)^2} = \mathrm{t}_{12}c$$

$$D_{13} = \sqrt{(x_1 - x)^2 + (y_1 - y)^2} - \sqrt{(x_3 - x)^2 + (y_3 - y)^2} = \mathrm{t}_{13}c$$

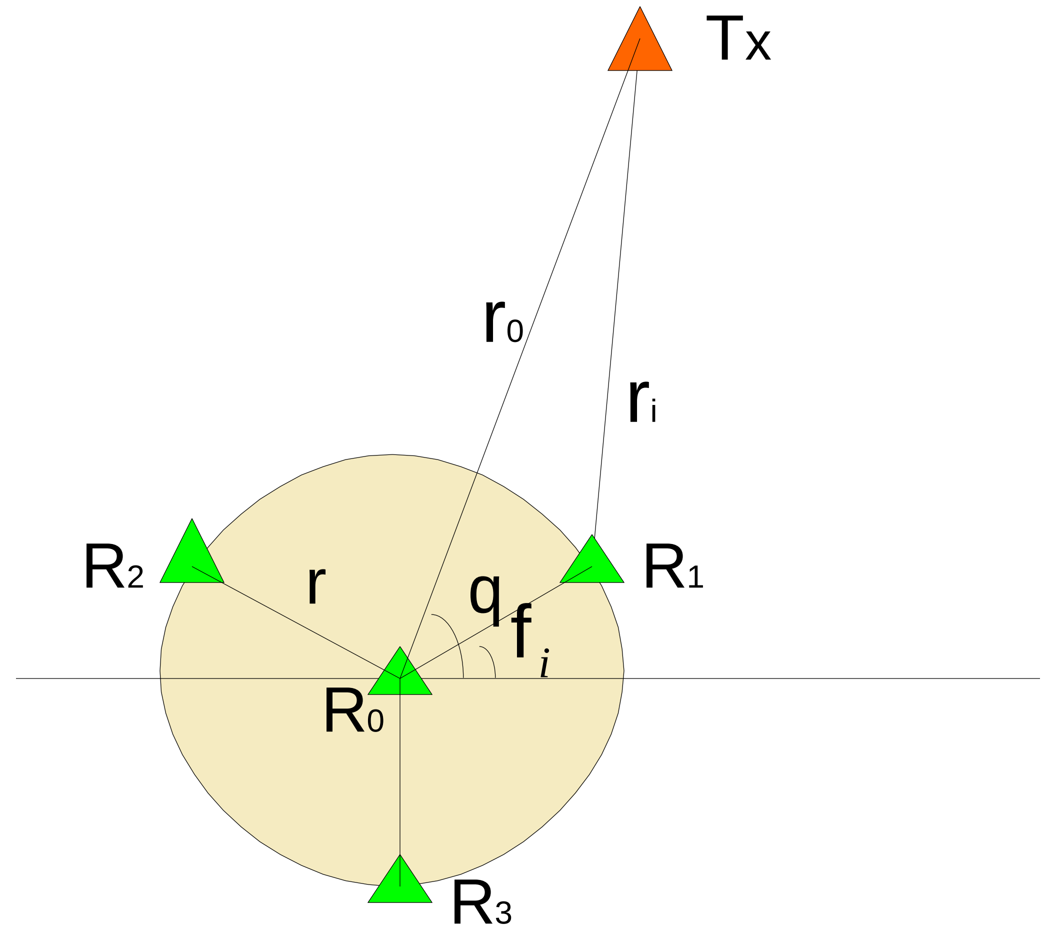
$$D_{23} = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_3 - x)^2 + (y_3 - y)^2} = \mathrm{t}_{23}c$$

TDOA Algorithm

- Taylor Series Expansion Least Squares Iterative Algorithm
(Initialization problem and convergence problem)

- Two-Stage Weighted Least Squares Algorithm
(one more receiver)

Resolution Analysis (setting)



Resolution Analysis (MSE)

$$\mathrm{MSE} = \frac{c^2 \sigma^2 r_0^2 \sum_{i=1}^{3} \left(a_i^2 + b_i^2 \right)}{r^2 \left(\sum_{i=1}^{3} a_i^2 \sum_{i=1}^{3} b_i^2 - \left(\sum_{i=1}^{3} a_i b_i \right)^2 \right)}$$

$$a_i = \cos \phi_i + \frac{r_i - r_0}{r} \cos \theta$$

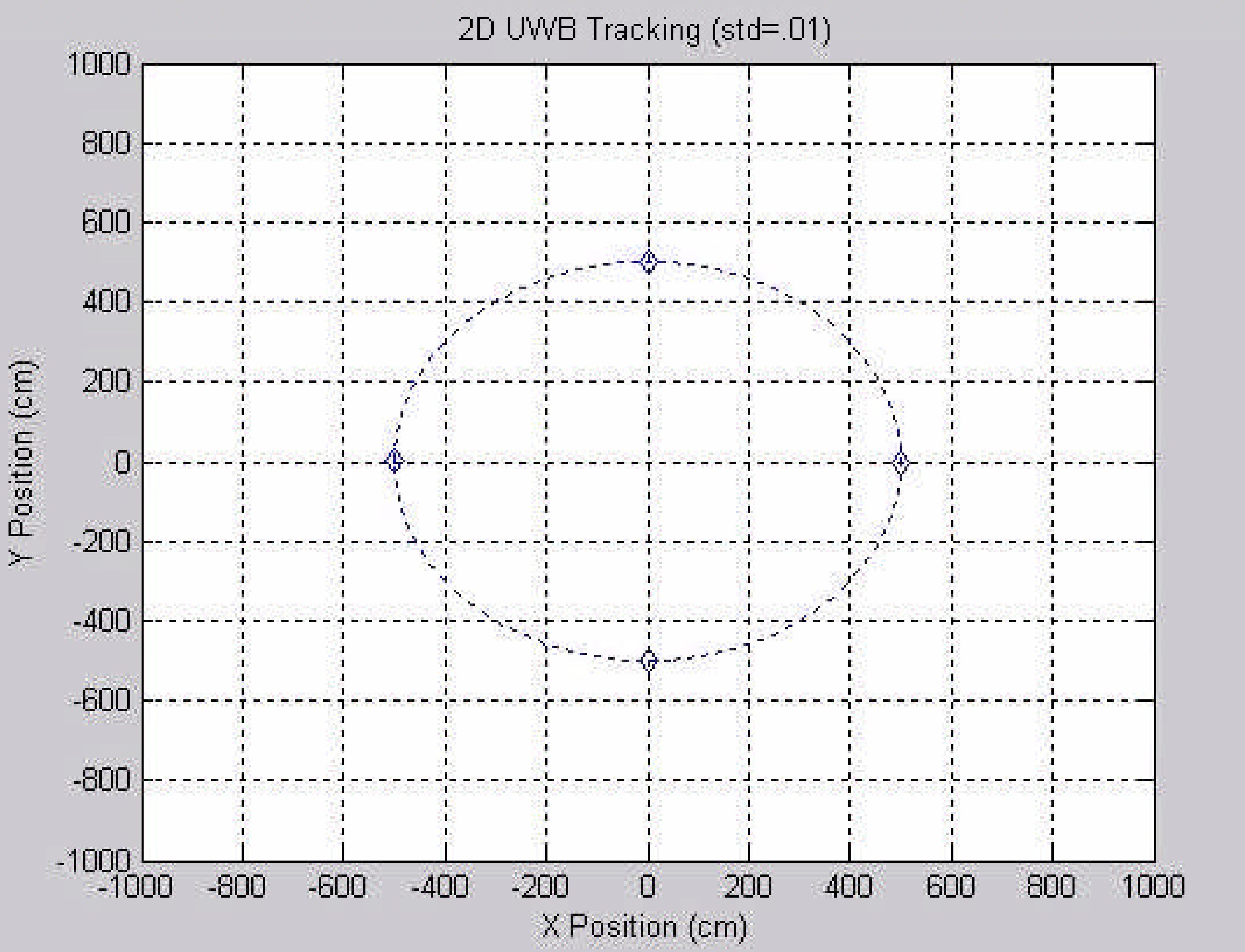
$$b_i = \sin \phi_i + \frac{r_i - r_0}{r} \sin \theta$$

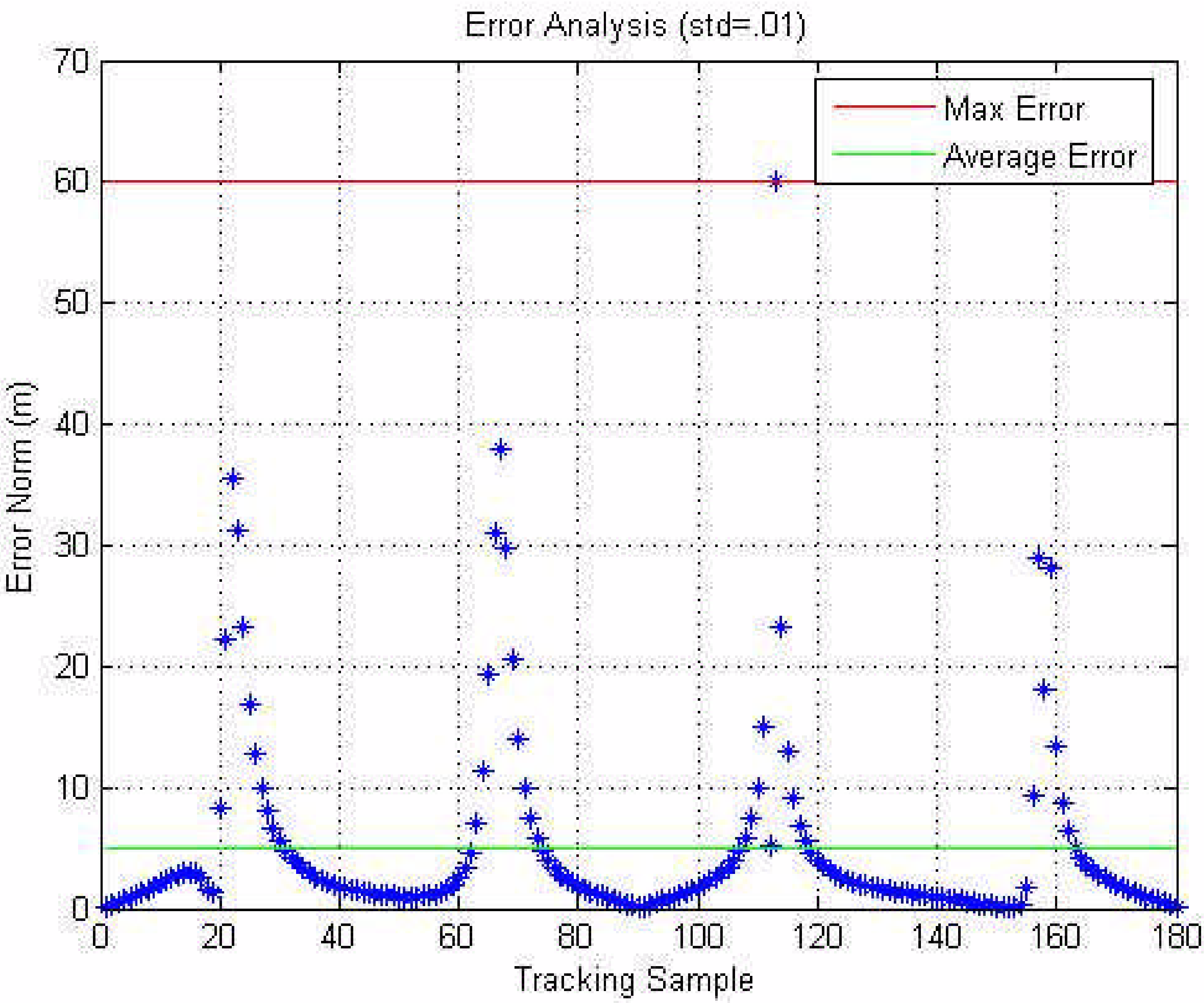
$$r_i = \sqrt{r_0^2 + r^2 - 2 r_0 r \cos(\theta - \phi_i)}$$

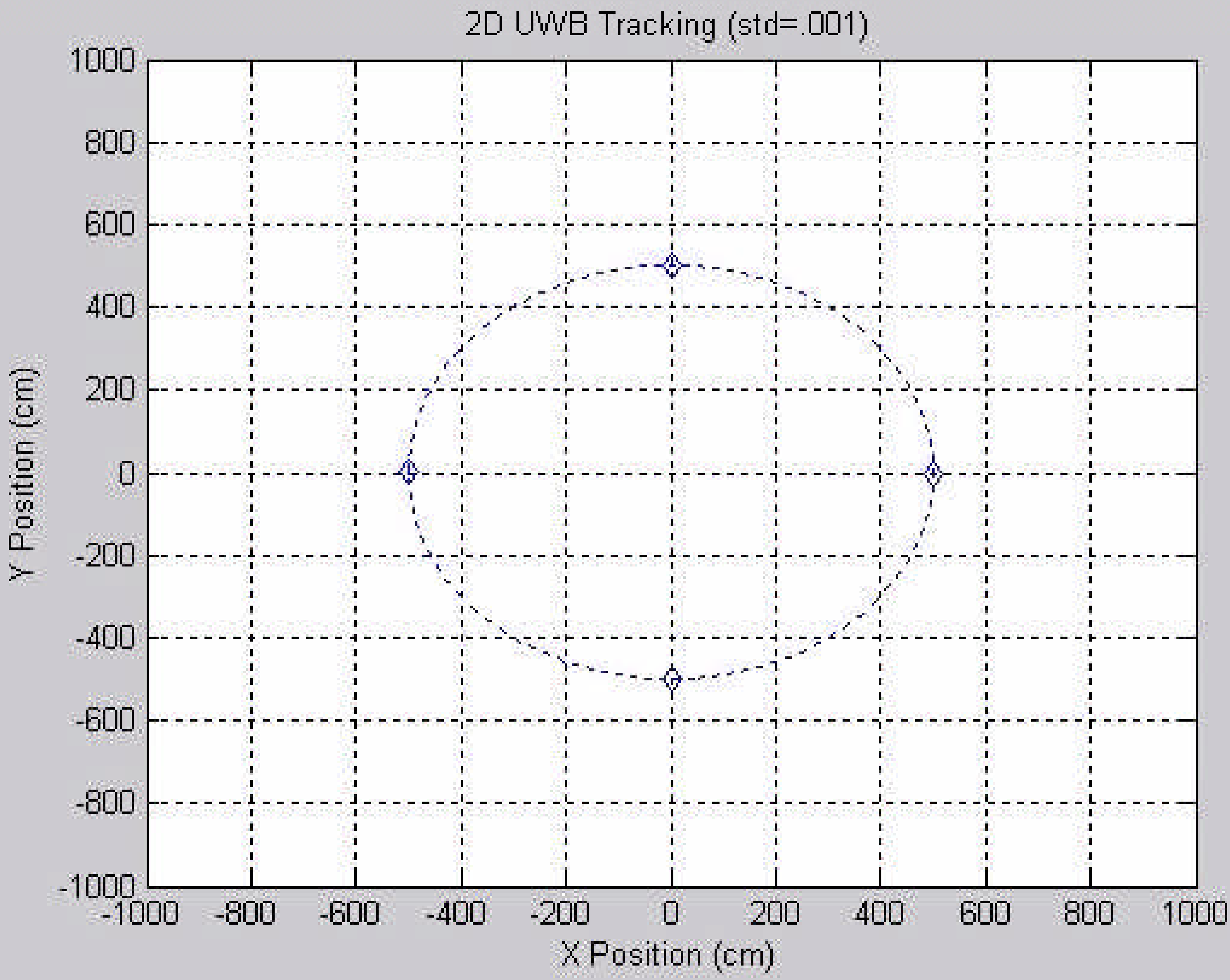
Tracking Simulation Demo

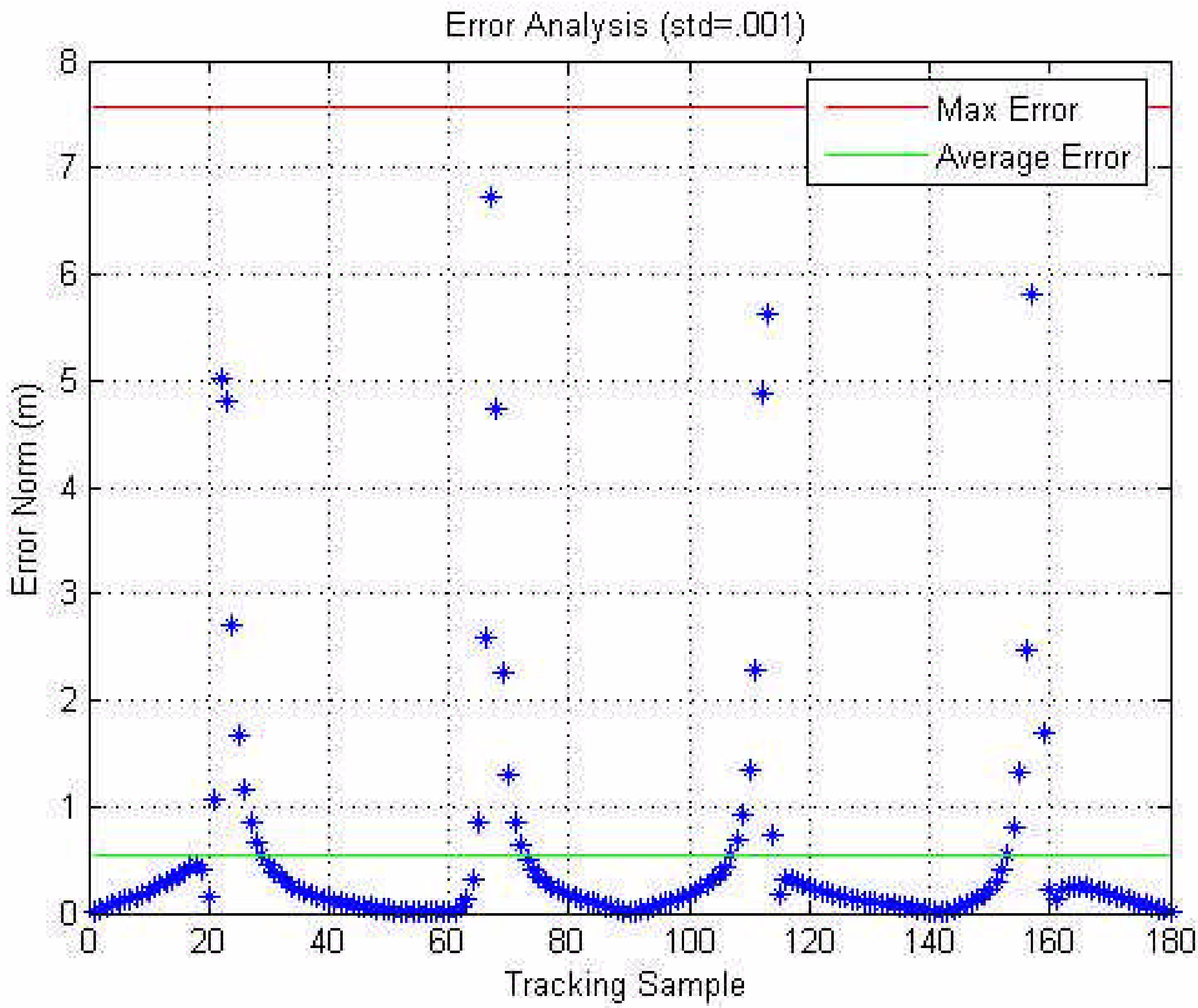
Orbit Tracking

- Tracking Resolution vs.TDOA noise
- Tracking Resolution vs. Receiver Configuration
- Tracking Resolution vs. Dynamic Reference


2D UWB Tracking (std=.01)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
X Position (cm)
Y Position (cm)


Error Analysis (std=.01)
Max Error
Average Error
Error Norm (m)
Tracking Sample
0
10
20
30
40
50
60
70
0
20
40
60
80
100
120
140
160
180

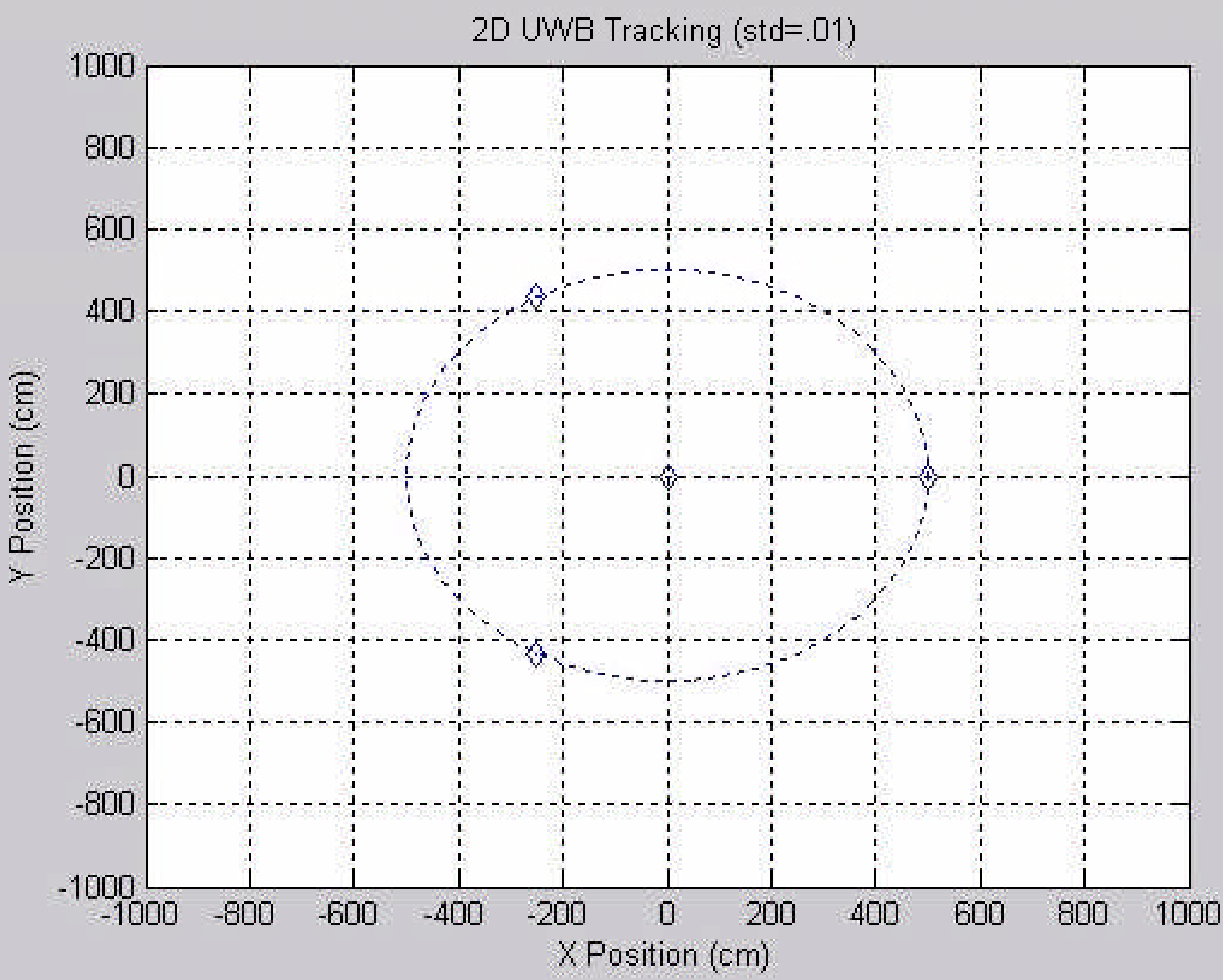

2D UWB Tracking (std=.001)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
Y Position (cm)
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
X Position (cm)

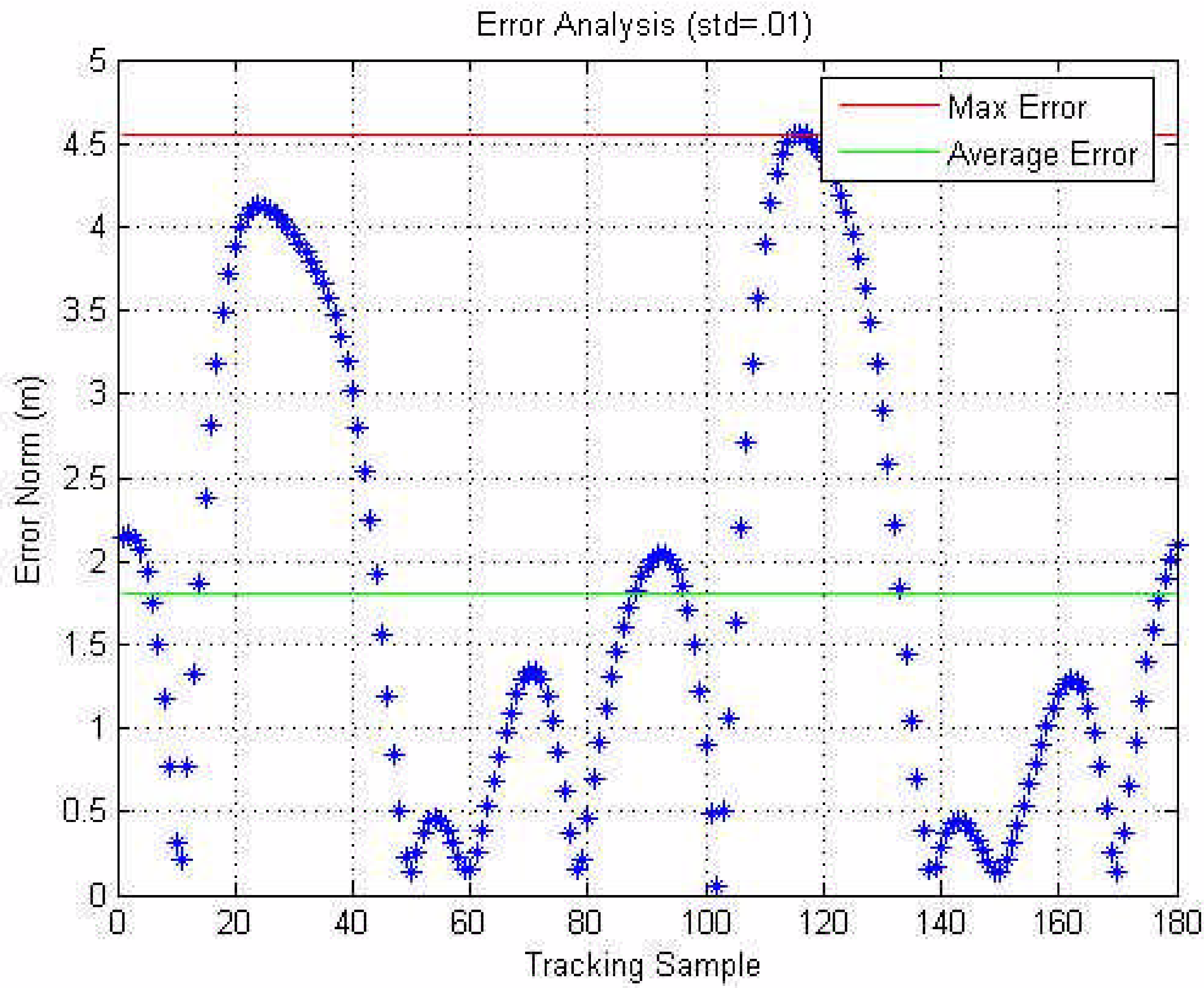

Error Analysis (std=.001)
Max Error
Average Error
Error Norm (m)
Tracking Sample
0
1
2
3
4
5
6
7
8
0
20
40
60
80
100
120
140
160
180

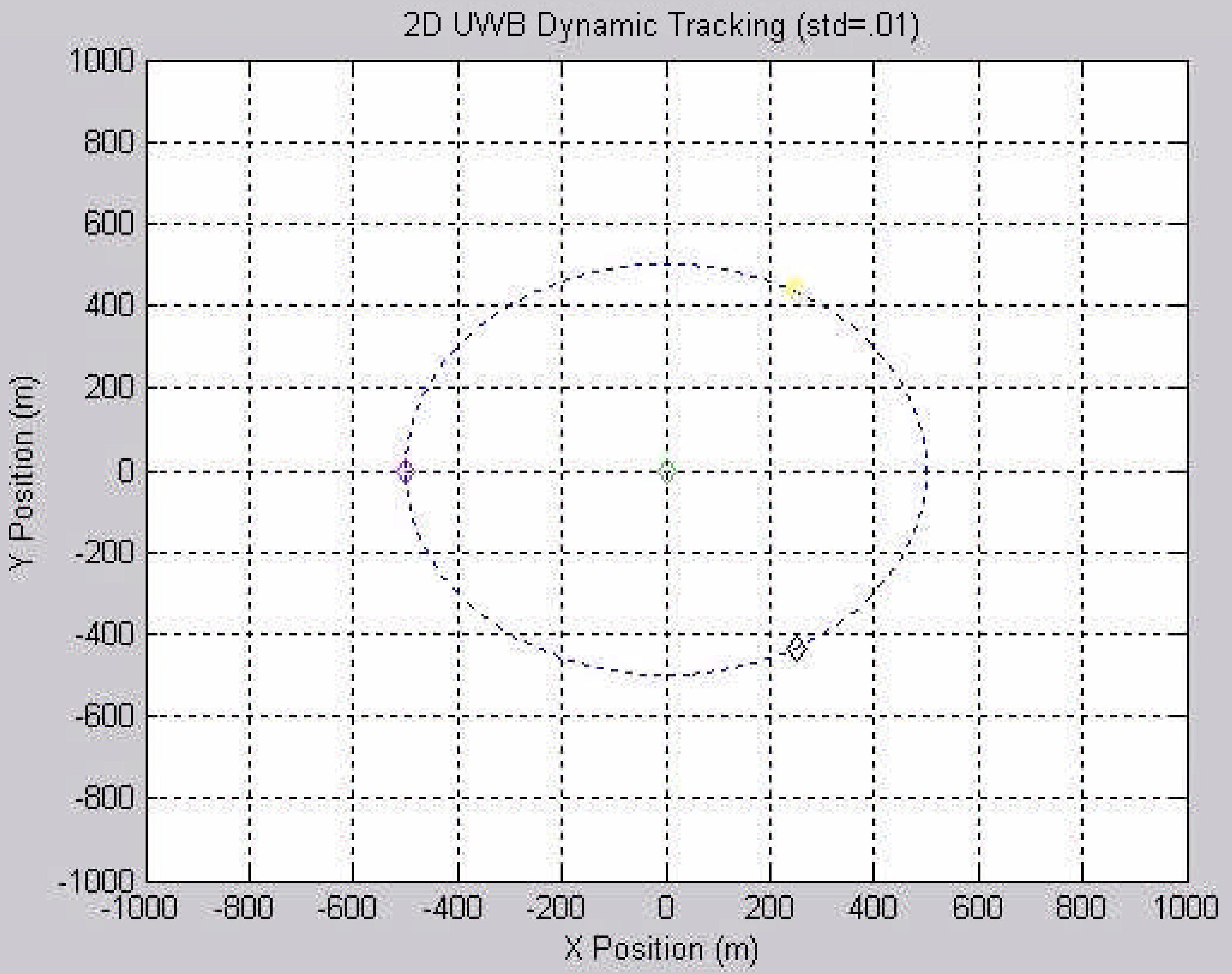
Error Analysis

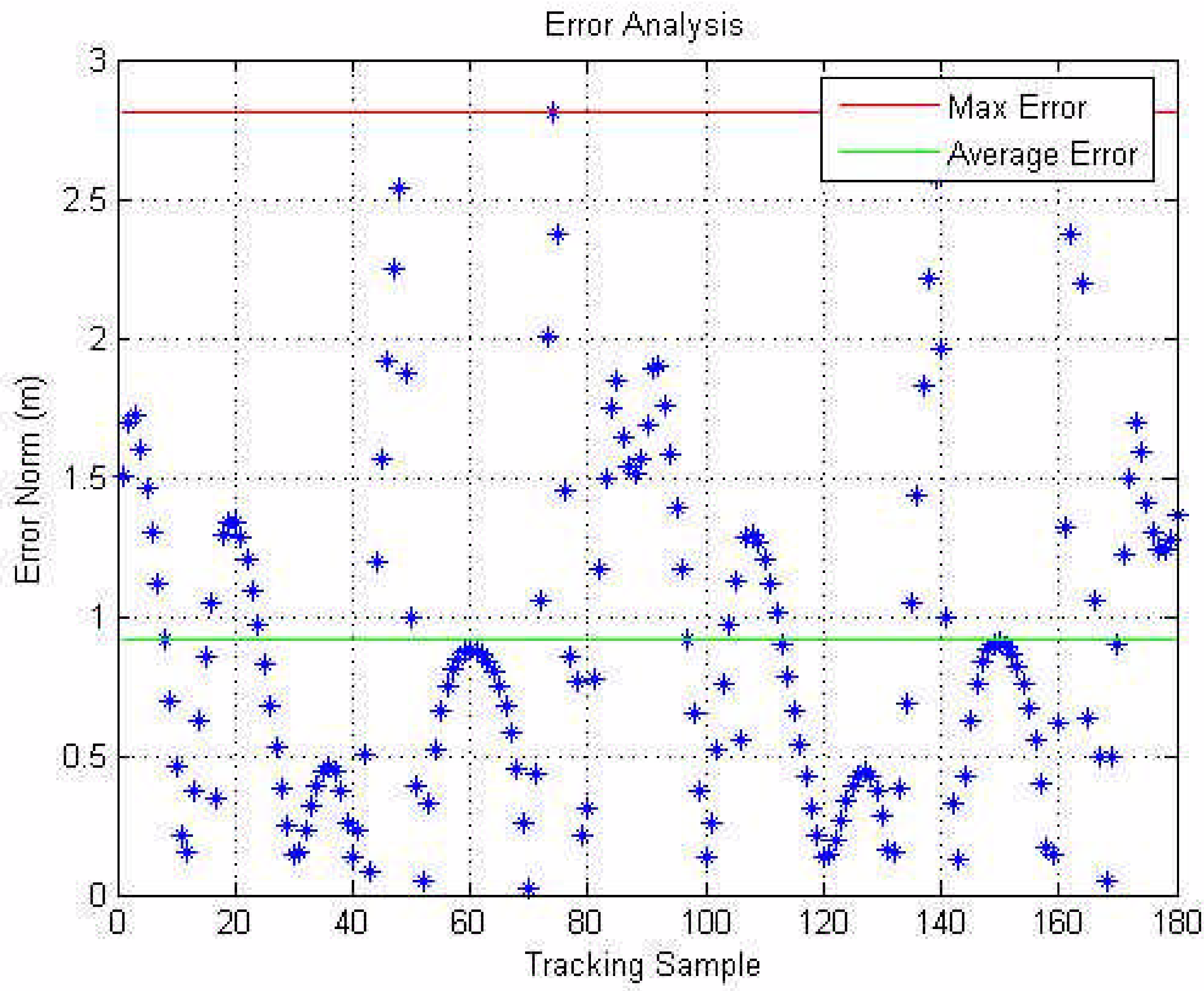
Standard Deviation of TDOA (ns)	Maximum Error (m)	Average Error (m)
0.01	**60.0461**	**4.9656**
0.001	**7.5611**	**0.5468**
0.0001	**0.7218**	**0.0544**

- The tracking error is linear to the standard deviation of TDOA data


2D UWB Tracking (std=.01)
Y Position (cm)
X Position (cm)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1000
-800
-600
-400
-200
0
200
400
600
800
1000


Error Analysis (std=.01)
Error Norm (m)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
20
40
60
80
100
120
140
160
180
Tracking Sample
Max Error
Average Error


2D UWB Dynamic Tracking (std=.01)
Y Position (m)
X Position (m)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1000
-800
-600
-400
-200
0
200
400
600
800
1000


Error Analysis
Max Error
Average Error
Error Norm (m)
Tracking Sample
0
0.5
1
1.5
2
2.5
3
0
20
40
60
80
100
120
140
160
180

TDOA Summary

- TDOA algorithm can achieve fine tracking resolution with UWB fine time resolution
- Tracking resolution is proportional to TDOA noise level
- Receiver antenna configuration matters
- Dynamic reference can improve the tracking resolution

Future Work

- AOA

  To study the scalable baseline configuration to increase the tracking coverage
- TDOA

  To improve the tracking resolution using enhanced algorithm (Totally Least Square Algorithm)

  To study the optimal receiving antenna configuration
- AOA and TDOA

  Extend the tracking dimension (from 2D to 3D)