HOME

Station-Keeping Requirements for Astronomical Imaging with Constellations of Free-Flying Collectors

Ronald J. Allen*

Space Telescope Science Institute, 3700 San Martin Drive, Baltimore MD 21218, USA

The requirements on station-keeping for constellations of free-flying collectors coupled as (future) imaging arrays in space for astrophysics applications are discussed. The typical *knowledge* precision required in the plane of the array depends on the angular size of the targets of interest; it is generally at a level of tens of centimeters for typical stellar targets, becoming of order centimeters only for the widest attainable fields of view. In the "piston" direction, perpendicular to the array, the typical *knowledge* precision required depends on the bandwidth of the signal, and is at a level of tens of wavelengths for narrow $\sim 1\%$ signal bands, beccoming of order one wavelength only for the broadest bandwidths expected to be useful. The significance of this result is that, at this level of precision, it may be possible to provide the necessary knowledge of array geometry without the use of signal photons, thereby allowing observations of faint targets. "Closure-phase" imaging is a technique which has been very successfully applied to surmount instabilities owing to equipment and to the atmosphere, and which appears to be directly applicable to space imaging arrays where station-keeping drifts play the same role as (slow) atmospheric and equipment instabilities.

I. Introduction

One of the major problems affecting the design of systems for high-resolution astronomical imaging using constellations of free-flying interferometers in space (e.g. Stellar Imager (SI), Infrared Terrestrial Planet Finder (TPF-IR), Far Infrared Interferometer in Space (SPECS), etc.) is the necessity to keep the individual elements of the constellation at their designated stations to a high degree of accuracy for extended periods of time. The current view is that the required precision transverse to the optical axis of the constellation is only of the order of a modest fraction of the diameter of the individual collector elements, but parallel to the optical axis (the "boresight", or "piston" direction) the required precision is currently expected to be a small fraction of a wavelength. Sophisticated radio + laser ranging systems may be adequate for station-keeping in the transverse direction, but more elaborate measures will be required to achieve the necessary precision in the parallel direction. Current thoughts for achieving the required level of "piston" accuracy include fine control using photons from the target itself; unfortunately this means that observations of faint targets may be impossible.

In this paper I first review the precision requirements in the transverse direction; the relevant scale length in this case depends on the wavelength and the size of the desired field of view, and for a representative example is measured in tens of centimeters. I then discuss the "piston" requirements and conclude that the relevant scale length is the correlation length of the signal, which is related to the signal wavelength and bandwidth; again, for a representative example, this is measured in units of tens of λ.

I then describe the concept of "closure phase" and its current application to synthesis imaging in radio and optical ground-based astronomy in the case where the interferometer data has poor fringe phase stability. This can happen for instance at low radio frequencies owing to the ionosphere, at optical and near-infrared wavelengths owing to the atmosphere, and at all wavelengths owing to insrumental drifts and instabilities. Transfer of the closure phase concept to the space astrophysics imaging problem appears straightforward,

*Astronomer, Science Division, Space Telescope Science Institute.

with station-keeping errors in the role of producing the instrumental phase instability, although many details still need to be considered. A rough example is sketched out in the form of a multi-channel strawman imaging mission similar to the Stellar Imager in dilute-aperture Fizeau mode, but applications to Michelson systems and systems with various degrees of "pupil densification" may also be feasible. If the idea proves viable in practice, the impact on the design of such systems in space could be substantial. On-board radio + laser ranging may then be adequate for controlling the piston errors and, since target photons are not needed for "guiding", faint targets would become accessible. The range of available astrophysics problems which such systems could address would then be greatly increased.

II. Accuracy requirements for Imaging

For illustration purposes one can think of the problem in terms of arranging a constellation of free-flying collectors on the surface of a (very) large sphere and combining the signals in a "classical" fashion at the focus; this "dilute aperture" imager is presently one of the designs being explored for the Stellar Imager (SI) concept mission. Let us define a cartesian coordinate system with x,y in the plane tangent to the (very-slightly-curved) spherical surface, and z along the optical axis. The collector elements in this example are distributed over a region on the sphere which is ~ 0.5 km in diameter; the focus of the system is at a central hub located ~ 65 km away along the optical axis.

A. Transverse direction

Let B denote the baseline distance between any two elements of the array. The fringe pattern formed by this component of the constellation in the focal plane when viewing a distant star located at the center of the field will have the approximate form at each wavelength λ of

$$P \approx P_0[1 + \cos(2\pi(B/\lambda)\theta)], \tag{1}$$

for θ in radians. Note that a typical system we might presently have in mind has $B/\lambda \sim 3 \times 10^9$ and $\theta \sim 3 - 100 \times 10^{-10}$ rads, or $\sim 60 - 2000\,\mu$as. This fringe pattern expands and contracts about the field center as B changes, and for a given (but in reality unknown) baseline error of δB the resulting fringe phase error grows with angular distance from the center of the field. The situation is sketched in Figure 1. The net phase change $\delta\phi$ in units of "turns" (one "turn" of phase corresponds to a path length difference of one full wavelength, or 360°) at a radius $\rho = N_f \times \lambda/B$ radians from the center of the field of view is:

$$\delta\phi = N_f \delta B/B \tag{2}$$

where N_f is the number of fringes in the angle ρ. If we require that the fringe phase error incurred by this *unknown* component of station "drift" be $\lesssim \lambda/Q$ where Q is defined as the "aperture quality factor" then the requirement is:

$$\delta B/B \lesssim 1/N_f Q. \tag{3}$$

As an example, suppose the source is a star like the sun and the instrument is intended to provide 30 independent resolution elements across the stellar diameter. It can be safely presumed that there is no signal outside a circle of radius $\rho = 15 \times \lambda/B_{max}$ rads from the center of the star, where B_{max} is the longest baseline in the constellation. For this example, $N_f \approx 15$. If we demand "optical quality" of, say, $\lambda/100$ in our imaging instrument, then $Q = 100$ and $\delta B/B \lesssim 1/1500$. For a maximum baseline of $B = 500$ meters, the largest *unknown* δB we can tolerate is 33 cm. Note that since $N_f \sim B$ for a given source size, δB is actually independent of B, so the required accuracy in this example is 33 cm for every baseline in the constellation. However, the requirement grows with the size of the target field of view in the sky. For instance, for a supergiant star the desired FOV could be a factor of 10 larger than the example calculated above, and the required tolerance on any baseline would be ~ 3 cm. But now such a constellation would provide 300 independent resolution elements across the star, and it may be more prudent to "shrink" the array by a factor 10 in order to return to the scale of the initial example and relax the station-keeping requirements accordingly.

It is interesting to ask what the limit on $\delta B/B$ might be for very large fields. Thee largest field of view will be limited by the diffraction pattern of the individual collector elements; this field is of angular radius

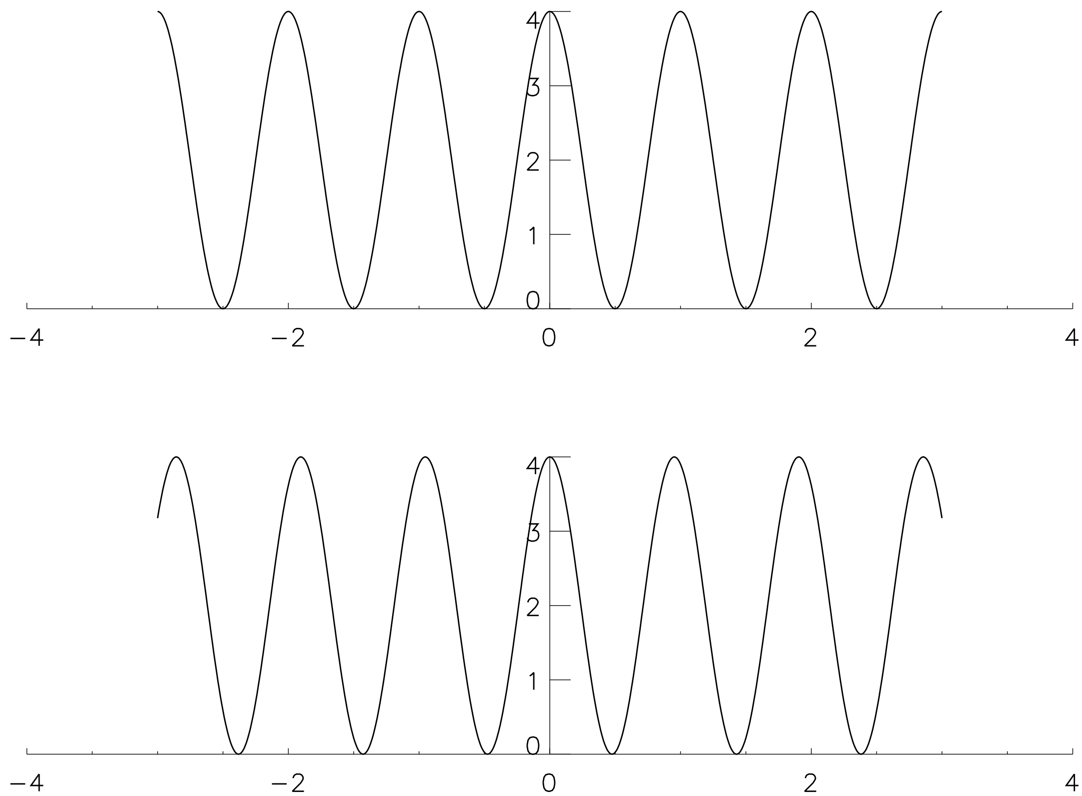

Figure 1. Top panel: fringe pattern for baseline B. Bottom panel: fringe pattern for a baseline which is in reality 5% larger. Notice the $\sim 0.15\lambda$ fringe phase error ($54^\circ \approx 1$ rad) at the edge of this (small) $3 \times \lambda$ field.

$\sim \lambda/a$ where a is the diameter of a collector element. For this field of view, $N_f \approx B/a$, so the maximum tolerable error is:

$$\delta B/B \lesssim 1/N_f Q \approx a/BQ; \tag{4}$$

i. e., $\delta B \lesssim a/Q$, which depends only on the size of the elements of the array, and is a result we could have obtained more directly from information theory considerations.

To conclude this section, I note that changes in operating wavelength have similar effects on the array reponse to those I have discussed above and can be modelled in the same way. One particularly deleterious effect of this is that the point spread function of a system with a finite spectral resolution will vary over the field of view, becoming radially elongated at large angular distances. Such effects were first analysed in detail by radio astronomers modelling the reponse of ground-based synthesis imaging arrays (see e.g. Thompson (1994)[6]).

B. Longitudinal direction

Station-keeping errors occurring in the z-direction are perhaps a little more obvious in their effect. Errors of δz directly translate to a fringe phase error of $2\delta z/\lambda$ rads in the dilute reflecting aperture (Fizeau interferometer) being modelled here, and half this much for a Michelson interferometer. In this case it is not an expansion or contraction of the fringe pattern about the field center, but a gross movement of the entire fringe pattern in a direction parallel to that of the baseline projection on the field of view. At first sight this is a serious problem, since it appears to require holding the station position to a tiny fraction of the wavelength. However, since it is the whole fringe pattern that moved and not the relative separation between two points in the field of view, the angular distance *measured in fringes* between the field center and the edge remains the same. In other words, the *relative* fringe phase is actually not sensitive to this shift. Figure 2 shows a sketch of the situation. If we can design our system and/or (post-) process the data in order to use this relative phase

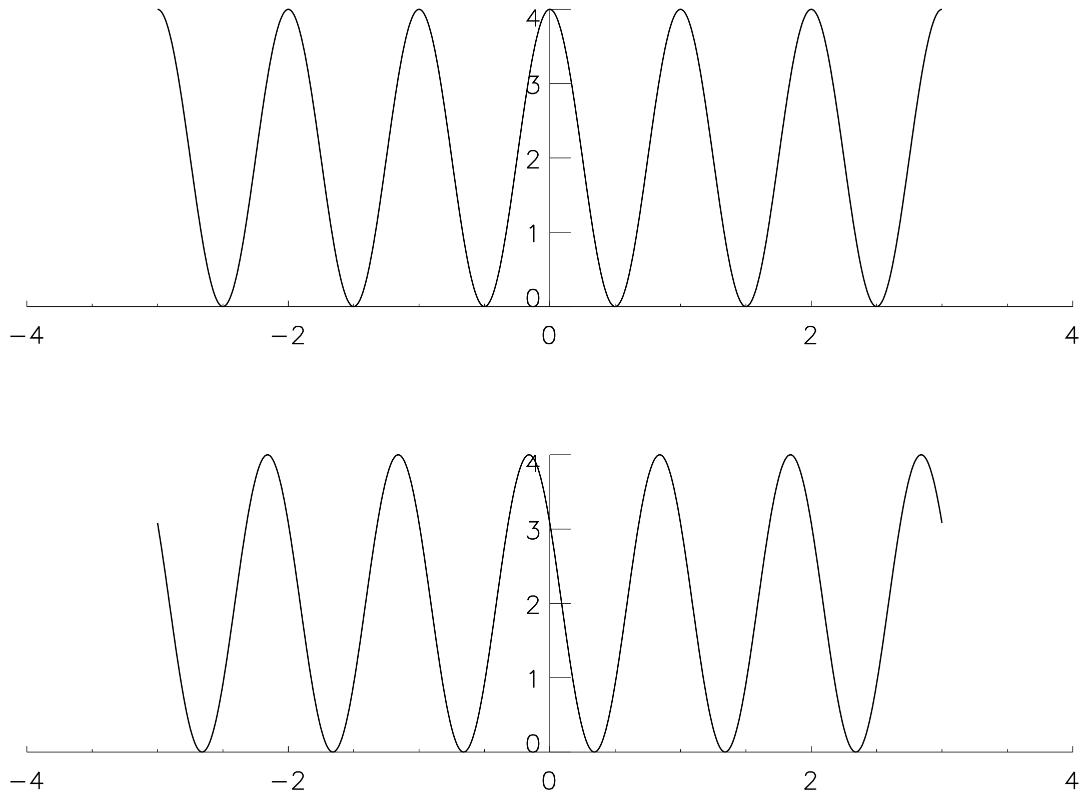

Figure 2. Top panel: original fringe pattern. Bottom panel: fringe pattern shifted by a phase error of $\delta\phi = 1$ rad, caused e.g. by a "piston" error of $\approx \lambda/12$ in a simple dilute reflecting Fizeau system.

insead of an absolute phase, we will be quite insensitive to "piston" errors, although we would lose the ability to measure the target position in an absolute sense. Except for precision wide-angle astrometry, no other applications in astrophysics require such absolute position information.

The real problem comes because of a loss of fringe amplitude at large piston errors owing to the finite bandwidth of the detection system; only if the bandwidth is infinitely narrow will the fringe pattern amplitude remain independent of δz as shown in Figure 2. For finite fractional bandwidths of $\delta\lambda/\lambda$, the fringe pattern has an effective width of $\approx \lambda^2/\delta\lambda$. This is the "correlation length" of the fringes. Figure 3 shows a sketch of this situation. If we can keep δz to some fraction of the correlation length, then the measurement of fringe amplitude and fringe phase can be made subject only to photon noise in the time scale of the drift in the "piston" position. Although this time scale is short in the case of ground-based observations through the earth's turbulent atmosphere, the time scale for a collector element to drift an appreciable fraction of a wavelength in space could be much longer, permitting longer coherent integration times on the fringes.

The problem now becomes to find a method of turning the recorded fringe amplitudes and (drifting) phases into an image in such a way as to be insensitive to shifts of the whole fringe pattern owing to piston errors. There are several related "post-processing" techniques which could be used. For instance, if the field of view contains a particularly bright star besides the target of interest, fringe phases could be "referenced" to this star. Another trick which was used first at radio wavelengths is to reference phases to a bright spectral line feature.

One quite general approach to this "unstable-phase" imaging problem which has been used with success for ~ 30 years uses the concept of "closure phase", which I will now summarize.

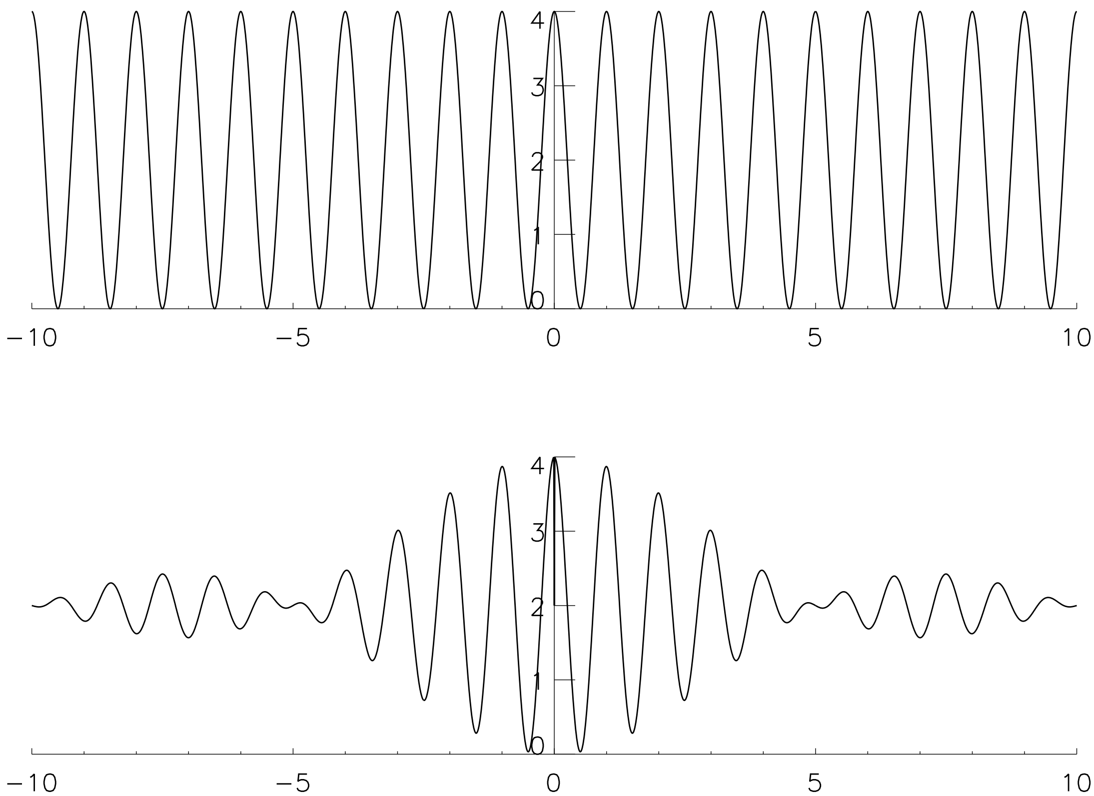

Figure 3. Top panel: Fringe pattern vs. piston error (in wavelengths) for a very narrow-band signal. Bottom panel: The same fringe pattern for a square bandpass of 20%. Note how the fringes disappear beyond $N_f \approx 5$.

III. Closure phase

In 1958, Jennison[1] presented a technique for measuring relative fringe phase which used three radio-linked collectors coupled as three interferometers operating at a wavelength of 2.4 meters over baselines up to ~ 10 km. Owing to a variety of instrumental problems related to the amplifiers and local oscillator electronics available at the time, the fringe phase between any two collectors was unstable and could normally not be measured; the source structure information had to be derived from the (squared) fringe amplitudes alone (a familiar situation in present-day ground-based optical interferometry). Jennison showed that, if the three observed fringe phases were summed, the resultant combined phase was insensitive to equipment instabilities. With this approach, Jennison & Latham (1959)[2] showed that the brightness distribution of the radio source Cygnus A, which until then was only known to be elongated, actually consisted of two separated sources of nearly equal brightness straddling a peculiar optical object tentatively identified at the time with two galaxies in collision. This was the first observation to reveal the double-lobed structure of a powerful radio galaxy.

Applications of this method to circumvent atmospheric phase instabilities in optical interferometry were described by Jennison (1961)[3] and, apparently independently, by Rogstad (1968).[5] The first use of the term "closure phase" which I have been able to find is in Rogers et al. (1974)[4] describing an application at radio wavelengths using very accurate, but independent, reference oscillators at the three stations in a so-called "very-long-baseline" interferometer array. Since that time, closure phase has been used extensively at radio, IR, and optical wavelengths, and there are many papers describing the subject, its virtues, and its limitations. For newcomers, the lectures presented at the Michelson Summmer schools by John Monnier and David Buscher in 2001, and by Peter Tuthill in 2003 are good sources (see http://msc.caltech.edu/school/2003 and links there).

IV. Conclusion

The requirements on station keeping for constellations of free-flying collectors coupled as (future) imaging arrays in space for astrophysics applications have been discussed. The typical *knowledge* precision required in the plane of the array depends on the angular size of the targets of interest; it is generally at a level of tens of centimeters for typical stellar targets, becoming of order centimeters only for the widest attainable fields of view. The requirements on *control* precision can be even less stringent, depending on the time scale over which the "knowledge" information is available. In the "piston" direction, perpendicular to the array, the typical *knowledge* precision required depends on the bandwidth of the signal, and is at a level of tens of wavelengths for narrow $\sim 1\%$ signal bands, beccoming of order one wavelength only for the broadest bandwidths expected to be useful. *Control* precision will likely be even less stringent, depending on the time scales involved, but a discussion of control issues lies outside this paper.

The significance of this result is that, at this relaxed level of precision, it may be possible to provide the necessary knowledge of array geometry without the use of signal photons, thereby allowing observations of faint targets.

Such constellations of free-flyers will produce images using various sophisticated reconstruction techniques which relax the level of precision required on the fringe phase of each component interferometer. "Closure-phase" imaging is one such technique which has been very successfully applied to surmount instabilities owing to equipment and to the atmosphere, and which appears to be directly applicable to space imaging arrays where station-keeping drifts play the same role as (slow) atmospheric and equipment instabilities. Among the many open questions to be investigated in connection with this idea, future work on this topic ought to include writing a computer-based imaging simulator for a typical free-flyer space array mission using closure phase reconstruction.

Acknowledgments

My thanks to my colleagues on the Stellar Imager "Optics" team, especially Dave Mozurkewich and Rick Lyon, for stimulating discussions. I am pleased to acknowledge the support of this work by the Space Telesope Science Institute, and by grants from the NASA-Goddard Space Flight Center.

References

[1] Jennison, R.C. 1958, Monthly Notices of the Royal Astronomical Society, 118, 276.

[2] Jennison, R.C., & Latham, V. 1959, Monthly Notices of the Royal Astronomical Society, 119, 174.

[3] Jennison, R.C. 1961, Proceedings of the Physical Society, 78, 596.

[4] Rogers, A.E.E, et al. 1974, Astrophysical Journal, 193, 293.

[5] Rogstad, D.H. 1968, Applied Optics, 7, 585.

[6] Thompson, A.R. 1994, in *Synthesis Imaging in Radio Astronomy*, eds. R.A. Perley, F.R. Schwab, & A.H. Bridle, ASP Conference Series, Vol. 6, (ASP, San Francisco), 32.