Structure Detection of Nonlinear Aeroelastic Systems with Application to Aeroelastic Flight Test Data: Part II

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NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03–0039–1 Date: February 7, 2003 Photo By: Alan Brown
NASA Dryden's highly–modified Active Aeroelastic Wing F/A–18A shows off its form during a 360–degree aileron roll during a research flight.

NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03–0039–14 Date: February 7, 2003 Photo By: Alan Brown
NASA's Active Aeroelastic Wing F/A–18A research aircraft rolls upside down during a 360–degree aileron roll on a test mission.

Outline

- Motivation
- Nonlinear Model Form
- Structure Detection
- Least Absolute Shrinkage and Selection Operator (LASSO)
- Objectives
- Results
  - Assess LASSO as a Structure Detection Tool: Simulated Nonlinear Models
  - Applicability to Complex Systems: F/A-18 Active Aeroelastic Wing Flight Test Data
- Conclusions

Motivation

- Parsimonious system description

- Black-box model

- Efficient control strategies

- Insight into functionality of system

Nonlinear Model Form

- Linear statistical model

$$z(n) = \sum_{j=1}^{p} \theta_j f(\varphi_j(n)) + e(n)$$

  - z: observed system output
  - θ_j: unknown system parameter
  - φ_j: regressor
  - e: independent Gaussian variable, zero-mean, constant variance σ^2
  - f: nonlinear mapping

- Let φ be described as:

$$\varphi(n) = [1, z(n-1), \cdots, z(n-n_z), u(n), \cdots, u(n-n_u), e(n-1), \cdots, e(n-n_e)]^T$$

  - Special case f polynomial: $u^2(n-3)$, $u(n)u(n-1)$, $z(n-1)z(n-2)$, $u^2(n-1)z(n-2)$
  - General case f: wide variety of nonlinear functions such as a sigmoid
  - NARMAX

Structure Detection

- NARMAX models described by few terms
- Maximum number of candidate terms:

$$
\begin{aligned}
p &= \sum_{k=1}^{l} p_k + 1 \\
p_k &= \frac{p_{k-1}(n_z + n_u + n_e + k)}{k}, \quad p_0 = 1
\end{aligned}
$$

  - Example: model of order: $O = [4\ 4\ 4\ 2] \Rightarrow p = 105$ candidate terms
  - The curse of dimensionality!

- Often leads to computationally intractable combinatorial optimisation problems

Several Fundamental Approaches to Structure Detection

- Exhaustive search
  - Every possible subset of the full model is considered
  - Requires large number of computations
- Covariance matrix, $\mathbf{P}_{\theta}$
  - Based on input-output data and estimated residuals to assess parameter relevance
  - Parameter variance estimates often inaccurate when the number of candidate terms large
- Bootstrap method
  - Numerical procedure for estimating parameter statistics
  - For convergence: number of data points needed *at least* 10 times square of initial number of candidate terms

LASSO

- Least absolute shrinkage and selection operator (LASSO)

$$\min_{\boldsymbol{\theta}} \frac{1}{2} \|(\mathbf{Z} - \boldsymbol{\Phi}\boldsymbol{\theta})\|_2^2 + \lambda \|\boldsymbol{\theta}\|_1$$

  – Least-squares like problem: addition of ℓ_1 penalty on parameter vector

- LASSO shrinks least-squares estimator towards 0, potentially sets $\theta_j = 0$ for some j

- Regularisation parameter $\mathbb{R} \ni \boldsymbol{\lambda} = [\lambda_{min}, \ldots, \lambda_{max}]$ controls the trade-off between approximation error and sparseness

- LASSO behaves as a structure selection instrument

Unique Optimum

Assumption 1. *Input signal is persistently exciting.*

Theorem 1. *If the excitation signal is persistently exciting, LASSO will have a unique optimum.*

Proof.

(i) Since the excitation signal is persistently exciting implies $\rightarrow \boldsymbol{\Phi}^T\boldsymbol{\Phi}$ is positive definite

(ii) As a result the first term of

$$\min_{\boldsymbol{\theta}} \frac{1}{2} \|(\mathbf{Z} - \boldsymbol{\Phi}\boldsymbol{\theta})\|_2^2 + \lambda \|\boldsymbol{\theta}\|_1$$

is a strictly convex function.

(iii) Since the second term is convex, it follows that the sum is strictly convex and a unique optimiser is guaranteed. $\square$

Convergence

Assumption 2. *Optimal regularisation parameter, λ^*, is known.*

Theorem 2. *If the excitation signal is persistently exciting and has a unique optimum, LASSO will converge to a unique global minimum.*

Proof. Since

$$\min_{\boldsymbol{\theta}} \frac{1}{2} \|(\mathbf{Z} - \boldsymbol{\Phi}\boldsymbol{\theta})\|_2^2 + \lambda \|\boldsymbol{\theta}\|_1$$

is strictly a convex optmisation problem the solution will converge to a unique global minimum. $\square$

Solution of LASSO

- Quadratic programming framework with slack variables

$$\min_{\mathbf{x}} \frac{1}{2}\mathbf{x}^T\mathbf{M}\mathbf{x} + \mathbf{c}^T\mathbf{x} \quad \text{such that } x_k \geq 0, \text{ and where,}$$

$$\mathbf{M} = \begin{bmatrix} \boldsymbol{\Phi}^T\boldsymbol{\Phi} & -\boldsymbol{\Phi}^T\boldsymbol{\Phi} \\ -\boldsymbol{\Phi}^T\boldsymbol{\Phi} & \boldsymbol{\Phi}^T\boldsymbol{\Phi} \end{bmatrix}, \ \mathbf{c} = \lambda\mathbf{1} - \begin{bmatrix} \boldsymbol{\Phi}^T\mathbf{Z} \\ -\boldsymbol{\Phi}^T\mathbf{Z}, \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} \boldsymbol{\theta}^+ \\ \boldsymbol{\theta}^- \end{bmatrix}$$

- Model parameters: $\boldsymbol{\theta} = \boldsymbol{\theta}^+ - \boldsymbol{\theta}^-$

- QP problem readily solved using standard optimisers

- Given suitable λ general structure computation problem can be solved

Selection of Regularisation Parameter: λ

- Method of cross-validation to estimate prediction error

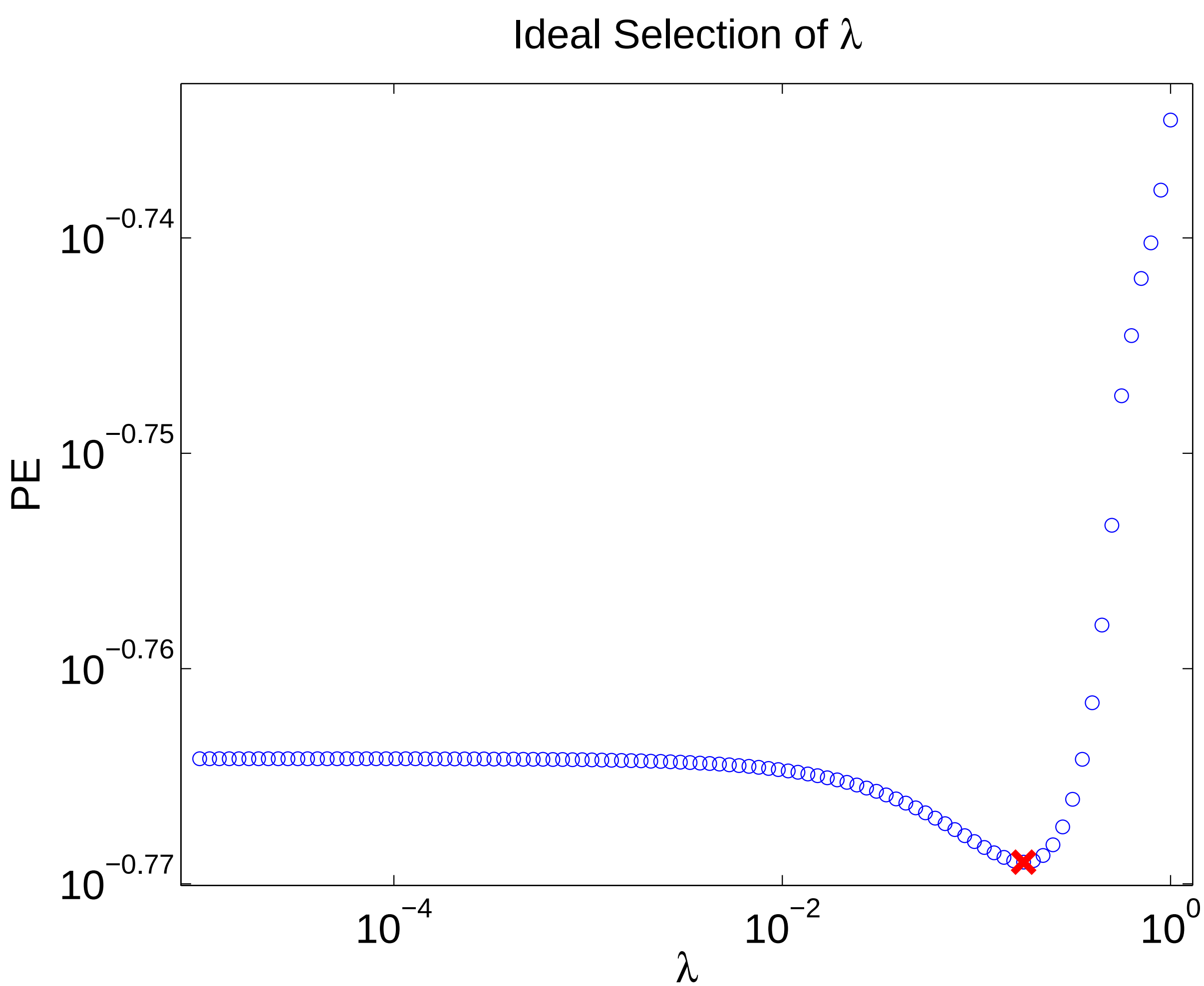
$$PE(\lambda) = E\left[\mathbf{Z} - \Phi\theta\right]^2$$

- Determined by numerically minimising the cross-validation error across a discrete set of logarithmically spaced λ values

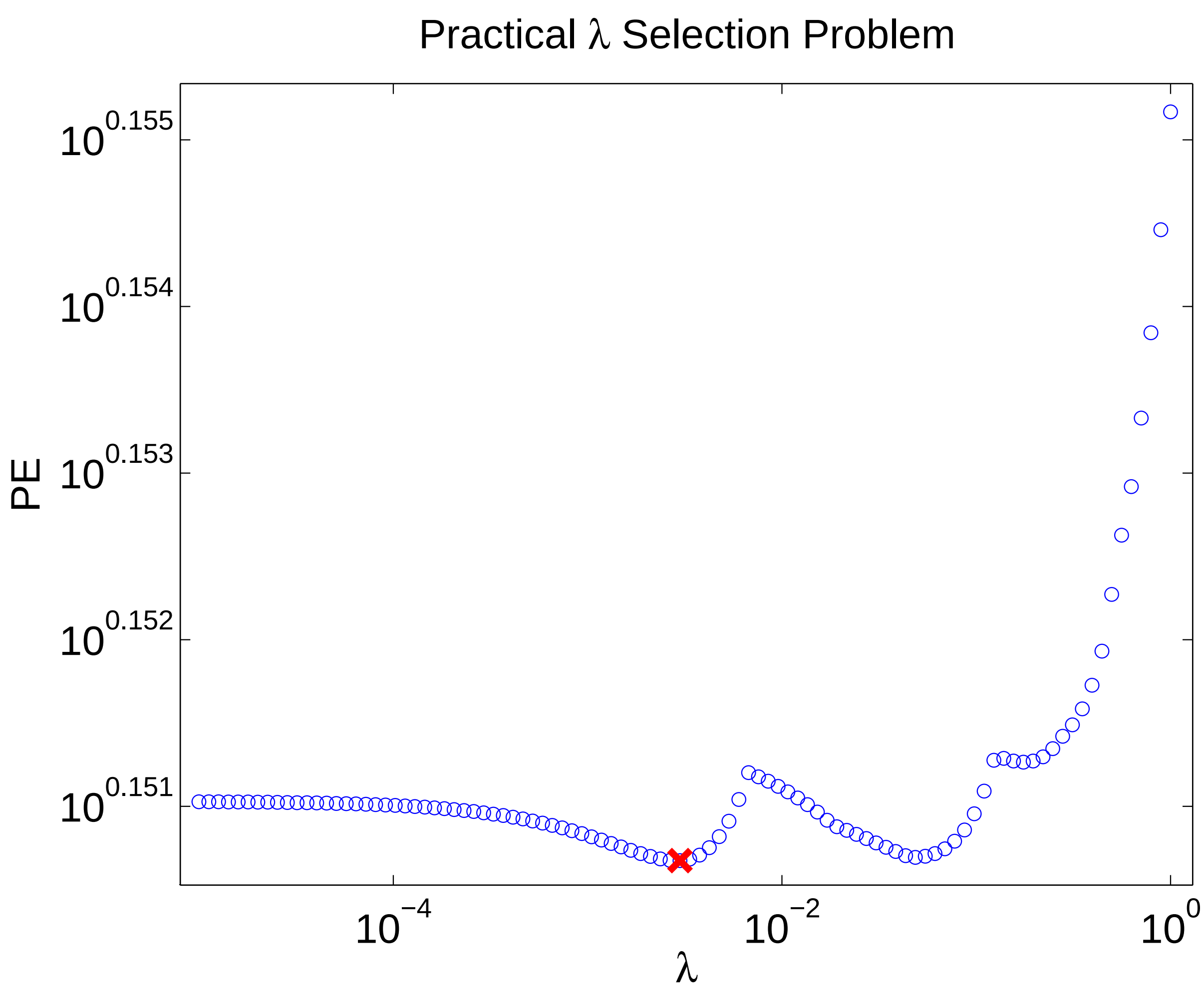
$$10^{\lambda_{min}} \leq \lambda \leq 10^{\lambda_{max}}$$

- Regularisation parameter, λ, is chosen to minimise this estimate

Ideal λ



Local Minima



Objectives

- Investigate LASSO as a structure detection tool
- Hypothesise useful for structure detection
- Performance evaluation

Simulated System

$$z(n) = 0.4[u(n-1) + u(n-1)^2 + u(n-1)^3] + 0.8z(n-1) - 0.8e(n-1) + e(n)$$

- Model order known: $O = [1\ 1\ 1\ 3]$
  - 35 candidate terms
  - True system has only 5 true terms

Simulations

- One thousand Monte-Carlo simulations
  - Input white, uniform distribution
  - Each output realisation had unique Gaussian distributed, white, zero-mean, noise sequence added
- Noise amplitude increased 5 dB increments, from 20 to 0 dB SNR
- $N_e = 667$ points for estimation and $N_v = 333$ for validation
- Regularisation parameter 1,000 logarithmically spaced λ's: $10^{-10} \leq \lambda \leq 10^{1.5}$
- Compare LASSO with covariance matrix, $\mathbf{P}_{\boldsymbol{\theta}}$ approach
  - Parameters tested for significance at 95% confidence-level

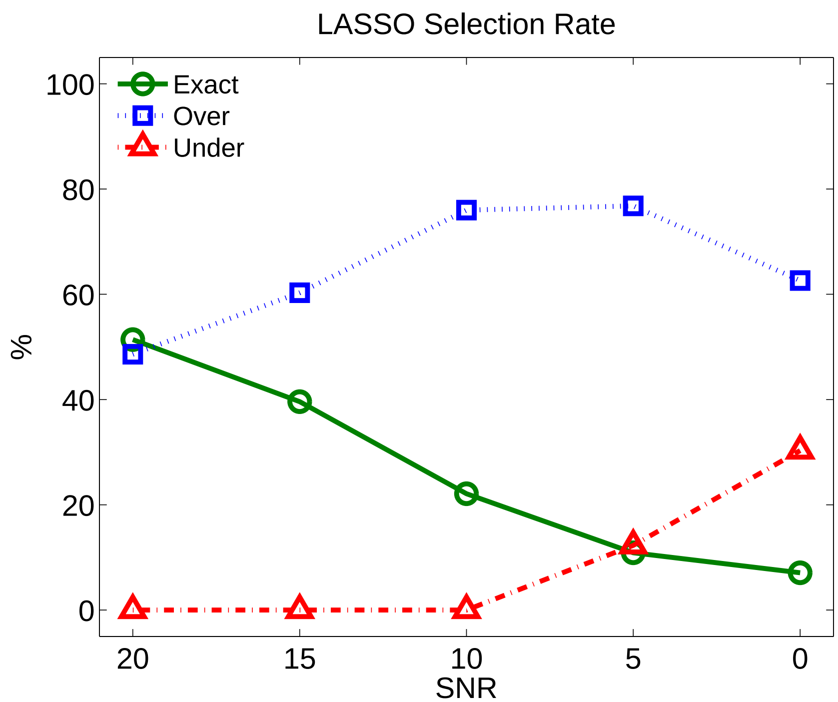
Results Classified into Three Categories

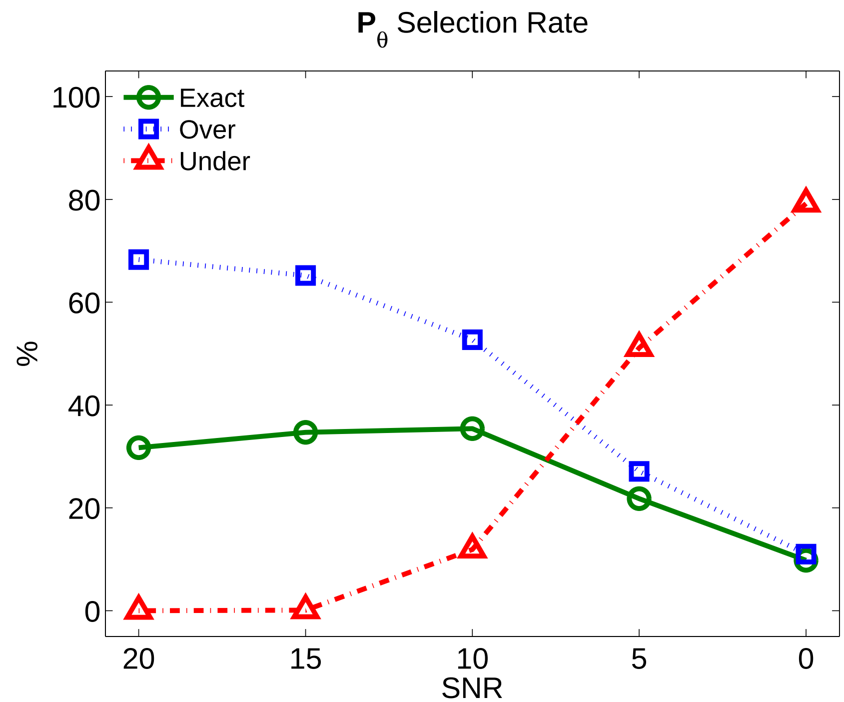
1. Exact Model: A model which contains only true system terms,

2. Over-modelled: A model with all its true system terms plus spurious parameters and

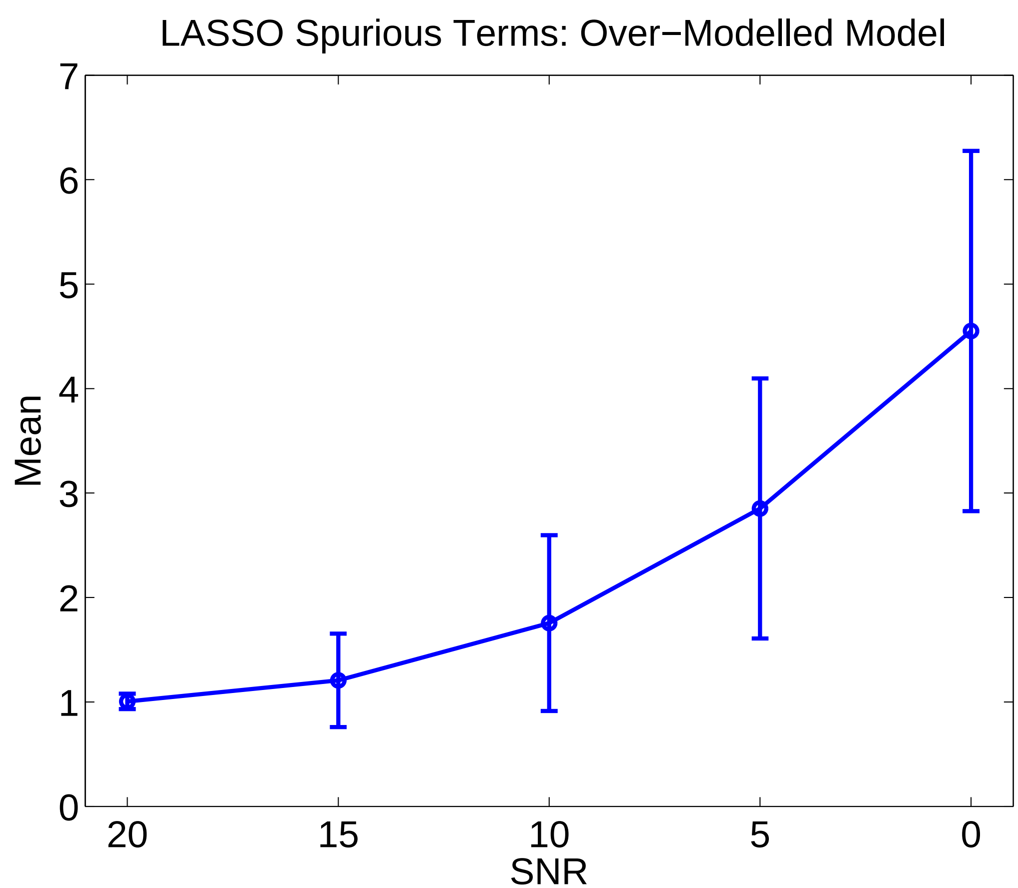
3. Under-modelled: A model without all its true system terms. An under-modelled model may contain spurious terms as well

Results: Selection Rate





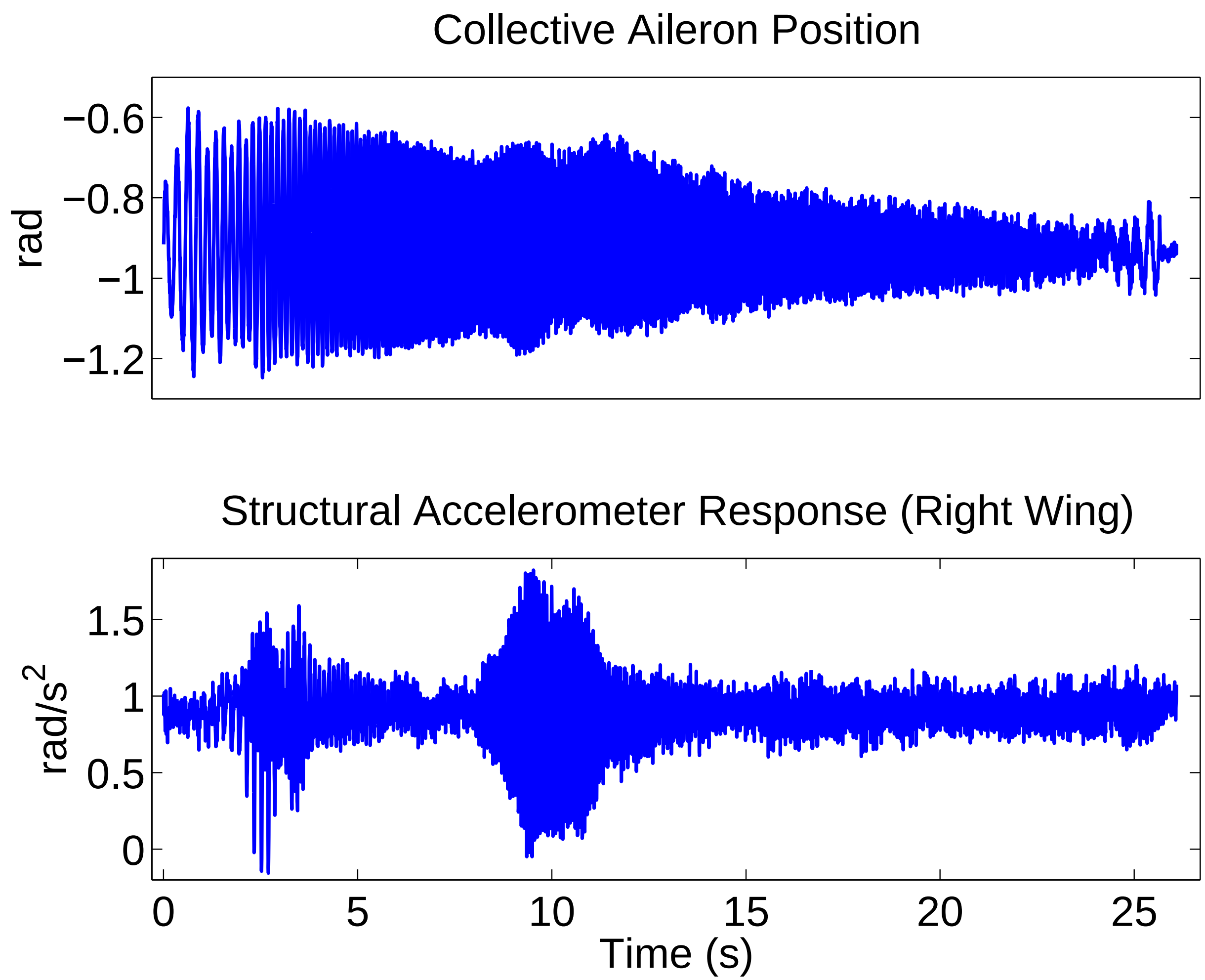
Results: Spurious Term Selection Rate



Experimental Aircraft Data

- Assumed model order: $O = [4\ 4\ 4\ 3]$:
  - Fourth-order dynamics selected because many aeroelastic structures are well defined by a fourth-order LTI system
  - Third-order nonlinearity selected because models of higher nonlinear order can often be decomposed to second or third-order
  - Full model description 560 candidate terms
- 1,000 logarithmically spaced λ's: $\lambda_{min} = -10$ and $\lambda_{max} = 1.0$
- Estimation $N_e = 5,200$: right wing & cross-validation $N_v = 5,200$: left wing

Identification Data

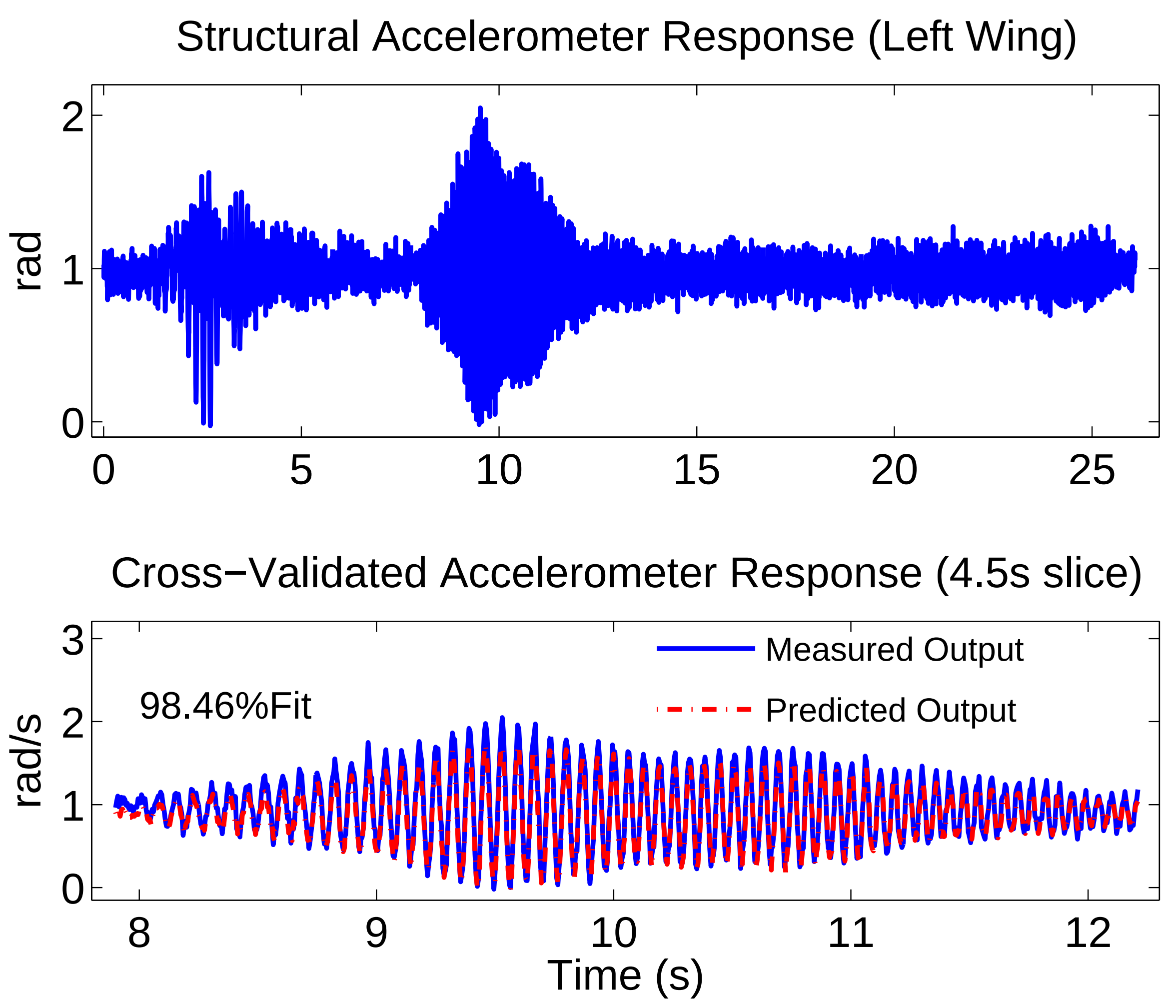


Computed Structure

- Contains 25 terms

$$
\begin{aligned}
z(n) &= \hat{\theta}_0 + \hat{\theta}_1 u(n-1) + \hat{\theta}_2 u(n-2) + \hat{\theta}_3 u(n-4) \\
&+ \hat{\theta}_4 u^2(n-1) + \hat{\theta}_5 u^2(n-2) + \hat{\theta}_6 u^2(n-4) \\
&+ \hat{\theta}_7 z(n-1) + \hat{\theta}_8 z(n-4) + \hat{\theta}_9 u^2(n-1) z(n-4) \\
&+ \hat{\theta}_{10} u^2(n-2) z(n-1) + \hat{\theta}_{11} u^2(n-4) z(n-4) \\
&+ \hat{\theta}_{12} z^3(n-1) + \hat{\theta}_{13} z^3(n-4) + \hat{\theta}_{14} \hat{\epsilon}(n-1) \\
&+ \hat{\theta}_{15} \hat{\epsilon}(n-4) + \hat{\theta}_{16} u^2(n-1) \hat{\epsilon}(n-4) \\
&+ \hat{\theta}_{17} u^2(n-2) \hat{\epsilon}(n-1) + \hat{\theta}_{18} u^2(n-4) \hat{\epsilon}(n-4) \\
&+ \hat{\theta}_{19} z^2(n-1) \hat{\epsilon}(n-1) + \hat{\theta}_{20} z(n-1) \hat{\epsilon}^2(n-1) \\
&+ \hat{\theta}_{21} \hat{\epsilon}^3(n-1) + \hat{\theta}_{22} z^2(n-4) \hat{\epsilon}(n-4) \\
&+ \hat{\theta}_{23} z(n-4) \hat{\epsilon}^2(n-4) + \hat{\theta}_{24} \hat{\epsilon}^3(n-4).
\end{aligned}
$$

Cross-Validation Data



Conclusions

- Novel approach for detecting the structure of highly over-parameterised nonlinear models in situations where other methods may be inadequate

- Practical significance in the analysis of aircraft dynamics during envelope expansion and could lead to more efficient control strategies

- Could allow greater insight into the functionality of various systems dynamics, by providing a quantitative model which is easily interpretable

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