Issues and Methods Concerning the Evaluation of Hypersingular and Near-Hypersingular Integrals in BEM Formulations

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It is known that higher order modeling of the sources and the geometry in BEM formulations is essential to highly efficient computational electromagnetics. However, in order to achieve the benefits of higher order basis and geometry modeling, the singular and near-singular terms arising in BEM formulations must be integrated accurately. In particular, the accurate integration of near-singular terms, which occur when observation points are near but not on source regions of the scattering object, has been considered one of the remaining limitations on the computational efficiency of integral equation methods. The method of singularity subtraction has been used extensively for the evaluation of singular and near-singular terms. Piecewise integration of the source terms in this manner, while manageable for bases of constant and linear orders, becomes unwieldy and prone to error for bases of higher order. Furthermore, we find that the singularity subtraction method is not conducive to object-oriented programming practices, particularly in the context of multiple operators.

To extend the capabilities, accuracy, and maintainability of general-purpose codes, the subtraction method is being replaced in favor of the purely numerical quadrature schemes. These schemes employ singularity cancellation methods in which a change of variables is chosen such that the Jacobian of the transformation cancels the singularity. An example of the singularity cancellation approach is the Duffy method, which has two major drawbacks: 1) In the resulting integrand, it produces an angular variation about the singular point that becomes nearly-singular for observation points close to an edge of the parent element, and 2) it appears not to work well when applied to nearly-singular integrals.

Recently, the authors have introduced the transformation

$$u(x') = \sinh^{-1}\frac{x'}{\sqrt{(y')^2 + z^2}}$$

for integrating functions of the form

$$\mathbf{I} = \int_{\mathcal{D}} \Lambda(\mathbf{r}') \frac{e^{-jkR}}{4\pi R}\, d\mathcal{D},$$

where $\Lambda(\mathbf{r}')$ is a vector or scalar basis function and $R = \sqrt{(x')^2 + (y')^2 + z^2}$ is the distance between source and observation points. This scheme has all of the advantages of the Duffy method while avoiding the disadvantages listed above. In this presentation we will survey similar approaches for handling singular and near-singular terms for kernels with $1/R^2$ type behavior, addressing potential pitfalls and offering techniques to efficiently handle special cases.