HOME

2nd International Symposium on Formation Flying Missions & Technologies, Sept. 14 – 16, 2004, Crystal City, Virginia

Relative Attitude Determination of Earth Orbiting Formations Using Global Positioning System Receivers

E. Glenn Lightsey*

The University of Texas at Austin, Austin, Texas, 78712-1085, U.S.A.

Satellite formation missions require the precise determination of both the position and attitude of multiple vehicles to achieve the desired objectives. In order to support the mission requirements for these applications, it is necessary to develop techniques for representing and controlling the attitude of formations of vehicles. A generalized method for representing the attitude of a formation of vehicles has been developed. The representation may be applied to both absolute and relative formation attitude control problems. The technique is able to accommodate formations of arbitrarily large number of vehicles.

To demonstrate the formation attitude problem, the method is applied to the attitude determination of a simple leader-follower along-track orbit formation. A multiplicative extended Kalman filter is employed to estimate vehicle attitude. In a simulation study using GPS receivers as the attitude sensors, the relative attitude between vehicles in the formation is determined 3 times more accurately than the absolute attitude.

I. Introduction

There are many satellite formation flying missions in which the coordinated control of the formation's attitude is proposed.[1] For example, it is necessary to reposition and reorient the satellites in a variable baseline interferometer application to image different targets. The size and cost of the individual satellites in the formation are commonly constrained to be as small as possible for the given mission requirements.

For Earth orbiting formations, it is efficient to use Global Positioning System (GPS) receivers as spacecraft sensors. The receivers can provide autonomous on-orbit position determination and time synchronization between satellites. Current GPS receiver technology conserves power and is well-suited to miniaturization. Dynamic filters can be used to provide estimates even when measurements are temporarily unavailable. In this manner, the useful range of spaceborne GPS receivers can be extended to altitudes above the GPS constellation, increasing the diversity of applications that can benefit from GPS devices.

It is also possible for GPS receivers to determine a vehicle's attitude. There are now several different methods by which the attitude information may be obtained. The most common method is a local interferometric one using multiple antennas on a single vehicle.[2] This technique requires the resolution of a carrier phase cycle ambiguity by external or other means.[3,4] Other methods employ antenna gain pattern mapping with signal to noise ratio measurements.[5] It is possible to use this approach with a single GPS antenna and another sensor, such as a magnetometer, to determine three-axis attitude.[6]

Since a GPS sensor will almost certainly be present on an Earth orbiting satellite for position and time determination, it would be advantageous to employ that device for attitude determination if it can reliably meet the mission attitude requirements. These requirements could be given in either absolute or relative attitude determination accuracy, depending on the mission.

It is well known that the relative position solution between two GPS receivers located in near proximity can be obtained more accurately than the absolute solution.[7,8] The greater accuracy is possible because

*Associate Professor, Department of Aerospace Engineering and Engineering Mechanics, 1 University Station, C0600, Austin, TX, 78712-1085, U.S.A. Email: lightsey@csr.utexas.edu

the most significant error sources are correlated between the different GPS measurements. When the range measurements are differenced, a more accurate relative solution is directly obtained.

For vehicle formations using GPS receivers, it might be expected that attitude measurement error cancellation would also occur between similarly designed vehicles operating in proximity with each other. If the measurement errors are highly correlated, relative attitude determination may be performed more accurately than absolute attitude determination.

Survey of Prior Work and Outline of Topics

Although formation orbit determination and control has been extensively studied, the formation attitude determination problem is somewhat less established. Scharf et al have compiled a survey of formation flying guidance and control applications with more than 100 references.[9] The papers that address formation attitude tend to focus primarily on control methods.[10–14] This paper approaches the problem from a different perspective: the formation attitude control is assumed and the sensor model is developed. First, a generalized framework is presented in which the formation's dynamics may be expressed. Then a quaternion attitude estimation algorithm is developed using GPS carrier phase measurements. The generalized framework and estimation algorithm are demonstrated using a Low Earth orbit two vehicle leader-follower formation example. The effect of having GPS correlated measurement errors is evident in the resulting improvement in relative attitude accuracy that is achieved.

II. Reference Frame Definitions for Vehicle Formations

To describe the dynamic state of a formation of vehicles, some conventions must be established in the definition and notation of frames of reference. The approach used in this paper is similar to one that was previously introduced by Xing.[15] The main feature of this approach is that it is generalized and scales easily with the number of vehicles in the formation.

The fundamental reference frame shown in figure 1 is an inertial reference specified as F_I. The formation target reference frame is given as F_T, which may be non-inertial. This frame defines a local reference for the entire formation. It does not have to be occupied by an actual satellite. F_{Ti} is the target reference frame for the i'th satellite in the formation. This is considered to be the satellite's desired position and orientation relative to the formation target reference frame. Finally, the i'th satellite's actual position and orientation is given by the F_{Bi} body reference frame, which is attached to the i'th vehicle's center of mass. In a regulator problem, the goal is to align F_{Bi} with F_{Ti} within some acceptable tolerance. A general vector $\underline{v}$ may be coordinated with respect to the i'th local body frame, F_{Bi}.

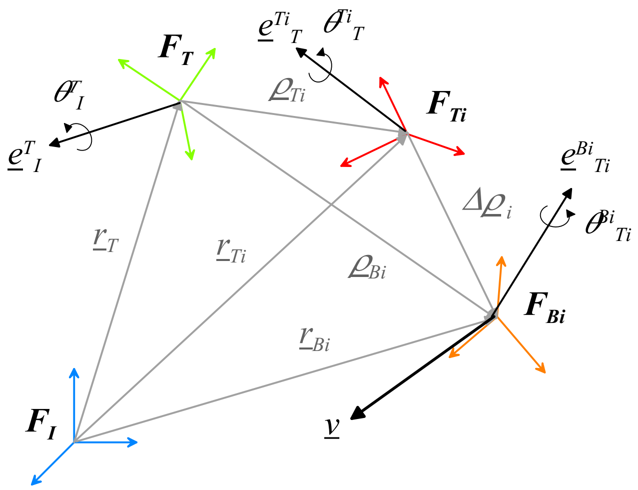


Figure 1. Formation Reference Frame Definitions.

In the notation of figure 1, the translational velocity of the i'th body frame relative to the i'th target frame is simply given by

$$\frac{{}^{Ti}d}{dt}(\Delta\underline{\rho}_i) \tag{1}$$

where $\Delta\underline{\rho}_i$ is the vector from the i'th target frame to the i'th body frame. The velocity of the body frame to the formation target frame can be expressed by adding translational and rotational terms

$$\frac{{}^{T}d}{dt}(\underline{\rho}_{Bi}) = \frac{{}^{T}d}{dt}(\underline{\rho}_{Ti}) + \frac{{}^{Ti}d}{dt}(\Delta\underline{\rho}_i) + \underline{\omega}^{TTi} \times \Delta\underline{\rho}_i \tag{2}$$

In this case, $\underline{\omega}^{TTi}$ is the angular velocity of the i'th target frame (F_{Ti}) relative to the formation target frame (F_T). The body frame's (F_{Bi}'s) velocity can be similarly expressed relative to the inertial frame (F_I) with the addition of two more terms

$$\frac{{}^{I}d}{dt}(\underline{r}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_T) + \frac{{}^{T}d}{dt}(\underline{\rho}_{Ti}) + \underline{\omega}^{IT} \times \underline{\rho}_{Ti} + \frac{{}^{Ti}d}{dt}(\Delta\underline{\rho}_i) + \underline{\omega}^{ITi} \times \Delta\underline{\rho}_i \tag{3}$$

The translational accelerations may be likewise derived. With the translational portion of the problem thus stated, it will no longer be considered in this paper, although its presence in the coupled formation control problem is assumed.

The attitude of a reference frame may be related to another by a unit quaternion, which is based upon Euler's theorem

$$q = \begin{bmatrix} \underline{q} \\ q_4 \end{bmatrix} = \begin{bmatrix} \underline{e}\sin(\phi/2) \\ \cos(\phi/2) \end{bmatrix} \tag{4}$$

where $\underline{e}$ is the $3x1$ axis unit vector and ϕ is the angle of rotation that relates the two frames.

Employing the definition of quaternion multiplication as stated by Shuster,[16]

$$q_A \otimes q_B = \begin{bmatrix} (q_{A4}\underline{q}_B + q_{B4}\underline{q}_A - \underline{q}_A \times \underline{q}_B)^T & , & q_{A4}q_{B4} - \underline{q}_A \cdot \underline{q}_B \end{bmatrix}^T \tag{5}$$

the quaternion conjugate,

$$\bar{q} = \begin{bmatrix} -\underline{q}^T & , & q_4 \end{bmatrix}^T \tag{6}$$

and the vector quaternion,

$$v = \begin{bmatrix} \underline{v}^T & , & 0 \end{bmatrix}^T \tag{7}$$

the *quaternion rotation operation* is defined

$$v_{Ti} \equiv q_{Ti}^{Bi} \otimes v_{Bi} \otimes \bar{q}_{Ti}^{Bi} \tag{8}$$

which takes a vector quaternion expressed in one frame and transforms it to another frame. Eq. (8) is equivalent to the well known direction cosine transformation matrix

$$\underline{v}_{Ti} = C_{Ti}^{Bi} \cdot \underline{v}_{Bi} \tag{9}$$

and in fact C_{Ti}^{Bi} may be derived from q_{Ti}^{Bi} as follows,[17]

$$C(q) = (q_4^2 - |\underline{q}|^2) \cdot I_{3x3} - 2q_4\left[\underline{q}\times\right] + 2\underline{q}\,\underline{q}^T \tag{10}$$

where I_{3x3} is the $3x3$ identity matrix and $\left[\underline{q}\times\right]$ is the cross product matrix

$$\left[\underline{q}\times\right] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{11}$$

Successive rotations are defined using the quaternion rotation operator listed in eq. (8). For example,

$$v_T = q_T^{Bi} \otimes v_{Bi} \otimes \bar{q}_T^{Bi} = q_T^{Ti} \otimes q_{Ti}^{Bi} \otimes v_{Bi} \otimes \bar{q}_{Ti}^{Bi} \otimes \bar{q}_T^{Ti} \tag{12}$$

It is evident that the quaternion q_{Ti}^{Bi} is a representation of the attitude of the i'th satellite local body reference frame (F_{Bi}) relative to the i'th satellite target reference frame (F_{Ti}). If the target reference frame tracks the desired satellite state, then the generalized vehicle attitude tracking problem can be given as

$$\left.\begin{matrix} \underline{q}_{Ti}^{Bi} \;,\; \underline{\omega}^{TiBi} & \rightarrow & \underline{0} \\ (q_4)_{Ti}^{Bi} & \rightarrow & \pm 1 \end{matrix}\right\} \text{ as } \; t \rightarrow \infty \tag{13}$$

There is a sign ambiguity on the scalar part of the quaternion in equation (13). An attitude performance vector for the i'th satellite may be established as

$$\underline{z}_i(t) = \begin{bmatrix} \underline{q}^T & , & |q_4| - 1 & , & \underline{\omega}^T \end{bmatrix}^T \tag{14}$$

Formulation of Optimal Attitude Controller

Using the performance vector defined in eq. (14), a regulator performance index may be specified as (for example)

$$J_i(t) = \frac{1}{2}\int_0^t (\underline{z}_i^T Q_i \underline{z}_i + \underline{u}_i^T R_i \underline{u}_i)dt \tag{15}$$

which incorporates $\underline{u}_i$ as the actuator control effort, and Q_i and R_i are gain weighting matrices. $\underline{z}_i$ may also include translational states if a coupled translational and rotational controller is required. Although not shown, J_i may include operational constraints using Lagrange multipliers and the calculus of variations.

The locally optimal controller $\mathbf{C}_i$ at the i'th satellite may be specified as

$$\mathbf{C}_i : \min_{t\to\infty} J_i(t) \tag{16}$$

Over the entire formation, the globally optimal controller $\mathbf{C}$ is given by

$$\mathbf{C} : \min_{t\to\infty} \sum_i J_i(t) \tag{17}$$

This completes the statement of the optimal control problem using the general framework presented. From this point forward, the paper focuses on the attitude determination problem, and the existence of a sufficiently optimal controller is presumed.

III. Attitude Estimation

The method of attitude representation chosen for this algorithm is the quaternion, as given in eq. (4). The quaternion is generally preferred for spacecraft applications because of its lack of singularities and its computational efficiency. However, there are only three degrees of freedom in the rotation group and so precautions must be taken to prevent the covariance matrix from becoming singular over time.

Following the method employed by Markley,[17] the true attitude is represented as a small error deviation (δq) from a reference attitude quaternion (q_{ref}), as expressed by the quaternion product

$$q(t) = \delta q(\underline{a}(t)) \otimes q_{ref}(t) \tag{18}$$

Note that the deviation vector $\underline{a}(t)$ has only three degrees of freedom and the components of $\underline{a}$ are called Modified Rodrigues Parameters[18]

$$\underline{a} \equiv 4\underline{e}\tan(\phi/4) \tag{19}$$

The factor of 4 causes the magnitude of $|\underline{a}| \approx \phi$ for small rotations. The second order approximation to the error quaternion is given by[17]

$$\delta q(\underline{a}) \approx \begin{bmatrix} \underline{a}/2 \\ 1 - |\underline{a}|^2/8 \end{bmatrix} \tag{20}$$

This method, known as the multiplicative extended Kalman filter (MEKF), avoids numerical stability problems that can occur from the normalization of the quaternion q.

Time Update

Attitude state propagation is performed using the kinematic quaternion relation

$$\dot{q}_{ref} = \frac{1}{2}\begin{bmatrix} \underline{\omega}_{ref} \\ 0 \end{bmatrix} \otimes q_{ref} \tag{21}$$

If the vehicle's attitude motion approximates that of a rigid body in an inertial reference frame, Euler's equations may be used to propagate $\underline{\omega}_{ref}$

$$\dot{\underline{\omega}}_{ref} = J^{-1}\left[(J\underline{\omega}_{ref}) \times \underline{\omega}_{ref} + \underline{M}_D + \underline{M}_C\right] \tag{22}$$

where it is assumed that the body frame (F_{Bi}) defines the principal axes of the vehicle. J is the $3x3$ diagonal matrix of principal moments of inertia, and $\underline{M}_D$ and $\underline{M}_C$ are disturbance and control moments, respectively.

Covariance propagation is performed according to the differential equation

$$\dot{P} = FP + PF^T + GQG^T \tag{23}$$

The linear approximation to the time derivative of $\underline{a}$ is[17]

$$\dot{\underline{a}} = \underline{f}(\underline{a}, t) \approx -\underline{\omega}_{ref} \times \underline{a} + \underline{\eta}(t) \tag{24}$$

and $\underline{\eta}(t)$ is a zero-mean white noise process.
The matrices listed in eq. (23) are then given as

$$F(t) \equiv \frac{\partial \underline{f}}{\partial \underline{a}} = -\left[\underline{\omega}_{ref} \times\right] \tag{25}$$

$$G(t) \equiv \frac{\partial \underline{f}}{\partial \underline{\eta}} = I_{3x3} \tag{26}$$

and

$$E\left[\underline{\eta}(t)\underline{\eta}^T(\tau)\right] = Q(t)\delta(t-\tau) \tag{27}$$

Measurement Update

Up to this point the filter design has been independent of the sensing technology that is needed to determine the attitude of the vehicle. If the vehicle is in Earth orbit, it is possible to sense its attitude with a GPS receiver.

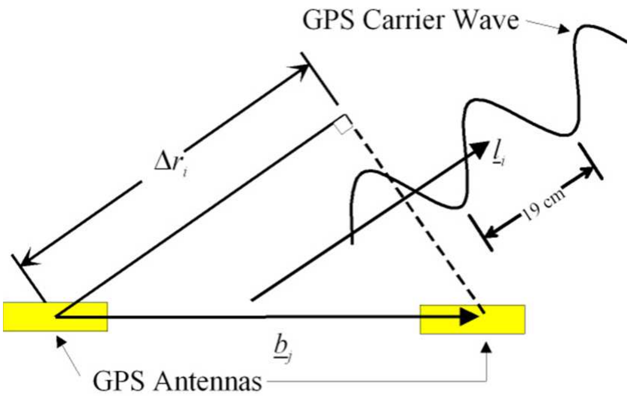


Figure 2. Carrier Phase Interferometry.

The method of attitude determination employed in this study is GPS interferometry using differential carrier phase measurements. It is assumed that the cycle ambiguities have been resolved, therefore the carrier phase measurements may be converted directly to range Δr measurements between antennas. The geometry of this measurement is shown in figure 2. The carrier phase measurement $\Delta\phi_{ij}$ is made between two antennas with baseline vector $\underline{b}_j$ tracking a GPS satellite with line of sight vector $\underline{l}_i$

$$\Delta\phi_{ij} = \Delta r_{ij} - k_{ij} + \beta_j + \nu_{ij} = \underline{l}_i \cdot \underline{b}_j - k_{ij} + \beta_j + \nu_{ij} \tag{28}$$

k_{ij} is the known single difference cycle ambiguity, β_j is a bias term, and ν_{ij} is measurement noise. The bias term is removed by differencing measurements along the same baseline j between GPS satellites i and k

$$\nabla\Delta\phi_{ijk} = \Delta\phi_{ij} - \Delta\phi_{kj} = \underline{b}_j \cdot (\underline{l}_i - \underline{l}_k) - (k_{ij} - k_{kj}) + (\nu_{ij} - \nu_{kj}) \tag{29}$$

Since the integers k are known, they may be incorporated into the carrier phase measurements and the measurement equation may be rewritten as

$$y = \underline{b}_j \cdot (\underline{l}_i - \underline{l}_k) + (\nu_{ij} - \nu_{kj}) = h + \nu \tag{30}$$

In order to evaluate the dot product in eq. (30), the vectors must be coordinated in the same reference frame. The line of sight vectors ($\underline{l}$) are known in the external reference frame, but the antenna baseline vectors ($\underline{b}$) are given in the body frame, so they must be transformed to evaluate the dot product

$$h = \left[C(q) \cdot \underline{b}_j\right]^T \cdot (\underline{l}_i - \underline{l}_k) \tag{31}$$

The quaternion q is now substituted using equations (18), (10), and (20) to give the linear approximation

$$C(q) = C(\delta q \otimes q_{ref}) = C(\delta q)C(q_{ref}) \approx (I_{3x3} - [\underline{a}\times])\, C(q_{ref}) \tag{32}$$

h may now be written in terms of the attitude deviation vector, $\underline{a}$

$$h(\underline{a}) \approx \left[(I_{3x3} - [\underline{a}\times])\, C(q_{ref}) \cdot \underline{b}_j\right]^T \cdot (\underline{l}_i - \underline{l}_k) \tag{33}$$

which may be rearranged as

$$h(\underline{a}) \approx \hat{h} + (\underline{l}_i - \underline{l}_k)^T \cdot \left(C(q_{ref}) \cdot \underline{b}_j\right) \times \underline{a} \tag{34}$$

where $\hat{h}$ is the expected measurement at the attitude given by q_{ref}. The measurement sensitivity matrix is

$$H^T \equiv \frac{\partial h}{\partial \underline{a}} = [(\underline{l}_i - \underline{l}_k)\times] \cdot \left(C(q_{ref}) \cdot \underline{b}_j\right) \tag{35}$$

The Kalman Gain matrix is computed as

$$K = P(-)H^T[HP(-)H^T + R]^{-1} \tag{36}$$

where R is the covariance of the zero mean measurement white noise, $E\left[\nu(t)\nu^T(\tau)\right] = R(t)\delta(t-\tau)$. The error state update is then completed

$$\hat{\underline{a}} = K(y - \hat{h}) \tag{37}$$

$$q_{ref}(+) = \delta q(\hat{\underline{a}}) \otimes q_{ref}(-) \tag{38}$$

The error covariance is updated according to

$$P(+) = (I_{3x3} - KH) \cdot P(-) \tag{39}$$

This completes the derivation of the vehicle attitude estimator. The filter will be adjusted for relative attitude observations in a later section.

IV. Relative Attitude Simulation

For the purposes of demonstrating the relative attitude concept using GPS receivers, a $6N$ degree of freedom simulation was developed, where N is the number of vehicles in the formation. In the example used in this paper, $N = 2$, however the results can be generalized to $N \geq 2$.

The translational dynamics are specified as a simple leader-follower arrangement with 2 satellites in the same orbit whose elements are specified by table 1. The anomaly of the follower satellite was adjusted to lag the leader satellite by a constant distance, $R = 100$ km. Since the focus of this study is on the attitude estimator performance, no perturbations are considered in this basic simulation.

Table 1. Orbit elements for leader satellite at $t = 0$.*

Element	Value	Element	Value
h	800.0 km	Ω	0
i	89.9 deg	ω	0
e	0	ν_0	0

* Follower satellite lags leader by 100 km.

The formation target reference frame (F_T) is assigned to the leader satellite with the $\underline{i}$-axis pointed along the zenith vector, the $\underline{j}$-axis is rotated 90 degrees in the direction of motion, and $\underline{i} \times \underline{j} = \underline{k}$. The frame rotates with the leader satellite's center of mass, and therefore defines a Local Vertical Local Horizontal (LVLH) reference for this circular orbit. The follower satellite is assigned a similarly defined LVLH target reference frame at its location (F_{T2}). The attitude of each vehicle's body axes (F_{B1} and F_{B2}) are given relative to their respective target reference frame (F_T and F_{T2}). Since the satellites are not in the same position, $F_T \neq F_{T2}$.

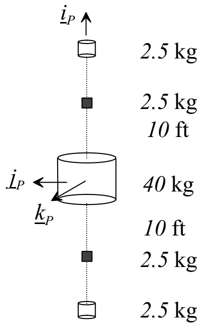


Figure 3. Gravity Gradient Satellite.

The rotational dynamics are specified as follows. A 50 kg, axisymmetric satellite is proposed as shown in figure 3. The satellite is a twin-boom configuration with each boom extending 3.05 m (10 ft) in length. The main body of the satellite is a 40 kg homogeneous cylinder with radius 0.23 m and height 0.38 m. The booms are modelled as lumped masses of 2.5 kg at the center of each span. Each boom tip is also modelled as a homogeneous cylinder, with mass 2.5 kg, radius 0.115 m, and height 0.19 m. The entire structure is assumed to be rigid and symmetric. The principal axes of the structure define the local body frame (F_B).

Using this simple mass model, the inertia properties are calculated as $J_x = 1.09$ kg m^2, and $J_y = J_z = 71.4$ kg m^2. The non-dimensional inertia ratios are

$$k_{pitch} = (J_y - J_x)/J_z = 0.985 \tag{40}$$

$$k_{roll} = (J_z - J_x)/J_y = 0.985 \tag{41}$$

$$k_{yaw} = (J_z - J_y)/J_x = 0 \tag{42}$$

According to linearized gravity gradient dynamic analysis, the vehicle attitude motion is represented with respect to the LVLH frame according to the following differential equations[19]

$$\texttt{Yaw:} \quad \frac{1}{k_{yaw}}\left[\ddot{\psi}_x - n\dot{\psi}_y\right] + \left[n\dot{\psi}_y + n^2\psi_x\right] = 0 + (M_{cx} + M_{dx}) \tag{43}$$

$$\texttt{Roll:} \quad -\frac{1}{k_{roll}}\left[\ddot{\psi}_y + n\dot{\psi}_x\right] + \left[n\dot{\psi}_x - 4n^2\psi_y\right] = 0 + (M_{cy} + M_{dy}) \tag{44}$$

$$\texttt{Pitch:} \quad \frac{1}{k_{pitch}}\left[\ddot{\psi}_z\right] + \left[3n^2\psi_z\right] = 0 + (M_{cz} + M_{dz}) \tag{45}$$

n is the angular rate, which is 0.595 rev/hr for this case. Control and disturbance moments could be applied to the right hand side of eqs. 43– 45 as shown, but will be assumed 0 for this analysis. The homogenous solution to the linearized gravity gradient equations of motion for small deflections is a pair of oscillators. For this configuration, the natural frequencies are

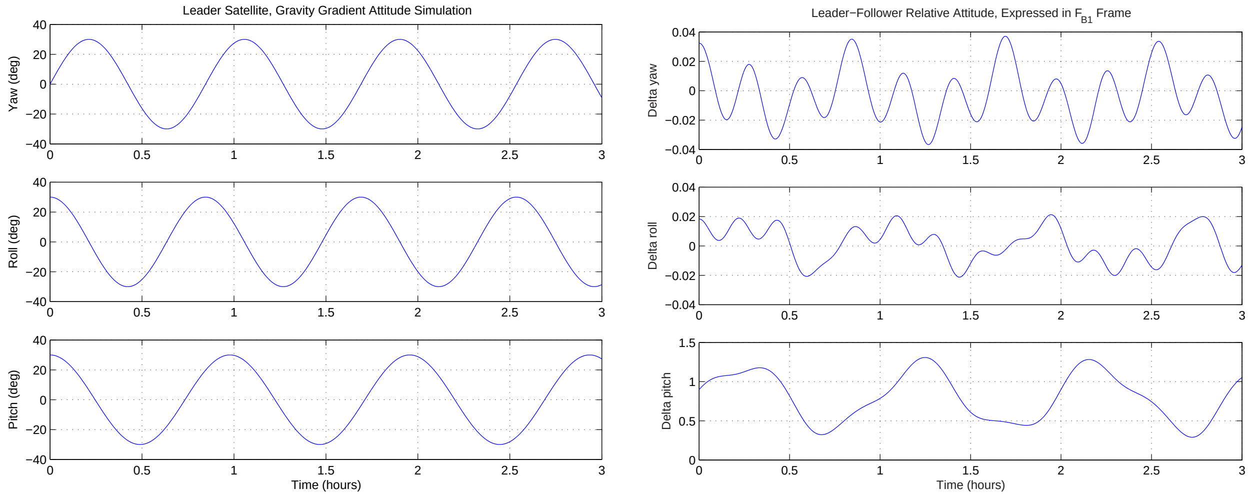
$$\omega_{pitch} = n(3k_{pitch})^{0.5} = (0.595)(3 \cdot 0.985)^{0.5} = 1.02 \texttt{ rev/hr} \tag{46}$$

$$\omega_{ry} = n(1 + 3k_{roll})^{0.5} = (0.595)(1 + 3(0.985))^{0.5} = 1.18 \texttt{ rev/hr} \tag{47}$$

The unforced response of the leader satellite is simulated with an initial condition offset of 30 degrees in pitch and roll relative to the target reference frame (q_T^{B1}). This response is shown in the left hand side of figure 4 on the next page. The follower satellite undergoes a very similar motion which is offset in phase angle by the difference in anomaly. The true relative attitude between the two vehicles is given by (see eq. 12 on page 3)

$$q_{B1}^{B2} = q_{B1}^{I} \otimes q_I^{B2} = \bar{q}_I^{B1} \otimes q_I^{B2} \tag{48}$$

The relative attitude motion between the leader and the follower is shown as a $3-2-1$ Euler angle sequence in the right hand of figure 4 on the next page. The Euler angles are extracted from the quaternion



(a) Leader Simulated Attitude.

(b) Leader-Follower Relative Attitude.

Figure 4. Simulated leader absolute attitude (q_T^{B1}) and leader-follower relative attitude (q_{B1}^{B2}).

q_{B1}^{B2}. Since the satellites are only 100 km apart, the relative attitude motion between the leader and follower satellites is small. The pitch bias is due to the difference in local vertical direction between the two satellites.

It is possible that the initial conditions of the gravity gradient motion between the satellites could be offset by different amplitudes and phase angles. In this case, there would almost certainly be larger relative motion between the two vehicles. However, due to the fact that the satellites have the same mass properties and orbits, it is expected that over time they will settle into gravity gradient motion of similar amplitude that is phase shifted by the value of the anomaly. This case is the one that is simulated.

V. Relative Attitude Results Using GPS Receivers

To simulate the performance of a GPS receiver in this application, the attitude estimator in section III processed simulated GPS measurements for each satellite undergoing the dynamics in section IV. The estimator is given simulated carrier phase measurements for all GPS signals that are within a 60 degree cone angle for each satellite's upward-facing axis of symmetry (the $\underline{i}_{B1}$ and $\underline{i}_{B2}$ directions). The carrier phase measurements are made from GPS antennas that lay along the vertices of a 1 m cube that is aligned with the satellite's body axes (3 total baselines per satellite). All antenna boresight directions are pointed along the vehicle axis of symmetry. Signal blockage due to structure is not considered, but carrier phase multipath is simulated in the form of correlated measurement noise.

To compute the relative attitude, an estimator is run independently for each satellite, producing the absolute attitude estimates, $\hat{q}_I^{B1}$ and $\hat{q}_I^{B2}$. The relative attitude estimate is then computed (compare to eq. 48 on the preceding page)

$$\hat{q}_{B1}^{B2} = \hat{q}_{B1}^{I} \otimes \hat{q}_I^{B2} = \overline{\hat{q}}_I^{B1} \otimes \hat{q}_I^{B2} \tag{49}$$

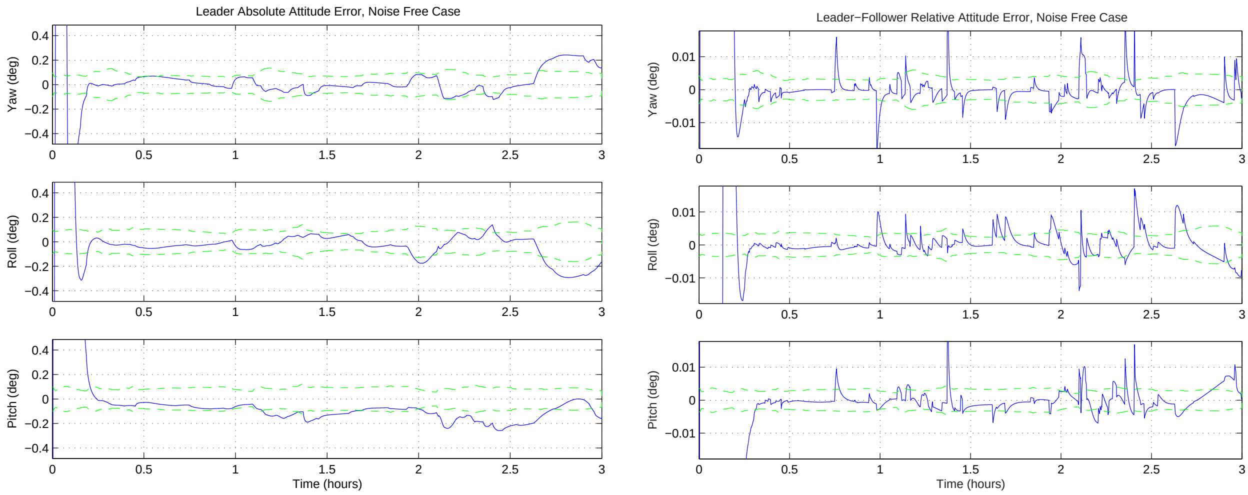
The resultant attitude error is then computed as follows. For the absolute attitude error of the leader satellite, the leader absolute error quaternion is given by

$$\hat{q}_{abserr} = q_{B1}^{I} \otimes \hat{q}_I^{B1} = \overline{q}_I^{B1} \otimes \hat{q}_I^{B1} \tag{50}$$

The relative attitude error of the follower satellite to the leader satellite is likewise specified

$$\hat{q}_{relerr} = q_{B2}^{B1} \otimes \hat{q}_{B1}^{B2} = \overline{q}_{B1}^{B2} \otimes \hat{q}_{B1}^{B2} \tag{51}$$

In each case, the error is reported as a $3-2-1$ Euler angle sequence, where the error Euler angles are extracted from the error quaternions q_{abserr} and q_{relerr}. Performance statistics are computed using the error Euler angles.



(a) Leader Attitude Error.

(b) Leader-Follower Relative Attitude Error.

Figure 5. Noise free absolute attitude error ($\hat{q}_{abserr}$) and leader-follower relative attitude error ($\hat{q}_{relerr}$).

Noise Free Simulation

Before introducing the effects of measurement noise, it is instructive to observe the estimator's performance in the noise free case. This may be considered as a best case bound on the performance for this algorithm.

The result, shown in figure 5, demonstrates the main thesis of this paper. The three-axis absolute attitude performance of 0.181 degrees root mean squared (RMS) is typical for this sensor under these dynamic conditions. There is a slight bias of absolute attitude in pitch. The estimator is limited by geometrical considerations associated with the number and direction of available measurements at each measurement epoch. Also, although the measurements are perfect, the estimator is not designed for this case. The absolute attitude performance could be improved by "tuning" the estimator for this special case, but this is not done since the actual application includes measurement noise.

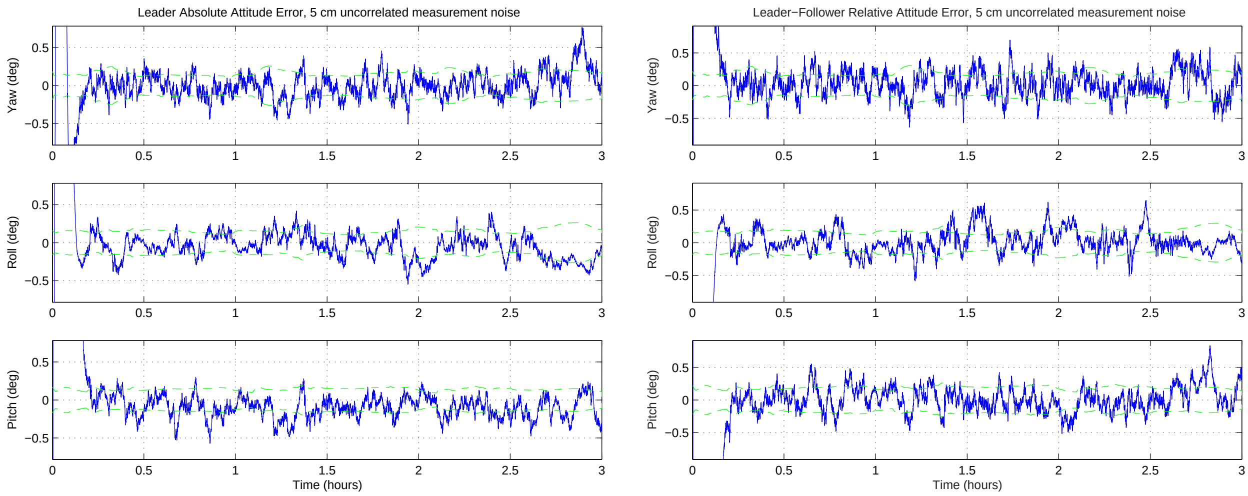
The relative attitude accuracy is strikingly better at 0.006 degrees RMS. Although the absolute attitude estimators for both satellites have errors associated with dynamics and measurements, these effects largely cancel out and leave a much more accurate relative state. The jumps that occur in figure 5b are associated with rising or setting GPS satellites. The jumps could be removed by adding logic to handle these cases.

Uncorrelated Measurement Noise Simulation

Every sensor is subject to measurement noise and GPS receivers are no exception. In the case of differential carrier phase measurements, the predominant source of noise is signal multipath due to reflections from nearby surfaces. Although deterministic in nature, multipath is difficult to model due to the high number of possible reflections. The stochastic signature of this noise is that of a time correlated random variable with a characteristic standard deviation (σ_n) and time constant (τ).[20]

In satellite applications, multipath can be a severe source of measurement noise. There are usually a large number of highly reflective surfaces in the vicinity of the GPS antenna. Multipath can be minimized by locating the antennas carefully to reduce reflections and by utilizing hardware practices such as the addition of antenna choke rings. Based on the results of previous orbit experiments, a standard deviation $\sigma_n = 5$ cm is considered to be typical for a Low Earth orbit satellite attitude application.[21]

In the worst case scenario of totally uncorrelated measurement noise (correlation coefficient $\rho = 0$), there is no benefit to differencing attitude solutions between satellites. In fact, relative attitude solution error is increased due to the independence of the noise sources. This case is shown in figure 6 on the next page. The introduction of measurement noise has increased the absolute attitude error to 0.295 degrees RMS, but the most significant development is that the relative attitude error has increased to 0.327 degrees RMS.



(a) Leader Attitude Error. (b) Leader-Follower Relative Attitude Error.

Figure 6. Uncorrelated noise absolute attitude error ($\hat{q}_{abserr}$) and leader-follower relative attitude error ($\hat{q}_{relerr}$).

Correlated Measurement Noise Simulation

It is hypothesized that for similar satellite designs that are operating in similar orbits and attitudes, error sources that are geometrically dependent such as multipath will be correlated to some extent. In order to investigate the effect of correlated measurement noise on the relative attitude solution accuracy, the carrier phase measurements are simulated with varying levels of correlated noise between the leader and follower satellites. As the correlated noise is added, the total noise is adjusted so that $\sigma_n = 5$ cm. Since the total noise level is the same in a probabilistic sense, it is possible to see the benefit of attitude differencing when correlated noise sources are present.

Figure 7 on the following page shows that this is the case. This plot shows the attitude performance when the multipath noise is highly correlated ($\rho = 0.9$) between satellites. The absolute attitude accuracy is approximately unchanged at 0.279 degrees RMS, but the relative attitude accuracy is improved at 0.084 degrees RMS.

These results are collected in table 2. An intermediate case for $\rho = 0.5$ is also listed. As expected, the relative attitude performance improves with the amount of multipath correlation.

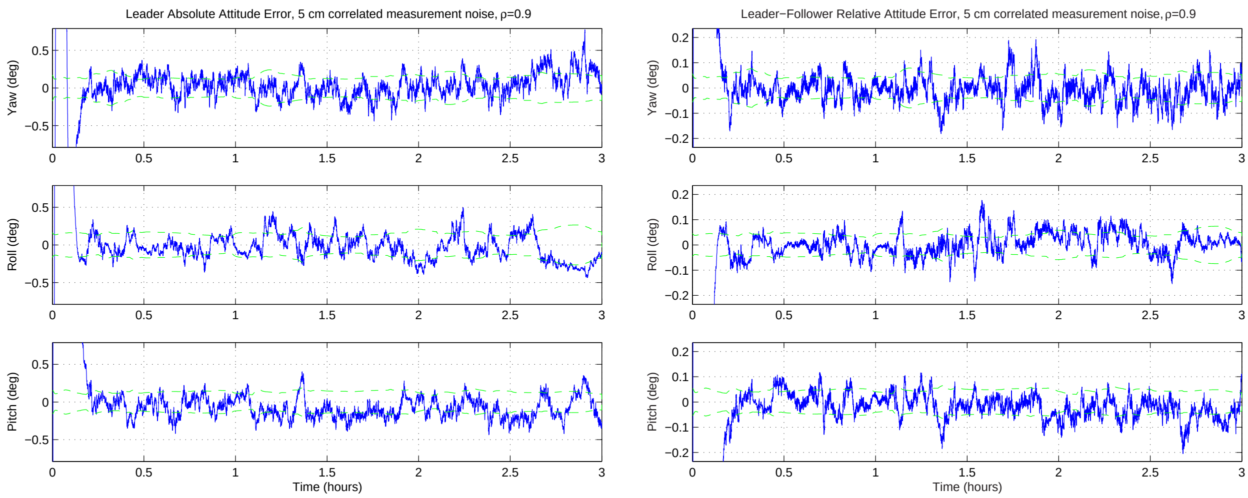
VI. Conclusion

This paper examined how GPS receivers may be employed as attitude sensors in Earth orbiting satellite formations. GPS receivers enjoy advantages as standalone navigation and attitude sensors when the size and cost of the vehicles are tightly constrained. It may be possible to meet the mission requirements in some cases with a minimal hardware complement of a single GPS receiver and possibly a simple additional sensor such as a magnetometer or sun sensor.

The mission presented here is hypothetical, but it is considered to be representative of several Earth

Table 2. Performance Summary for Attitude Simulation.

Case	Comment	σ_n (cm)	ρ	θ_{abserr} (deg RMS)	θ_{relerr} (deg RMS)
1	Noise Free	0	–	0.181	0.006
2	Uncorrelated	5	0	0.295	0.327
3	Somewhat Correlated	5	0.5	0.291	0.210
4	Highly Correlated	5	0.9	0.279	0.084



(a) Leader Attitude Error. (b) Leader-Follower Relative Attitude Error.

Figure 7. Correlated noise ($\rho = 0.9$) absolute attitude error ($\hat{q}_{abserr}$) and leader-follower relative attitude error ($\hat{q}_{relerr}$).

orbiting formation applications. The design of a GPS attitude estimator is codified using a multiplicative extended Kalman filter (MEKF) method. The typical performance of such a system is demonstrated in the presence of measurement noise. When the measurement noise is correlated among the satellites in the formation (correlation coefficient $\rho > 0.5$), improvements in the relative attitude accuracy can be obtained. For the case considered in this study, highly correlated GPS measurements ($\rho = 0.9$) produced more than a threefold improvement in relative attitude accuracy.

Although GPS measurements will never be fully correlated, it may be possible to create a high level of correlation by using the same satellite design and controlling the vehicle attitude so that the geometry of the multipath reflections is similar for each satellite. Attitude control is not addressed in this paper, but a general framework for evaluating the optimality of a formation attitude control system is given.

This work represents an early step in the design of formation attitude estimation systems. In this study, the GPS measurements were processed separately for each satellite and the attitude solutions were differenced. Although good results were obtained, additional work could improve the performance of the relative estimation algorithm by centrally processing all of the available measurements. This would allow the estimator to take maximum advantage of all the correlations in the measurements. It would also be valuable to demonstrate the performance of this system using a real-time hardware-in-the-loop test facility. Experience has shown that hardware tests can expose implementation issues that are not apparent in a simulation study. These additional investigations are left as future work.

Acknowledgment

This work was funded by NASA Goddard Space Flight Center grant number NCC5-688. The author gratefully acknowledges this support.

References

[1]Burns, R., McLaughlin, C. A., Leitner, J., and Martin, M., "Techsat 21: Formation Design, Control, and Simulation," *2000 Aerospace Conference Proceedings*, IEEE, Big Sky, MT, 2000, pp. 19–25.

[2]Cohen, C. E., "Attitude Determination," *Global Positioning System: Theory and Applications, Volume II*, edited by B. W. Parkinson and J. J. J. Spilker, Vol. 164, AIAA, New York, 1996, pp. 519–538.

[3]Teunissen, P. J. G., Jonge, P. J. D., and Tiberius, C. C. J. M., "Performance of the LAMBDA Method for Fast GPS Ambiguity Resolution," *Navigation: Journal of the Institute of Navigation*, Vol. 44, No. 3, 1997, pp. 373–383.

[4]Lightsey, E. G. and Crassidis, J. L., "Real-Time Attitude-Independent GPS Integer Ambiguity Resolution," *The John L. Junkins Astrodynamics Symposium*, American Astronautical Society, College Station, TX, 2003.

[5]Axelrad, P. and Behre, C. P., "Satellite Attitude Determination Based on GPS Signal-to-Noise Ratio," *Proceedings of*

the IEEE, Vol. 7, No. 1, 1999, pp. 122–144.

[6]Madsen, J. and Lightsey, E. G., "Robust Spacecraft Attitude Determination Using Global Positioning System Receivers," *Journal of Spacecraft and Rockets*, Vol. 41, No. 4, 2004, pp. 635–643.

[7]Ebinuma, T., Bishop, R. H., and Lightsey, E. G., "Hardware in the Loop GPS Test Facility for Spacecraft Autonomous Rendezvous," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 3, 2003, pp. 425–433.

[8]Busse, F. D., How, J. P., and Simpson, J., "Demonstration of Adaptive Extended Kalman Filter for Low-Earth-Orbit Formation Estimation Using CDGPS," *Navigation: Journal of the Institute of Navigation*, Vol. 50, No. 2, 2003, pp. 79–93.

[9]Scharf, D. P., Hadaegh, F. Y., and Ploen, S. R., "A Survey of Spacecraft Formation Flying Guidance and Control (Part I): Guidance," *Proceedings of the American Control Conference*, IEEE, Denver, CO, 2003, pp. 1733–1738.

[10]Manikonda, V., Arambel, P. O., Gopinathan, M., Mehra, R. K., and Hadaegh, F. Y., "A Model Predictive Control-based Approach for Spacecraft Formation Keeping and Attitude Control," *Proceedings of the American Control Conference*, AACC, San Diego, CA, 1999, pp. 4258–4262.

[11]Lawton, J., Beard, R. W., and Hadaegh, F. Y., "Elementary Attitude Formation Maneuvers via Leader-Following and Behavior-Based Control," *American Institute of Aeronautics and Astronautics Guidance, Navigation, and Control Conference*, AIAA, Denver, CO, 2000, pp. 1414–1424.

[12]Wang, P. K. C., Hadaegh, F. Y., and Lau, K., "Synchronized Formation Rotation and Attitude Control of Multiple Free-Flying Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 1, 1999, pp. 28–35, also AIAA Paper 97–3753, Aug. 1997.

[13]Luquette, R. J. and Sanner, R. M., "A Nonlinear, Six-Degree of Freedom, Precision Formation Control Algorithm, Based on Restricted Three Body," *26th Annual American Astronautical Society Guidance and Control Conference*, AAS, Breckenridge, CO, 2003.

[14]Schwartz, J. L., *The Distributed Spacecraft Attitude Control Simulator: From Design Concept to Decentralized Control*, Ph.D. thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA, July 2004.

[15]Xing, G. Q., Parvez, S. A., and Folta, D., "Design and Implementation of Synchronized Autonomous Orbit and Attitude Control for Multiple Spacecraft Formation Using GPS Measurement Feedback," *2000 Space Flight Mechanics Conference*, AAS, Clearwater, FL, 2000, pp. 115–134.

[16]Shuster, M. D., "A Survey of Attitude Representations," *Journal of the Astronautical Sciences*, Vol. 41, No. 4, 1993, pp. 439–517.

[17]Markley, F. L., "Attitude Error Representations for Kalman Filtering," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 2, 2003, pp. 311–317.

[18]Wiener, T. F., *Theoretical Analysis of Gimballess Inertial Reference Equipment Using Delta-Modulated Instruments*, Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, MA, March 1962.

[19]Sidi, M. J., *Spacecraft Dynamics and Control, A Practical Engineering Approach*, Cambridge University Press, 1997.

[20]Gaylor, D. E., *Integrated GPS/INS Navigation System Design for Autonomous Spacecraft Rendezvous*, Ph.D. thesis, The University of Texas at Austin, Austin, TX, Dec. 2003.

[21]Gomez, S., "Flying High, GPS on the International Space Station and Crew Return Vehicle," *GPS World*, Vol. 13, No. 6, 2002, pp. 12–20.

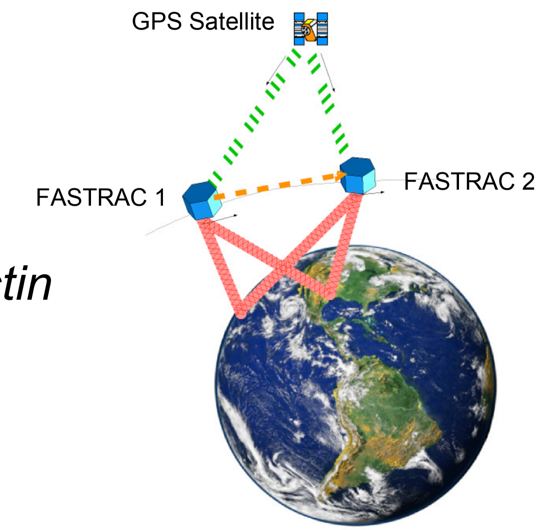
HOME

Relative Attitude Determination of Earth Orbiting Satellites Using GPS Receivers

E. Glenn Lightsey

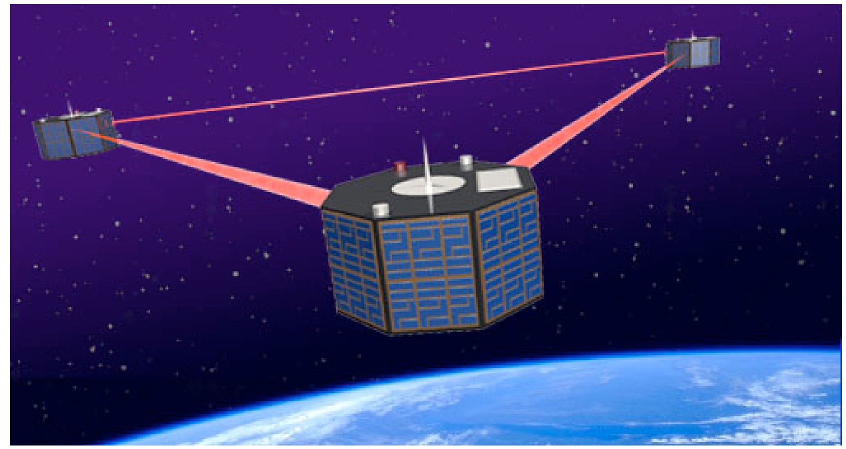
Associate Professor

The University of Texas at Austin



Introduction and Motivation

- Formation Guidance is usually separated into two tasks
  - Position trajectory determination and guidance
  - Attitude trajectory determination and guidance



- For most formation missions, these objectives are connected and should be considered simultaneously
  - Mission objectives require simultaneous navigation and attitude targeting of all vehicles in the formation
  - Attitude constraints may affect navigation guidance (e.g. thruster firing)

Prior Research

- Hundreds of references (e.g. **Scharf ’03**) on formation navigation estimation and guidance

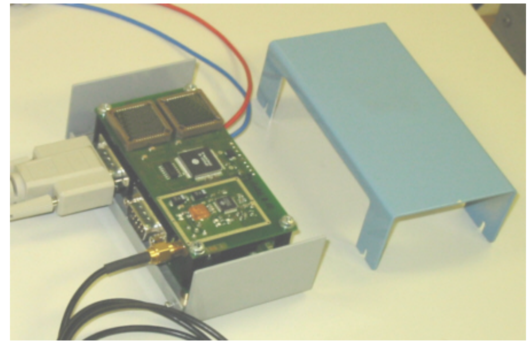
- Substantially less research (~10 papers) on formation attitude
    - Most focus on guidance (e.g. **Manikonda ’99, Lawton ’00, Wang ’99**)
    - Less emphasis on specific sensor technology, estimation methods, and achievable performance

- GPS receivers have been used for on-orbit navigation and attitude determination of individual spacecraft (e.g. **Cohen ’96, Gomez ’02**)

- How can GPS attitude technology be applied to LEO formation attitude determination, specifically *relative* attitude determination?

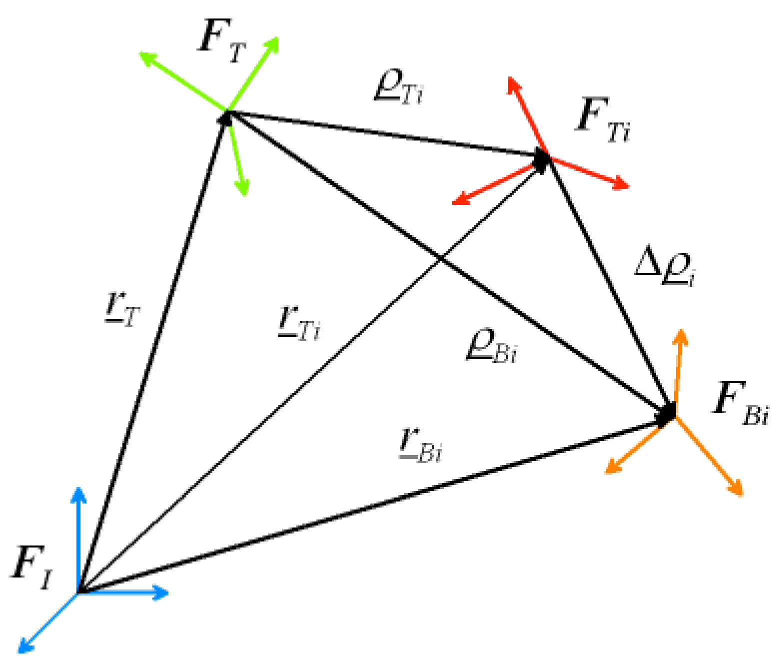
Why Use GPS?

- It is compact
- It is relatively cheap



- It is probably already on the satellite
- It is reasonably mature and well understood
- It is a stepping stone to other sensors and orbit types (e.g. Libration point orbiting formations)

Formation Reference Frame Definitions



(ref. Xing '00)

- $\boldsymbol{F_I}$: $(\underline{i}_I, \underline{j}_I, \underline{k}_I)$ = inertial reference frame
- $\boldsymbol{F_T}$: $(\underline{i}_T, \underline{j}_T, \underline{k}_T)$ = formation target RF
- $\boldsymbol{F_{Ti}}$: $(\underline{i}_{Ti}, \underline{j}_{Ti}, \underline{k}_{Ti})$ = *i*'th satellite target RF
- $\boldsymbol{F_{Bi}}$: $(\underline{i}_{Bi}, \underline{j}_{Bi}, \underline{k}_{Bi})$ = *i*'th satellite body cm RF

- The formation target is specified wrt to the inertial reference by $\underline{r}_T$.
- The *i*'th satellite target is specified wrt to the formation target by □$_{Ti}$.
- The *i*'th satellite body cm RF is specified wrt to the *i*'th satellite target by □□$_i$.

Translational Kinematics of *i*'th Satellite

Relative to *i*'th Satellite Target RF Velocity Result:

$$\frac{{}^{Ti}d}{dt}(\square\underline{\square}_i)$$

Relative to Formation Target RF Velocity Result:

$$\frac{{}^{T}d}{dt}(\underline{\square}_{Bi}) = \frac{{}^{T}d}{dt}(\underline{\square}_{Ti}) + \frac{{}^{Ti}d}{dt}(\square\underline{\square}_i) + \square^{TTi}\,\square\square\underline{\square}_i$$

Relative to Inertial RF Velocity Result:

$$\frac{{}^{I}d}{dt}(\underline{r}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_{T}) + \frac{{}^{T}d}{dt}(\underline{\square}_{Ti}) + \square^{IT}\,\square\underline{\square}_{Ti} + \frac{{}^{Ti}d}{dt}(\square\underline{\square}_i) + \square^{ITi}\,\square\square\underline{\square}_i$$

Translational Kinematics of *i*'th Satellite

Relative to *i*'th Satellite Target RF Acceleration Result:

$$\frac{{}^{Ti}d^2}{dt^2}(\Box\underline{\Box}_i)$$

Relative to Formation Target RF Acceleration Result:

$$\frac{{}^{T}d^2}{dt^2}(\underline{\Box}_{Bi}) = \frac{{}^{T}d^2}{dt^2}(\underline{\Box}_{Ti}) + \frac{{}^{Ti}d^2}{dt^2}(\Box\underline{\Box}_i) + 2\Box^{TTi}\Box\frac{{}^{T}d}{dt}(\Box\underline{\Box}_i) + \frac{{}^{T}d}{dt}(\Box^{TTi})\Box\Box\underline{\Box}_i + \Box^{TTi}\Box(\Box^{TTi}\Box\Box\underline{\Box}_i)$$

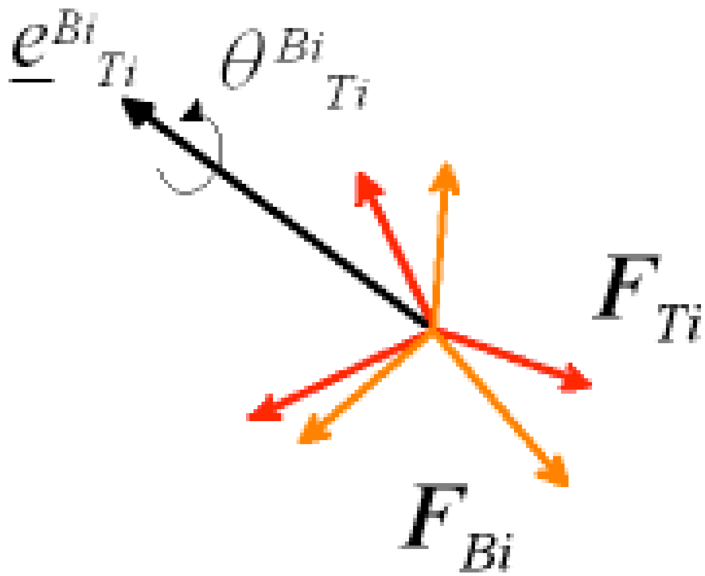
Relative to Inertial RF Acceleration Result:

$$\frac{{}^{I}d^2}{dt^2}(\underline{r}_{Bi}) = \frac{{}^{I}d^2}{dt^2}(\underline{r}_{T}) + \cdots$$

$$\frac{{}^{T}d^2}{dt^2}(\underline{\Box}_{Ti}) + 2\Box^{IT}\Box\frac{{}^{T}d}{dt}(\underline{\Box}_{Ti}) + \frac{{}^{I}d}{dt}(\Box^{IT})\Box\underline{\Box}_{Ti} + \Box^{IT}\Box(\Box^{IT}\Box\underline{\Box}_{Ti}) + \cdots$$

$$\frac{{}^{Ti}d^2}{dt^2}(\Box\underline{\Box}_i) + 2\Box^{ITi}\Box\frac{{}^{T}d}{dt}(\Box\underline{\Box}_i) + \frac{{}^{I}d}{dt}(\Box^{ITi})\Box\Box\underline{\Box}_i + \Box^{ITi}\Box(\Box^{ITi}\Box\Box\underline{\Box}_i)$$

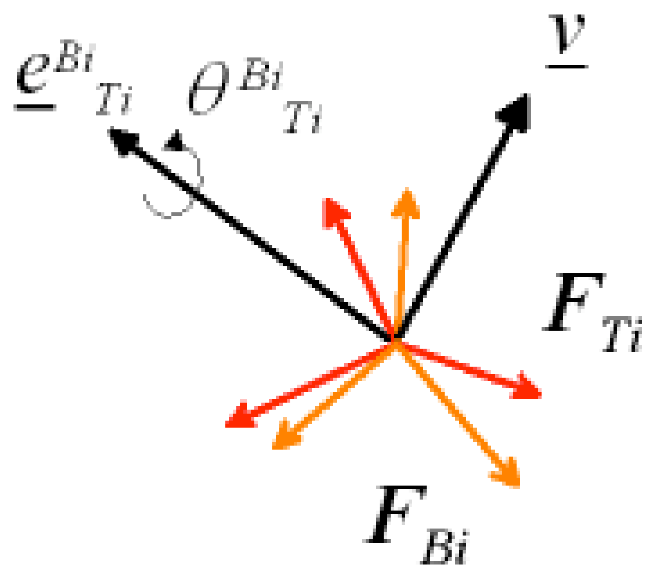
Euler Rodrigues Parameters



- $\boldsymbol{F_{Ti}}$: ($\underline{i}_{Ti}$, $\underline{j}_{Ti}$, $\underline{k}_{Ti}$) = *i*'th satellite target RF
- $\boldsymbol{F_{Bi}}$: ($\underline{i}_{Bi}$, $\underline{j}_{Bi}$, $\underline{k}_{Bi}$) = *i*'th satellite body cm RF
- Euler Rodrigues Parameters Definition
  - $\underline{q}$ = $\underline{e}$ sin(□ / 2) = [q_1 , q_2 , q_3]^T
  - q_4 = cos(□ / 2)
  - Quaternion q = [$\underline{q}$, q_4]^T

- Unity Norm Constraint: ‖ ‖
- Time Derivative:
- Euler's Equation: $J\square + \square\square J\square = \underline{M}_d + \underline{M}_c$

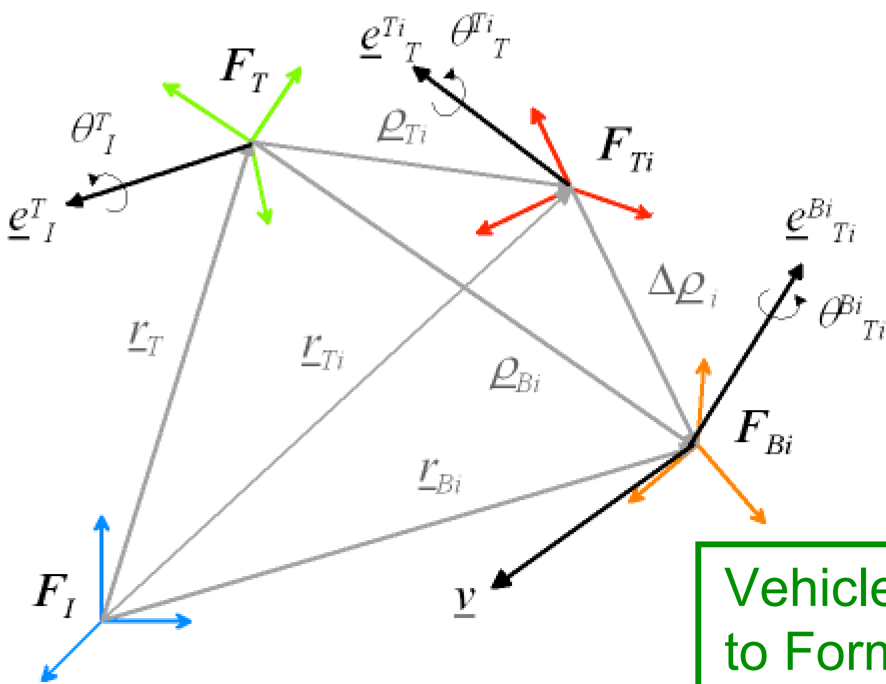
Quaternions as a Rotation Operator



- $\boldsymbol{F_{Ti}}$: ($\underline{i}_{Ti}$, $\underline{j}_{Ti}$, $\underline{k}_{Ti}$) = *i*'th satellite target RF
- $\boldsymbol{F_{Bi}}$: ($\underline{i}_{Bi}$, $\underline{j}_{Bi}$, $\underline{k}_{Bi}$) = *i*'th satellite body cm RF
- Vector $\underline{v}$ may be coordinated in *Ti* or *Bi*
- Define vector quaternion $v = [\ \underline{v}\ , 0\ ]^T$

- Quaternion Multiplication:
- Quaternion Conjugate:
- Quaternion Rotation Operator:

ER Parameters, Successive Rotations



Vehicle Attitude Relative to *i*'th Satellite Target RF: q^{Bi}_{Ti}

Vehicle Attitude Relative to Formation Target RF:

Vehicle Attitude Relative to Inertial RF:

Generalized Vehicle Attitude Tracking

Design a control law ***C*** subject to constraints and disturbances ***D*** such that:

Vehicle Attitude Tracking Relative to *i*'th Satellite Target RF:

Vehicle Attitude Tracking Relative to Formation Target RF:

Vehicle Attitude Tracking Relative to Inertial RF:

Generalized Attitude Performance Index

- Define Attitude Performance Vector for the *i*'th Vehicle:

- Generalized Attitude Performance Index is given by:

$$J_i(t) = \frac{1}{2}\int_0^t \left(\underline{z}_i^{\mathrm{T}} Q_i \underline{z}_i + \underline{u}_i^{\mathrm{T}} R_i \underline{u}_i\right) dt$$

$\underline{u}_i$ is the actuator control effort
Q_i and R_i are gain weighting matrices

- $\underline{z}_i$ may be augmented with translational states if desired

- J_i may include Lagrange multipliers to include constraints

Formation Control Law Design

- The Locally Optimal Control Law for the *i*'th vehicle is given by:

$$\boldsymbol{C_i} : \min_{t \to \infty} J_i(t)$$

- The Globally Optimal Control Law for the formation is given by:

$$\boldsymbol{C} : \min \sum_i J_i(t \to \infty)$$

Multiplicative Extended Kalman Filter

Motivation: **(ref. Markley '03, Lefferts '82)**

- Rotation group possesses 3 degrees of freedom
- Quaternions are defined as 4-tuples with unity norm constraint
- If using an additive EKF, maintaining unit norm is difficult, e.g.:

Solution:

- Use MEKF, where perturbation is a small angle ($\underline{a}$) correction with 3 degrees of freedom from a reference attitude (q_{ref}):

- $\underline{a}$ is the Modified Rodrigues Parameter:

MEKF Time Update Equations

- Use Euler's Equations to propagate state:

Principal Axes Assumed

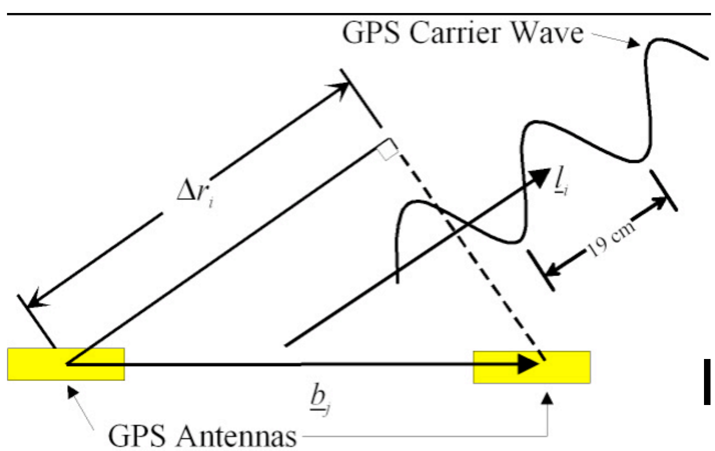
- Quaternion Kinematic Relation

- Covariance Propagation:

Vehicle Attitude Sensing Using GPS

- Raw Measurements Available for Attitude Determination
  - Signal to Noise Ratio : Provides Coarse Attitude Information and assists with Carrier Phase Ambiguity Resolution
  - Differential Carrier Phase : Provides Sub Degree Attitude Estimation using an array of GPS Antennas attached to a rigid platform
- Additional Sensors
  - Magnetometer : Provides Local Magnetic Field Measurements
- Any Combination of These Sensors May Be Used, Additional Sensors May Be Added if Available

Diff. Carrier Phase Measurement Model



$$\square r = \underline{l}_i \cdot \underline{b}_j$$

$$\square\square_{ij} = \square r \square k_{ij} + \square_j$$

$$\square\square\square_{ijk} = \square\square_{ij} \square\square\square_{kj} = \underline{b}_j \cdot \left(\underline{l}_i \square \underline{l}_k\right) \square \left(k_{ij} \square k_{kj}\right)$$

If integer is known, it may be removed:

$$y = \square\square\square_{ijk} + \left(\square_{ij} \square\square_{kj}\right)$$

$$h = \underline{b}_j \cdot \left(\underline{l}_i \square \underline{l}_k\right)$$

$\underline{b}$ and $\underline{l}$ must be coordinated in the same reference frame, say Inertial (I) or GPS frame:

Diff. Carrier Phase Measurement Model (2)

Write $C(q)$ as a perturbation to a reference quaternion

Allows measurement model for h to be written as

Measurement sensitivity matrix

MEKF Measurement Update

Kalman Gain Matrix

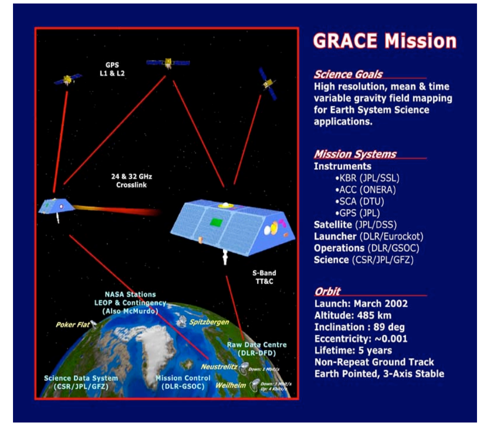
Attitude Error State Update

Covariance Measurement Update

6N DOF LEO Formation Simulation

- Rotational and Translational States Simulated
- Sensor Measurements Simulated Based on True Position and Attitude Plus Sensor Noise (e.g., multipath)
- Attitude Estimation Performed on Simulated Measurements
- Gravity Gradient Attitude Dynamics Simulated
- No Transmit Latency

Leader-Follower 2-node Formation



- Formation Target RF ($\boldsymbol{F_T}$) mean elements:

  h_{per} = 800 km   □ = 0
  i = 89.9 deg   □ = 0
  e = 0 $□_0$ = 0

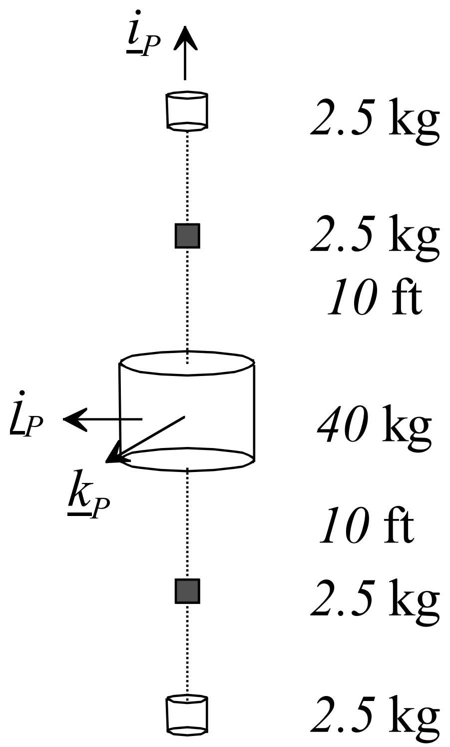
- 'Leader' Satellite Tracks $\boldsymbol{F_T}$ RF

- 'Follower' Satellite Tracks $\boldsymbol{F_{T2}}$ RF at R = 100 km From Leader (RFs rotate at orbit rate)

- All Tracked RFs ($\boldsymbol{F_T}$ and $\boldsymbol{F_{T2}}$) are aligned to Local Vertical

- Initial Simulation: Two Body (Earth/Satellite) Point Mass Central Force Motion, Gravity Gradient Torques Only

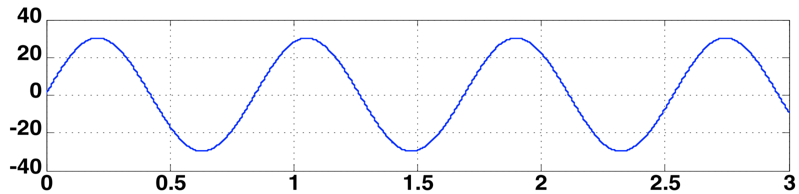
Satellite Mass Properties

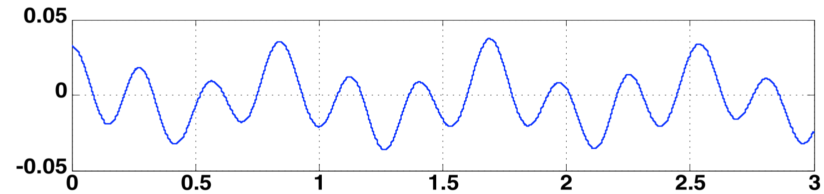


- Axisymmetric, Twin Boom Design
- Total Satellite Mass = 50 kg
- I_x = 1.09 kg m^2, I_y = I_z = 71.4 kg m^2
- $k_{pitch} = k_{roll} = 0.985$
- $k_{yaw} = 0$

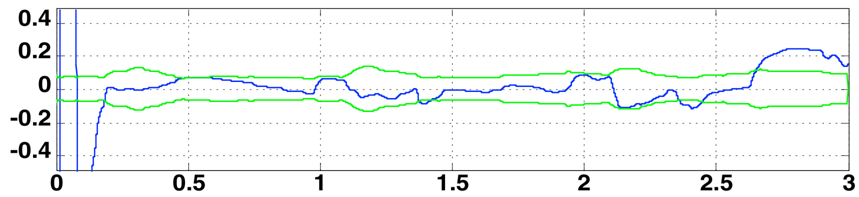
- Assumed rigid structure, zero inertia products
- 4 antenna array, 1x1x1 m Cube (blockage not considered)

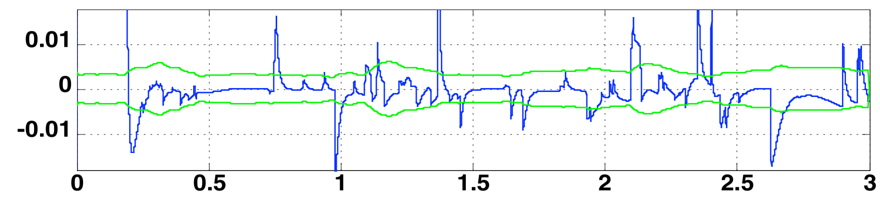
Gravity Gradient Dynamics





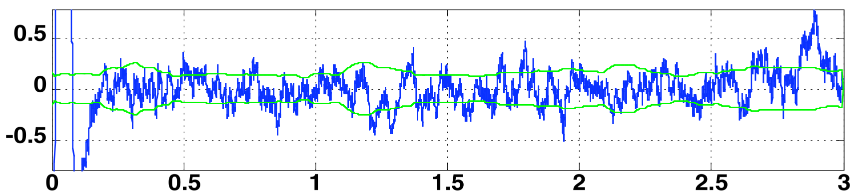
Noise Free Attitude Performance

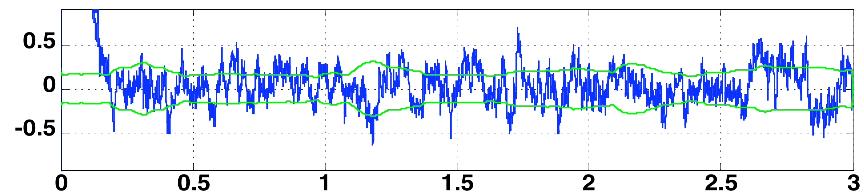




Angle (deg)	Leader Absolute Error			L-F Relative Error		
	Mean	□	RMS	Mean	□	RMS
Yaw	.022	.085	.107	.000	.004	.004
Roll	-.040	.093	.097	.000	.003	.003
Pitch	-.106	.062	.168	.000	.003	.003
Three Axis	**.115**	**.140**	**.181**	**.000**	**.006**	**.006**

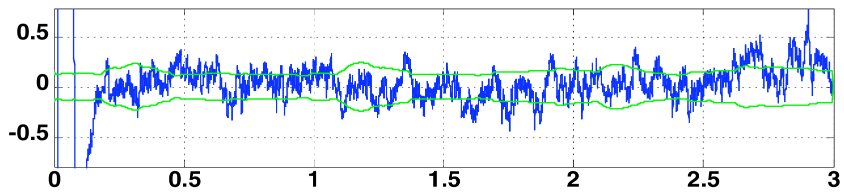
5 cm Uncorrelated Measurement Noise

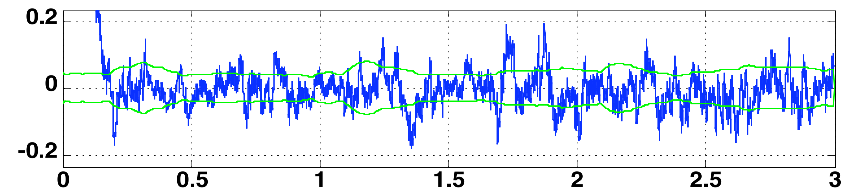




Angle (deg)	Leader Absolute Error			L-F Relative Error		
	Mean	□	RMS	Mean	□	RMS
Yaw	.008	.166	.174	.026	.196	.222
Roll	-.055	.159	.214	.019	.178	.197
Pitch	-.102	.143	.245	.017	.189	.206
Three Axis	**.116**	**.270**	**.294**	**.036**	**.325**	**.327**

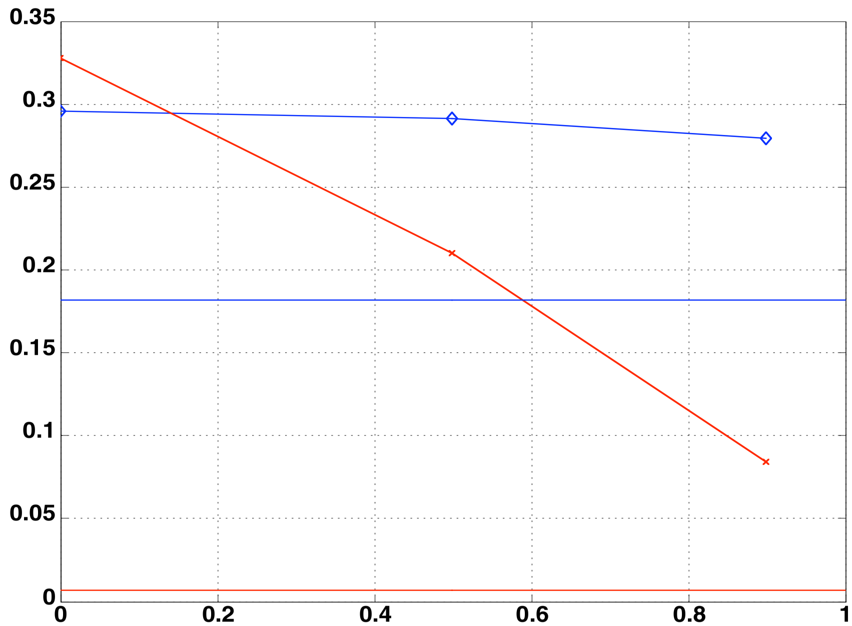
5 cm Correlated Meas. Noise, □ = 0.9





Angle (deg)	Leader Absolute Error			L-F Relative Error		
	Mean	□	RMS	Mean	□	RMS
Yaw	.042	.155	.197	.009	.031	.040
Roll	-.036	.160	.196	.008	.032	.033
Pitch	-.081	.137	.218	.011	.055	.066
Three Axis	**.098**	**.261**	**.279**	**.031**	**.078**	**.084**

Summary of Correlated Noise Performance



Case	Comment	$\square_{n}$ (cm)	$\square$	$\square_{abserr}$ (deg)	$\square_{relerr}$ (deg)
1	Noise Free	0	--	.181	.006
2	Uncorrelated	5	0	.295	.327
3	Somewhat Correlated	5	0.5	.291	.210
4	Highly Correlated	5	0.9	.279	.084

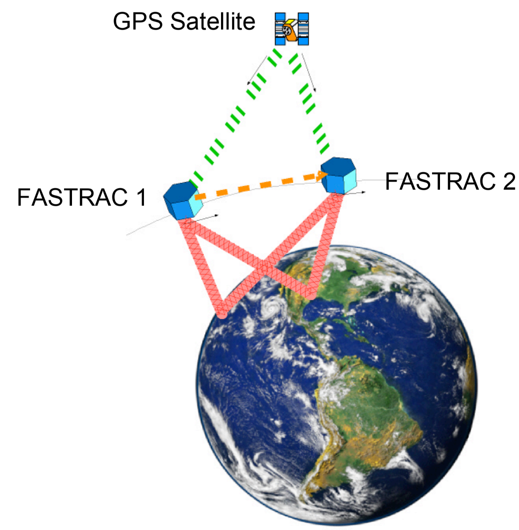
Conclusions

- **An architecture has been proposed for formation flying**
  attitude determination problems
- **A Multiplicative Extended Kalman Filter has been developed**
  using GPS carrier phase measurement updates
- **The MEKF was applied to a simulated LEO Leader-Follower Formation**
  as a demonstration for relative attitude determination
- **An improvement in relative attitude accuracy is possible**
  when the measurement noise is correlated (□ >= 0.5)
- **Arc-minute level relative attitude determination is possible using GPS**
  when the measurement noise is highly correlated (□ >= 0.9)

Ideas for Future Research

- **Extending the Range of Application using new sensors**
  - Interplanetary and Libration Formations: Cross-links
  - LEO Formations: GPS SNR + Magnetometer
- **Combine Measurements in a Centralized Optimal Filter**
  - Take Full Advantage of Measurement Correlations
- **Host Algorithms on Suitable Sensor Hardware Platform**
  - GPS Receiver, or RF/Optical Transceiver
  - Demonstrate Via Hardware Simulation
- **Consider Integrated Formation Sensor & Controller Problem**
  - Design of Mission-Specific Guidance Laws
  - Coupled Navigation and Attitude Guidance Problems

Questions and Comments



Translational Kinematics of *i*'th Satellite

Relative Velocity Derivation:

$$\underline{\square}_{Bi} = \underline{\square}_{Ti} + \square\underline{\square}_{i}$$

$$\frac{{}^{T}d}{dt}(\underline{\square}_{Bi}) = \frac{{}^{T}d}{dt}(\underline{\square}_{Ti} + \square\underline{\square}_{i}) = \frac{{}^{T}d}{dt}(\underline{\square}_{Ti}) + \frac{{}^{Ti}d}{dt}(\square\underline{\square}_{i}) + \square^{TTi}\,\square\square\underline{\square}_{i}$$

Inertial Velocity Derivation:

$$\underline{r}_{Bi} = \underline{r}_{T} + \underline{\square}_{Bi} \quad , \quad \underline{\square}_{Bi} = \underline{\square}_{Ti} + \square\underline{\square}_{i}$$

$$\frac{{}^{I}d}{dt}(\underline{r}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_{T} + \underline{\square}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_{T}) + \frac{{}^{T}d}{dt}(\underline{\square}_{Bi}) + \square^{IT}\,\square\underline{\square}_{Bi}$$

$$\frac{{}^{I}d}{dt}(\underline{r}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_{T}) + \frac{{}^{T}d}{dt}(\underline{\square}_{Ti} + \square\underline{\square}_{i}) + \square^{IT}\,\square(\underline{\square}_{Ti} + \square\underline{\square}_{i})$$

$$\frac{{}^{I}d}{dt}(\underline{r}_{Bi}) = \frac{{}^{I}d}{dt}(\underline{r}_{T}) + \frac{{}^{T}d}{dt}(\underline{\square}_{Ti}) + \square^{IT}\,\square\underline{\square}_{Ti} + \frac{{}^{T}d}{dt}(\square\underline{\square}_{Ti}) + \square^{IT}\,\square\square\underline{\square}_{i}$$

$$\frac{{}^{T}d}{dt}(\square\underline{\square}_{Ti}) = \frac{{}^{Ti}d}{dt}(\square\underline{\square}_{i}) + \square^{TTi}\,\square\square\underline{\square}_{i}$$

Extended Kalman Filter

(ref. Markley '03)

Model:

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t),t) + \underline{w}(t)\,, \quad E[\underline{w}(t)\underline{w}^T(\tau)] = \mathbf{Q}(t)\delta(t-\tau)$$

$$y = h(\underline{x}(t)) + \nu(t)\,, \quad E[\nu(t)\nu^T(\tau)] = r(t)\delta(t-\tau)$$

Propagation:

$$\dot{\hat{\underline{x}}}(t) = \underline{f}(\hat{\underline{x}}(t),t)$$

$$\mathbf{A}\left(\hat{\underline{x}}(t)\right) = (\partial \underline{f} / \partial \underline{x})|_{\underline{x}=\hat{\underline{x}}}$$

$$\dot{\mathbf{P}}(t) = \mathbf{A}(\hat{\underline{x}}(t))\mathbf{P}(t) + \mathbf{P}(t)\mathbf{A}^{\mathbf{T}}(\hat{\underline{x}}(t)) + \mathbf{Q}$$

Update:

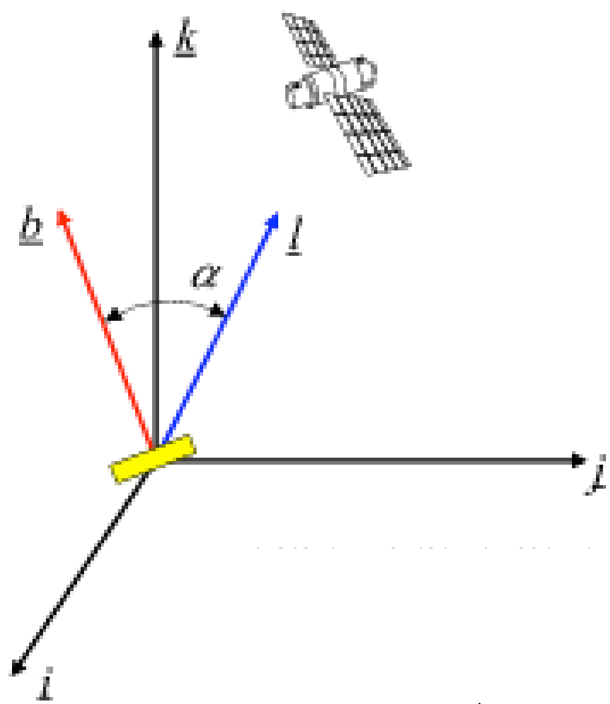
$$\mathbf{H}\left(\hat{\underline{x}}(-)\right) = \left(\partial h / \partial \underline{x}\right)|_{\underline{x}=\hat{\underline{x}}(-)}$$

$$\underline{K} = \mathbf{P}(-)\mathbf{H}^{\mathbf{T}}\left(\hat{\underline{x}}(-)\right)\left[\mathbf{H}\left(\hat{\underline{x}}(-)\right)\mathbf{P}(-)\mathbf{H}^{\mathbf{T}}\left(\hat{\underline{x}}(-)\right) + r\right]^{-1}$$

$$\hat{\underline{x}}(+) = \hat{\underline{x}}(-) + \underline{K}\left[y - h\left(\hat{\underline{x}}(-)\right)\right]$$

$$\mathbf{P}(+) = \left[\mathbf{I} - \underline{K}\mathbf{H}\left(\hat{\underline{x}}(-)\right)\right]\mathbf{P}(-)$$

SNR Measurement Model



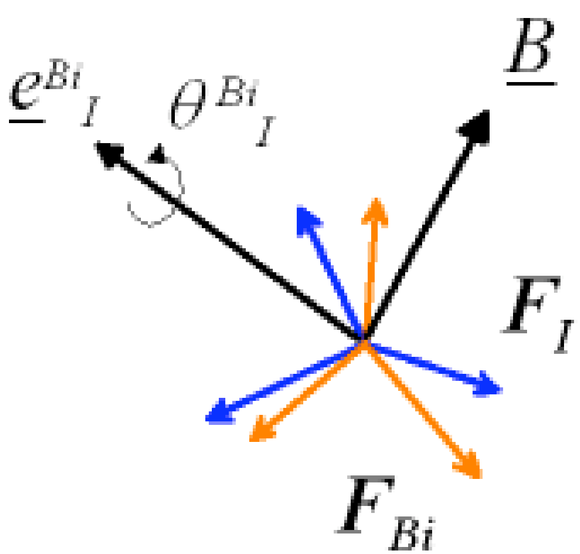
$$y = \square = f(\text{SNR}) + \square$$

$$h = \cos(\square) = \underline{b} \cdot \underline{l} = b_x l_x + b_y l_y + b_z l_z$$

$\underline{b}$ and $\underline{l}$ must be coordinated in the same reference frame, say Inertial (I) or GPS frame:

Result:

Magnetometer Measurement Model



Magnetometer measures local $\underline{B}$-field in body frame (*Bi*), which is approximately known in inertial (*I*) or other frame.

Vector measurement may therefore be used to estimate vehicle attitude:

$$\underline{y} = \underline{B}_{Bi} + \square$$

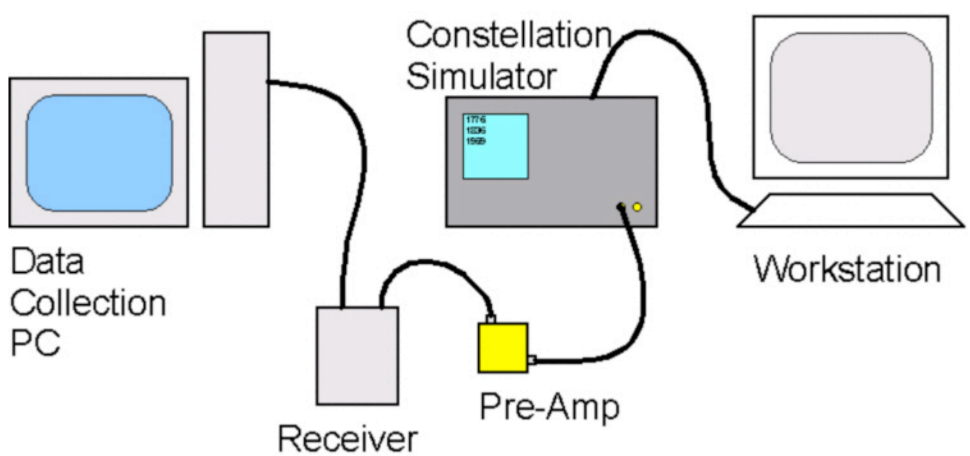
Magnetometer Measurement Model

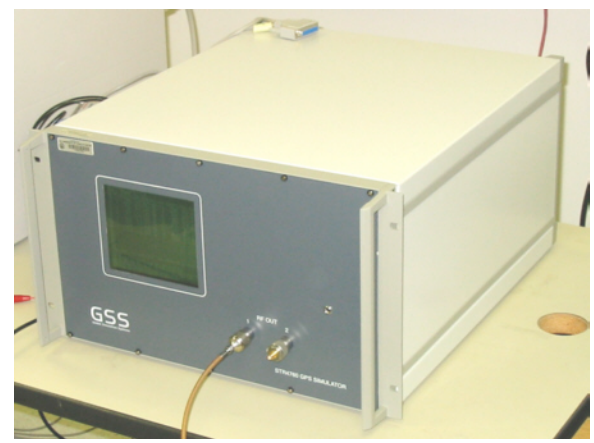
Result:

Note some sign differences
are due to different direction of rotation
for quaternion

Hardware in the Loop Real-Time Testing

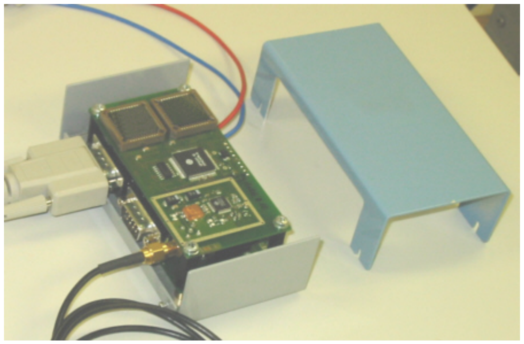
- GPS Constellation Simulator
  - -2x12 RF Hardware Simulation of GPS Signal
  - -Dual Frequency Capable (e.g. JPL's BlackJack)
  - -Clinical Test Environment Allows Determination of Receiver Performance





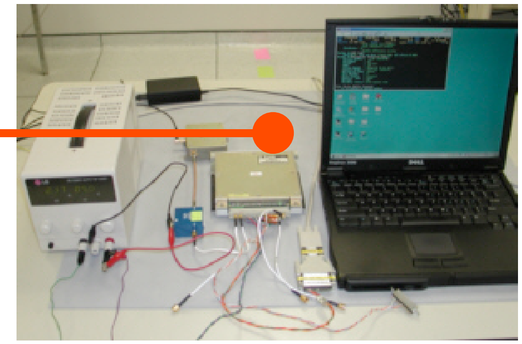
UT Space Receiver Test Resources

Orion



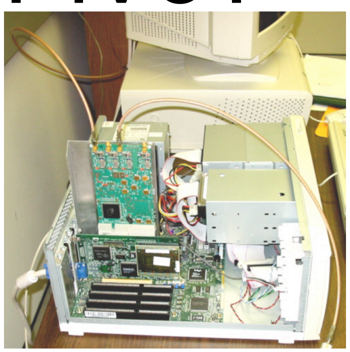
- 1 RF Coarse SNR + Carrier Phase
- Modifiable Code Version D07M (Full Software Access)
- SNR + Magnetometer Attitude Code Being Developed for FASTRAC

Force-19



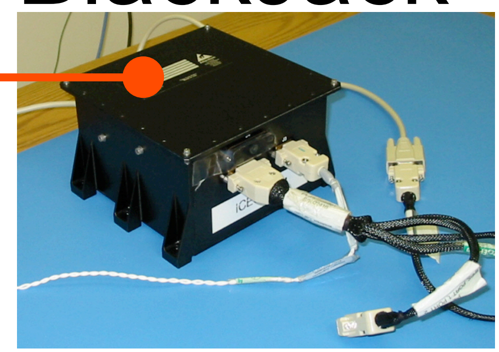
- SIGI Attitude Code maintained at JSC (Access Questionable)
- 4 RF Carrier Phase + Coarse SNR
- 1 RF No Longer Works on UT Model

PiVoT



- 1 RF Good SNR Measurement Ability
- Unknown Carrier Phase Measurement
- UT Version Not Used Since 2003

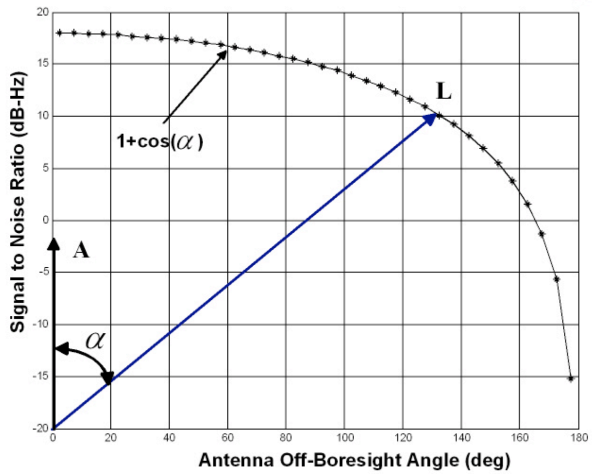
BlackJack



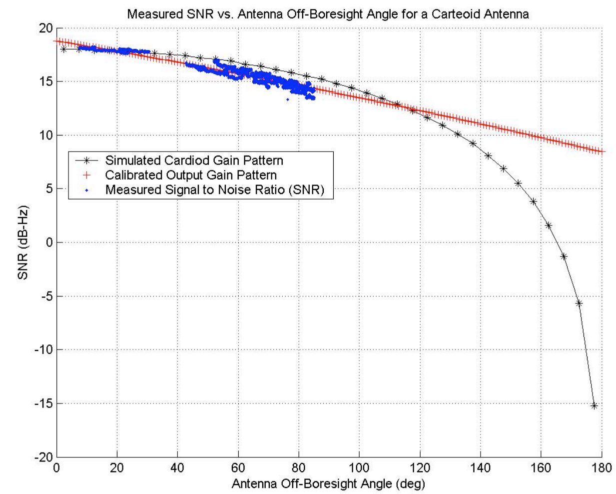
- 1 RF Good SNR + Carrier Phase
- Dual Frequency/patented tracking
- 2 RF Version Possible
- Expensive (~$100k's) and Power Hungry (~10 W)
- Would have to approach JPL for permission to develop (have source)

Orion SNR Measurement Limitations

Theoretical (Cardioid)



Measured Using STR4760



- Calibration Mapping Function Routinely Performed, _but_
- Orion SNR Measurement is Poorly Quantized and Contains Errors
- Poor Gain Sensitivity at Highest SNR Levels (Antenna Pattern)

- PiVoT : Best SNR Measurement Resolution, but unknown status

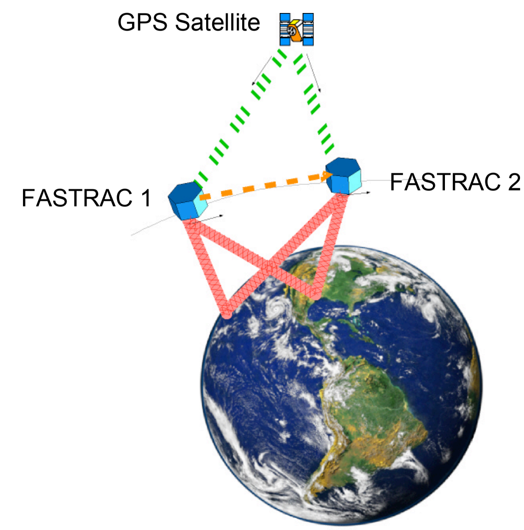
Status of Orion Hardware Demo

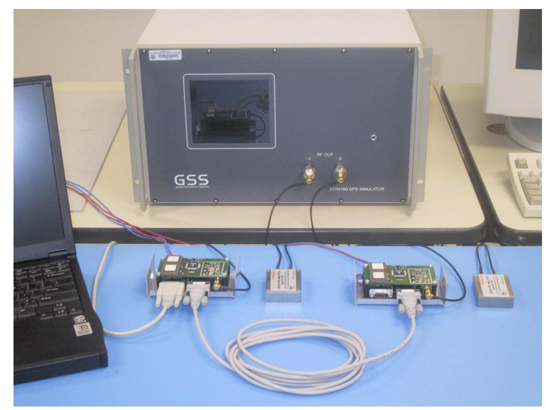
- SNR + Magnetometer Attitude Algorithm Currently Being Embedded on Orion receiver for FASTRAC (summer), *but*

- Expected Demo performance less than Simulation Due to available Orion Receiver limitations

- Algorithm Demo completion expected end of summer 2004 (after end of this task)

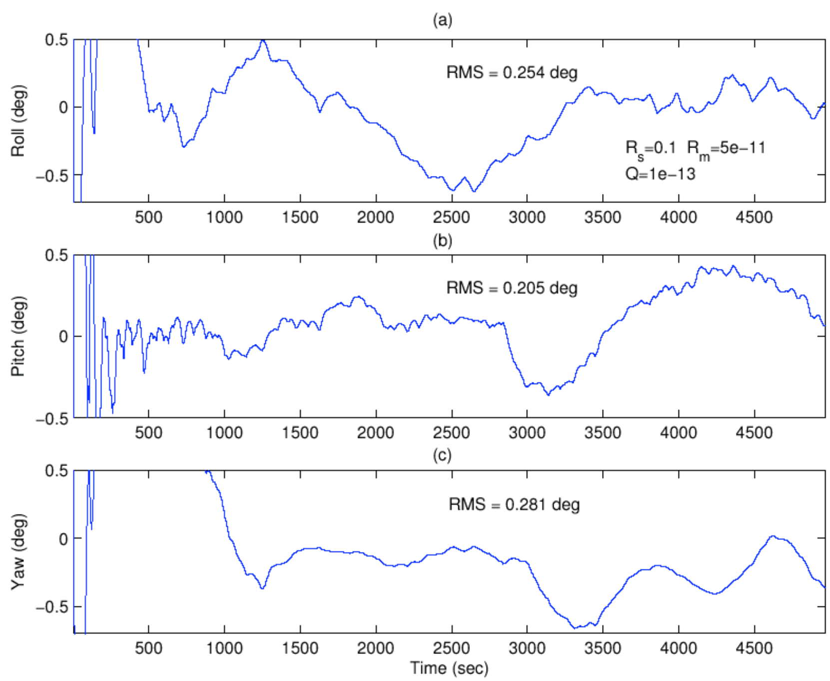
- Test Results Should be Available By September 2004 Formations Conference





SNR/Magnetometer, Prior Results (2003)

- Single Antenna GPS + Magnetometer Attitude Determination Using Signal To Noise Ratio Pattern Matching



Conclusions

- Signal strength measurements can be used to resolve integers
- New algorithm more robust than previous
- Better signal strength measurements would improve resolution further
- Magnetometer (or other sensor) can also resolve integers