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GENERALIZED SEPARATION OF AN OBJECT JETTISONED FROM THE ISS

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ABSTRACT

The International Space Station (ISS) Program faces unprecedented logistics challenges in both upmass and downmass. Some items employed on the ISS exterior present significant technical issues for a controlled de-orbit on either the shuttle or an expendable supply vehicle. Such manifest problems arise due to structural degradation, insufficient containment of hazardous pressures or contents, excessive size, or some combination of all of these factors. In addition, the mounting hardware and other flight service equipment to manifest the returned equipment must itself be launched, competing with other upmass. EVA techniques and equipment to successfully contain and secure such problematic equipment result in numerous significant risks to the spacewalking crews and cost and schedule risks to the program. The ISS Program office has therefore developed a policy that advises the jettison of the most problematic objects. Such jettisoned items join a small family of nearly co-planar orbital debris objects that threaten the ISS on several timescales, besides threatening all satellites with perigee below the ISS orbit and the general human population on Earth. This analysis addresses the governing physics and the ensuing risks when an object is jettisoned. It is shown that there are four time domains which must be considered, each with its own inherent problems, and that a ballistic solution is usually possible that satsfies all constraints in all domains.

PROBLEM DOMAINS

The problem space for relative motion of a jettisoned object divides naturally into four time domains and regions of concern:

1) within the first seconds of trajectory, the direct motion away from the jettison point must not intersect with structure (i.e.: don't throw the object at a solar array, or at any other structure).

2) throughout the first orbit, (~5400 seconds) the Clohessy-Wiltshire solutions to the Hill equations determine a looping arc or relative motion that must clear the ISS structure by minimum specified margins in every dimension.

3) over all subsequent orbits until its decay (weeks to months of activity), the jettisoned object will exhibit increasing effects of differential drag relative to the parent spacecraft. These effects, which when coupled with the relative motion resulting from the initial jettison and from subsequent parent spacecraft maneuvers, create a complicated relative trajectory. Because the orbits are nearly co-planar, the trajectory of a jettisoned object remains problematic to the ISS, particularly during subsequent reboost activities

(contingency or planned) whose delta-V is comparable to or greater than the accumulated delta-V between the ISS and the object. This period occurs as long as the apogee of the object is above the perigee of the ISS. In some cases, this period can last for weeks or months. The extended free-flight phase of the object places it in the catalogue of general space debris in low earth orbit, thus posing an incremental risk to spacecraft with perigees lower than the ISS.

4) the final (essentially random) re-entry of the object presents a potential hazard to the general population of the planet. Although this has a specific event horizon of only a few minutes, and can be roughly predicted within a few days of final re-entry, at the time of jettison the exact re-entry is uncertain within several days or weeks. Thus the analysis must cover average population conditions within the latitudes bounded by the orbit inclination.

PROBLEM CONSTRAINTS

The jettison policy must assure minimum risk in each of these four major problem domains. The policy must also account for numerous practical constraints. The constraints affecting the problem are as follows:

a) The act of jettison must be within the gross motor skills of the crew: The range of view from a spacesuit helmet, poor tactile sense through the gloves, and fatigue can make the physical act of jettison more complicated than simple “shirtsleeve” environments would indicate. However, after much discussion and practical experience on air bearing floors, hydrolab tests, and real space experience, it has been agreed that in a simple “chest pass” 2-handed push, it is certain that the crew can control the trajectory of an object within a cone of half-cone 35 degrees from a target vector near normal to the chest. It therefore becomes a safety constraint to show that in the geometry of the EVA release operation, no ISS structure is within 40 degrees of the chest normal vector at the moment of jettison, allowing for a 5 degree margin. For aft-end jettisons, this is trivial, bit for mid-body jettisons, all structure must be taken into account, and depending upon ballistic requirements, the ISS attitude may have to be adjusted align the clearance corridor into the proper jettison direction.

b) The physical limits of motion and loading of the EVA crewmember and his/her equipment limit the total momentum that can be imparted: particularly to large objects. The foot restraints have bending and sheer limits (some enforced by load relief mechanisms) that reduce the peak force on the jettisoned object to be below 85 Newtons. Over a full arm extension of only 50 cm, a maximum total imparted energy of only 22 joules is possible, based upon all other mechanical constraints. This limit enforces moderate speeds on massive objects.

c) The geometry and construction of the jettisoned object determines handling, tracking, and relative ballistic number. If an object’s handling points are away form the center of mass, it is probable that a portion of the imparted momentum will go into rotation of the object, and not into relative linear motion. Most EVA objects have handholds that bracket the center of mass, which reduce this issue, but it must be accounted for in particularly unwieldy objects. It can be shown that for an object with mass M, characteristic length L, and moment of inertia I defined as:

$$I \equiv \frac{M\ L^2}{b}$$

(where b is a constant derived form the geometry (12 for long sticks, 2.5 for spheres), then accounting for rotation imparted by pushing at some distance *r* from the center of mass of the object, we get the maximum jettison speed of:

$$V_{jettison} = \frac{V_{push}}{\left[1 + \frac{br^2}{L^2}\right]}$$

for crew push-off speed of V_{push}. Note that this is independent of object mass. Generally, the value r is small compared to L, and thus $V_{jettison}$ is nearly V_{push}. However, this energy partitioning effect must be considered for large awkward objects.

It is further a structural requirement that the object have enough structural integrity to be handled with momentary acceleration during the jettison act. Lastly, it must have enough radar cross section to allow it to be tracked by the US Space Command (important in the third problem domain).

d) Program rules impose allowable clearances from the structure. The object must maintain a monotonically increasing separation from the release point for the first half orbit. It is preferred but not required to clear the ISS along the direction of initial jettison (the minus V-bar for aft jettison, positive Vbar for a posigrade jettison) by 25 meters if jettisoned from somewhere other than the end of the station. (In the case of end-outwards jettison, the acceptable clearance is obviously zero.) It must then clear in the vertical plane by 50 meters during the first orbit. It is required to clear in the V-Bar axis by more than 200 meters at the first and any subsequent crossing. The direction of jettison relative to the VBar is the dominant means to satisfy all geometry issues in problem domains 1 and 2, so long as trivially small velocity is achieved. The "rule of halves" governs subsequent orbits, saying that (except for final intentional proximity operations, dockings, and captures) any ballistic operation cannot bring spacecraft closer than half their previous distance within any next orbit. This is driven by drag effects and reboosts.

e) The future reboosts (planned and contingency) of the spacecraft complicate the relative motion. The ISS must regularly reboost to achieve altitude and phasing targets. In addition, it must always be prepared on a few days' notice to execute a debris avoidance maneuver of typically 0.5m/sec. Objects with higher ballistic number than the ISS are problematic in this regard. If such an object is jettisoned aft, the ISS must execute a posigrade burn within a few days after jettison to avoid drifting back into the jettisoned object, and must continue to do so until drag effects have pulled the object's apogee below the ISS perigee. The ISS must accumulate delta V faster than it is eroded by the drag difference between itself and the aft-jettisoned object. Similarly, if the object is jettisoned posigrade, the ISS must avoid the case where it accelerates into the jettisoned object in a future scheduled or unscheduled burn. (The sum of ISS delta V posigrade burns must not exceed the combined dV of the forward jettison velocity plus the accumulated delta V from drag effects.) Since by constraint (b) some large objects cannot achieve enough delta V to exceed the debris avoidance maneuver minimum requirements, recontact is a short-term potential issue for posigrade jettisons. Outrunning the object can be a long-term issue for retrograde jettisons. For the one object encountered so far that falls in the high B_N category, the ISS ballistics team has elected to jettison it retrograde, and to plan the ballistics such that a significant burn is desirable within days after the jettison EVA. This plan will shorten the total time that the object remains a threat for recontact with the ISS, by allowing a significant orbital phase separation early after jettison, and giving the object a maximum time to sink permanently below ISS operational perigee before the two return to the same orbit phasing ($\Delta X=0$).

f) The uncertainties in total drag of the ISS and of the jettisoned object will define significant dispersions in required delta-velocity over the orbital lifetime of the object. Again, this is problematic only for items of comparable or larger ballistic number to that of the ISS, since low bnallistic number items (always jettisoned aftwards) will continue to separate even in light atmospheres. The growth in delta velocity between the ISS and the jettisoned object is linear in the density of the surrounding atmosphere. Since atmospheric density varies significantly over the period in question, a range of future relative velocities must be calculated. Further, although the ISS ballistic number is known, it is variable with time as the arrays track the beta angle of the sun. Moreover, it is difficult to know exactly how the jettisoned object will spin (or stabilize), giving a range of possible ballistic numbers. Worst of all, some objects may be prone to future leaks of internal fluids, leading to propulsive events of their own. Thus, region 3 of the problem space encompasses the greatest uncertainty in ISS operations, and ultimately places the need to track the jettisoned object as one of the key portions of the policy.

g) The object must conform to NASA standard 1740.14 (section 7.1) which states any object re-entry must pose no more than 10^{-4} chance of human injury. For a random re-entry, the surviving debris cross section must be shown to be small enough that the chances of hitting a person in the average population density within the orbit latitude band lie below this threshold. (Typically, the cross section must be less than 8 m^2 to meet this criterion.) Otherwise, special review of the object in question must be pursued and it's jettison specifically approved by the administrator, if competing risks merit its continued consideration as a jettison candidate.

GENERAL SOLUTION

A jettisoned object has three factors governing its future separation:

1) its initial relative velocity (including X, Y, and Z components) imparted at the instant of jettison,

2) its ongoing differential drag, which causes it to decelerate at a different rate than the parent spacecraft (say, the International Space Station (ISS)).

3) the future propulsive events of the ISS (and potentially, of the object, particularly if gas leakage is a possibility).

The solutions to these three influences can be solved separately and then linearly superimposed to create a complete prediction of the trajectory.

Note that the first and third factors are completely independent of any geometric or mass properties of the ISS or the jettisoned object, and are purely dependent on orbital mechanics and initial separation velocity magnitude and angle.

The second factor accounts completely for all environmental, geometric and mass properties. The solution of separation due to initial jettison velocity is solved first, and the case of relative drag is treated as a perturbation. As a special case, we address the case where the two effects work in opposite directions (i.e., they ultimately cancel each other out) to develop a time constant for recontact before a reboost is initiated.

PART 1: Relative Motion Due to an Initial Jettison Velocity

Generally, a jettisoned object has a finite initial velocity with components $\dot{X}_o, \dot{Y}_o, \dot{Z}_o$.[1] The general relative motion of a second object in ISS coordinates is solved by the Clohessy-Wiltshire solutions to the Hill Equations [Ref. 1, 2], which linearize the difference in orbital motions of objects in similar orbits (say, the ISS and a jettisoned object) in the local coordinate system of one of the objects (say, the ISS):

$$\ddot{X} + 2\omega\dot{Z} = f_x \quad \textbf{(1a)}$$

$$\ddot{Y} + \omega^2 Y = f_y \quad \textbf{(1b)}$$

$$\ddot{Z} - 2\omega\dot{X} - 3\omega^2 Z = f_z \quad \textbf{(1c)}$$

where ω is the orbital frequency of the ISS orbit, and the functions f_i are zero for non-propulsive operations.

►**Note**: ω is nearly constant to within 1% accuracy over all credible ISS altitudes, at 0.000144 radians/sec, and is therefore be treated as this constant in these analyses. Due to much larger uncertainties in atmospheric densities and ballistic numbers, there is no engineering value in calculating a more precise value of ω at any specific jettison event.

In the absence of any drag or propulsive effects, the functions f_i are all zero, and the Clohessy-Wiltshire solution of these equations is:

$$X(t) = \left(6Z_o + \frac{4\dot{X}_o}{\omega}\right)\sin(\omega t) + \frac{2\dot{Z}_o}{\omega}\cos(\omega t) - \left(6\omega Z_o + 3\dot{X}_o\right)t + \left(X_o - \frac{2\dot{Z}_o}{\omega}\right) \quad \textbf{(2a)}$$

$$Y(t) = Y_o\cos(\omega t) + \frac{\dot{Y}_o}{\omega}\sin(\omega t) \quad \textbf{(2b)}$$

$$Z(t) = \frac{\dot{Z}_o}{\omega}\sin(\omega t) - \left(3Z_o + \frac{2\dot{X}_o}{\omega}\right)\cos(\omega t) + \left(4Z_o + \frac{2\dot{X}_o}{\omega}\right) \quad \textbf{(2c)}$$

The first derivatives (velocities) are:

$$\dot{X}(t) = \left(6\omega Z_o + 4\dot{X}_o\right)\cos(\omega t) - 2\dot{Z}_o\sin(\omega t) - \left(6\omega Z_o + 3\dot{X}_o\right) \quad \textbf{(3a)}$$

$$\dot{Y}(t) = -Y_o\omega\sin(\omega t) + \dot{Y}_o\cos(\omega t) \quad \textbf{(3b)}$$

$$\dot{Z}(t) = \dot{Z}_o\cos(\omega t) + \left(3\omega Z_o + 2\dot{X}_o\right)\sin(\omega t) \quad \textbf{(3c)}$$

A typical trajectory is shown in Figure 1. Although the initial X velocity is negative, the object quickly loops around forward of the release point, continuing in looping curves characteristic of orbital relative motion.

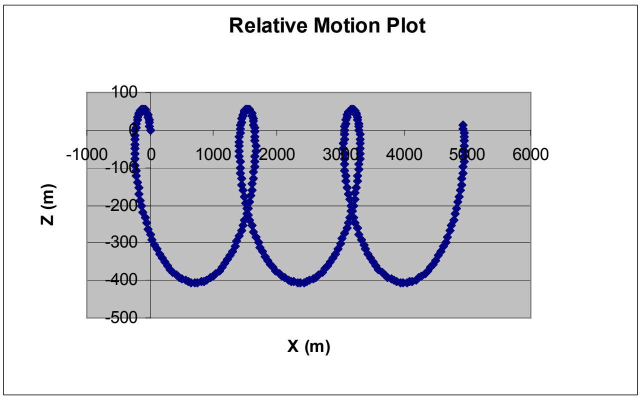

Figure 1: Typical orbital relative motion

First-Orbit Separation in the Vertical (Z) Axis

It is evident from equation **(2c)** and from Figure 1 that in the absence of drag of future propulsive effects, the value of Z repeats exactly at orbital frequency ω. We are only concerned with the clearance of Z when it first reaches the ISS Z axis (X=0 point, directly under the release point), because the object continues to separate ahead of the ISS on the remainder of the first, and throughout every subsequent orbit. We can thus ignore separation in Z after the direct nadir crossing. This direct nadir crossing can be derived by setting X=0 in equation **(2a)**, and solving for the time t_z at which that occurs:

$$X(t_z) = 0 = \left(6Z_o + \frac{4\dot{X}_o}{\omega}\right)\sin(\omega t_z) + \frac{2\dot{Z}_o}{\omega}\cos(\omega t_z) - \left(6\omega Z_o + 3\dot{X}_o\right) * t_z + \left(X_o - \frac{2\dot{Z}_o}{\omega}\right) \quad \textbf{(4)}$$

Under the assumption that Z_o and X_o are zero (release point is defined as the center of the

[1] (The convention for this paper and for the ISS is that X is in the V-bar, Y is in the H-bar, and Z is in the R-bar vector)

coordinate system, and is very close to the c.g. of the ISS)[2], the equation **(4)** reduces to:

$$4\dot{X}_o \sin(\omega t_z) - 3\dot{X}_o \omega t_z = 2\dot{Z}_o\left(1-\cos(\omega t_z)\right) \quad \textbf{(5)}$$

Rearranging:

$$\frac{\dot{Z}_o}{\dot{X}_o} = F_1(\omega t_z) \equiv \frac{\left(4\sin(\omega t_z) - 3\omega t_z\right)}{2\left(1-\cos(\omega t_z)\right)} \quad \textbf{(5a)}$$

$\dot{X}_o$ and $\dot{Z}_o$ are respectively the horizontal and vertical components (cosine and sine projections) of the same velocity V_o, (which is in turn the projection of the total velocity V_T into the orbital plane). V_o orients at an angle Φ relative to the aft (–X axis) vector in the XZ plane. From equation **(5a)** we see that:

$$\tan(\Phi) = F_1(\omega t_z) \quad \textbf{(5b)}$$

or

$$\Phi = F_2(\omega t_z) \equiv \arctan(F_1) \quad \textbf{(5c)}$$

plotted below:

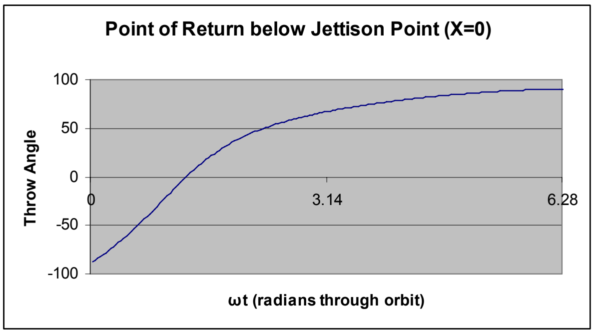

Figure 2: $\Phi = F_2(\omega t_z)$

$F_2(\omega t_z)$ defines a characteristic curve for the condition *X*=0 (the first nadir crossing of an aft-jettisoned object). Note that this curve is monotonic, and thus for any jettison angle Φ a *unique* value of ωt_z can thus be determined. (We find the inverse function of F_2 to get ωt_z as a function of Φ.) i.e.,

$$(\omega t_z) = F_3(\Phi) \quad \textbf{(6)}$$

[2] Because the ISS is such a large spacecraft, this is not always the case. Variations in Z are generally of more interest than in X, because of their impact on the net *shapes* of the curves. Variations in X only *offset* the solution slightly forward or aft of the c.g. Typically, the planned jettison velocity swamps any perturbations from jettison location offset in Z, but this is always checked after the basic solution has been worked to see if the assumption remains valid.

This pure function of Φ allows us to uniquely find the time of crossing below the release point, dependent only upon the *angle* of release, not the magnitude of the release velocity. Generally, $F_3(\Phi)$ is easier solved with a lookup table from equation **(5c)** and subs, than is convenient to derive geometrically. With the unique value of ωt_z that we generate (look up) from our proposed jettison angle Φ, we insert into equation **(2c)** to determine the separation in Z when the object passes directly below the jettison point.

$$Z(t_z) = \frac{V_o}{\omega}\sin(\Phi)\sin(\omega t_z) - \frac{2V_o}{\omega}\cos(\Phi)\cos(\omega t_z) + \frac{2V_o}{\omega}\cos(\Phi) \quad \textbf{(7)}$$

or:

$$Z = V_o\left[\frac{\sin(\Phi)\sin(F_3(\Phi)) - 2\cos(\Phi)\left[\cos(F_3(\Phi)) - 1\right]}{\omega}\right] \quad \textbf{(7a)}$$

Since ω is known to be essentially constant, we see that we can define $F_4(\Phi)$ such that:

$$Z = V_o * F_4(\Phi) \quad \text{or, } \mathbf{F_4(\Phi) = Z/V_o} \quad \textbf{(7b)}$$

where F_4 is dependent *only* on release angle.

►Note: The vertical separation *Z* for any specified jettison angle is thus directly linear in the jettison velocity V_o. A jettison of 2 cm/sec at angle J will clear exactly twice as far below the ISS as a 1 cm/sec jettison at exactly the same angle J.

$F_4(\Phi)$ is plotted below, and shows that for very small values of V_o, in jettison angles between +/- 70°, the object will clear well below the desired point 50 meters below the ISS. For instance, an initial jettison velocity V_o at 0.1 meters per second at say, 50 degrees positive (upwards) compared to pure retrograde motion, will clear the ISS vertically by at least (0.1)*F_4(50) = 140 meters, even before any drag effects contribute to further separation. It can be seen that for even a five centimeters per second jettison velocity, the minimum target separation distance of 50 meters is achieved if the value of $F_4(\Phi)$ is > 1000, which occurs for jettison angles $-23<\Phi<84$. It is interesting to note that nadir separation is maximized for any given velocity if the object is jettisoned at an upwards angle of 45 degrees. This is helpful for crew trying to avoid structure when jettisoning, since from the top side of the ISS one is helped by jettisoning upwards. (From the nadir one is already below structure, and downwards jettison simply adds to that separation with any finite angle.)

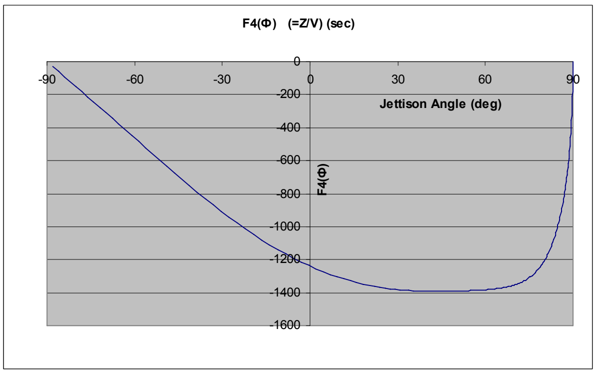

Figure 3: F4(Φ)= Z/Vo (sec)

Because it is a goal to have the object's apogee fall below the ISS perigee in the shortest possible time, it can be seen by inspection of equation 2c that, to the extent possible based on direct contact (domain 1) concerns, it is advantageous to aim the jettison as closely into the horizontal (XY) plane as possible. Figure 3 illustrates that in the condtiton of jettison in the horizontal plane, if possible, requires only 4 cm/sec aftwards component. To clear the 50 meter nadir criterion.

First-Orbit Separation in the Horizontal (Y) Direction

We note from equation **(2b)** that the solution for horizontal separation is a simple sinusoidal motion at orbital frequency, meaning that the jettisoned object crosses the orbit plane of the ISS exactly twice per orbit on the exact half period ($t_y=\pi/\omega$) marks, but is otherwise separated from the vehicle in the horizontal direction. Because the Z axis crossing varies with jettison angle, there is only a single point solution (at an upwards jettison angle of $+67^o$) where the orbital plane crossing and the Z axis crossing occur simultaneously. It is thus evident that with finite velocity in the Y direction and moderate in-plane vertical component of jettison, the total separation from the ISS c.g. will generally be larger at the Z axis crossing than the in-plane solution would suggest: i.e., there will generally be finite separation in Y at any particular X=0 (Z axis) event t_z, except at throw angle of projected angle 67^o in the orbital plane, when $t_y = t_z$.

First-Orbit Separation in the Longitudinal (X) Direction

It is evident from the non-oscillating term in equation **(2a)** that for any finite initial retrograde (i.e., negative) separation velocity $\dot{X}_o < -2\omega Z_o$,[3] the propagated $\dot{X}(t)$ has an average positive component over every orbit. Thus, *X(t)* generally increases with time (moves forward of the ISS with each orbit), with an oscillating component at the orbital period superimposed. Therefore, unless drag forces accumulate to counter the initial jettison velocity, if a jettisoned object is shown to clear forward of the ISS after one orbit, it will remain clear of the ISS for all subsequent orbits.[4]

We can determine the minimum separation in X by similar logic to the derivation of the Z separation. Note in general, for every Z=0 (X axis) crossing (say at time t_x), because the orbital pattern repeats in exactly orbital frequency, we see another crossing at time $t_x + (2\pi/\omega)$.

Longitudinal Separation with Initial Downwards Jettison Velocity Component

It can be seen by inspecting Figure 1 that an object thrown with an upwards component will cross the X axis aft of the ISS before looping around to the front of the station, while an object thrown with a downwards component of velocity will arc under and forward of the ISS without crossing the negative X axis after the point of release. Thus for objects thrown with a downwards component, the *minimum* separation forward of the ISS will come at time $t_x = (2\pi/\omega)$, (one full orbital period, or nearly 5400 seconds after jettison) while for objects thrown with an upwards component, we need to locate the aft-most crossing and propagate $+(2\pi/\omega)$ seconds forward of that. (It is instructive to note that these moments coincide exactly with the moments when Y also equals zero)

Recalling equation **(2a):** and setting $(\omega t_x) = 2\pi$:

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[3] $2\omega Z_o$ is typically small compared to typical jettison velocity magnitudes, so this relation is almost always true

[4] Assuming that the ballistic number of the object is equal to or greater than the ISS, and thus that drag effects do not decelerate the ISS to a subsequent re-rendezvous with the jettisoned object. We will treat the special case where this is not so in the final section.

$$X(t_x) = -3\dot{X}_o t_x \quad \textbf{(8)}$$

Thus, an aft-jettisoned object with any downward component will clear forward of its jettison point exactly three times its afterwards jettison velocity times its orbital period. Assuming that the clearance distance X is required to be 200 meters forward of the jettison point, the aft component of the jettison velocity ($-V_o\cos(\Phi)$) should therefore be:

$$\frac{X(t_x)}{3t_x} = \frac{-200m}{3 * 5400\,\text{sec}}$$

$$= \dot{X}_o \quad \textbf{(9)}$$

$$= -0.0123\ \text{m/sec}$$

to clear 200 meters forward of the ISS c.g. in LVLH flight. This trivial speed can be achieved in virtually all cases, and is the least constraining portion of the ballistics. Recall that for the minimum 50 meter separation in the Z-axial direction, several centimeters per second were required.

Longitudinal Separation with Initial Upwards Jettison Velocity Component

To find the Z=0 crossing aft of the ISS (at some time t_a)for an object released with an ***upward*** component, we substitute into equation **(2c):**

$$Z(t_a) = 0 = \frac{\dot{Z}_o}{\omega}\sin(\omega t_a) - \left(3Z_o + \frac{2\dot{X}_o}{\omega}\right)\cos(\omega t_a) + \left(4Z_o + \frac{2\dot{X}_o}{\omega}\right) \quad \textbf{(10)}$$

where with the same approximations and substitutions made in the earlier derivation, we see:

$$\Phi = F_5(\omega t_a) \equiv \arctan\left[\frac{2(1-\cos(\omega t_a))}{\sin(\omega t_a)}\right] = \arctan\left[2\tan\left(\frac{\omega t_a}{2}\right)\right] \quad \textbf{(11)}$$

and, as in equations (5) and (6), we can assert:

$$(\omega t_a) = F_6(\Phi) \quad \textbf{(12)}$$

F6(Φ) is plotted below:

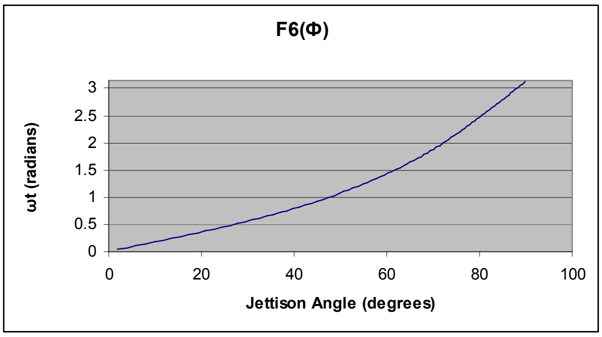

Figure 5: F6(Φ) locates the phase ωta of the aft X-axis crossing

Substituting our known ωt_a into equation **(2a)** we get the location for the closest aft X crossing:

$$X(t_a) = \left(6Z_o + \frac{4\dot{X}_o}{\omega}\right)\sin(\omega t_x) + \frac{2\dot{Z}_o}{\omega}\cos(\omega t_a) - \left(6\omega Z_o + 3\dot{X}_o\right)t_a + \left(X_o - \frac{2\dot{Z}_o}{\omega}\right) \quad \textbf{(13)}$$

Taking the usual approximations:

$$X(t_x) = \frac{4V_o}{\omega}\cos(\Phi)\sin(\omega t_x) + \frac{2V_o}{\omega}\sin(\Phi)\cos(\omega t_x) - (3V_o)\cos(\Phi)t_x - \frac{2V_o}{\omega}\sin(\Phi) \quad \textbf{(14)}$$

$$X(t_a) = \left(\frac{V}{\omega}\right)\begin{Bmatrix}\cos(\Phi)[4\sin(F_6(\Phi)) - 3F_6(\Phi)] \\ + 2\sin(\Phi)[\cos(F_6(\Phi)) - 1]\end{Bmatrix}$$

(15a)

Again recognizing that ω is virtually constant over all conditions:

$$\therefore X(t_a) = V_o * F_7(\Phi) \quad \textbf{(15b)}$$

$F_7(\Phi)$ is plotted below:

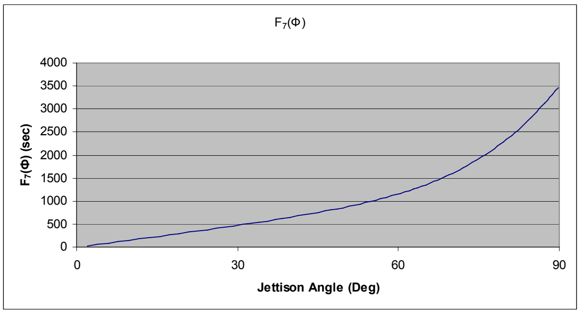

Figure 6: F7(Φ) defines the ratio X/Vo of the aft X-axis crossing of an object jettisoned at an upwards angle.

From a jettison angle of zero to +45 degrees, the curve is nearly linear (to within 4.5% accuracy) such that the aft crossing is at

$$X_{aft} = -(Vo*16*\Phi) \quad \textbf{(16)}$$

where Φ is in degrees, Vo is in meters per second, and X is in meters. The minimum separation forward of the release point will happen exactly one period later than this aft crossing point, and is the linear sum of the aft separation and 3 times the period times the magnitude of the aftwards jettison velocity, as shown in equation **(8)**.

The combined effects of velocity and jettison angle on forward longitudinal separation are plotted below. The range of parameters for which separation is less than the assumed minimum of 200 meters is shown in the black region. The contour lines are in 200 meter increments. Clearly, any small aftwards velocity within credible release angles will cause many hundreds of meters of forward separation, growing to more than a kilometer for velocities as low as 10 cm/sec:

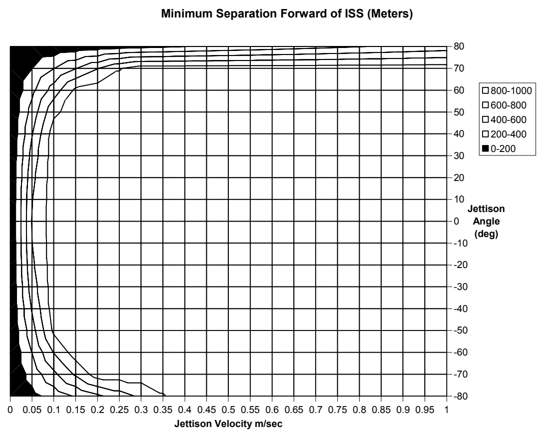

Figure 7: Longitudinal separation in X

Example:

An object is jettisoned at 0.10 m/sec, at an upwards elevation of +20 degrees and a lateral angle of +30 degrees. What are the distances from the ISS at the closest forward X-axis crossing, and the nadir crossing?

Solution:

1) Calculate V_o:

$Vo = V_T\cos(30) = .0866$m/sec

2) Calculate $Y(t)$:

$Y(t) = (1/\omega)V_T\sin(30)*\sin(\omega t)$ = **347.2*sin(ωt) meters**

3) Locate aft crossing point:

Equation **(16)** yields an aft separation of $X=-(V_o *16*20)=$ -27.7 meters at the aft X-axis crossing

4) Locate forward crossing point (1 orbit beyond aft crossing):

From equation **(8),** we see that the minimum forward separation will be

-27.7m+3*(0.0866m/sec)*cos(20°)*5400sec=**1290 m**.

5) Locate nadir crossing:

From equation **(7b)** and the associated plot of F_4 at value (Φ=20) ($F_4(20)$ =1355) we see that the vertical separation at the crossing below the initial release point will be:

$Z_o = V_o*F_4(20)$ = 0.0866m/sec*1355sec= **117.3 meters**.

6) Locate separation in Y at the nadir crossing (X=0) point.

We see as well from $F_3(\omega t)$ (see Figure 2) that the nadir crossing occurs at almost exactly $(\omega t)=(\pi/2)$. Therefore, from step 2 above we see that $Y(t)$ at the nadir crossing is **+347.2 meters.**

7) Determine the total distance to ISS at the nadir crossing point:

The total separation at the nadir crossing is the vector sum of the separations in Z and Y when the object returns to the X=0 coordinate:

Separation = $((347.2)^2+(117.3)^2)^{1/2}$ = **366.5 meters**

Thus, for a given location and surrounding ISS geometry, it is possible to quickly evaluate the ability to meet the constraints of problem domains 1 and 2.

PART 2: DRAG EFFECTS

Individual Object Drag

Any object in low earth orbit (LEO) will decelerate due to its own drag with the relation:

$$\frac{dV}{dt} \equiv a = -\left[\frac{C_D A}{m}\right] * \frac{\rho}{2} * V^2 \qquad \textbf{(17)}$$

where $\boldsymbol{\rho}$ is the density at altitude (typically well above 10^{-12} kg/m^3, which is the density at 400km altitude at F(10.7)=75, i.e., solar minimum conditions) and $\boldsymbol{V}$ is the velocity in orbit, typically about 7800 m/sec, $\boldsymbol{A}$ is the projected area (in m^2)of the object in the V-bar, $\boldsymbol{m}$ is its mass in kg, and $\boldsymbol{C_D}$ is its (dimensionless) coefficient of drag, typically 2 or greater. Recently, the Coefficient of Drag for the ISS has been established to be 2.07.

A *Ballistic Number* (B_N) in units of kg/m^2 is defined for each object

$$B_N = \left[\frac{m}{C_D A}\right] \qquad \textbf{(18)}$$

Note that any non-spherical object will have a *range* of ballistic numbers that depend upon the orientation of the object with respect to the flight vector (and hence, upon the projected area of the object). Generally then, when studying relative ballistics of two objects, one must select extremes of projected areas to show clearance of the entire range of Ballistic Number possibilities, unless the attitudes of the objects are assured. In the screening analysis, the term "ballistic number" is taken to be the most extreme B_N in the possible range, picked to establish the limiting case. Note that the ISS has a ballistic number that varies substantially throughout any one orbit, as a function of solar array angle to the velocity vector.

Equation **(17)** reduces to a general deceleration of greater than 0.00006/ B_N m/sec^2 for typical orbital parameters of velocity and minimum density of 10^{-12} kg/m^3.

$$a = V^2 \rho / B_N \approx 63000000 \rho / B_N \qquad \textbf{(19)}$$

where ρ is in kg/m^3 and a is in m/sec^2. With a ballistic number of approximately 200kg/m^2, and density of about $5*10^{-12}$ kg/m^3, the ISS decelerates at approximately $15x10^{-7}$ m/sec^2, losing about 13 cm/sec per day, or about 3.9 m/sec/month.

Differential Drag

Two objects of differing ballistic number (say ISS with B_{N1} and a jettisoned object with B_{N2} at zero initial relative velocity) will decelerate with a rate difference (*differential* drag acceleration, or Δa_D) of

$$\Delta a_D = \frac{dV_{relative}}{dt} = \ddot{X}_D = \left(-\frac{\rho}{2B_{N1}} * V^2\right) - \left(-\frac{\rho}{2B_{N2}} * V^2\right) \qquad \textbf{(20)}$$

$$\Delta a_D = \ddot{X}_D = \left(B_{N2}{}^{-1} - B_{N1}{}^{-1}\right) * \frac{V^2 \rho}{2} \qquad \textbf{(21)}$$

which we can reduce by equation **(17)** to be:

$$\Delta a_D = 6.3*10^7 \rho \left(B_{N2}{}^{-1} - B_{N1}{}^{-1}\right) \qquad \textbf{(22)}$$

Adding in the effects of orbital mechanics, a general separation of two objects decelerating relative to each other can be derived [Ref.3] for small decelerations:

$$\Delta X_D = \frac{-\Delta a_D * \left(8 - 8\cos(\omega t) - 3(\omega t)^2\right)}{2\omega^2} \qquad \textbf{(23)}$$

$$\Delta Z_D = 2\Delta a_D * \frac{(\omega t - \sin(\omega t))}{\omega^2} \qquad \textbf{(24)}$$

Note that both Z and X have terms in ωt that quickly dominate the small cyclic variations (caused from initial separation of coincident bodies as the lower ballistic number object first drags backwards). After the initial "slingshot" behavior seen with more impulsive delta V's, the curve quickly settles into a parabolic shape associated with constant decay. The generic curve in *XZ* space for early times is shown in Figure 8 below:

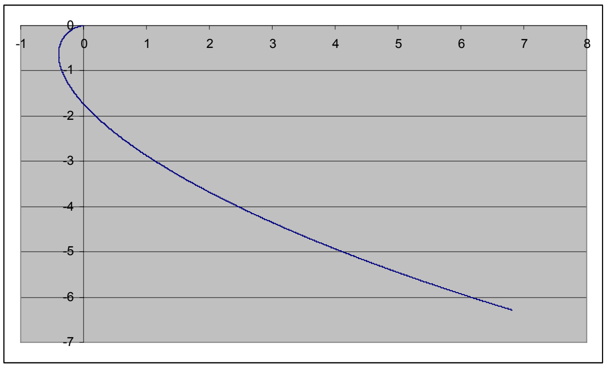

Figure 8: Generic (Dimensionless) X-Z profile due to Differential Drag

The curve in Figure 8 scales linearly with *($\Delta a_D/\omega^2$)* (where Δa_D for any object relative to ISS is typically of the order 10^{-7} m/sec^2 and ω^2 is $2.07*10^{-8}$), yielding a scalar multiplier of on the order of 5 by which the values in the plot can be multiplied to evaluate actual distances (in meters). In the long term, it is evident that :

$$\Delta X_D = \frac{3\omega^2 \Delta Z_D{}^2}{8\Delta a_D} \qquad \textbf{(25)}$$

Note that as Δa_D becomes *very* small (caused by comparable ballistic number, low atmospheric density, or both), the change in Z for any change in X becomes proportionately smaller. Given that we have seen that large amplitudes in Z around the X axis are possible, it can take a very long time for the object to sink such that its apogee is lower than the perigee of the ISS, and recontact becomes a possibility if the relative acceleration can cancel the jettison velocity.

Note also that the actual value of Δa_D is very hard to estimate exactly, both due to the attitude-dependent nature of both ballistic numbers and the uncertainty in atmospheric density at the time and altitude of release. However, we can establish target values easily.

Although an exact prediction is hard based upon true knowledge of the objects and the atmosphere, we can note several features of equations **(23)** and **(24):**

a) After *t=0*, the separation in *Z* is always positive: the object with larger drag is *always* trending below the object with lower drag.
b) The vertical and horizontal separations are each linear with Δa_D, therefore the general shape of the *X* vs. *Z* plot is universal. Only the scale changes.

c) The separation in X always starts out negative (the high drag object starts by moving aft of the low drag object), and then goes positive (forward of the low drag object). The point of maximum negative separation occurs when the term $\left(8-8\cos(\omega t)-3(\omega t)^2\right)$ is at a maximum, which occurs when $(\omega t)=$ 1.275, or 20.3% of the way through the orbit from the moment of separation, where X characteristic is .395. The point when the object returns to a point directly below the release point occurs when (ωt) =1.831, or 29.1% of the way through the orbit, with the value of the Z characteristic = -1.73.
d) In the long term, when the harmonic terms are small compared to the linearly increasing terms, $\Delta X_D \approx 1.5\,\Delta a_D t^2$, $\Delta Z_D \approx \frac{-2\Delta a_D t}{\omega}$, which defines a parabola.
e) There is no component in the Y direction. Drag has no "lift" component, and cannot deflect the object out of its orbital (XZ) plane.

From the above observations and from equation **(22)**, it is possible to determine the required ballistic number difference to achieve the desired 50-meter nadir separation based solely upon drag. As noted above, the nadir separation below the jettison point has a value of 1.73* $(\Delta a_D/\omega^2)$ where

$$\Delta a_D = \left(B_{N2}^{-1} - B_{N1}^{-1}\right) * \frac{V^2\rho}{2} \quad \textbf{(26)}$$

Thus we see that the nadir crossing will occur at a distance:

$$Z = 1.73 * \Delta\left(B_N^{-1}\right)\rho * \frac{V^2}{2\omega^2}$$
$$= 0.865 * \Delta\left(B_N^{-1}\right)\rho R^2 \quad \textbf{(27)}$$

where R is the radius from the spacecraft to the center of the earth (~6,780,000 meters) so:

$$Z = 0.865 * \Delta\left(B_N^{-1}\right)\rho * 4.596 * 10^{13}$$
$$= 3.976 * 10^{13}\Delta\left(B_N^{-1}\right)\rho$$

From this, we can derive a minimum $\Delta\left(B_N^{-1}\right)$ for any given separation and atmospheric density. For example, if the density is 10^{-12} kg/m^3, we see that the required $\Delta\left(B_N^{-1}\right)$ is 50/39.6 = 1.263 m^2/kg. This is a very diaphanous structure, considering that the ISS is approximately 0.005 m^2/kg. Even towels (occasionally used during EVA to wipe hazardous surfaces) would have to orient broadside to the velocity vector to approach this value. Thus, it is assumed that no object will naturally separate within the ISS guidelines, and should be positively jettisoned. Further, the drag effects for most credible bodies after one orbit are very small compared to the ballistic effects of the imparted jettison velocity, and so are typically ignored for the first orbit calculation of trajectory (domains 1 and 2). Cumulatively, the drag effects dominate the equation in domain 3.

COMBINATION OF EFFECTS

Jettison of Objects with <u>Lower</u> Ballistic Number than the ISS.

It is intuitive (and obvious from equation **(24)**) that if any object has a lower ratio of mass to area than the space station (B_{N2} is smaller than B_{N1}) then such an object will *continue* to decelerate relative to the ISS after an aftwards jettison, achieving progressively lower orbit and spiraling well below and in front of the ISS. The differential drag effect increases the net separation from the ISS at all times, and is a benefit to the separation in X and Z. The drag effect is dominated by the jettison effect in the early phase of the separation, as long as the jettison velocity is more than a few cm per second. We have seen in the example that large separations (many times larger than required) are expected for only 10 cm/sec jettison velocity. By comparison, 10 cm/sec is similar to the deceleration of the station itself over an entire day, due to drag effects.

Jettison of Objects with <u>Higher</u> Ballistic Number Than the ISS

We face a potential problem if a high B_N object is jettisoned retrograde from the ISS, but decelerates more slowly than the ISS. In such a case, the natural drag difference will ultimately decelerate the ISS more than the jettisoned object, and at some point the steady-state effects of equation **(23)** will cancel those of equation **(2a),** i.e.:

$$\Delta a_D t > \dot{X}_o \quad \textbf{(28)}$$

Case A : Aft Jettison of Very Hogh Ballistic Number Objects

The ISS Trajectory Operations and Planning Officers (TOPO team) have procedures in place to execute a debris avoidance maneuver for any object that can be tracked in orbit, but such procedures need a minimum 2-day period to identify the need for, plan, and execute debris avoidance maneuver. In addition, at low jettison speeds, it will conservatively take about two days to clearly establish the ballistics of the separated object using ground radars, maning that four days is the minimum acceptable predicted time of return to the vicinity of the ISS before a reboost can or must be performed, and two days (to establish proper range) where it should be performed to achieve long-term separation. Thus, if we set t = 2 days =172,800 seconds, we can derive a minimum aft jettison velocity component for such an object, so that the trajectory team can respond if necessary:

$$\dot{X}_o > \Delta a_D * 172800 \quad \textbf{(29)}$$

from Equation **(22):**

$$\dot{X}_o > 172800 * 63000000 \rho \Delta \left(B_N^{-1} \right) \quad \textbf{(30)}$$

$$\dot{X}_o > 1.0884 * 10^{13} \rho \Delta \left(B_N^{-1} \right) \quad \textbf{(31)}$$

In the limiting case of an infinitely massive, zero-area (javelin-like) object that does not decelerate at all, we see the equation reduce to the simple deceleration of the ISS accumulating a retrograde velocity matching the initial jettison velocity:

$$\dot{X}_o > 1.0884 * 10^{13} \frac{\rho}{B_{N(ISS)}} \text{ (m/sec)} \quad \textbf{(32)}$$

All other objects will need smaller jettison velocities for 2-day separation rate reversals.

With the B_N of the ISS approximately 160 kg/m^3, we get worst-case minimum separation velocity of a low ballistic number object in a maximum atmosphere of 10^{-11} kg/m^3 of:

$$\begin{aligned} \dot{X}_o &> 1.0884 * 10^{13} * 10^{-11} * .006 \\ &= 0.653 m / \sec \end{aligned} \quad \textbf{(33)}$$

It can be seen form constraint (b) that this value can be hard to achieve for particularly massive objects.

At an atmosphere of 10^{-12}kg/m^3, the minimum value is 1/10 of the 10^{-11}kg/m^3 case, or 6.5 cm/sec.

Forward Jettison of Very High Ballistic Number Objects

Since the natural tendency of the ISS is to decelerate relative to an object with higher ballistic number, we can consider posigrade jettison (intentional velocity into the V-bar) for such objects. This *might* a desirable option if a large delta-V cannot be guaranteed for the jettisoned object. However, by constraint (b), delta-V on very massive objects can be substantially less than the delta-V required for a debris avoidance maneuver. Thus, the ISS is in some risk for several days of outrunning the jettisoned object if a contingency burn is required, and the delta V budget must be managed in such a way as to never close the gap during subsequent phasing burns for other programmatic goals. The *intent* of the forward jettison is to cause an ever-growing separation of the higher-orbiting object above and aft of the ISS. By the time the ISS could lap one orbit and re-encounter the object, it would have witnessed one or more routinely scheduled reboosts, which occur on average every 60 days. During those boosts, the ISS would close some of the gap towrds the higher B^N object, and it is imperative not to 'outrun' it.. Since relative drag rate can be up to 0.31 m/sec/day for the ISS, the phasing relative to an infinite-mass object would be one full orbit when time t is such that:

$$\begin{aligned} \Delta X &= 2\pi R_{earth} = 4.26 * 10^7 \, meters \\ &\equiv 1.5 * \Delta a_D t^2 \\ &= 1.5 * 6.3 * 10^7 * \rho B_{N(ISS)} * t^2 \end{aligned} \quad \textbf{(34)}$$

or

$$t = \sqrt{\frac{0.4733}{\rho B_N}} \approx 2.8 * 10^6 \sec \approx 1 month \quad \textbf{(35)}$$

minimum for a jettisoned object of infinite mass in the maximum atmosphere of 10^{-11} kg/m^3. Objects of finite low ballistic number, and/or in lower density atmospheres, can go many months or years before being lapped by the ISS. Considering though, that aft jettison would have been ruled out because of lack of adequate jettison speed, one faces a long-term restriction in not accumulating more delta-V than has been generated between the object and the ISS until the orbit can be cleanly rasied above that of the jettisoned object. Thus, it is generally accepted (after mch rigorous debate) that aft jettison is *usually* the correct answer, even for massive objects. A reboost is typically scheduled shortly after a planned EVA jettison of such a massive object, just to give maximum flexibility on future ballistic planning.

Jettison of Objects with Ballistic Number Comparable to the ISS

If the object's ballistic number is at all comparable to the ISS, then the small difference in their ballistic numbers becomes a dominant driver of equation **(31)**, and since either jettison direction yields a long time before revisit, the responsible action is to jettison retrograde so that the object re-enters the atmosphere sooner. This is particularly true if a large delta-V (on the order of 1 m/sec) can be imposed. The difference of 1 m/sec retrograde vs. posigrade leads to an orbital lifetime difference of approximately two months for the jettisoned object.

CONCLUSION

Aft jettison is recommended for almost all expendable EVA objects. In extreme case of very dense and massive objects (with B_N higher than that of the ISS, for which large delta-V cannot be guaranteed) forward jettison is a possibility, but not necessarily the best option, because future reboost needs may overrun the object. These cases need to be carefully examined. In all high ballsitic number cases examined to date, aft jettison was ultimately the chosen option, making it (to date) the only means of jettsion in the ISS program for all objects.

Aftwards jettison of objects from the ISS is guaranteed to clear the station by more than the required 50 meters nadir and 200 meters forward thresholds over a wide range of jettison angles and trivially small speeds. Total speeds as low as 5 cm/sec over large jettison angles will meet the criteria. Only a 1.2 cm/sec aft (-X) velocity component is sufficient to clear 50 meters nadir and 200 meters forward of the ISS, allowing for a wide choice of possible jettison angles to clear immediately surrounding structure in the initial outbound trajectory.

For very massive objects with potentially tiny jettison speeds, it can be appropriate to loft diagonally away from the X axis at moderate angles (up to 70 degrees, 45 being optimum) to assure that the target 50 meter nadir separation is achieved, as this is generally the more stringent threshold. When jettison speeds of only a few centimeters per second can be assured, it is appropriate to loft the object biased (as much as the initial path outwards will safely allow) towards the aft horizontal plane to minimize the time that it remains a threat to ISS operations, and to minimize its time in orbit. I.e., the path will have a Z velocity component only large enough to clear structure on the initial (domain 1) departure, and all other constraints are easily met.

Almost no credible EVA-retrievable equipment will have a ballistic number high enough to guarantee its separation from the ISS solely by drag effects outside the 50-meter-nadir 200-meter-forward minimum clearance zone.

Because a jettisoned object represents an ongoing threat to other orbiting spacecraft until its decay, and because its future path is highly dependent upon uncertain parameters of ballistic number and atmospheric density, it is required that any such object must have a radar cross section large enough to be tracked by ground assets.

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