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Matlab Stability and Control Toolbox: Trim and Static Stability Module

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Abstract

This paper presents the technical background of the Trim and Static module of the Matlab© Stability and Control Toolbox (MASCOT). This module performs a low-fidelity stability and control assessment of an aircraft model for a set of flight critical conditions. This is attained by determining if the control authority available for trim is sufficient and if the static stability characteristics are adequate. These conditions can be selected from a prescribed set or can be specified to meet particular requirements. The prescribed set of conditions includes horizontal flight, take-off rotation, landing flare, steady roll, steady turn and pull-up/push-over flight, for which several operating conditions can be specified. A mathematical model was developed allowing for six-dimensional trim, adjustable inertial properties, asymmetric vehicle layouts, arbitrary number of engines, multi-axial thrust vectoring, engine(s)-out conditions, crosswind and gyroscopic effects.

Nomenclature[1]

a_{ij}	jth orientation angle of the ith engine
$\mathcal{B}$	Body reference frame
$\mathcal{E}$	Earth reference frame
$\mathcal{F}$	Fuselage reference frame
$\mathbf{h}_i$	angular momentum of the ith engine
$\mathbf{h}_{ji}$	j-term of the angular momentum of the ith engine
I	Identity matrix
$\widetilde{I}$	Inertia tensor
$L^{\mathcal{AC}}$	Transformation from frame $\mathcal{C}$ to frame $\mathcal{A}$
n	Load factor
N_j	Normal force at jth landing gear
0	Nominal center of gravity
$\mathcal{P}_i$	ith Propulsion reference frame
$\mathbf{r}$	Residue
$\mathbf{r}_{m/n}$	Position vector of m relative to n
$\widehat{\mathbf{s}}$	Trimmed state
$\mathbf{s}$	State vector
V	Magnitude of the aircraft's velocity relative to the wind
$\overline{V}$	Target speed relative to ground
$\mathcal{W}$	Wind reference frame
δ	Control surface deflections
γ	Target angle of climb
μ	Friction coefficient tire-runway

[1]See reference [1] for additional symbols.

ν Wind velocity vector
ω Angular velocity vector
$\tilde{\omega}$ Desired magnitude of the angular velocity (scalar)
σ Target course angle
τ Bank angle
ξ Geometrical variables
(i) Referring to the ith component of a vector

Subscript

$\mathbf{a}$ Referring to an aerodynamic term
CG Center of gravity
$\mathbf{g}j$ Referring to the jth contact force
$\mathbf{i}$ Referring to a inertial term
$\mathbf{p}i$ Referring to the ith engine
$\mathbf{w}$ Weight

1 Introduction

Aircraft stability and control requirements are crucial for safety and performance. These requirements, however, are evaluated quantitatively only in the later stages of the vehicle's design cycle. Unfortunately, at that stage, the basic aerodynamic characteristics are costly to change, resulting in an aircraft that is either overly conservative or that requires substantial stability and control augmentation to correct deficiencies. The flight control system is commonly expected to rectify, if possible, the legacy of stability and control deficiencies left by the aircraft designer. Several studies [2–4] on stability and control during conceptual design are available. However, most of the methods are ad-hoc or limited in their generality, extendability, and scope. Literature on high-fidelity analysis is plentiful, but the methods can not easily be adapted to the early phases of the vehicle's design.

The Matlab© Stability and Control Toolbox (MASCOT) is being developed by the National Institute of Aerospace and the Dynamic Systems and Control Branch of NASA Langley Research Center. MASCOT integrates stability and control considerations into a multi-disciplinary conceptual design framework. In addition, MASCOT has been conceived and developed into a tool that is well suited to assess stability and control characteristics at progressive fidelity levels during the design cycle. Studies on the aircraft's control authority and flying qualities are made in a variable-fidelity environment, where the refinement of the models and of the resulting assessments progresses as the design cycle evolves from the conceptual to the detailed design phase. MASCOT is organized into five interconnected modules. The Trim and Static (TS) Module generates a low-fidelity assessment for a set of Flight Critical

Conditions. The TS Module determines if the control authority available for trim is sufficient and if the static stability characteristics are acceptable. The Open-Loop Dynamics (OD) module, generates a medium-fidelity assessment where a dynamic analysis of the linear and non-linear open-loop system is carried out. This includes time-based simulations and sensitivity analyses. The Closed-Loop Dynamics (CD) module generates a high-fidelity assessment of the linear and non-linear closed-loop system. This implies the integration of user-designed controllers, the ability to do conventional control studies and the assessment of the vehicle flying qualities. The Sizing and Control Allocation (SA) module integrates MASCOT into a multi-disciplinary vehicle's sizing framework. This allows the sizing of control surfaces, engines and horizontal and vertical stabilizers. The SA module also supports performance and maneuverability analyses and control allocation studies. Finally, the Uncertainty Management (UM) module accounts for the effects of low- and medium-fidelity input data. This paper describes only the TS module.

This paper is organized as follows. Section 2 presents the fundamental mathematical model and implementation details used by all the modules of MASCOT. Section 3 details the trim analysis procedure and section 4 presents the static stability analysis capability. Finally, some concluding remarks are provided in section 5.

2 Aircraft Dynamics

Derivation of the equations of motion from first principles is available in multiple references [1, 5]. This section presents relevant information, including some mathematical preliminaries, on the manner in which the equations are implemented. Omitted definitions and developments can be easily found in the literature.

2.1 Frames of Reference

Let $\mathbf{x}^{\mathcal{A}}$, $\mathbf{y}^{\mathcal{A}}$, and $\mathbf{z}^{\mathcal{A}}$ denote the members of the orthonormal basis of frame $\mathcal{A}$. The main frames to be used are:

2.1.1 Flat Earth ($\mathcal{E}$)

Frame fixed to Earth where $\mathbf{x}^{\mathcal{E}}$ and $\mathbf{y}^{\mathcal{E}}$ span the horizontal plane while $\mathbf{z}^{\mathcal{E}}$ points downward. This frame is assumed to be inertial.

2.1.2 Fuselage ($\mathcal{F}$)

Frame fixed to the fuselage with origin at the arbitrary point O fixed relative to the fuselage, $\mathbf{y}^{\mathcal{F}}$ aligned with the right wing and $\mathbf{x}^{\mathcal{F}}$ aligned with the forward longitudinal direction of the vehicle.

2.1.3 Body ($\mathcal{B}$)

Frame fixed to the fuselage with origin at the center of gravity CG, with the same orientation as $\mathcal{F}$.

2.1.4 Stability ($\mathcal{S}$)

Frame fixed to the fuselage with origin at O[2] and $\mathbf{x}^{\mathcal{S}}$ in opposition to the projection, onto the $\mathbf{x}^{\mathcal{B}}\mathbf{z}^{\mathcal{B}}$ plane, of the wind velocity relative to the vehicle. $\mathbf{z}^{\mathcal{S}}$ points downward when $\mathbf{x}^{\mathcal{S}}$ is in the horizontal plane.

2.1.5 Wind ($\mathcal{W}$)

Frame fixed to the fuselage with origin at O and $\mathbf{x}^{\mathcal{W}}$ in opposition to the velocity of the wind relative to the vehicle. $\mathbf{z}^{\mathcal{W}}$ points downward when $\mathbf{x}^{\mathcal{W}}$ is in the horizontal plane.

2.1.6 Propulsion of the ith engine ($\mathcal{P}_i$)

Frame fixed to the ith engine having $\mathbf{x}^{\mathcal{P}_i}$ aligned with its direction of thrust. $\mathbf{z}^{\mathcal{P}_i}$ points downward when $\mathbf{x}^{\mathcal{P}_i}$ is in the horizontal plane.

2.2 Transformations Between Reference Frames

Rotation matrices of direction cosines are defined as

$$L_x(\cdot) \triangleq \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\cdot) & \sin(\cdot) \\ 0 & -\sin(\cdot) & \cos(\cdot) \end{bmatrix} \tag{1}$$

$$L_y(\cdot) \triangleq \begin{bmatrix} \cos(\cdot) & 0 & -\sin(\cdot) \\ 0 & 1 & 0 \\ \sin(\cdot) & 0 & \cos(\cdot) \end{bmatrix} \tag{2}$$

$$L_z(\cdot) \triangleq \begin{bmatrix} \cos(\cdot) & \sin(\cdot) & 0 \\ -\sin(\cdot) & \cos(\cdot) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Relations among frames of reference are shown in Figure 1. The arguments

[2] The origin O is commonly chosen to be at the CG, making the frames $\mathcal{F}$ and $\mathcal{B}$ coincide. This is a particular case of the one considered here. The framework chosen allows performing studies where the location of the CG varies, without altering the procedure by which the aerodynamic forces and torques are calculated.

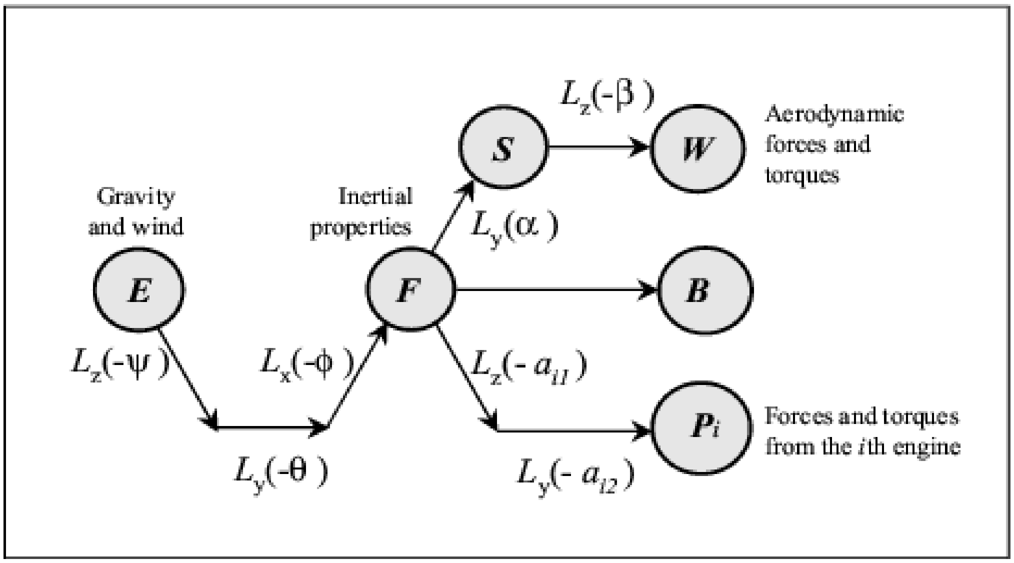

Figure 1. Frames of reference. Comments refer to quantities usually described in such a frame.

of the rotation matrices are the Euler angles, ψ, θ, ϕ; the angle of attack α, the sideslip angle β, and the thrust vectoring angles of the ith engine a_{i1} and a_{i2}. The matrix transformation between two frames can be formed by multiplying the transformation matrices corresponding to the reverse of the ordered sequence of rotations that relate the frames [5]. For instance, the transformation from the Wind frame $\mathcal{W}$ to the Earth frame $\mathcal{E}$ is given by $L^{\mathcal{EW}} = L^{\mathcal{EF}} L^{\mathcal{FS}} L^{\mathcal{SW}} = L_z(-\psi)L_y(-\theta)L_x(-\phi)L_y(\alpha)L_z(-\beta)$. The reverse transformation is given by the inverse matrix, e.g. $L^{\mathcal{WE}} = (L^{\mathcal{EW}})^{-1}$. Note that since a translation relates frames $\mathcal{F}$ and $\mathcal{B}$, no rotation matrix is indicated. Figure 2 shows the frames' orientation with respect to each other.

2.3 Preliminaries

Constituents of the equations of motion are briefly introduced next.

2.3.1 Aerodynamics

Let $k^{\mathcal{B}}$ be a component of a force or torque applied to the vehicle by the air. A linear aero-model, based on small perturbation theory, uses

$$k^{\mathcal{B}} = k_0^{\mathcal{B}} + \sum_i \frac{\partial k^{\mathcal{B}}}{\partial x_i} \Delta x_i \tag{4}$$

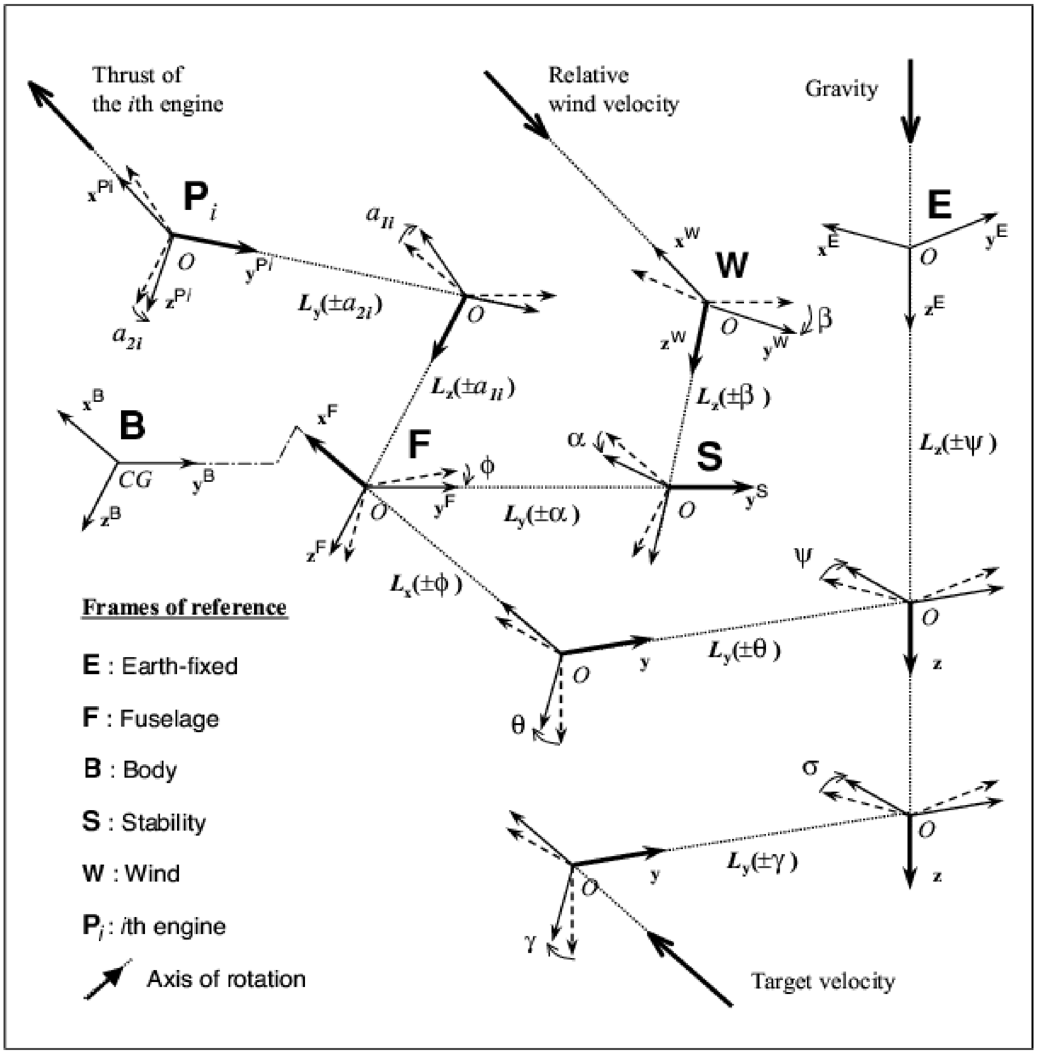

Figure 2. Reference Frames.

where $k_0^{\mathcal{B}}$ is a dimensional aero-coefficient, x_i is a state variable, $\partial k^{\mathcal{B}}/\partial x_i$ is a dimensional stability or control derivative, and Δx_i is the perturbation of the ith state variable from its value at trim. Evaluating this expression requires the transformation of coefficients and derivatives from $\mathcal{W}$ to $\mathcal{B}$ and their dimensionalization [1, 5]. Low-fidelity aero-models usually retain a subset of the states in Equation (4). For instance, while longitudinal forces/torques are usually assumed to be dependent on u, w (or α), q, $\dot{w}$ (or $\dot{\alpha}$), and δ_e (elevator deflection) only, lateral dynamics are usually assumed to be dependent on v (or β), p, r, δ_a (aileron deflection), and δ_r (rudder deflection). A general aerodynamic model takes the form

$$k^{\mathcal{B}}(h, V, \alpha, \beta, p, q, r, \dot{V}, \dot{\alpha}, \dot{\beta}, \dot{p}, \dot{q}, \dot{r}, \delta, \dot{\delta}, \xi, \dot{\xi})$$

where h is the altitude, δ contains the variables that parameterize the control surface deflections and propulsion settings, and ξ contains the variables that parameterize geometrical changes in the vehicle's layout, e.g. landing gear retracted or deployed. The aerodynamic forces and torques in $\mathcal{B}$ are given by

$$\begin{aligned} \mathbf{f_a}^{\mathcal{B}} &= \mathbf{f_a}^{\mathcal{W}} \\ \mathbf{t_a}^{\mathcal{B}} &= \mathbf{t_a}^{\mathcal{W}} + \mathbf{r}^{\mathcal{W}}_{O/cg} \times \mathbf{f_a}^{\mathcal{W}} \end{aligned} \tag{5}$$

where $\mathbf{r}^{\mathcal{F}}_{O/cg}$ is the position vector of point O relative to the CG.

2.3.2 Weight

The force and torque resulting from gravity are

$$\begin{aligned} \mathbf{f_w}^{\mathcal{B}} &= L^{\mathcal{B}\mathcal{E}}\,[0, 0, mg]^T \\ \mathbf{t_w}^{\mathcal{B}} &= \mathbf{0} \end{aligned} \tag{6}$$

where m is the vehicle's mass and g is the gravitational constant.

2.3.3 Propulsion

The force and torque generated by the ith engine are given by

$$\begin{aligned} \mathbf{f_p}_i{}^{\mathcal{B}} &= L^{\mathcal{B}\mathcal{P}_i}\mathbf{f_p}_i{}^{\mathcal{P}_i} \\ \mathbf{t_p}_i{}^{\mathcal{B}} &= \mathbf{t_p}_i{}^{\mathcal{F}} + \mathbf{r}^{\mathcal{F}}_{O/cg} \times \mathbf{f_p}_i{}^{\mathcal{B}} \end{aligned} \tag{7}$$

where

$$\mathbf{t_p}_i{}^{\mathcal{F}} = L^{\mathcal{F}\mathcal{P}_i}\left(\mathbf{t_p}_i{}^{\mathcal{P}_i} + \mathbf{r}^{\mathcal{P}_i}_{e_i/O} \times \mathbf{f_p}_i{}^{\mathcal{P}_i}\right)$$

and $\mathbf{r}^{\mathcal{P}_i}_{e_i/O}$ is the position vector of the ith engine relative to point O.

2.3.4 Contact forces

The force and torque generated by the ground on the jth landing gear are given by

$$\begin{aligned} \mathbf{f_g}_j{}^{\mathcal{B}} &= \mathbf{f_g}_j{}^{\mathcal{F}} \\ \mathbf{t_g}_j{}^{\mathcal{B}} &= \left(\mathbf{r}^{\mathcal{B}}_{O/cg} + \mathbf{r}^{\mathcal{F}}_{cp_j/O}\right) \times \mathbf{f_g}_j{}^{\mathcal{B}} \end{aligned} \tag{8}$$

where $\mathbf{r}^{\mathcal{F}}_{cp_j/O}$ is the position vector of the contact point between the ground and the jth landing gear relative to point O.

2.3.5 Kinematics

The relation between the Euler angles and the angular velocity is given by

$$\omega^{\mathcal{B}} \triangleq \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \sin(\phi)\cos(\theta) \\ 0 & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{9}$$

2.3.6 Inertia

Denote by $\mathbf{v}_{cg}^{\mathcal{B}} = [u, v, w]^T + L^{\mathcal{BE}}\nu^{\mathcal{E}}$ the vehicle's velocity relative to ground, where $\nu^{\mathcal{E}}$, the wind velocity, is assumed to be constant. The inertial force is given by

$$\mathbf{f_i}^{\mathcal{B}} = m\left(\mathbf{a}_{cg}^{\mathcal{B}} + \omega^{\mathcal{B}} \times \mathbf{v}_{cg}^{\mathcal{B}}\right) \tag{10}$$

where $\mathbf{a}_{cg}^{\mathcal{B}} = [\dot{u}, \dot{v}, \dot{w}]^T + \dot{L}^{\mathcal{BE}}\nu^{\mathcal{E}}$ is the acceleration of the vehicle relative to the ground in frame $\mathcal{B}$, and $\omega^{\mathcal{B}}$ is the angular velocity of frame $\mathcal{B}$ relative to $\mathcal{E}$. Notice that Coriolis and the relative acceleration terms are neglected due to the flat-earth assumption. The velocity and acceleration can be written in terms of the wind-relative speed V, the angle of attack α, the side-slip angle β, and their time derivatives according to

$$[u, v, w]^T = V\left[\cos(\alpha)\cos(\beta), \sin(\beta), \sin(\alpha)\cos(\beta)\right]^T$$

The inertial torque is given by

$$\mathbf{t_i}^{\mathcal{B}} = \widetilde{\mathbf{I}}^{\mathcal{B}}\dot{\omega}^{\mathcal{B}} + \omega^{\mathcal{B}} \times \left(\widetilde{\mathbf{I}}^{\mathcal{B}}\omega^{\mathcal{B}} + \sum_i \mathbf{h}_i{}^{\mathcal{B}}\right) \tag{11}$$

where $\widetilde{\mathbf{I}}^{\mathcal{B}}$ is the inertia tensor, $\omega^{\mathcal{B}} = [p, q, r]^T$ is the angular velocity, $\dot{\omega}^{\mathcal{B}} = [\dot{p}, \dot{q}, \dot{r}]^T$ is the angular acceleration and $\mathbf{h}_i{}^{\mathcal{B}}$ is the angular momentum of the ith engine. These terms can be calculated from

$$\begin{aligned} \mathbf{h}_i{}^{\mathcal{B}} &= L^{\mathcal{FP}_i}\mathbf{h_1}_i{}^{\mathcal{P}i} + \mathbf{h_2}_i{}^{\mathcal{F}} \\ \widetilde{\mathbf{I}}^{\mathcal{B}} &= \widetilde{\mathbf{I}}^{\mathcal{F}} + m\left[(\mathbf{r}_{cg/O}^{\mathcal{B}} \cdot \mathbf{r}_{cg/O}^{\mathcal{B}})I - \mathbf{r}_{cg/O}^{\mathcal{B}}\mathbf{r}_{cg/O}^{\mathcal{B}}{}^T\right] \end{aligned}$$

where I is the identity matrix while $\mathbf{h_1}_i{}^{\mathcal{P}i}$ and $\mathbf{h_2}_i{}^{\mathcal{F}}$ are terms used to describe the angular momentum of the rotating components of the ith engine. Usually, one of these two vectors is zero. If the axis of rotation of the engine moves relative to the fuselage $\mathbf{h_2}_i{}^{\mathcal{F}} = \mathbf{0}$, e.g. rotating nacelles; otherwise $\mathbf{h_1}_i{}^{\mathcal{F}} = \mathbf{0}$, e.g. fixed nacelles.

2.3.7 Navigation

The navigation equation is given by

$$\mathbf{v}_{cg}^{\mathcal{E}} = L^{\mathcal{EB}}\mathbf{v}_{cg}^{\mathcal{B}} \tag{12}$$

2.4 Equations of Motion

Define the target velocity vector as

$$\overline{\mathbf{v}}^{\mathcal{E}} \triangleq L_z(-\sigma)L_y(-\gamma)\left[\overline{V},0,0\right]^T$$

where $\overline{V}$ is the target speed relative to ground, σ is the desired course angle and γ is the desired angle of climb. Figure 2 shows the relations among frames and relevant quantities. The force, torque and navigation equations can be written as

$$\mathbf{r}_1 = \mathbf{f_a}^{\mathcal{B}} + \mathbf{f_w}^{\mathcal{B}} + \sum_i \mathbf{f_p}_i^{\mathcal{B}} + \sum_j \mathbf{f_g}_j^{\mathcal{B}} - \mathbf{f_i}^{\mathcal{B}} \tag{13}$$

$$\mathbf{r}_2 = \mathbf{t_a}^{\mathcal{B}} + \sum_i \mathbf{t_p}_i^{\mathcal{B}} + \sum_j \mathbf{t_g}_j^{\mathcal{B}} - \mathbf{t_i}^{\mathcal{B}} \tag{14}$$

$$\mathbf{r}_3 = \left[\sigma - \tan^{-1}\left(\frac{\mathbf{v}_{cg}^{\mathcal{E}}(2)}{\mathbf{v}_{cg}^{\mathcal{E}}(1)}\right), \gamma - \tan^{-1}\left(\frac{-\mathbf{v}_{cg}^{\mathcal{E}}(3)}{\sqrt{\mathbf{v}_{cg}^{\mathcal{E}}(1)^2 + \mathbf{v}_{cg}^{\mathcal{E}}(2)^2}}\right), \|\mathbf{v}_{cg}^{\mathcal{E}}\| - \overline{V}\right]^T \tag{15}$$

where specific references to vector components are given in Equation (15), e.g. $\mathbf{v}_{cg}^{\mathcal{E}}(1)$ is the first component of $\mathbf{v}_{cg}^{\mathcal{E}}$. We will refer to $\mathbf{r}_k$ for $k = 1, 2, 3$; as residues. Providing that Equation (9) holds, the force and torque equations are satisfied when $\mathbf{r}_1 = \mathbf{0}$ and $\mathbf{r}_2 = \mathbf{0}$ respectively. The vehicle's velocity will match the target velocity when $\mathbf{r}_3 = \mathbf{0}$. The complete set of differential equations that describe the vehicle's dynamics is given by Equations (9,12,13,14).

3 Trim Analysis

Trim analysis requires searching for the states that satisfy a simplified realization of the equations of motion. This search is to be done for all flight critical conditions. A flight critical condition is defined as the combination of a flight condition and an operating condition. Flight sets of interest are introduced next.

3.1 Flight Conditions

A flight condition is an instantaneous flight maneuver prescribed by Equations (13,14,15) with $\mathbf{r}_k = \mathbf{0}$ for $k = 1, 2, 3$; along with some kinematic and kinetic constraints. Physical interpretations of flight conditions of interest and the corresponding constraints are presented next.

3.1.1 Horizontal Flight

This flight condition occurs when the vehicle's velocity relative to ground is horizontal. This implies $\gamma = 0$ while $\omega^{\mathcal{B}}$, $\dot{\omega}^{\mathcal{B}}$, ${\mathbf{f_g}_j}^{\mathcal{F}}$ for all j, and $\mathbf{a}_{cg}^{\mathcal{B}}$ are zero.

3.1.2 Take-Off Rotation

This flight condition occurs when the vehicle, moving at lift-off velocity on the ground, starts rotating about the axis of rotation of the main landing gear. At that instant, the normal forces at the wheels whose axes do not coincide with the axis of rotation are zero. This condition implies $\theta = 0$, $\gamma = 0$, $\ddot{\theta} > 0$ and $\omega^{\mathcal{B}} = \mathbf{0}$. In addition, the constraints $f_j \leq \mu N_j$, where f_j is the friction force, μ is the friction coefficient between the tire and the runway and N_j is the normal force; bound the components of ${\mathbf{f_g}_j}^{\mathcal{F}}$.

3.1.3 Landing Flare

This flight condition occurs when the vehicle, flying at approach speed, starts a rotation that raises its nose. This is done to prevent touching down at high sink rates. This condition implies $\ddot{\theta} > 0$, $\omega^{\mathcal{B}} = \mathbf{0}$ and ${\mathbf{f_g}_j}^{\mathcal{F}} = \mathbf{0}$ for all j landing gear.

3.1.4 Steady Roll

This flight condition occurs when the vehicle rolls at constant angular velocity. This implies $\omega^{\mathcal{B}} = [p, 0, 0]^T$ while $\dot{\omega}^{\mathcal{B}}$, ${\mathbf{f_g}_j}^{\mathcal{F}}$ for all j and $\mathbf{a}_{cg}^{\mathcal{B}}$ are zero. Another roll condition of interest is given by $\omega^{\mathcal{B}} = L^{\mathcal{BW}} [\tilde{\omega}, 0, 0]^T$, case in which the axis of rotation coincides with the direction of the aircraft's velocity relative to the wind.

3.1.5 Steady Turn

This flight condition occurs when the vehicle turns at a constant, vertical angular velocity. This implies $\omega^{\mathcal{B}} = L^{\mathcal{BE}} [0, 0, \tilde{\omega}]^T$, $\mathbf{v}_{cg}^{\mathcal{E}}(3) = 0$, $\gamma = 0$, $\mathbf{a}_{cg}^{\mathcal{B}} = L^{\mathcal{BE}}(\omega^{\mathcal{E}} \times \mathbf{v}_{cg}^{\mathcal{E}})$, $\dot{\omega}^{\mathcal{B}} = \mathbf{0}$ and ${\mathbf{f_g}_j}^{\mathcal{F}} = \mathbf{0}$ for all j. If the resultant of the weight and the inertial force is in the $\mathbf{x}^{\mathcal{B}}\mathbf{z}^{\mathcal{B}}$ plane, condition commonly refereed to as coordinated turn, the approximation $\omega^{\mathcal{B}} = \tan(\tau)g/V [-\theta, n\cos(\phi), 1/n]^T$, where τ is the bank angle [6], and n is the load factor; is commonly assumed for $\nu^{\mathcal{E}} = \mathbf{0}$. Note that wind velocity does not contribute to the centripetal acceleration of the turn.

3.1.6 Pull-Up/Push-Over

This flight condition occurs when the vehicle flies at the bottom or the top of a curved path and the centripetal acceleration remains vertical. These conditions imply $\gamma = 0$, $\mathbf{v}_{cg}^{\mathcal{E}}(3) = 0$ and

$$\omega^{\mathcal{B}} = \frac{(n-1)g}{V} L^{\mathcal{BW}} [0, -1, 0]^T$$

while ${\mathbf{f_g}_j}^{\mathcal{F}} = \mathbf{0}$ for all j and $\mathbf{a}_{cg}^{\mathcal{B}} = L^{\mathcal{BE}} [0, 0, (n-1)g]^T$. Non-maneuvering, i.e. $n = 1$, pull-up, i.e. $n > 1$, push-over, i.e. $n < 1$, and ballistic, i.e. $n = 0$, flight conditions are special cases of interest. Note that wind velocity does not contribute to the centripetal acceleration.

3.2 Operating Conditions

An operating condition determines the circumstances occurring during a given flight condition. Engine(s) out, crosswind, and crabbed configurations exemplify this aspect. This is attained by means of additional constraints, e.g. an engine out condition requires ${\mathbf{f_p}_i}^{\mathcal{F}} = \mathbf{0}$, ${\mathbf{f_t}_i}^{\mathcal{F}} = \mathbf{0}$, ${\mathbf{h_1}_i}^{\mathcal{P}i} = \mathbf{0}$ and ${\mathbf{h_2}_i}^{\mathcal{F}} = \mathbf{0}$ for a given i; and parameter assignments, e.g. crosswind requires $\nu^{\mathcal{E}} \neq \mathbf{0}$. We will refer to quasi-static FCC when the intended acceleration is non-zero.

3.3 State Partition and Trim Search

Denote by $\mathbf{s}$ a vector containing all the state variables. The state variables are the Euler angles $\{\psi, \theta, \phi\}$, the velocity components $\{V, \alpha, \beta\}$[3], the control surface deflections $\{\delta\}$, the angular velocity components $\{p, q, r\}$, the propulsion setting $\{a_{1i}, a_{2i}, {\mathbf{f_p}_i}^{\mathcal{F}}, {\mathbf{t_p}_i}^{\mathcal{F}}\}$ for all engines, the contact force $\{{\mathbf{f_g}_j}^{\mathcal{F}}\}$ at all landing gears, and the quasi-static terms $\{\dot{u}, \ddot{\theta}\}$.

Equations (13,14,15) define a coupled system of nine scalar equations. For a given FCC, the vehicle will be trimmed when there exist a realization of the state, denoted by $\widehat{\mathbf{s}}$, that leads to $\mathbf{r}_k(\widehat{\mathbf{s}}) = \mathbf{0}$ for $k = 1, 2, 3$. The trim conditions found using the TS module can be used as reference conditions for the sizing and control allocation module (SA) for both low and progressive fidelity models. This will allow for the evaluation of critical maneuverability requirements.

The search for the trimmed states $\widehat{\mathbf{s}}$ is done by minimizing the sum of the norms of the residues. When the number of states exceeds nine variables, the resulting system of equations is under-determined. To prevent solving such a system and to pose flight critical conditions that impose stringent demands on the control solution, a partition of the state vector is required. Hence, the state vector is partitioned into three groups: the group of unknowns, the group of constants and the group of related variables. The latter group contains all the state variables assuming a value equal to the value of an unknown. There is an equality associated to each member of this group. Each flight

[3]Equivalently, the aircraft's velocity relative to the wind can be prescribed by $\{u, v, w\}$

condition set, with the exception of the user defined case, has a prescribed state partition. Special care should be given when $\mathbf{s}$ is partitioned by the User since the resulting FCC might be physically meaningless.

4 Static Stability Analysis

The Trim and Static (TS) module performs a static stability analysis for each FCC, by calculating equivalent stiffnesses, dampings, and static margins. The signs of the stiffnesses determine the direction of the initial restoring force from a perturbed state. The sign of the damping terms indicate if energy is flowing either from or to the vehicle. The static margin is a metric that assess the CG location in pitch by measuring the closeness to neutral stability. Notice that a static stability analysis refers to the instantaneous response of the vehicle from a perturbed state in the vicinity of $\widehat{\mathbf{s}}$ and does not constitute a stability analysis in the dynamic sense. Statically unstable vehicles could be dynamically stable and vice versa. Details on the static metrics are given below.

4.1 Equivalent Stiffnesses

Static stability in roll, pitch and yaw are attained when the following inequalities are satisfied

$$\left(\frac{\partial \mathbf{r}_2(1)}{\partial \phi}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{16}$$

$$\left(\frac{\partial \mathbf{r}_2(2)}{\partial \alpha}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{17}$$

$$\left(\frac{\partial \mathbf{r}_2(3)}{\partial \beta}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{18}$$

4.2 Equivalent Dampings

Static stability in roll, pitch and yaw requires

$$\left(\frac{\partial Cl}{\partial \hat{p}}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{19}$$

$$\left(\frac{\partial Cm}{\partial \hat{q}}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{20}$$

$$\left(\frac{\partial Cn}{\partial \hat{r}}\right)_{\mathbf{s}=\widehat{\mathbf{s}}} < 0 \tag{21}$$

where Cl, Cm, and Cn are the non-dimensional aerodynamic coefficients for rolling, pitching and yawing respectively, and $\hat{p}$, $\hat{q}$ and $\hat{r}$ are the non dimensional components of the angular velocity. Notice that these dynamic derivatives are required to build the linear aerodynamic model in Equation (4).

4.3 Longitudinal Static Margin

The static margin is determined by the longitudinal distance between the CG and the vehicle's neutral point. The neutral point is defined as the location where the equivalent pitch stiffness is zero, i.e. where Equation (17) no longer holds. The static margin is provided as a percentage of the mean aerodynamic chord. Usually, subsonic transport vehicles have positive margins while fighters have negative margins.

5 Concluding Remarks

This paper presents the mathematical background of the Trim and Static Module of MASCOT. Even though this module generates low-fidelity stability and control assessments, the developments presented also support the medium- and high-fidelity tasks performed by other modules. MASCOT assessments should not only evolve in fidelity as the design cycle does but more importantly, they should be performed according to the resources and information available. For instance, if the conceptual designer is not willing to spend more than few minutes in the evaluation of a concept, longer turn-around times will certainly render this tool impractical. These requirements as well as the underlying consistency among the variable fidelity assessments constitute the main challenges of this endeavor.

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14. ABSTRACT

This paper presents the technical background of the Trim and Static module of the Matlab© Stability and Control Toolbox. This module performs a low-fidelity stability and control assessment of an aircraft model for a set of flight critical conditions. This is attained by determining if the control authority available for trim is sufficient and if the static stability characteristics are adequate. These conditions can be selected from a prescribed set or can be specified to meet particular requirements. The prescribed set of conditions includes horizonta flightl, take-off rotation, landing flare, steady roll, steady turn and pull-up/ push-over flight, for which several operating conditions can be specified. A mathematical model was developed allowing for six-dimensional trim, adjustable inertial properties, asymmetric vehicle layouts, arbitrary number of engines, multi-axial thrust vectoring, engine(s)-out conditions, crosswind and gyroscopic effects.

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