# COMPARISON OF THE RADIATIVE TWO-FLUX AND DIFFUSION APPROXIMATIONS

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## Abstract

Approximate solutions are sometimes used to determine the heat transfer and temperatures in a semitransparent material in which conduction and thermal radiation are acting. A comparison of the Milne-Eddington two-flux approximation and the diffusion approximation for combined conduction and radiation heat transfer in a ceramic material was preformed to determine the accuracy of the diffusion solution. A plane gray semitransparent layer without a substrate and a non-gray semitransparent plane layer on an opaque substrate were considered. For the plane gray layer the material is semitransparent for all wavelengths and the scattering and absorption coefficients do not vary with wavelength. For the non-gray plane layer the material is semitransparent with constant absorption and scattering coefficients up to a specified wavelength. At higher wavelengths the non-gray plane layer is assumed to be opaque. The layers are heated on one side and cooled on the other by diffuse radiation and convection. The scattering and absorption coefficients were varied. The error in the diffusion approximation compared to the Milne-Eddington two flux approximation was obtained as a function of scattering coefficient and absorption coefficient. The percent difference in interface temperatures and heat flux through the layer obtained using the Milne-Eddington two-flux and diffusion approximations are presented as a function of scattering coefficient and absorption coefficient. The largest errors occur for high scattering and low absorption except for the back surface temperature of the plane gray layer where the error is also larger at low scattering and low absorption. It is shown that the accuracy of the diffusion approximation can be improved for some scattering and absorption conditions if a reflectance obtained from a Kubelka-Munk type two flux theory is used instead of a reflection obtained from the Fresnel equation. The Kubelka-Munk reflectance accounts for surface reflection and radiation scattered back by internal scattering sites while the Fresnel reflection only accounts for surface reflections.

# Comparison of the Radiative Two-flux and Diffusion Approximations

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# Objective

Determine the error in using the diffusion solution compared to the Milne-Eddington two-flux solution

Determine if using a reflectance from the Kubelka-Munk 2-flux theory could reduce the diffusion solution error

# Approach

A parametric study of a one dimensional semitransparent gray plane layer and a non-gray layer on a substrate was performed

Fresnel and Kubelka-Munk reflectance were used

Baseline   $a = 0.13\ cm^{-1}$ and $\sigma_s = 94.38\ cm^{-1}$

Bond coat emissivity changed from 0.7 to 0.3

Cutoff wavelength 5 μm for the layer on a substrate

# Approximations

Diffusion approximation

- simplest
- radiation treated as a diffusion process
- radiation is absorbed  emitted and reflected at the surface

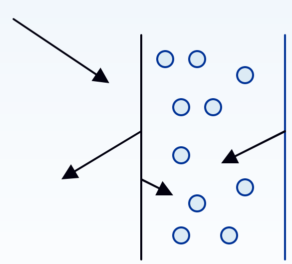
Two-flux approximation

- more complicated requires computer solution
- there is a radiative flux traveling in positive and negative x directions and act at distance
- radiation is absorbed and emitted inside the material
- surface reflection still occur

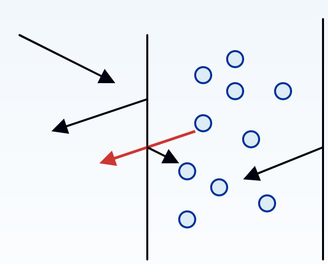
# Reflections

Fresnel reflections $\rho^o$ and $\rho^i$ are surface reflections and are functions of refractive index.

Kubelka-Munk reflectivity takes into account Fresnel surface reflections and the radiative energy scattered out of the layer. It is a function of $\rho^o$, $\rho^i$, a, $\sigma_s$, and D

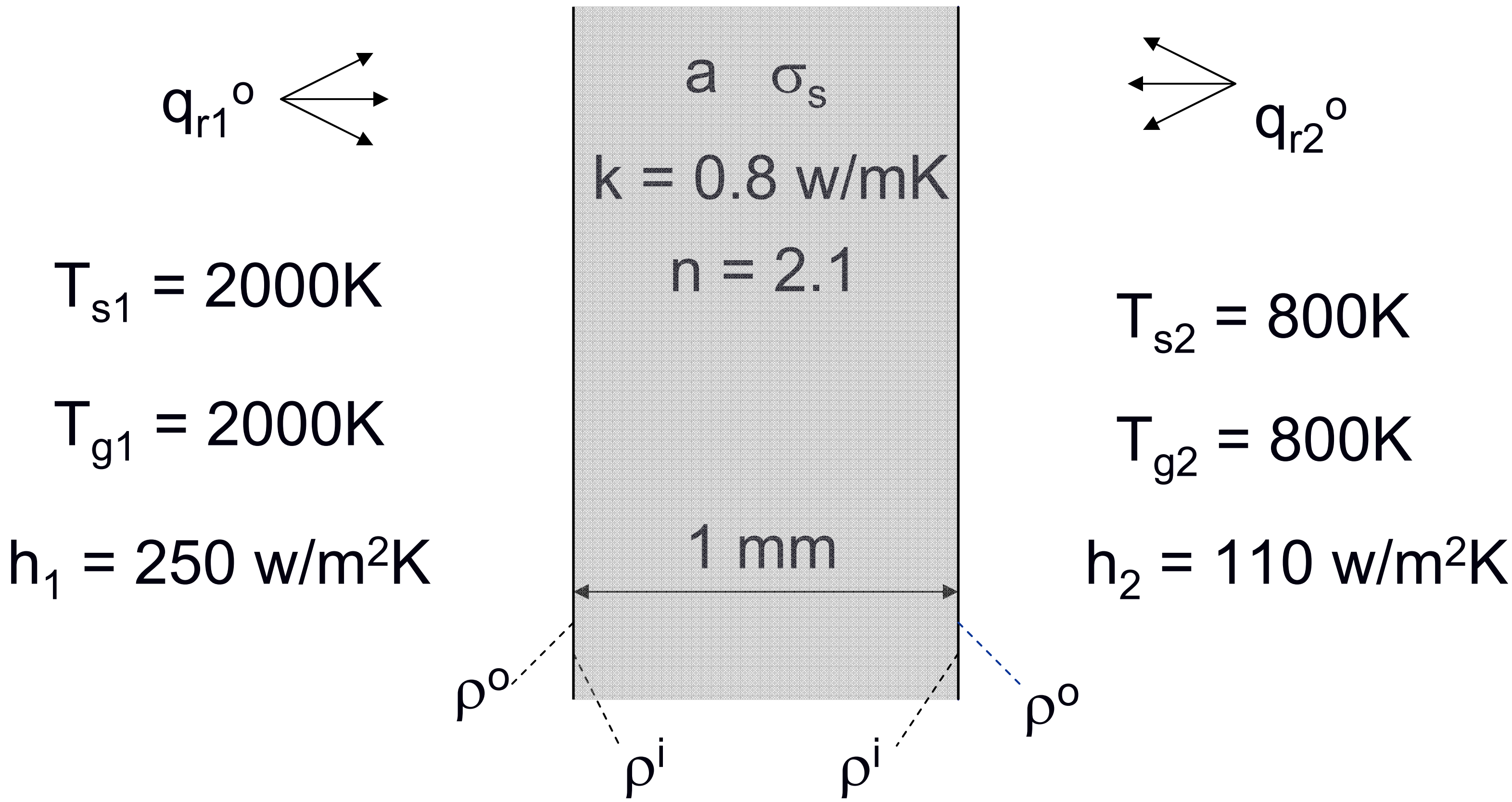


Fresnel



Kubelka -Munk

# Heat Transfer Model



# Comparison of Exact Radiative Transfer and Milne-Eddington Two-Flux Approximation

$$\textbf{Percent difference} = \left( \frac{T_{\text{two-flux solution}} - T_{\text{exact solution}}}{T_{\text{exact solution}}} \right) \cdot 100$$

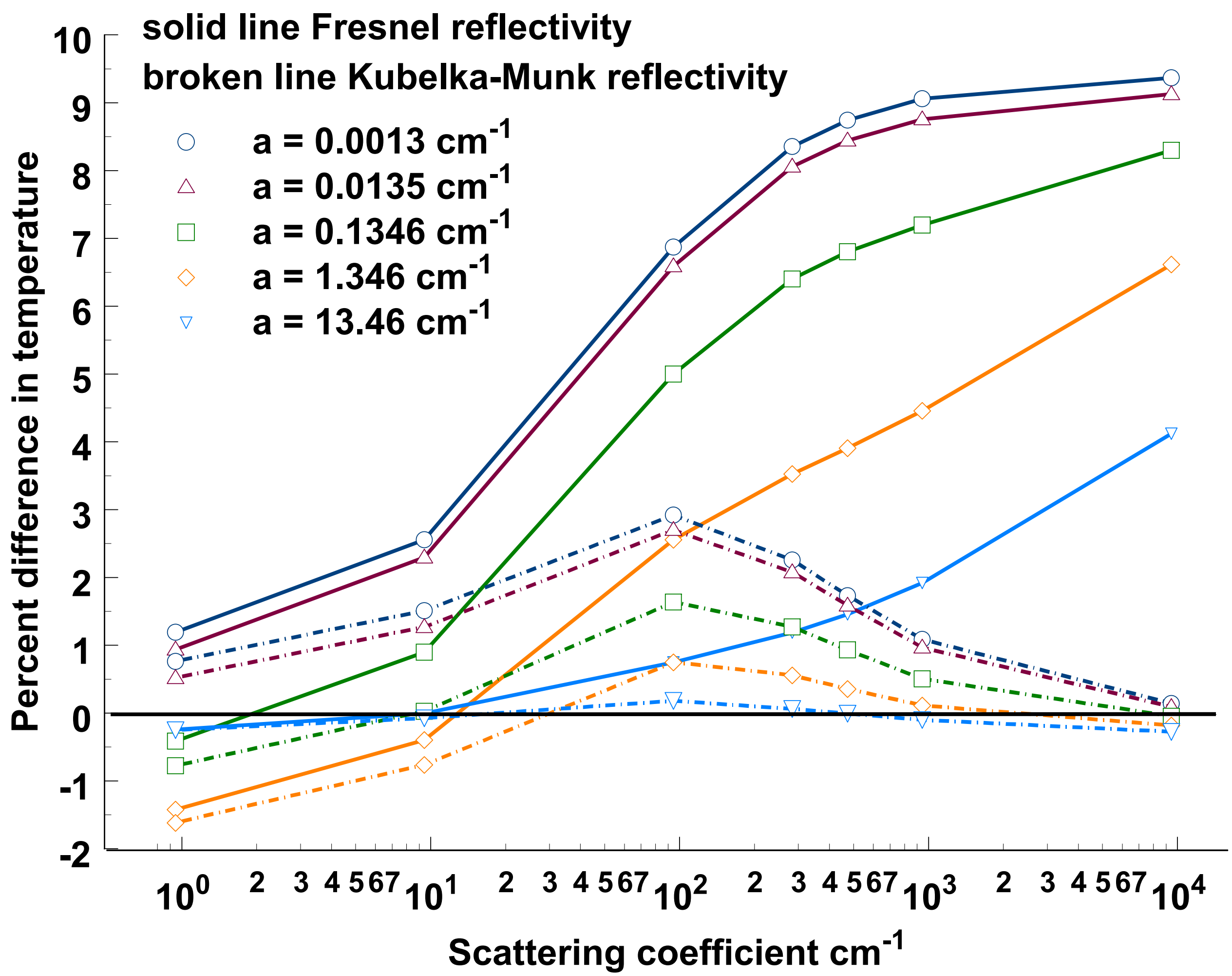
Plane Layer

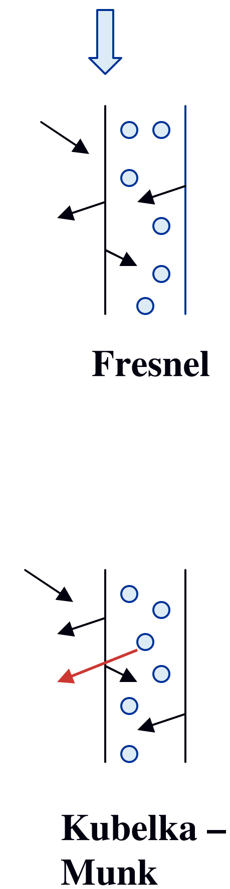
| | |
|---|---|
| Front surface temperatures | 0.075 – 0.28% |
| Back surface temperature | -0.44 – 0.21% |
| Heat flux | 0.14 – 3.6% |

# Comparison of the Diffusion and Milne-Eddington Two-Flux Approximation

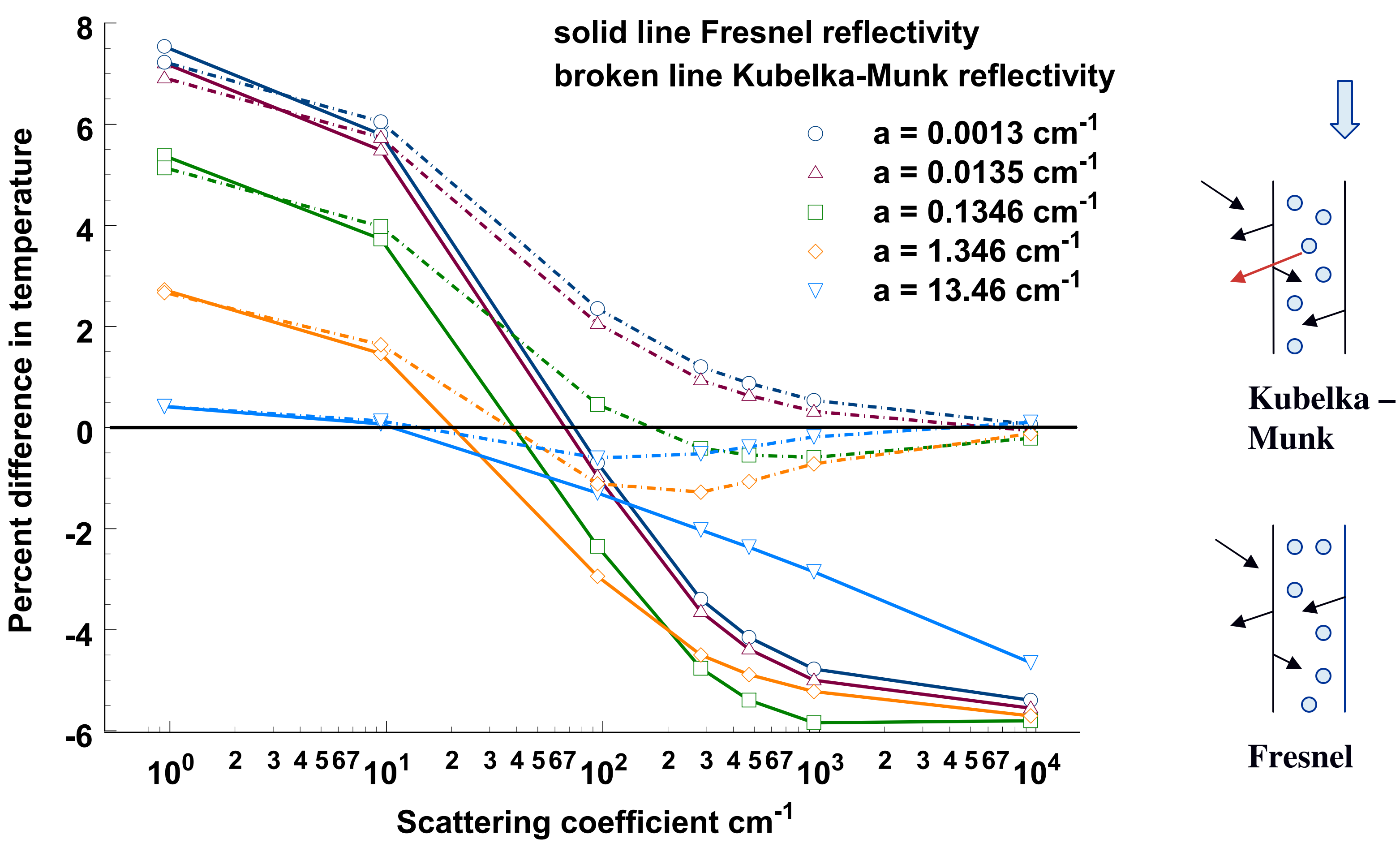
$$\textbf{Percent difference} = \left( \frac{T_{\text{diffusion}} - T_{\text{two-flux solution}}}{T_{\text{two-flux solution}}} \right) \cdot 100$$

# Percent Difference between Diffusion and Two-flux Solutions for Front Surface of Plane Layer

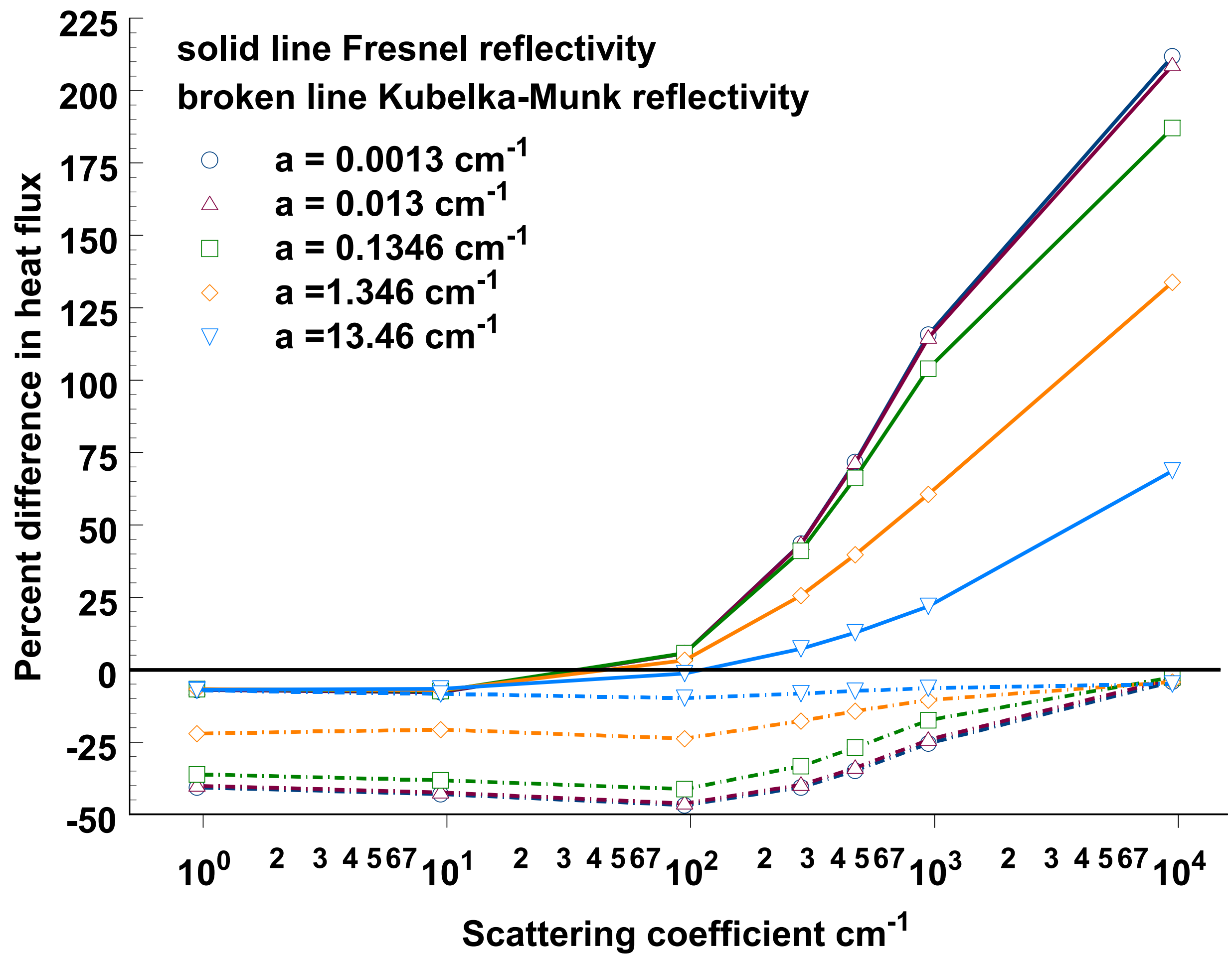


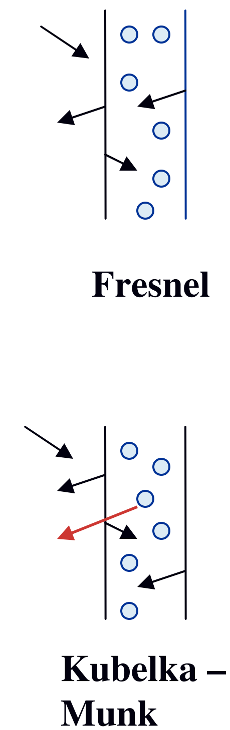



# Percent Difference between Two-flux and Diffusion Solutions for Back Surface of Plane Layer

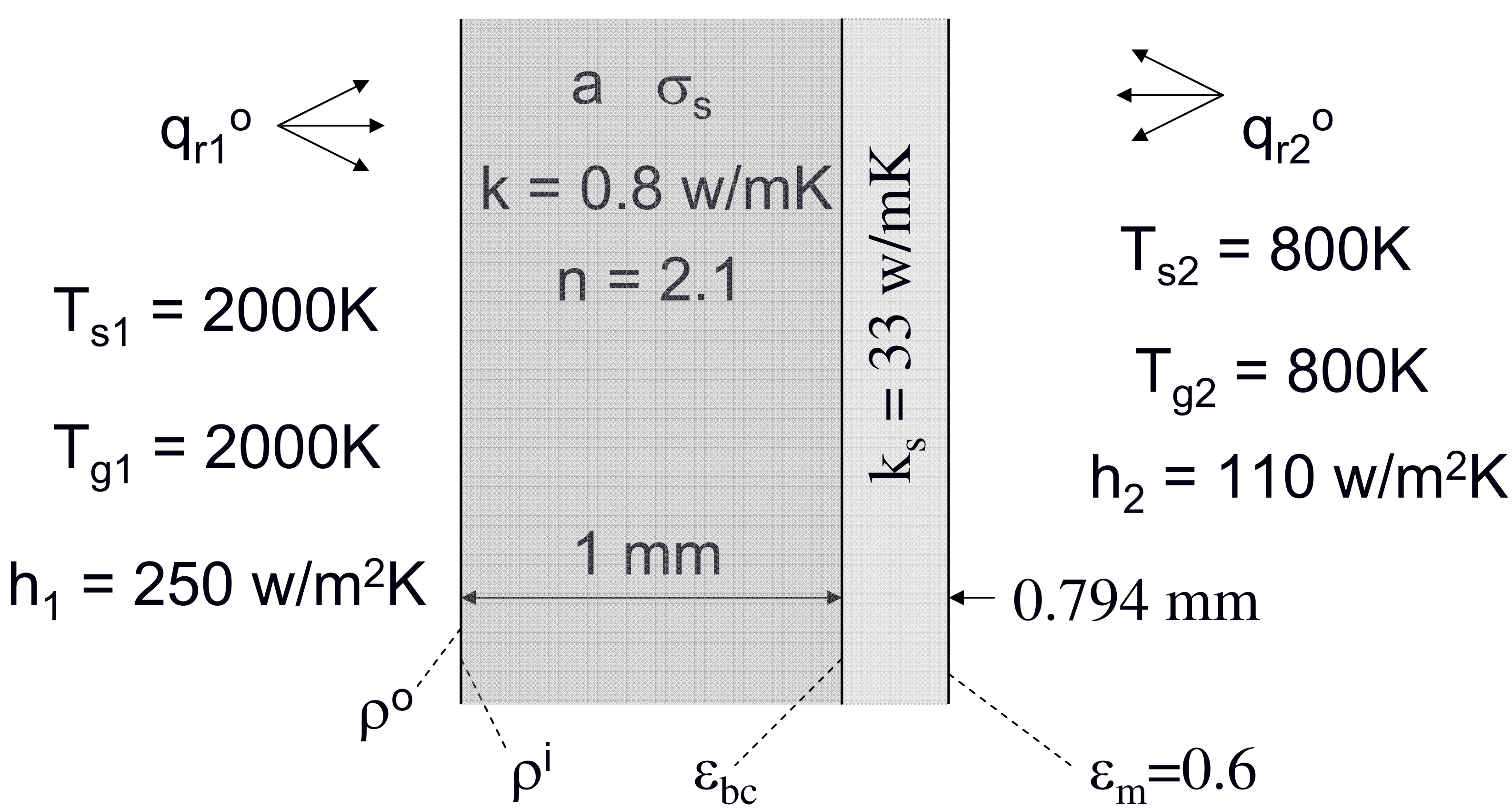


# Percent Difference in Heat Flux between the Diffusion and Two-flux Solutions for a Plane Layer



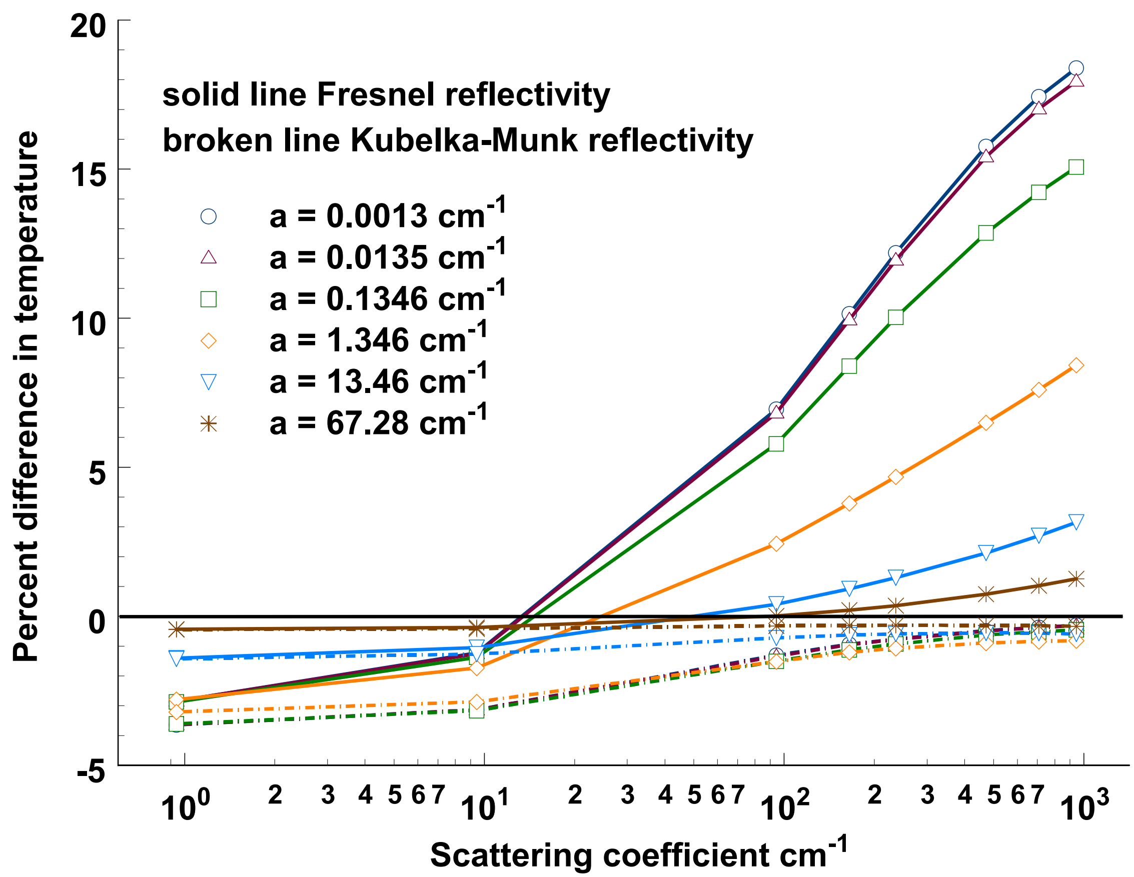



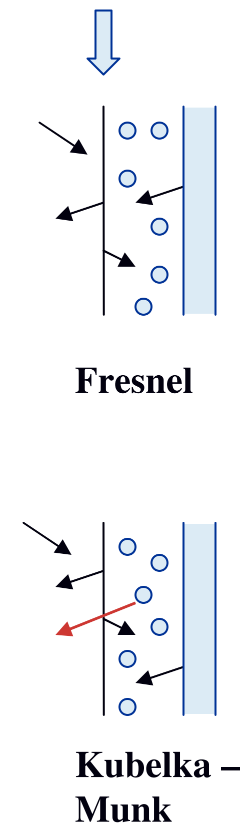
# Heat Transfer Model



# Percent Difference in Temperatures between Two-flux and Diffusion Solutions for Front Surface of a Layer

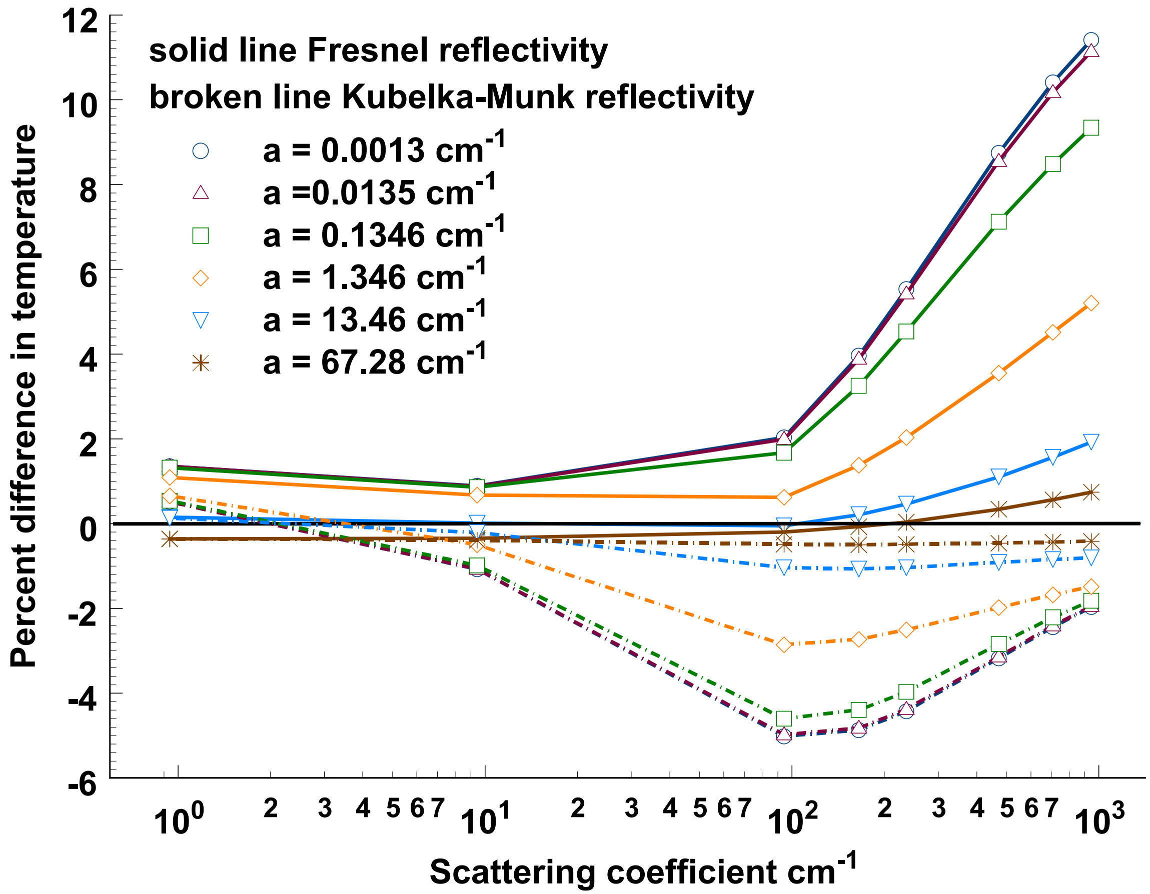
## $\lambda_c = 5.0\ \mu m$ and $\varepsilon_{bc} = 0.7$

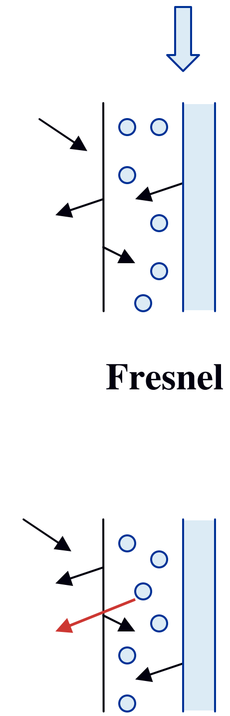


# Percent Difference in Temperatures between Two-flux and Diffusion Solutions for Front of Substrate

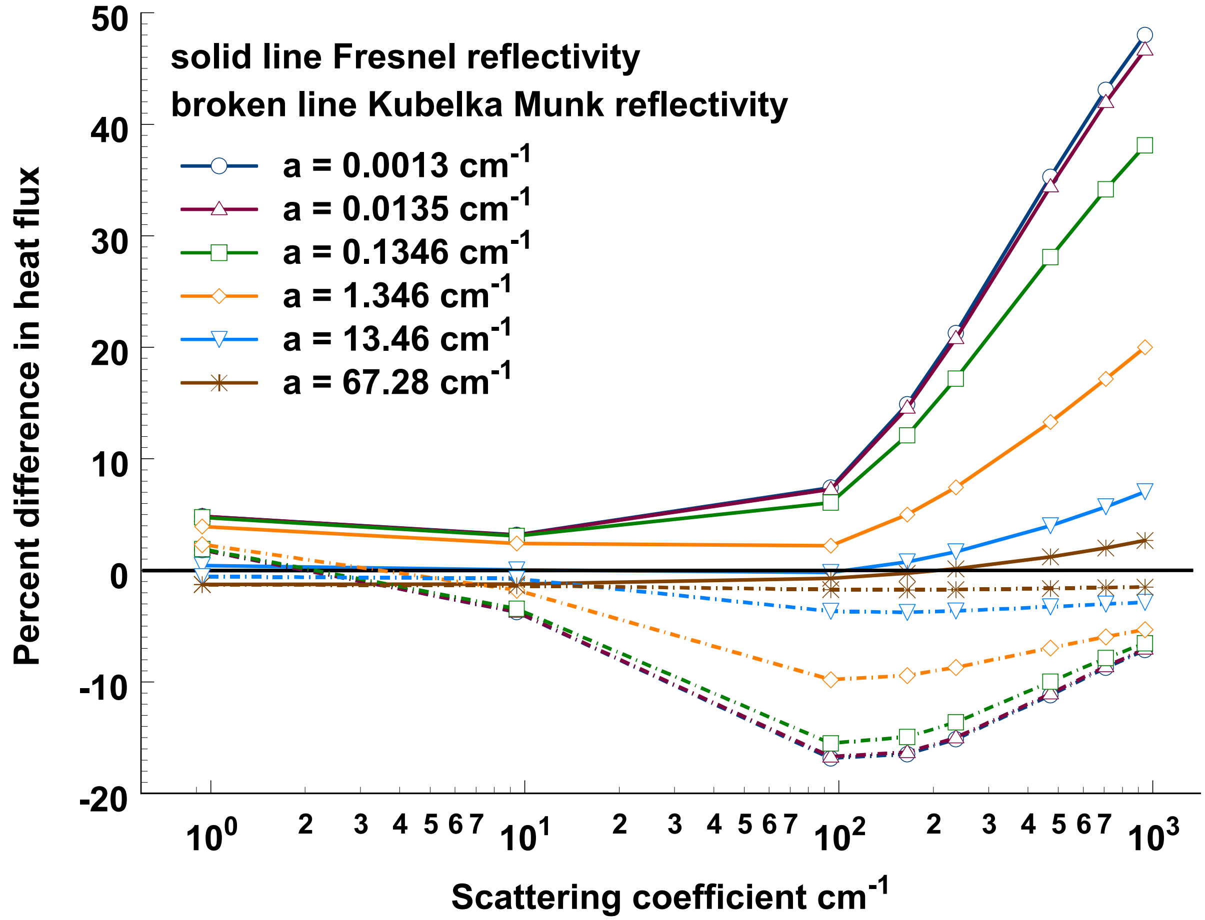
## $\lambda_c = 5.0\mu m$ and $\varepsilon_{bc} = 0.7$



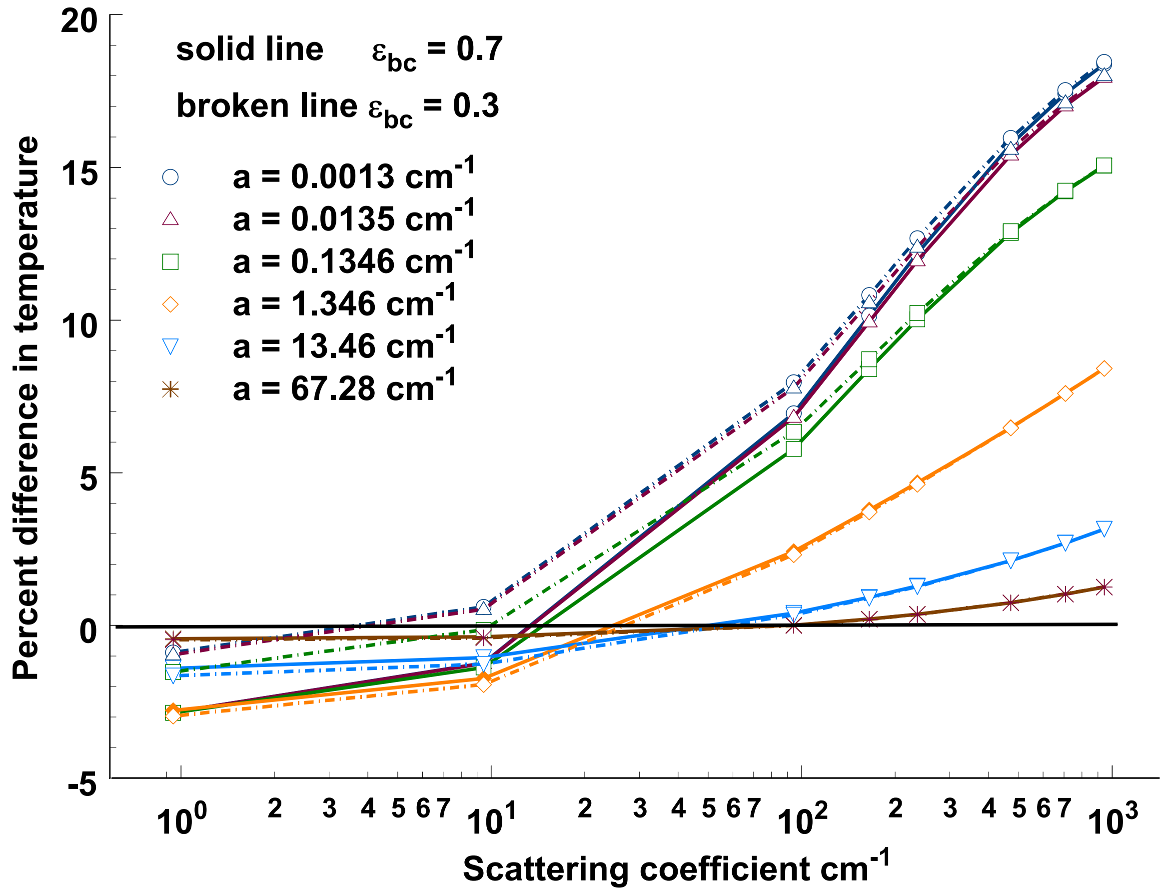
# Percent Difference in Heat Flux between the Diffusion and Two-flux Solutions for a Layer with a Substrate

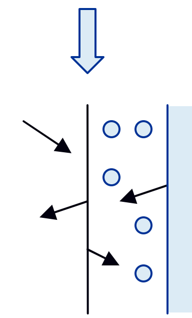
## $\lambda_c = 5.0\mu m$ and $\varepsilon_{bc} = 0.7$



# Effect of Bond Coat Emissivity on the Percent Difference in Temperature between Two-Flux and Diffusion Solutions

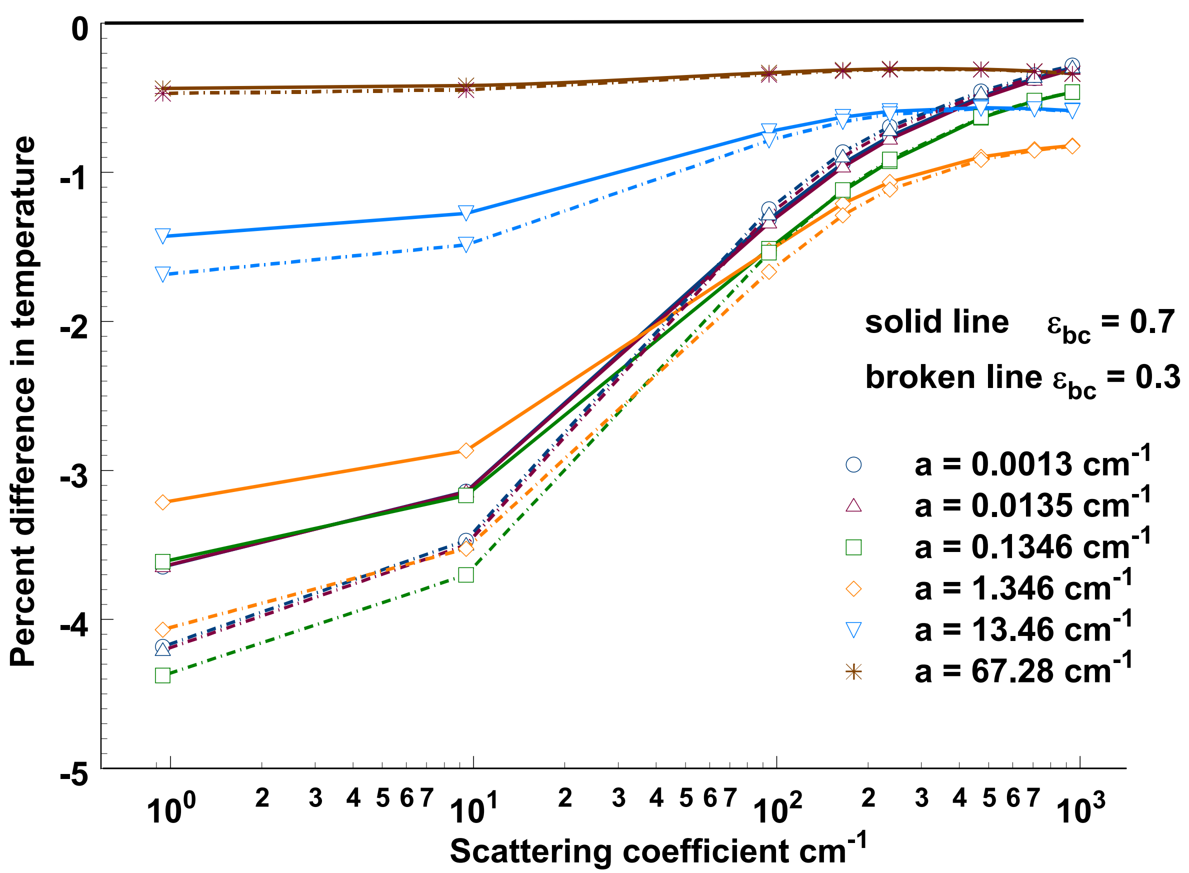
**Front Surface of Layer    Fresnel Reflectivity    $\lambda_c = 5.0\mu m$**

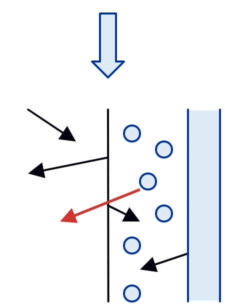


# Effect of Bond Coat Emissivity on the Percent Difference in Temperature between Two-Flux and Diffusion Solutions

**Front Surface of Layer    K-M Reflectivity    $\lambda_c = 5.0\mu m$**

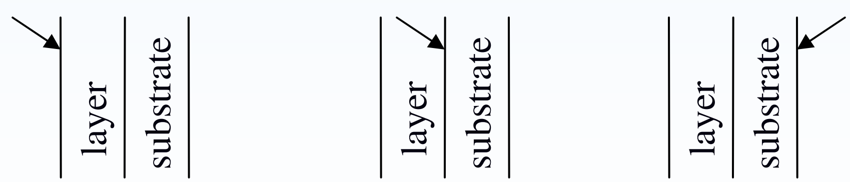


Kubelka – Munk

# Maximum Change in Percent Difference for a Change in Bond Coat Emissivity from 0.7 to 0.3

| | Front of layer | Front of substrate | Back of substrate | Heat flux |
|---|---|---|---|---|
| Fresnel reflection | -1.96 | -2.98 | -2.94 | -11.33 |
| K-M reflection | 0.86 | -1.11 | -1.90 | -4.04 |



# Conclusions

The percent error using the diffusion solution is lowest for all scattering considered when the absorption is high

The largest errors occur mostly for high scattering and low absorption when the Fresnel surface reflection is used

The error at high scattering can be reduced by using a Kulbelka-Munk reflectance

For a layer with a substrate the Kubelka-Munk reflectance is able to handle a change in bond coat emissivity better

# Generalized Conclusions

For the heating conditions presented here the absorption thickness ($a \cdot D$) not the optical thickness [$(a + \sigma_s) \cdot D$] is the important factor in determining the use of the diffusion approximation.  Need a large absorption thickness.

The worst conditions to use the diffusion approximation are for high scattering thicknesses ($s_\sigma \cdot D$) with no absorption or low absorption thickness and only surface reflections accounted for.

Kubelka-Munk reflectivity can reduce the diffusion solution approximation error for large scattering thicknesses