The Cosmic Battery Revisited

Ioannis Contopoulos,[1] Demosthenes Kazanas,[2] and Dimitris M. Christodoulou[3]

ABSTRACT

We reinvestigate the generation and accumulation of magnetic flux in optically thin accretion flows around active gravitating objects. The source of the magnetic field is the azimuthal electric current associated with the Poynting-Robertson drag on the electrons of the accreting plasma. This current generates magnetic field loops which open up because of the differential rotation of the flow. We show through simple numerical simulations that what regulates the generation and accumulation of magnetic flux near the center is the value of the plasma conductivity. Although the conductivity is usually considered to be effectively infinite for the fully ionized plasmas expected near the inner edge of accretion disks, the turbulence of those plasmas may actually render them much less conducting due to the presence of anomalous resistivity. We have discovered that if the resistivity is sufficiently high throughout the turbulent disk while it is suppressed interior to its inner edge, an interesting steady-state process is established: accretion carries and accumulates magnetic flux *of one polarity* inside the inner edge of the disk, whereas magnetic diffusion releases magnetic flux *of the opposite polarity* to large distances. In this scenario, magnetic flux of one polarity grows and accumulates at a steady rate in the region inside the inner edge and up to the point of equipartition when it becomes dynamically important. We argue that this inward growth and outward expulsion of oppositely-directed magnetic fields that we propose may account for the ~ 30 min cyclic variability observed in the galactic microquasar GRS1915+105.

Subject headings: accretion, accretion disks—MHD—plasmas—stars: magnetic fields

[1]Research Center for Astronomy, Academy of Athens, GR-11527 Athens, Greece.
Email: icontop@academyofathens.gr

[2]NASA/GSFC, Code 665, Greenbelt, MD 20771, USA.
Email: kazanas@milkyway.gsfc.nasa.gov

[3]Math Methods, 54 Middlesex Turnpike, Bedford, MA 01730, USA.
Email: dimitris@mathmethods.com

1. Introduction

The issue of the orgin of cosmic magnetic fields, despite much progress in the field, remains an open issue in astrophysics. The problem is basically of a topological nature, because the magnetic field, being a solenoidal vector $\mathbf{B} = \nabla \times \mathbf{A}$ ($\mathbf{A}$ is the magnetic vector potential), is necessarily absent in a homogeneous and isotropic universe. For the same reason, the magnetic field cannot be generated by the potential fluid motions that result from the growth of scalar inhomogeneities in such a universe, as current models contend. As such, the evolution of the magnetic field is akin to that of vorticity $\omega = \nabla \times \mathbf{v}$ ($\mathbf{v}$ is the flow velocity), a fact that has been noticed by at least one (to our knowledge) paper in the literature (see Eqs. [4] and [7] in Kulsrud et al. [1997]). It is also of interest to note that the source of both ω and $\mathbf{B}$ is proportional to $\nabla p \times \nabla \rho / \rho^2$ (p is the pressure and ρ is the density), a quantity which is nonzero only in the presence of nonbarotropic fluids (Kulsrud et al. [1997] consider as such curved shocks and photoheating). This source term, while generally small, is nonetheless of great importance because in its absence the initial values of ω, $\mathbf{B} = 0$ are preserved by the evolution equations. It is generally referred to as the *Biermann battery* and leads to magnetic fields that are usually quite weak initially ($\sim 10^{-20}$ G) but thought to be subsequently amplified to the observed mean galactic values ($\sim 10^{-6}$ G) by dynamo processes.

Some years ago, we proposed a source-term alternative to the term of the Biermann battery (Contopoulos & Kazanas [1998], hereafter CK). More specifically, we posited that the Poynting-Robertson radiation force acting predominantly on the electrons of an accretion flow around an active astrophysical source may generate toroidal electric currents sufficiently large to support poloidal magnetic fields that in certain situations could approach equipartition values. The Poynting-Robertson source term is proportional to $\nabla \times (\ell \omega)$, where $\ell = L \sigma_{\mathrm{T}} / 4\pi r m_e c^3$ is the usual accreting source compactness (L is the source luminosity, σ_T is the Thompson cross-section, m_e is the electron mass, c is the speed of light, and r is the distance from the center); as such, besides its much larger magnitude compared to the term of the Biermann battery, this term is additionally important because it provides a hitherto unexplored, direct coupling between the magnetic field and the vorticity equations of motion.

In our earlier work (CK) we concentrated our attention on the effects of the Poynting–Robertson source term on the evolution of the magnetic field within the volume of the disk alone, by solving the corresponding (induction) equation in that region. Our study showed that, in a highly conducting plasma, currents and fields grow only for a restricted time of the order of the accretion time, and thus they quickly saturate to values well below equipartition (but still much higher than those induced by the Biermann battery). However, for plasma

conductivity below a certain critical value (and a free inner boundary condition that allows the accumulation of magnetic flux interior to that point), we found a different regime in which the magnetic field accumulated around the gravitating object grows at a constant rate; we then showed that in several astrophysically interesting systems, magnetic fields may reach even equipartition values within astrophysically relevant time scales.

This 'unrestricted' field growth has its origin on the fact that the magnetic field loops, generated by the toroidal Poynting-Robertson current inside the disk, open up to infinity once they reach the surface of the disk because of its differential rotation (and the ideal MHD conditions outside the disk). This opening up, coupled with sufficiently low conductivity, allows the separation of *inward advected* field of one polarity from the *outward diffusing* field of the opposite polarity. This effect of magnetic loop opening above and below the disk is central in our mechanism and it has been clearly described in our earlier paper. Our work was criticized by Bisnovatyi-Kogan, Lovelace & Belinski (2002), who, arguing along the lines of earlier work by Bisnovatyi-Kogan & Blinnikov (1977), claimed that the magnetic field grows only over an accretion time scale or so and saturates at values well below equipartition. In support of their view they provided an exact solution of the induction equation with the Poynting-Robertson term included that indeed exhibited the claimed saturation in the ideal MHD limit, while in an Appendix to their paper they argued that their results would not change significantly for finite plasma conductivity.

In our opinion, the reason for the difference in the conclusions of the two treatments lies in the consideration of the inner boundary condition to the solution of essentially the same equation. In addition, the one-dimensional character of the model used, although it showed that unrestricted field growth can take place, did not help the reader to visualize the actual geometry of the magnetic field on the poloidal plane. We therefore decided to return to this problem and perform more sophisticated two-dimensional numerical simulations with a variety of boundary conditions which, we hope, will convince the reader that the mechanism described in our original paper can produce astrophysically interesting results. In § 2 we describe the setup of our simulations and in § 3 we present the results of our calculations. Finally, in § 4 our results are discussed and some conclusions are drawn.

2. Simulation Setup

Our simulations improve on those of CK by extending the treatment to two spatial dimensions while incorporating the essential elements of the cosmic battery scenario. Our treatment is still linear in that the dynamics of the accretion disk are prescribed and we solve for the magnetic field evolution; we expect that this simplified treatment will serve as

a guide for our upcoming detailed, resistive, numerical MHD simulations of the combined magnetic field and vorticity equations that is in preparation.

Our comptuational domain consists of two regions: a viscous, resistive, geometrically thick accretion disk of specified flow velocity field and an overlying force-free magnetosphere where ideal MHD conditions apply. Inside the disk, the magnetic field generated through the Poynting-Robertson azimuthal drag on plasma electrons is dynamically insignificant (at least initially), hence magnetic field lines follow the flow as dictated by the time-dependent induction equation

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{L\sigma_T v_\phi}{4\pi r^2 ce}\hat{\phi} + \mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} \tag{1}$$

(see also CK and Bisnovatyi-Kogan, Lovelace & Belinski 2002). Here, $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field, e is the electron charge, and η is the magnetic diffusivity. The Poynting-Robertson radiation force on electrons is equal to $-L\sigma_T v_\phi/(4\pi r^2 c)\hat{\phi}$. Henceforth, we will work in a spherical system of coordinates (r, θ, ϕ). We assume the disk to be Keplerian ($v_\phi \propto v_K$) and thick (height $h \simeq r$), i.e. very similar to an ADAF (Narayan & Yi 1994)[1]. The remaining unknown quantity relevant to our calculation is the radial velocity v_r which (for $h \simeq r$) is given by

$$v_r = -\alpha v_K \ , \tag{2}$$

where α is the usual accretion disk parameter.

In an α–disk of the type considered here, it is natural to assume that the flow turbulence that is responsible for the viscosity required for accretion is also responsible for other dissipative processes, namely the anomalous plasma resistivity or equivalently anomalous magnetic diffusivity (see e.g. Heitsch & Zweibel 2003). This can be expressed through the introduction of the magnetic Prandtl number $\mathcal{P}_m$ in the expression for the anomalous magnetic diffusivity

$$\eta \sim \mathcal{P}_m r |v_r| \tag{3}$$

(Reyes-Ruiz & Stepinski 1996). In what follows, $\mathcal{P}_m$ is taken to be a free parameter. Once $\mathbf{v}$, L and η are specified, eq. (1) can be directly integrated to yield the magnetic field as a function of time and position in the interior of the accretion flow. Note that the disk's differential rotation will continually wind up the field in the azimuthal direction.

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[1]The picture one may have in mind is that of small scale turbulent magnetic fields building-up to equipartition values due to the magneto-rotational instability. This effectively thermalizes the kinetic energy of the azimuthal motion on time scales not much longer than the local free fall time, thus leading to a disk of height $h \simeq r$, which is precisely the ADAF geometry we consider.

The Poynting-Robertson source of poloidal electric current extends up to the surface of the disk, and hence magnetic field loops emerge into a magnetosphere above and below the disk (note that the Poynting-Robertson effect is of course at work everywhere, only its effectiveness drops quickly with distance from the central luminosity source). The physical conditions in the magnetosphere are assumed to be very different in that the dominant dynamical factor is the magnetic field and not the plasma inertia, as is the case in the disk interior[2]. Under the magnetospheric ideal MHD conditions, electric currents develop that quickly establish a force-free magnetic field equilibrium. In particular, the azimuthal winding of the field described above will generate electric currents that will flow along the magnetic loops and will tend to open them up. It has been shown in the solar–photosphere literature (Aly 1984) that there exists a critical amount of winding in the azimuthal direction, beyond which each loop will quickly break into two disconnected open parts with their footpoints on the rotating disk.

In reality, the disk and the magnetosphere evolve at the same time, however the evolution speed in the magnetosphere (the Alfven speed $v_A \sim c$) is much higher than the evolution speeds within the disk ($v_K, v_r \ll v_A$). We decided not to follow the full evolution of these loops in the magnetosphere and consider instead a sequence of force-free MHD equilibria above and below the disk, characterized by

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \ . \tag{4}$$

We would like to acknowledge here that force-free is an approximation of the physical conditions in the magnetosphere above the disk corona. Under conditions of axisymmetry, eq. (4) can be rewritten in spherical coordinates as

$$\frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{r^2 \tan\theta}\frac{\partial\Psi}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2} = -I\frac{\mathrm{d}I}{\mathrm{d}\Psi} \ , \tag{5}$$

We have introduced here the magnetic flux function $\Psi \equiv r\sin\theta A_\phi$ and the poloidal electric current distribution $I = I(\Psi)$. The three field components are then given by

$$B_r = \frac{1}{r^2\sin\theta}\frac{\partial\Psi}{\partial\theta} \ , \tag{6}$$

$$B_\theta = -\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial r} \ , \tag{7}$$

$$B_\phi = \frac{I}{r\sin\theta} \ . \tag{8}$$

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[2]We assume the simplest possible configuration, one with minimal magnetospheric plasma loading. We also neglect the corona region above and below the disk which is believed to be the origin of disk wind outflows. The study of the plasma dynamics in that region is outside the scope of our present investigation, and we do refer the interested reader to the relevant literature (e.g. Krasnopolsky, Li & Blandford 2003).

Eq. (5) is an elliptic equation with well-defined boundary conditions (the field distribution $\Psi(r,\theta)|_{\text{surface}}$ on the surface of the disk) that allows us to determine the magnetic field configuration in the disk's atmosphere.

As we argued above, the magnetic field loops get wound up by the disk's differential rotation up to a critical amount of winding, where all loops break into two disconnected open parts. Under the ideal force-free MHD conditions that we have assumed for the disk magnetosphere, this critical configuration contains a certain distribution $I(\Psi)$ of poloidal electric current along the magnetic field. It is easy to check that $I(\Psi)$ is a monotonically growing function of Ψ, since in that case the Lorentz force $(\nabla \times \mathbf{B}) \times B_\phi \hat{\phi}$ acts to open up the magnetic loop. The exact functional form depends on the detailed disk-magnetosphere interaction. We decided to choose the following simple form

$$I(\Psi) = -\lambda \Psi(2 - \Psi/\Psi_{\max}) \,, \tag{9}$$

known to us from a different problem of magnetospheric field line opening, the problem of the axisymmetric pulsar magnetosphere (Contopoulos, Kazanas & Fendt 1999). The constant of proportionality λ is determined by the requirement that all magnetospheric field lines open up to infinity because of the disk differential rotation. $\Psi_{\max}$ is the maximum value of·the magnetic flux function on the surface of the disk.[3] Our choice simplifies considerably the numerical calculation but it is obviously not unique. As we shall see, such a distribution of poloidal electric current does indeed lead to a largely open magnetospheric configuration.

Inside the disk, the azimuthal field winding is limited by field diffusion as described by the poloidal component of eq. 1. One may easily check that, in the region outside r_{in} where η is of the same order as rv_r, the azimuthal field is limited to a value of the order of v_K/v_r times its poloidal value. Outside the disk, the azimuthal field winding is limited by the opening-up of the coronal loops to a value of the same order as its poloidal value (e.g. Aly 1984). We thus expect a small discontinuity in the azimuthal field in a transition region across the surface of the disk.

The problem is now fully specified and the numerical simulations proceed as follows:

1. At each timestep, we evolve the magnetic field in the interior of the disk through eq. (1).

2. We obtain the distribution of $\Psi(r,\theta)|_{\text{surface}}$.

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[3]The magnetic field is generated in the disk interior, and the total amount of magnetic flux crossing the surface of the disk is zero, hence $\Psi(r,\theta)|_{\text{surface}}$ has a maximum value $\Psi_{\max}$ at a certain distance, and decreases again to zero farther out.

3. This distribution is used as a boundary condition for the calculation of a force-free ideal MHD magnetospheric equilibrium via eq. (5).

4. The above sequence of steps is repeated for each subsequent timestep.

Before we proceed with the results of our numerical integrations, we would like to note that the magnetic-field configurations in the disk and magnetosphere are closely coupled. It is obvious that the field generated in the disk is the source of the magnetospheric field. What is not obvious though it that, in the presence of magnetic field diffusion, the magnetospheric field also acts back on the disk's magnetic field: the opening up of the magnetospheric field generates field bending and field tension on the surface of the disk, and as a result, the field diffuses into the interior of the disk, in the direction that will release the magnetic tension. This back–reaction of the magnetosphere onto the disk's magnetic field has not been taken into account by Bisnovatyi-Kogan, Lovelace & Belinski (2002) in their criticism of our original paper. This effect is a central element in our cosmic battery mechanism.

3. Numerical Integration

We will work in renormalized variables where distances, times, and magnetic fields are rescaled, respectively, to the inner radius r_{in}, the dynamical time $t_{in} \equiv r_{in}/v_{K,in}$ at the inner radius, and the magnetic field $B_{P-R} \equiv t_{in} \times L\sigma_T v_{K,in}/(4\pi ce r_{in}^3)$ that is generated at the inner radius within a dynamical time, viz.,

$$r/r_{in} \rightarrow r \ , \quad t/t_{in} \rightarrow t \ , \quad B/B_{P-R} \rightarrow B \ . \tag{10}$$

As was discussed above, our simulation setup allows us to study any accretion disk model in conjunction with any magnetic diffusivity distribution in the disk. The main elements of the cosmic battery mechanism may, however, be demonstrated clearly by using the following simple flow pattern:

$$v_\phi = \begin{cases} r^3 & r < 1, \ 60° < \theta < 120° \\ r^{-1/2} & r \geq 1, \ 60° < \theta < 120° \\ 0 & \text{otherwise} \end{cases}$$

$$v_r = -0.1 v_\phi$$

$$v_\theta = 0.$$

We emphasize once again that the flow pattern dictated by the specific accretion disk model is independent of the Poynting-Robertson generated magnetic field, as long as the latter

remains below equipartition values. This allows us to decouple the magnetic field evolution from the flow kinematics.

The anomalous magnetic diffusivity η is calculated from eq. (3) at all distances. We assume that interior to $r = 1$ the accretion disk terminates and the velocity profiles are different from those for $r > 1$. The behavior given above corresponds qualitatively to settling on a stellar surface, although one could also produce the appropriate behavior for accretion onto a black hole. As the flow rotation decreases to match that of the underlying object, the shear decreases, and the flow becomes laminar (in the case of a black hole, the flow again becomes laminar interior three Schwarzschild radii because matter is in free fall there). Laminar flow implies the decay of the MHD turbulence that led to the anomalous disk magnetic diffusivity, and therefore we recover perfect field-plasma coupling, i.e. $\eta \approx 0$ inside $r < 1$. This turns out to be important in obtaining the increase in magnetic flux found in CK.

The poloidal magnetic field evolution in the disk is determined by the ϕ–component of eq. (1) written in dimensionless form as

$$\frac{\partial \Psi}{\partial t} = \frac{v_\phi}{r} + r v_r B_\theta - \mathcal{P}_m r v_r \left[\frac{\partial}{\partial r}(r B_\theta) - \frac{\partial B_r}{\partial \theta} \right] , \tag{11}$$

which was integrated for 100 dynamical times. In Fig. 1, we plot the resulting poloidal magnetic field configurations for various values of the magnetic Prandtl number $\mathcal{P}_m$ assumed constant throughout the disk. The proportionality constant in the expression for the magnetospheric poloidal electric current (eq. [9]) is set equal to $\lambda = 2.5$ because we found that, for smaller values, a large fraction of closed magnetic field loops remains in the magnetosphere. For small values of the magnetic Prandtl number, the magnetic field near the inner edge of the disk grows for about one accretion time ($\sim \alpha^{-1}$ in dimensionless time units), and reaches an asymptotic value of the order of α^{-1} (Fig. 2). The total magnetic flux accumulated around the center is also of the same order. The field O-point in the disk is displaced from its original position around $r = 1$ to near the inner radial boundary of our computational domain. In the high diffusivity limit, magnetic-field diffusion wins over accretion, the field O-point moves outside $r = 1$, and the magnetic field generated by the Poynting-Robertson source eventually saturates in this case as well.

These results are not in contradiction with those of CK because CK assumed a free inner boundary for the flow that can only absorb the incident B_z–flux. To simulate this situation, we obtained a second set of solutions in which we set $\mathcal{P}_m = 0$ inside $r = 1$, a reasonable condition, as was discussed above. Our results are shown in Figs. 3 and 4. For low values of the Prandtl number, $\mathcal{P}_m \ll 1$, the field evolution is similar to the former case, namely the field saturates within a few accretion times. However, beyond a critical value of $\mathcal{P}_m \sim 1$,

we obtain a very different behavior, with new magnetic field loops centered around $r = 1$ continuously being generated. These loops open up because of the disk's differential rotation and accretion carries inward magnetic flux of one polarity, whereas diffusion acts to remove to large distances flux of the opposite polarity. The high conductivity at $r \leq 1$ can then hold any magnetic flux that enters this region. It is interesting that the growth of B_z inside $r = 1$ may also contribute to the suppression of MHD turbulence in that region, as seen in recent numerical simulations (Camenzind, personal communication). We thus recover the effect discovered by CK, namely that for values of the magnetic diffusivity higher than a critical value $\mathcal{P}_m \sim 1$, the accumulated magnetic field grows without limit.

Finally, in order to demonstrate that the opening up of magnetic field lines is of central importance in this mechanism, we obtained one more set of solutions in which we set $\lambda = 0$, i.e. we ignored the magnetospheric field twisting because of the disk's differential rotation. In that case (Figs. 5 and 6), magnetic-field lines in the magnetosphere remain closed and the magnetic field again saturates to a finite value well below equipartition.

4.   Discussion and Conclusions

We presented above a two-dimensional analysis of the cosmic battery mechanism first proposed in CK, paying particular attention to the criticism of Bisnovatyi-Kogan, Lovelace & Belinski (2002); while still simplified, our present treatment represents a significant improvement over our earlier work, mainly because of its two-dimensional character which is essential in capturing the structure and evolution of the magnetic field on the poloidal plane.

The two-dimensional models confirm our previous results. When we use appropriate values of the plasma conductivity and the proper magnetospheric boundary conditions, these models produce configurations in which the magnetic flux interior to the accretion disk's inner edge r_{in} increases linearly with time. At the same time, the present results complement our previous investigation and delineate conditions under which such field growth is possible. We have thus found that, for steady magnetic-field growth near the central object of an accretion disk, we need large diffusivity in the disk and a region near the central object in which the diffusivity practically drops to zero. The zero-diffusivity condition in the central region proves to be necessary because then the magnetic flux that crosses into radii $r < r_{in}$ cannot diffuse back out and becomes effectively trapped near the central object (a finite diffusivity in the central region would eventually allow for the flux to leak out to $r > r_{in}$, saturating thus the field growth, as was argued in the Appendix of Bisnovatyi-Kogan, Lovelace & Belinski [2002]). We believe that the zero-diffusivity condition is reasonable in the central region where the accretion disk does not extend and the associated dissipative processes are

thus not operative.

In our present simplified picture, we have ignored the transition region between the conducting interior (where η is effectively zero and the field is greatly amplified from a field a fraction of a Gauss to a field six or more orders of magnitude higher) and the diffusive exterior. In reality, any small amount of diffusivity present will 'smooth out' the abrupt field transition. Magnetic flux will continue to accumulate as long as the thickness of the transition region remains smaller than r_{in}. Were the thickness of the transition region to become of the same order as r_{in}, the mechanism would then saturate, and the field would diffuse outward accross r_{in} at the same rate that it is brought in. If we take into account the Spitzer collisional diffusivity estimated as $\eta_{Spitzer} \sim 10^3 T_6^{1.5} cm^2 s^{-1}$, where T_n is the plasma temperature in units of 10^n K (e.g. Zombeck 1992), the field growth would saturate when the ratio $B(r < r_{in})/B(r > r_{in})$ reaches a value of the order of $v_r r_{in}/\eta_{Spitzer} > 10^{12}$. We conclude that the Spitzer resistivity does not place any practical limit to the growth of the field interior to r_{in}.

Finally, we conclude that this battery mechanism could be important in accounting for the observed magnetic fields in several astrophysical sites as outlined in CK. Around a black hole in particular, our mechanism generates a magnetic field of the right large–scale dipolar topology required for the Blandford-Znajek mechanism of electromagnetic energy extraction from a spinning black hole to work (Blandford & Znajek [1977]; Blandford [2002]). In this respect we would like to point the reader's attention to the intriguing possibility that these considerations may in fact apply to understanding the low frequency cyclic variability observed in the galactic microquasar GRS 1915+105 (e.g. Pooley & Fender [1998]; Ueda *et al.* [2002]). A distinguishing characteristic of this source are its X-ray/IR/radio flares that repeat over time scales of $\sim 20 - 40$ minutes and lead to outflows with $\Gamma \simeq 2 - 3$. It is interesting to note that if magnetic fields of order $B_z \simeq 0.1$ G generated by the mechanism proposed herein in one accretion time of ~ 0.01 sec (in a model with $M = 10 M_{\odot}$, $L = 0.1 L_{Eddington}$, and $\alpha = 0.1$) were to increase linearly with time as envisioned in this work, then after a period of ~ 1000 seconds they would achieve values which could easily account for the observed flares and outflows.

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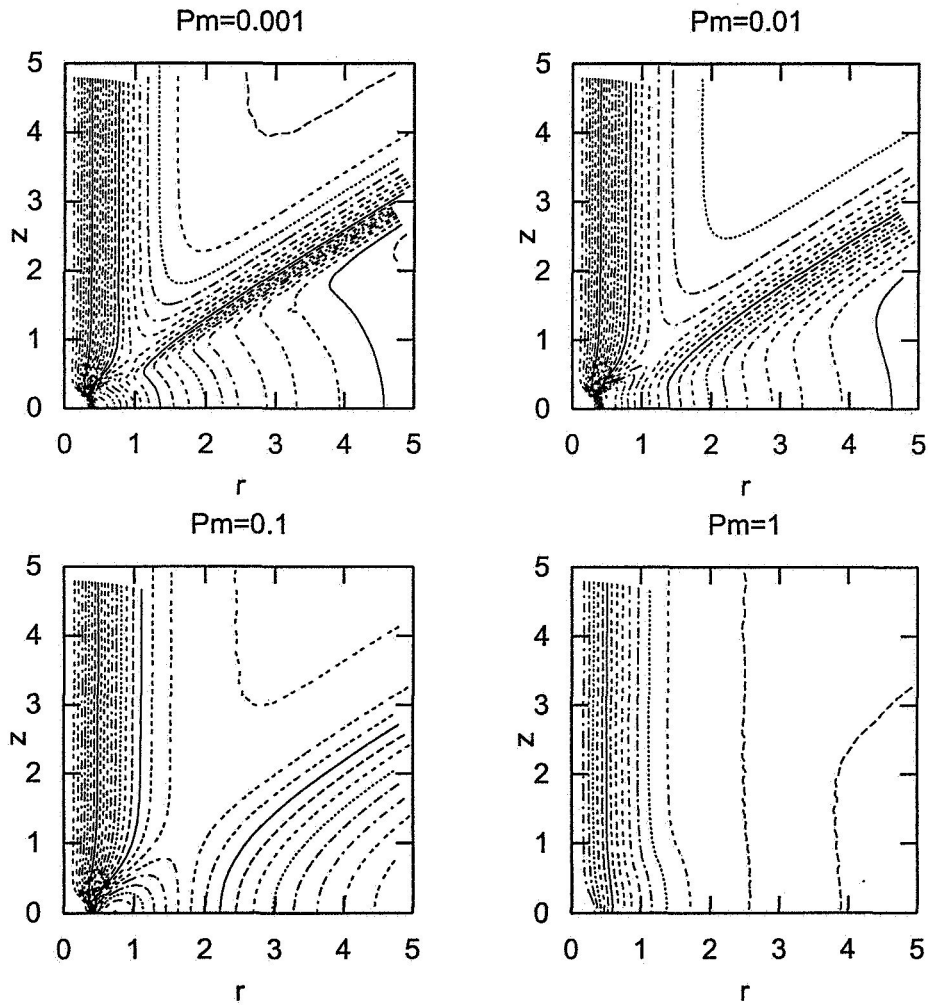
Fig. 1.— We plot the poloidal magnetic field structure after 100 dynamical times for various values of the Prandtl number $\mathcal{P}_m$. $\alpha = 0.1$. The disk extends from $60° < \theta < 120°$, and is surrounded by a force-free atmosphere. Magnetic field lines that enter the atmosphere are wound up by the disk differential rotation and open up. A poloidal electric current distribution as defined by Eq. 9 is established in the magnetosphere.
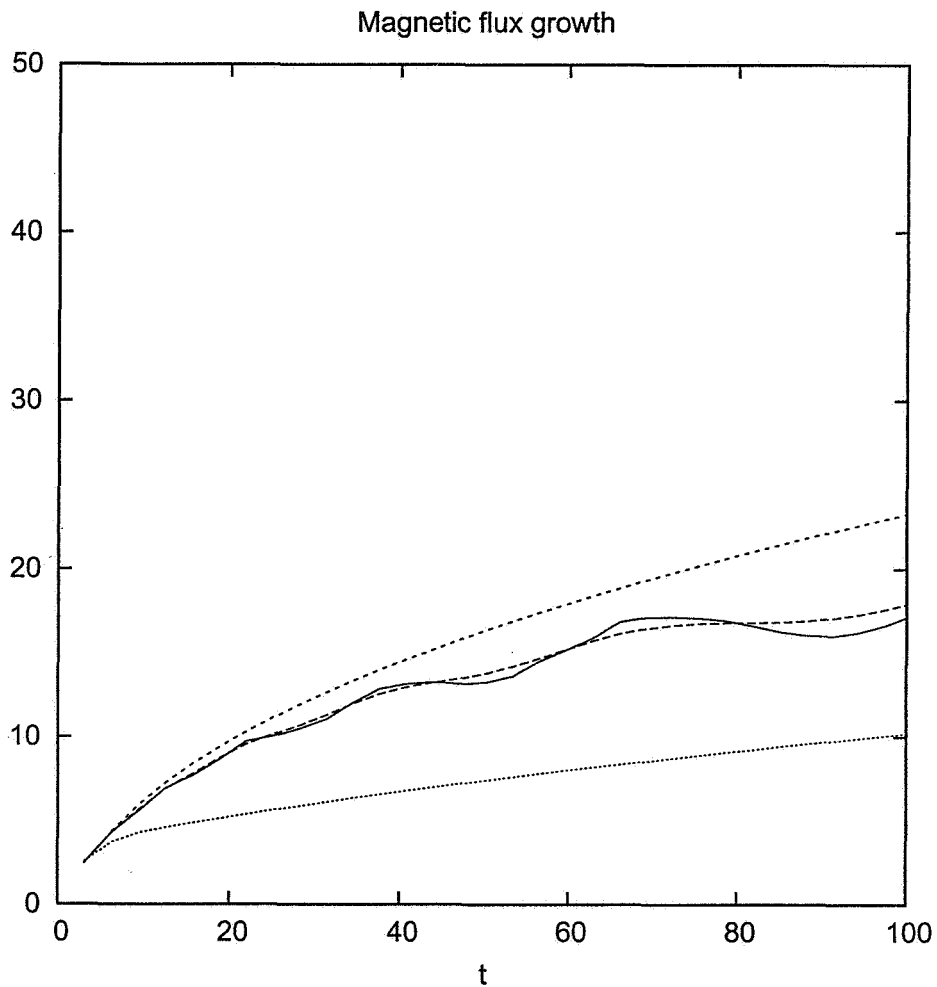
Fig. 2.— We plot the evolution of the magnetic flux Ψ contained within $r = 1$ on the disk midplane for various values of the Prandtl number ($\mathcal{P}_m = 0.001, 0.01, 0.1, 1$ for the solid, dashed, short-dashed and dotted line respectively). In all cases, the flux accumulation saturates.
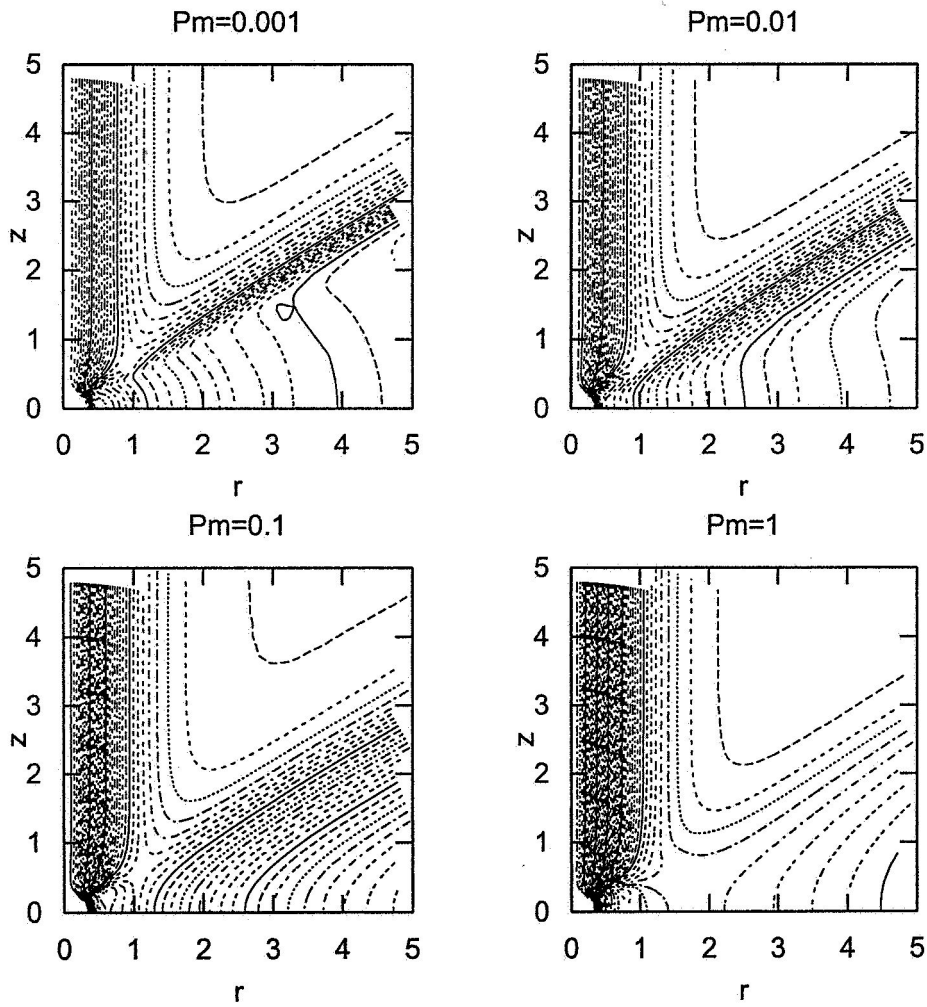
Fig. 3.— Same as Fig. 1 only now we set $\eta = 0$ inside $r = 1$. We see here the effect first discovered in CK, namely that for values of the magnetic diffusivity higher than a critical value the accumulated magnetic field grows without limit.
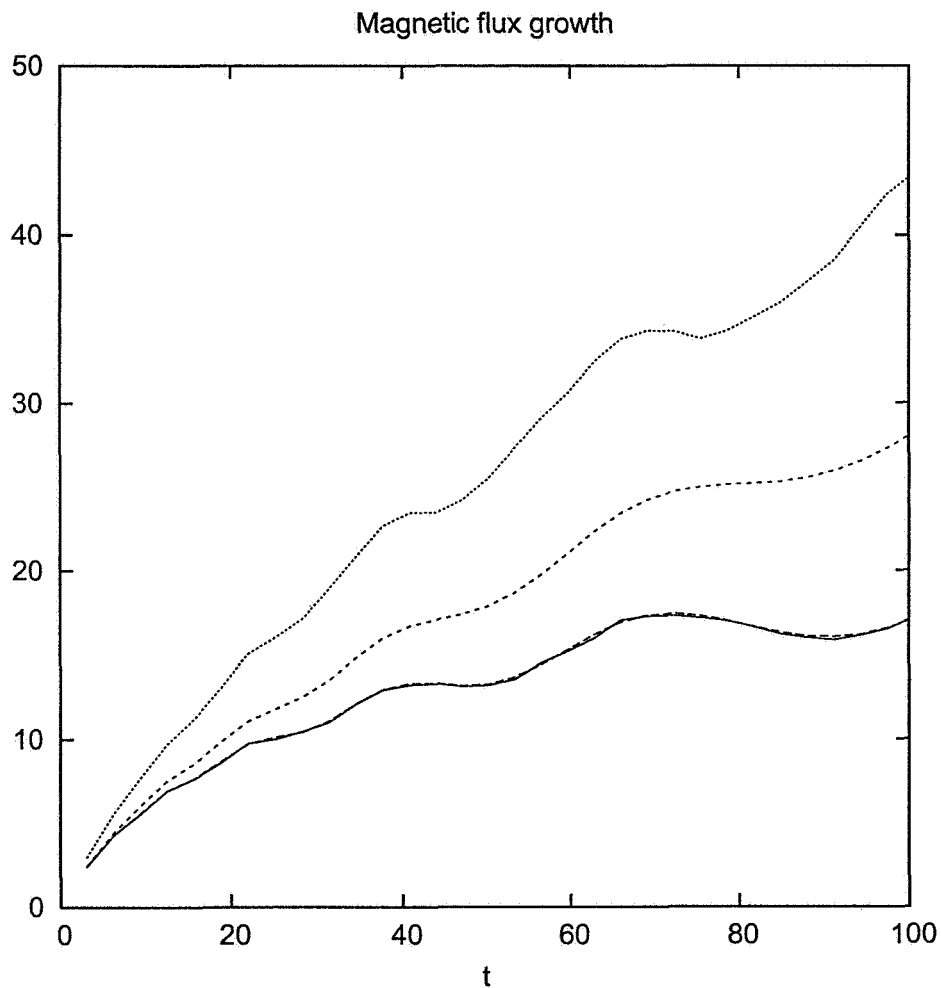
Fig. 4.— Same as Fig. 2 for values of the Prandtl number $\mathcal{P}_m = 0.001$, 0.01, 0.1, 1 for the solid, dashed, short-dashed, and dotted line respectively. For low values of the Prandtl number, the mechanism saturates as before. Above a critical value of the Prandtl number $\mathcal{P}_m \approx 1$, however, we observe unlimited steady field growth.
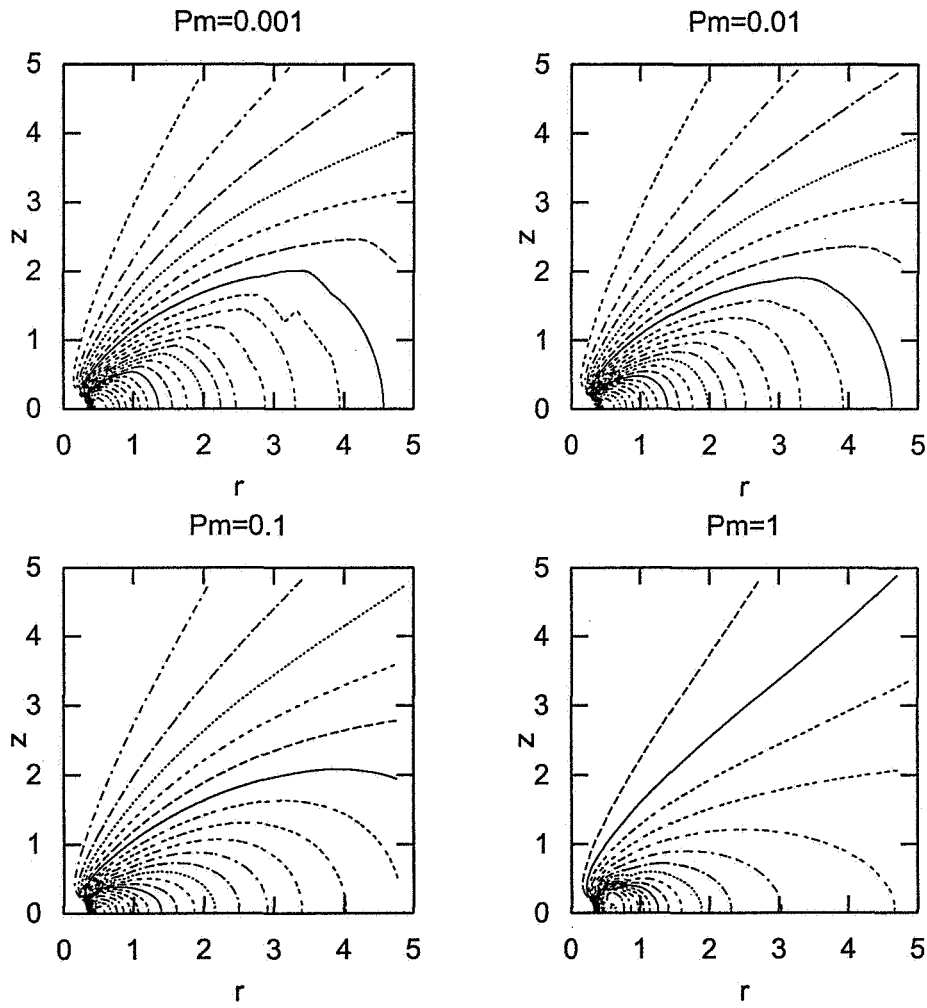
Fig. 5.— Same as Fig. 3 only now we set $I(\Psi) = 0$ in the magnetosphere, and the magnetic field loops do not open up. The effect shown in Fig. 3 disappears.
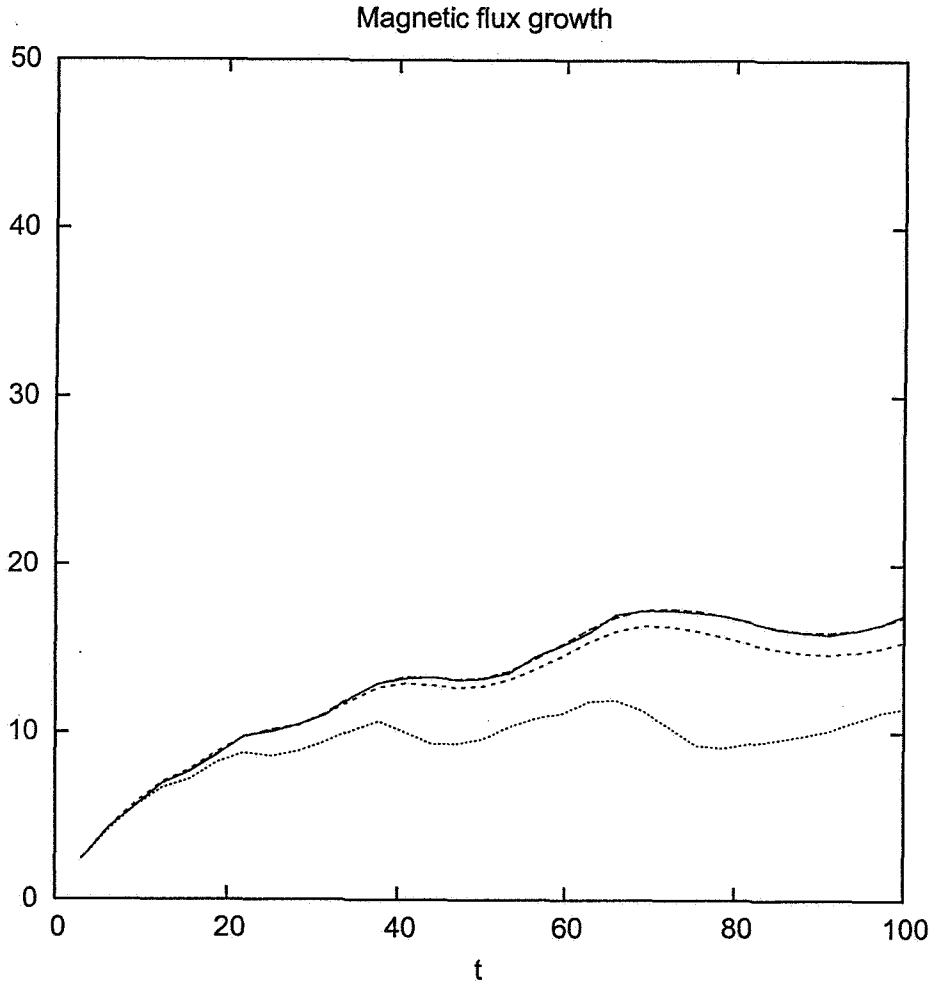
Magnetic flux growth

Fig. 6.— Same as Fig. 4 for values of the Prandtl number $\mathcal{P}_m$ = 0.001, 0.01, 0.1, 1 for the solid, dashed, short-dashed, and dotted line respectively. $I(\Psi) = 0$. The magnetic field again saturates.