Accommodating Sensor Uncertainty in the Cones Method: Polycones and Fuzzycones

Joseph A. Hashmall
a.i.solutions, Inc.
10001 Derekwood Lane, Suite 215
Lanham, MD 20706
joseph.hashmall@ai-solutions.com

ABSTRACT

The "cones method" is an analytical algorithm to combine a pair of angle observations into a common vector. Two new algorithms have been developed to optimize the "cones method" solutions when more than two observation angles are available and when estimates of measurement uncertainties can be made. The polycones algorithm consists of determining a simple weighted average of the solution vectors over all possible pairs of measurements with the weights determined from the measurement uncertainty. The Fuzzycones method finds the vector of maximum probability. Both of these methods have been implemented and tested and both reduce errors in computed vector positions.

INTRODUCTION

The "cones method" is a simple technique for determining a vector direction based on measured angles between the desired vector and at least two known reference vectors at a common time (Reference 1). For spacecraft attitude analysis, the method is most commonly used in the following computations:

1. Determining the inertial frame orientation of the satellite spin axis from measured angles between two or more known body frame directions and their corresponding reference objects (Sun, Stars, Earth, etc.).
2. Determining the body frame Sun direction from measurements from two or more "cosine law" Sun detectors.

The method consists of finding the vector described by the intersection of cones, each of which is a measured angle from a known axis. Although there is some description of the error distribution of vectors determined using the cones method (Reference 2), there has been little analysis of the optimum solution if more than two measurements are available.

APPROACH

Polycones

The cones method generally produces two possible solution vectors for a pair of measurements. The correct solution must be selected using additional information. If n measurements ($n > 2$) are available a total of $n(n-1)$ solutions can be found using all possible pairs. The ambiguity of which of the two solutions for each pair of measurements is correct is removed because the correct solutions will be near each other, while the incorrect ones will not.

The two solution vectors obtained using independent angle measurements (ρ_i and ρ_j) from sensors with non-parallel axes, are designated $\hat{U}_{ij}$ and $\hat{V}_{ij}$, where the correct solution is designated $\hat{V}_{ij}$. Designating the ith measurement uncertainty as σ_i, the ad hoc approximation is made that the weight of the solution $\hat{V}_{ij}$ is given by:

$$w_{ij} = \frac{1}{\sigma_i \sigma_j} \tag{1}$$

and the overall solution vector is given by a simple weighted average of the correct solutions:

$$\vec{V} = \frac{\sum_i \sum_{j \neq i} w_{ij} \hat{V}_{ij}}{\sum_i \sum_{j \neq i} w_{ij}} \tag{2}$$

Fuzzycones

The measurement ρ_i is the angle from a known axis. Designating the azimuth and coelevation angles of this axis direction by (Φ_i and Θ_i) then the relative probability that the measurement will represent a vector with azimuth and coelevation of ϕ_i and θ_i is given by:

$$P_i(\theta, \phi) = N_i \exp\left(-\frac{(\Gamma_i - \rho_i)^2}{2\sigma_i^2} \right) \tag{3}$$

where

$$\Gamma_i = \cos^{-1}(\cos\Theta_i \sin\theta_i + \sin\Theta_i \sin\theta_i \cos(\Phi_i - \phi_i)) \tag{4}$$

and N_i is a normalization constant selected so that the integral of P_i over all space is one.

The relative probability that the solution from simultaneous of measurements is at the position (ϕ, θ) is given by:

$$P(\theta, \phi) = \prod_i P_i \tag{5}$$

The maximum probability solution is found at the maximum of P.

RESULTS

Polycones and Fuzzycones were tested using simulated and on-orbit spacecraft data.

Results from a single set of simulations are presented here. A Sun sensor with four cosine law detectors was simulated. The detectors were each 45 degrees from a symmetry axis and situated uniformly around this axis at 90 degree intervals. The Sun position was simulated at a uniform distribution of locations within 45 degrees of the sensor symmetry axis. A Monte Carlo simulation of detector measurements was performed using approximately five million simulated cases. The error model for the sensor included a normally distributed error on the angular measurement and a second normally distributed error on the detector current. Since the current is proportional to the cosine of the angle, the total error distribution is a function of the angle.

A more complete error model including the effect of reflections from the lit-Earth was also considered.

The distribution of angles between the solved and "truth" Sun positions is shown in Figure 1. For comparison, the distributions of two "cones method" solutions are also shown. The one designated with the subscript "cones" uses all pairs of detectors. The one with the subscript "0" uses the cones solution from the two measurements with smallest uncertainties. For a "cones method" standard deviation of 3.82 degrees, Polycones has an uncertainty of 2.85 degrees, while the Fuzzycones uncertainty is 2.70 degrees.

Both new algorithms provide a significant improvement over the cones method—polycones providing the greater improvement. Polycones has the additional advantage that there is no need for resolution of the ambiguity between pairs of solutions.

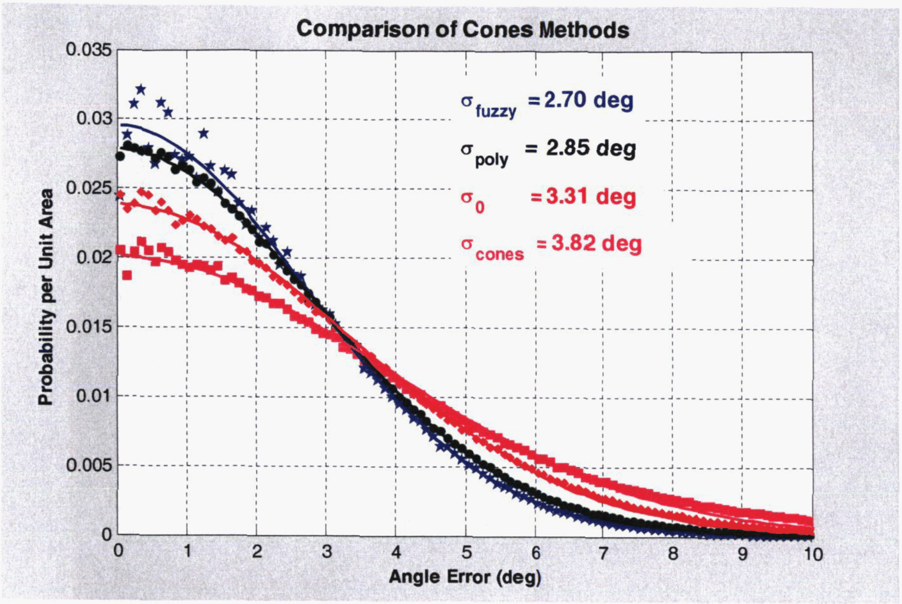

Figure 1. Comparison of Cones Methods

REFERENCES

1. J. R. Wertz (ed.), *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Co., Dordrecht, Holland, 1985, pp 364-370

2. ibid., pp 156-158