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COMPARISON OF A SIMPLE PATCHED CONIC TRAJECTORY CODE TO COMMERICIALLY AVAILABLE SOFTWARE

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Often in spaceflight proposal development, mission designers must evaluate numerous trajectories as different design factors are investigated. Although there are numerous commercial software packages available to help develop and analyze trajectories, most take a significant amount of time to develop the trajectory itself, which isn't effective when working on proposals. Thus a new code, PatCon, which is both quick and easy to use, was developed to aid mission designers to conduct trade studies on launch and arrival times for any given target planet. The code is able to run quick analyses, due to the incorporation of the patched conic approximation, to determine the trajectory. PatCon provides a simple but accurate approximation of the four body motion problem that would be needed to solve any planetary trajectory.

PatCon has been compared to a patched conic test case for verification, with limited validation or comparison with other COTS software. This paper describes the patched conic technique and its implementation in PatCon. A description of the results and comparison of PatCon to other more evolved codes such as AGI's Satellite Tool Kit and JAQAR Astrodynamics' Swingby Calculator is provided. The results will include percent differences in values such as C3 numbers, and Vinfinity at arrival, and other more subjective results such as the time it takes to build the simulation, and actual calculation time.

INTRODUCTION

In any proposal, time is of the essence and this hold especially true for spaceflight proposals, where the time between the announcement of opportunity (AO) and the delivery of the proposal is on the order of a few months. Even with advanced knowledge of an impending AO release, the expense and time necessary to assemble and develop a new high fidelity simulation, precludes the simulation development. In most cases the early phase of concept development is focused on determining the launch and arrival pair, the launch mass, and departure energy (C3). It is critical that the development of the

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trajectory be determined in an efficient manner to enable definition of the spacecraft configuration and preliminary subsystem definitions. Also as the concept definition phase progresses, sensitivity studies on launch mass and specific launch and arrival dates may be needed.

After realizing that the currently available software would take several days to develop a single trajectory, with limited capability to conduct trade studies, it was determined that a new code could be developed, allowing these trade studies to be easily executed. At first, the code was going to analytically determine the equations of motion, but it was determined that a patched conic approximation would be the quickest to code up and run, thus PatCon was developed

DEVELOPMENT OF PATCON

Description of the Patched Conic Approximation

A planetary trajectory is a four-body motion involving the Earth, the target planet, the Sun and the spacecraft. It would be possible and proper to use an N-body simulation to study the trajectory; however the patched conic technique is a reasonable approximation of this four-body motion with sufficient accuracy for almost all purposes. The patched conic approximation subdivides the planetary mission into three distinct trajectories and patches them together to create a single trajectory path. The patched conic approach is typically limited to impulsive maneuvers. The three stages include:

1) The departure stage: A departure hyperbola in which the two pertinent bodies are Earth and the spacecraft, with Earth at the focus. The influences of the Sun and target planet are ignored.

2) The cruise stage: A transfer ellipse in which the two bodies of interest are the Sun and the spacecraft, with the Sun at the focus. The influences of Earth and the target planet are neglected.

3) The arrival stage: An arrival hyperbola in which the two bodies of interest are the target planet and the spacecraft, with the target planet at the focus. The influences of the Sun and Earth are neglected.

An arbitrary sphere of influence, R_s, is defined about the Earth and the target planet. The transfer ellipse is then "patched" to the hyperbolas at the boundaries of the spheres of influence, where R_s is the distance that the gravitational attractions of the planet and the Sun are approximately equal. Inside the sphere of influence, spacecraft times and positions are calculated on the departure or arrival hyperbola. Outside the sphere, times and positions are calculated on the transfer ellipse.

Patched Conic Procedure

The procedure for the patched conic approximation includes four steps:

The first is to pick a launch date and arrival date and accurately determine the position of the Earth and the target planet on the chosen dates. It should be noted that it is very important to obtain accurate locations of the planets as a small error has a great affect on the results.

The second step is to design a transfer ellipse from Earth to the target planet. The transfer ellipse must contain the Earth's position at launch and the planet's position at arrival. The time of flight on the transfer arc must be equal to the time between the launch and arrival dates. This is an iterative process in which different transfer ellipses are defined by an arbitrary selection of the longitude of the line of apsides. In other words the two planets positions define two points on a family of transfer ellipses, each defined by the radial position of the line of apsides. Selection of the position of the line of apsides defines the true anomaly for each planet position and thus the time of flight between points. The trial and error process then becomes a series of selections of line of apsides positions and subsequent calculations of times of flight can be calculated on the resulting transfer ellipse. Where the required time of flight on the transfer ellipse is equal to the number of days between the Julian dates of launch and arrival.

The third step is to design the departure hyperbola such that it will deliver the spacecraft to the transfer ellipse. This is where the plane change is made to take advantage of the energy economy of combining the plane change with injection. As a result, the transfer ellipse is not in the ecliptic plane; it is in an intersection plane which contains the center of mass of the Sun, the Earth at launch, and target planet at arrival, see figure 1. The departure hyperbola has a periapsis radius equal to the radius of the parking orbit and a V_∞ magnitude and direction which are calculated to place the spacecraft on the proper transfer ellipse.

The final step is to design the approach hyperbola and the arrival mission. The objective of arrival targeting is to place the incoming asymptote in the desired location relative to the target planet. The design of the approach hyperbola is similar to the design of the departure hyperbola. The first step is to determine the inclination of the transfer plane with respect to the orbital plane of the target planet, and then the V_∞ at the target planet.

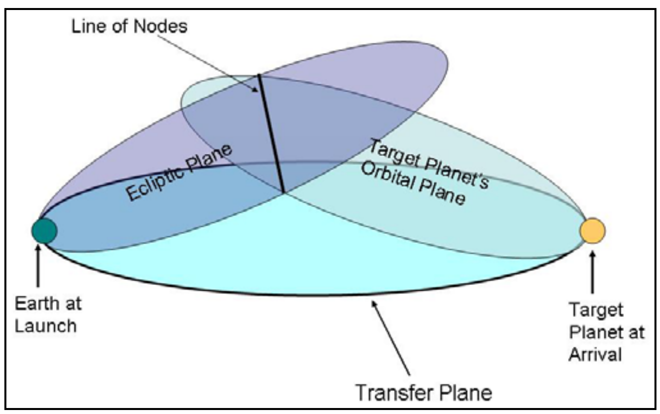

Figure 1. Transfer Plane

Development of the PatCon

PatCon was initially developed in FORTRAN for only Type I trajectories. Throughout the development, the code was continually verified with an example given in Charles Brown's Spacecraft Mission Design[1]. The example used was a mission to Venus, launching on April 8, 1988 and arriving July 26, 1988. With the detailed example given, nearly all values could be verified throughout the development. This was extremely helpful in catching small mistakes such as angle conversions and getting signs correct. The first step is to obtain the orbital elements of Earth and the target planet for the desired launch and arrival date pair. Table 1 lists the needed parameters.

Table 1
NEEDED ORBITAL ELEMENTS

Date of departure/arrival in Julian Date
Eccentricity
Semi-major Axis
Longitude of periapsis
Ascending Node
Gravitational Constants
Inclination
Position of planets at departure/arrival (True Anomaly)
Radius of arrival planet

Most of these values are easily obtained, but time was spent to ensure the accuracy of each value as it was observed that a small difference in either the position of the planets or the longitude of periapsis resulted in large errors in the calculations of the transfer ellipse. Once these values were obtained, the planet's true anomaly, the velocity of the planet with respect to the Sun, and the flight angle of the planets were calculated, which are needed for subsequent calculations of the excess velocity. The calculation of the transfer ellipse was an iterative process that started with an initial value of 0° for the true anomaly for earth. Then the target planet's true anomaly was calculated by using the longitudes (the longitude of periapsis plus true anomaly) of the planets to determine how many degrees the planet was ahead or behind compared to Earth. The eccentricity of the transfer ellipse is calculated using the true anomaly and position of the two planets. The travel time of the transfer ellipse is calculated by first determining the time since periapsis for the two planets and then subtracting the two to get the transfer time, which was then compared to the needed flight time. This iteration process continues until the transfer time equals the needed flight time, thus providing the orbital elements for the transfer ellipse. This method does not provide the inclination of the transfer ellipse which is found using spherical trigonometry. Using the calculated eccentricity of the transfer ellipse and the true anomalies of the planets for the transfer ellipse, the velocity and flight angle of the spacecraft was determined at the respective planets. C3 and V_∞ is then determined by the Law of Cosines using the planets' and the spacecraft velocities and flight angles. Finally the semi-minor axis and the angle of asymptote of the departure and arrival hyperbolas were calculated using C3 and $V_{\infty.}$

This was all completed and verified in FORTRAN, then translated to MATLAB to take advantage of some of MATLAB's intrinsic capabilities. After the completion of the translation, modifications were done to include Type II trajectories. At the present time, PatCon is hard-coded with Mars as the target planet. So the only needed inputs are the launch and arrival dates, and the exact position of the planets at these times. If another planet other then Mars is wanted as the target planet, the user needs to update the values listed in table 1 within the program.

VERIFICATION OF PATCON

A single test case was used to compare PatCon to several different COTs software. The sample case was a mission to Mars with a launch date of November 17, 2011 and arrival date of June 11, 2012, see Figure 2 for a pictorial of the trajectory. There are two different software programs that were used to verify PatCon. The first one is AGI's Satellite Tool Kit which is a large COTS software used mostly during the mission implementation phase with high fidelity capabilities for orbital design and analysis. The second software used for comparison is JAQAR Astrodynamics' Swing-by Calculator, which is more simplified then STK but can be downloaded off the internet[3] for free. Since different software, including PatCon, has different outputs, the one parameter that always is listed is C3, as this value is vital when it comes to designing the trajectories, as it is used to determine the launch mass and launch vehicle that is needed. Thus, C3 will be the one value that is compared in all the different software. Other values such as excess velocity will be compared. Some of the results from PatCon are listed in table 2.

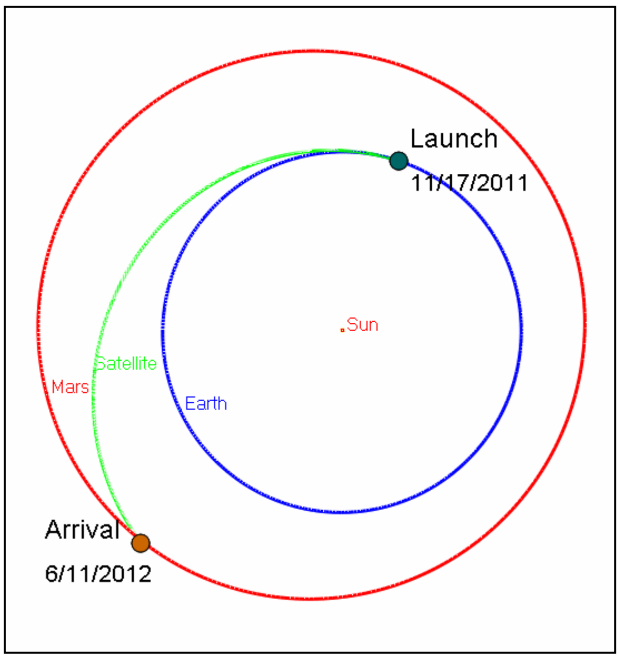

Figure 2. Trajectory of Test Case.

Table 2
RESULTS OF PATCON- EARTH TO MARS. TYPE 11 TRAJECTORY

Launch Date	November 17, 2011
Arrival Date	June 11, 2012
C3	13.1072 km^2/sec^2
Departure Velocity	33.56 km/sec
Arrival Velocity	20.97 km/sec
Departure Excess Velocity	3.62 km/sec
Arrival Excess Velocity (V_∞)	5.49 km/sec

Comparison to AGI's Satellite Tool Kit (STK)

STK is one of the larger commercial software packages that offer a set of software tools designed to support space-based systems from concept, design, and deployment through real-time operations[2]. To develop this type of scenario in STK extensive knowledge of its Astrogator tool and how its target sequence works is needed. The setup includes developing 3 targeting sequences: the first to target the needed outgoing asymptote right ascension and declination to get to Mars; the second to target the B-Plane and the correct arrival date; finally the correct inclination and radius of periapsis. Once completed, a report on all the target sequences is printed (see Appendix A) and the results compared to the output of PatCon. As can be seen in the results in Appendix A, STK has a C3 value of 13.23 km^2/sec^2, which is a difference of about 1% from PatCon. STK also has a departure excess velocity of 3.63 km/sec and an arrival excess velocity of 5.97 km/sec. This is a difference of 0.2% and 8.14% respectively. The large difference in the arrival excess velocity is due to the fact that PatCon cannot target a specific inclination, which affects the arrival flight path angle which is needed for the calculations in the excess velocity.

Comparison to JAQAR Astrodynamics' Swing-by Calculator

JAQAR's Swing-by Calculator is a tool to find trajectories from a departure planet to an arrival planet via multiple swing-bys. The Swing-by Calculator features optimization of interplanetary transfer to any body in the solar system[3]. The inputs are similar to PatCon, and include the launch and arrival date, what target planets, some final orbit parameters and predicted swingby dates and distances. The results are shown in Appendix B. The swing-by calculator results include a C3 value of 13.28 $km^2/sec^2$with a difference of 1.33% compared to PatCon. The departure velocity is 33.65 km/sec which is a difference of 0.26%. The arrival excess velocity is 5.95 km/sec with a difference of 7.73%, again noting that the arrival flight path angles are different which would lead to slightly different excess velocities.

CONCLUSIONS

The complete results are listed in table 2. After comparison to other COTS software it can be seen that PatCon values agree within reason. Again it should be noted that it is believed that the large difference in the arrival velocity values is due to the difference in the different flight paths used in the different software, and since flight path values is what is needed to calculate arrival velocity, it is believed this is where the discrepancy lies. As for the development and calculation time needed to obtain these results STK took the longest, on the order of a day to complete the scenario, the Swing-by calculator was much quicker and comparable to Pat_Con. Though STK scenario's do take time to set up their graphics are superb, and the ability to link to Matlab and other software is very helpful when running real time scenarios and/or doing systems engineering. Though the Swing-by Calculator does not have these features, the ease of use is very helpful especially for systems engineers that are not completely trained in orbital mechanics, and the capability to produce pork chop plots (aka plots of C3 for different launch and arrival dates) is very helpful. Finally as PatCon is the simplest to use and execute which makes it very easy to run trade studies on launch and arrival dates, it is limited to early conceptual analyses only.

Table 3
COMPARISON OF RESULTS: MISSION TO MARS

	PatCon	**STK**	**Swing-by Calculator**
C3	13.1072 km^2/sec^2	13.23 km^2/sec^2	13.28 km^2/sec^2
Departure Velocity	33.56 km/sec	**N/A**	33.65 km/sec
Arrival Velocity	20.97 km/sec	**N/A**	**N/A**
Departure Excess Velocity	3.62 km/sec	3.63 km/sec	**N/A**
Arrival Excess Velocity (V_∞)	5.49 km/sec	5.97 km/sec	5.95 km/sec

ACKNOWLEDGEMENTS

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2. STK Space Systems. (2007). Retrieved 05 January, 2007, from STK Web site: http://www.agi.com/solutions/applicationAreas/spaceSystems.cfm
3. JAQAR Swing-by Calculator (SBC). Retrieved 05 January, 2007, from JAQAR Web site: http://www.jaqar.com/

4. **APPENDIX A- STK Results**

Astrogator MCS Target Sequence Summary

MCS Segment Type: TargeterSequence
Name: Target Sequence
User Comment: -Targeter Sequence Description-

Sequence Start: 16 Nov 2011 23:59:59.000 UTCG; 2455882.49998843 UTC Julian Date
Sequence Stop: 10 Jun 2012 23:59:58.967 UTCG; 2456089.49998804 UTC Julian Date

<<<< Start of Sequence: Target Sequence >>>>

MCS Segment Type: InitialState
Name: Target Sequence.Initial State
User Comment: -Initial State Description-

Satellite State at End of Segment:

UTC Gregorian Date: 16 Nov 2011 23:59:59.000 UTC Julian Date: 2455882.49998843
Julian Ephemeris Date: 2455882.50074287
Time past epoch: 3.43181e+008 sec (Epoch in UTC Gregorian Date: 1 Jan 2001 00:00:00.000)

State Vector in Coordinate System: Earth Inertial

Parameter Set Type: Cartesian

X:	6252.0065675902570000 km	Vx:	-2.0340379745960071 km/sec
Y:	-2282.0050993558611000 km	Vy:	0.7424321519930859 km/sec
Z:	1283.0240078144591000 km	Vz:	11.2320990448373600 km/sec

Parameter Set Type: Keplerian

sma:	-30122.5668883252290000 km	RAAN:	339.947733455389 deg
ecc:	1.2250140243717345	w:	10.91149339885557 deg
inc:	89.99999581738042 deg	TA:	0 deg

Parameter Set Type: Spherical

Right Asc:	339.9477342617047 deg	Horiz. FPA:	-4.919920522888878e-015 deg
Decl:	10.91149339885554 deg	Azimuth:	4.259630457956097e-006 deg
|R|:	6777.9999999488155000 km	|V|:	11.4389057577833050 km/sec

Other Hyperbolic Orbit Parameters :

Ecc. Anom:	0 deg	Mean Anom:	0 deg
Long Peri:	350.8592268542445 deg	Arg. Lat:	10.91149339885557 deg
True Long:	350.8592268542445 deg	Vert FPA:	90 deg
Ang. Mom:	77532.90322566974 km^2/sec	p:	15081.1450570777290000 km
C3:	13.23261869673158 km^2/sec^2	Energy:	6.616309348365788 km^2/sec^2
Vel. RA:	159.9477117587936 deg	Vel. Decl:	79.08850660114362 deg
Rad. Peri:	6777.9999999488145000 km	Vel. Peri:	11.4389057577833060 km/sec

Rad. Apo: -67023.1337765992790000 km
Excess Vel: 3.637666655526806 km/sec
Time Past Periapsis: 0 sec
Time Past Ascending Node: 113.6005525832507 sec
Beta Angle (Orbit plane to Sun): 64.0571324536142 deg
Mean Sidereal Greenwich Hour Angle: 55.7018970603081 deg

Geodetic Parameters:
Latitude: 11.04333713233443 deg
Longitude: -75.6063621701999 deg
Altitude: 400.6416237537594000 km
Geocentric Parameters:
Latitude: 10.97547200185943 deg
Longitude: -75.6063621701999 deg

MCS Segment Type: Propagate
Name: Target Sequence.ToMarsPeriapsis
User Comment: -Propagation Description-

Propagator model used: Heliocentric_Point_of_Mass (Heliocentric Point Mass)

Stopping Condition Information (Gregorian Date [Julian Date]):
10 Jun 2012 23:59:58.967 [2456089.49998804]: Stopped on: Periapsis; Run Sequence STOP

Propagation Statistics:
Number of steps: 808
Average step size: 21491.1 sec
Largest step size: 86400 sec
Smallest step size: 0.092317 sec

Satellite State at End of Segment:

UTC Gregorian Date: 10 Jun 2012 23:59:58.967 UTC Julian Date: 2456089.49998804
Julian Ephemeris Date: 2456089.50074249
Time past epoch: 3.61066e+008 sec (Epoch in UTC Gregorian Date: 1 Jan 2001 00:00:00.000)

State Vector in Coordinate System: Mars Inertial

Parameter Set Type: Cartesian

X:	-2014.7134164123536000 km	Vx:	-5.0780517531498361 km/sec
Y:	1303.5148309707643000 km	Vy:	3.2848286044845225 km/sec
Z:	3200.1812569580079000 km	Vz:	-4.5349526653385777 km/sec

Parameter Set Type: Keplerian

sma:	-1198.7373153711928000 km	RAAN:	327.1005827834625 deg
ecc:	4.3367799523187527	w:	126.8641594225972 deg
inc:	89.99748446678056 deg	TA:	359.9999330256832 deg

Parameter Set Type: Spherical

Right Asc:	147.0972280310976 deg	Horiz. FPA:	-5.442474248493144e-005 deg
Decl:	53.13590747807553 deg	Azimuth:	179.9958068781558 deg
|R|:	3999.9226420292193000 km	|V|:	7.5592661181737295 km/sec

Other Hyperbolic Orbit Parameters :

Ecc. Anom:	-5.29614999895243e-005 deg	Mean Anom:	-0.0001767208714098072 deg
Long Peri:	93.96474220605973 deg	Arg. Lat:	126.8640924482805 deg
True Long:	93.96467523174289 deg	Vert FPA:	90.00005442474249 deg
Ang. Mom:	30236.47970319378 km^2/sec	p:	21346.7069667955400000 km
C3:	35.72790414722518 km^2/sec^2	Energy:	17.86395207361259 km^2/sec^2
Vel. RA:	147.1024690388994 deg	Vel. Decl:	-36.86414683161546 deg
Rad. Peri:	3999.9226420269979000 km	Vel. Peri:	7.5592661181745173 km/sec
Rad. Apo:	-6397.3972727693836000 km	Excess Vel:	5.977282337921237 km/sec

Time Past Periapsis: -0.0006185651758413287 sec
Time Past Descending Node: -647.5367067149058 sec
Beta Angle (Orbit plane to Sun): -20.3592930390122 deg
Mean Sidereal Greenwich Hour Angle: 259.730764576634 deg

Planetodetic Parameters:
Latitude: 53.440318453274 deg
Longitude: 141.5895847447389 deg
Altitude: 617.0449283390713600 km

Planetocentric Parameters:
Latitude: 53.13847941545803 deg
Longitude: 141.5895847447389 deg

APPENDIX B-Swing-by Calculator

Minimum C3 Departure planet:
Departure planet(3): 2011-11-17
C3 value_Departure = 13.2843844330952 km^2/s^2
RA_departure = 158.946748691266 deg
Decl_departure = 24.0071091773929 deg
V_departure = (-27.9294800703636,16.8525554656063,8.27029030074363)

Arrival planet(4) : 2012-6-11
Transfer time = 207 days
C3 value_Planet = 35.5001213948431 km^2/s^2
Vinf_arrival = (-5.89346315382902,-0.871173681718069,-0.0909388013152288)
DV = 4.42084316820946 km/s
B-plane magnitude = 7822.13830467858 km

Total C3 = 48.7845058279383 km^2/s^2
Total DV = 8.06561811779074 km/s
Duration: 207 days

Final orbit parameters (Planet Equatorial of Date):
Rp = 6708.2 km
Ra = 6708.2 km
Inclination = 23.2062907334617 deg
RAAN = 212.62606526428 deg

Vinf pl. Equ. of Date= (-4.6120382266237,2.95247861173145,-2.34778509201187)

****
Minimum C3 Arrival Planet:
Departure planet(3): 2011-11-17
C3 value_Departure = 13.2843844330952 km^2/s^2
RA_departure = 158.946748691266 deg
Decl_departure = 24.0071091773929 deg
V_departure = (-27.9294800703636,16.8525554656063,8.27029030074363)

Arrival planet(4) : 2012-6-11
Transfer time = 207 days
C3 value_Planet = 35.5001213948431 km^2/s^2
Vinf_arrival = (-5.89346315382902,-0.871173681718069,-0.0909388013152288)
DV = 4.42084316820946 km/s
B-plane magnitude = 7822.13830467858 km

Total C3 = 48.7845058279383 km^2/s^2
Total DV = 8.06561811779074 km/s
Duration: 207 days

Final orbit parameters (Planet Equatorial of Date):
Rp = 6708.2 km
Ra = 6708.2 km
Inclination = 23.2062907334617 deg
RAAN = 212.62606526428 deg
Vinf pl. Equ. of Date= (-4.6120382266237,2.95247861173145,-2.34778509201187)