Accurate State Estimation and Tracking of a Non-Cooperative Target Vehicle

Julie K. Thienel*, John M. VanEepoel†
NASA Goddard Space Flight Center, Flight Dynamics Analysis Branch
Greenbelt, Maryland, 20771, USA

Robert M. Sanner‡
University of Maryland, Aerospace Engineering Department, College Park, Maryland, 20742, USA

Autonomous space rendezvous scenarios require knowledge of the target vehicle state in order to safely dock with the chaser vehicle. Ideally, the target vehicle state information is derived from telemetered data, or with the use of known tracking points on the target vehicle. However, if the target vehicle is non-cooperative and does not have the ability to maintain attitude control, or transmit attitude knowledge, the docking becomes more challenging. This work presents a nonlinear approach for estimating the body rates of a non-cooperative target vehicle, and coupling this estimation to a tracking control scheme. The approach is tested with the robotic servicing mission concept for the Hubble Space Telescope (HST). Such a mission would not only require estimates of the HST attitude and rates, but also precision control to achieve the desired rate and maintain the orientation to successfully dock with HST.

I. INTRODUCTION

Vehicles designed to rendezvous and dock with existing space missions are being considered for a number of scenarios. In-space assembly is considered a fundamental requirement to accomplish space exploration goals.1,2 ‘Space Tug’ concepts are being designed to provide servicing and refuelling for in-flight missions.3,4 Kimura, et al present a demonstration mission for on-orbit maintenance of satellites.5 In 2004 former NASA Administrator Sean O’Keefe asked the HST program to investigate robotic servicing of the HST to extend the science life.6 In all cases, the chase vehicle must have knowledge of the target vehicle state and must perform precise control in order to safely dock with the target vehicle. When the target vehicle is non-cooperative, meeting the knowledge and control requirements becomes much more challenging.

This work applies an approach to estimate the attitude and rates of a non-cooperative target vehicle, and then uses the attitude and rate estimates as the desired state of the chase vehicle tracking control algorithm. The target vehicle attitude estimate is assumed to be provided by a vision or feature-based sensor. The target rate is determined with the nonlinear estimation approach presented in reference 7. The estimator is exponentially stable in the absence of any measurement errors, and remains robust to bounded perturbations resulting from uncertainties in the measured target attitude. The estimator provides the desired rate for the nonlinear passivity-based control scheme presented in reference 8. The nonlinear controller is asymptotically stable in the absence of any disturbances. The actual attitude of the chase vehicle is assumed to be available from an accurate sensor such as a star tracker. The chase vehicle rates are provided by calibrated gyros.

The stability of the nonlinear control algorithm, given bounded estimates of the desired state of the chase vehicle, is explored. We also consider the effects of additional error sources, such as errors in both the target and chase vehicle inertia, and noise in the chase vehicle gyros. The approach is tested with the HST robotic servicing mission concept, referred to here as the Hubble Robotic Vehicle (HRV). The HRV must be capable
of docking with HST, regardless of the orientation or rotation rate of HST, including the scenario in which the HST batteries have died and HST is tumbling.

The next section gives an overview of the mathematical terms. Then the nonlinear estimation algorithm is summarized, followed by a summary of the nonlinear control algorithm. We then present the results of several scenarios, applied to the HST-HRV mission concept. Finally conclusions are given along with future considerations.

II. ATTITUDE AND ANGULAR RATE DEFINITIONS

The attitude of a spacecraft can be represented by a quaternion, consisting of a rotation angle and unit rotation vector $\vec{e}$, known as the Euler axis, and a rotation $\phi$ about this axis so that

$$q = \begin{bmatrix} \vec{e} \sin\left(\frac{\phi}{2}\right) \\ \cos\left(\frac{\phi}{2}\right) \end{bmatrix} = \begin{bmatrix} \vec{e} \\ \eta \end{bmatrix}$$

where $q$ is the quaternion, partitioned into a vector part, $\vec{e}$, and a scalar part, $\eta$. The target attitude quaternion is designated as $q_t$, which defines the rotation from inertial to the target spacecraft body coordinates. The chase vehicle attitude quaternion is designated as $q_c$.

The rotation, or attitude, matrix can be computed from the quaternion components as

$$R = R(q) = (\eta^2 - \vec{e}^T \vec{e})I_3 + 2\vec{e}\vec{e}^T - 2\eta S(\vec{e})$$

where $I_3$ is a 3x3 identity matrix and $S(\vec{e})$ is a matrix representation of the vector cross product operation.

Note also that $R(q)\vec{e} = \vec{e}$. The derivative of $R(q)$ is given as

$$\dot{R}(q) = -S(\omega)R(q)$$

where $\omega$ is the angular velocity in body coordinates.

A relative rotation between coordinate frames is computed as

$$\tilde{q}_{rel} = \begin{bmatrix} \tilde{e} \\ \tilde{\eta} \end{bmatrix} = q_1 \otimes q_2^{-1} = \begin{bmatrix} \eta_2 I - S(\vec{e}_2) & -\vec{e}_2 \\ \vec{e}_2^T & \eta_2 \end{bmatrix} \begin{bmatrix} \vec{e}_1 \\ \eta_1 \end{bmatrix}$$

Using the definition given in equation 2, the relative attitude quaternion from the chase vehicle body coordinates to the target body coordinates is then

$$\tilde{q}_{ct} = q_t \otimes q_c^{-1}$$

The angular velocity of the target vehicle body coordinates with respect to inertial space, resolved in target body coordinates, is designated as $\omega_t$. Similarly the angular velocity of the chase vehicle in chase vehicle body coordinates is designated as $\omega_c$.

III. TARGET VEHICLE NONLINEAR ANGULAR VELOCITY ESTIMATOR

The angular velocity estimator is intended for the scenario in which the target vehicle is non-cooperative. For example, in the HST robotic servicing mission the estimator would be used in the event that the HST batteries have died and no telemetry is available from HST. The chase vehicle is equipped with an accurate quaternion star tracker, which provides $q_c$, and a sensor system which produces a measurement of the relative quaternion, $\tilde{q}_{ct}$. The unknown target vehicle angular velocity is estimated in the inertial coordinate system through the estimation of the target vehicle inertial angular momentum. The target vehicle angular velocity in body coordinates is computed by a transformation of the inertial angular velocity. The development and stability analysis of the algorithm are provided in reference 7. The algorithm is summarized here.
The system equations consist of the kinematic equation for the target vehicle attitude quaternion and Euler's equation for the target vehicle given in inertial coordinates

\[
\dot{Q}(q_t) = \frac{1}{2} Q(q_t) \omega_t = \frac{1}{2} Q(q_t) R_t \omega_{i,t} = \frac{1}{2} Q(q_t) I^{-1}_t R_t \dot{h}_{i,t}
\]

where \( I_t \) is the target vehicle inertia matrix in body coordinates, assumed to be constant. Note that \( I_{i,t} = R_t^T I_t R_t \), where \( I_{i,t} \) is the inertia matrix in inertial coordinates and \( R_t \) is the target vehicle attitude matrix defining the transformation from inertial to target body coordinates. \( T_{i,t} \) is the external torque acting on the target vehicle, resolved in inertial coordinates, and

\[
Q(q_t) = \begin{bmatrix}
\eta_t I_3 + S(\epsilon_t) \\
-\epsilon_t^T
\end{bmatrix} = \begin{bmatrix}
Q_1(q_t) \\
-\epsilon_t^T
\end{bmatrix}
\]  

(4)

where, by inspection, \( Q_1(q_t) = \eta_t I_3 + S(\epsilon_t) \). The predicted target vehicle quaternion as defined as

\[
\hat{q}_t = \begin{bmatrix}
\hat{\epsilon}_t \\
\hat{\eta}_t
\end{bmatrix}
\]

The attitude error is defined as the relative orientation between the predicted attitude \( \hat{q}_t \) and the measured attitude, \( q_t \), computed from equation 2 from the measured relative attitude quaternion and the measured chase vehicle attitude quaternion. The estimator attitude error is

\[
\tilde{q}_t = \begin{bmatrix}
\tilde{\epsilon}_t \\
\tilde{\eta}_t
\end{bmatrix} = q_t \otimes \hat{q}_t^{-1}
\]  

(5)

The state estimators for the HST attitude and angular momentum are defined as

\[
\dot{\hat{q}}_t = \frac{1}{2} Q(\hat{q}_t) R(\hat{q}_t)^T [I_t^{-1} R_t \hat{h}_{i,t} + k \hat{\epsilon}_t \text{sign}(\hat{\eta}_t)]
\]

(6)

\[
\dot{\hat{h}}_{i,t} = T_{i,t} + \frac{\beta}{2} R_t^T I_t^{-1} \hat{\epsilon}_t \text{sign}(\hat{\eta}_t)
\]

(7)

The term \( R(\hat{q}_t)^T \) in equation 6 transforms the angular velocity terms from the body frame to the predicted attitude frame. The gain \( k \) is chosen as a positive constant. Similarly, the learning rate, \( \beta \), is also a positive constant. Essentially, \( \hat{q}_t \) is a prediction of the attitude at time \( t \), propagated with the kinematic equation using the estimated angular momentum.

The error equations are given as

\[
\dot{\tilde{q}}_t = \frac{1}{2} Q(\tilde{q}_t) (I_t^{-1} R_t h_{i,t} - I_t^{-1} R_t \hat{h}_{i,t} - k \tilde{\epsilon}_t \text{sign}(\tilde{\eta}_t))
\]

(8)

Let \( \tilde{h}_{i,t} = h_{i,t} - \hat{h}_{i,t} \). The derivative of \( \tilde{h}_{i,t} \) is

\[
\dot{\tilde{h}}_{i,t} = -\frac{\beta}{2} R_t^T I_t^{-1} \tilde{\epsilon}_t \text{sign}(\tilde{\eta}_t)
\]

(9)

Note that the equilibrium states for 8 and 9 are

\[
\begin{bmatrix}
\tilde{q}_t^T \\
\tilde{h}_{i,t}^T
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & \pm 1 & 0 & 0 & 0
\end{bmatrix}
\]

In the absence of any errors, equations 8 and 9 are exponentially stable, i.e. \( \tilde{\omega}_t \rightarrow 0 \) exponentially fast.\(^7\)

Reference 7 examines the stability of the nonlinear estimator given errors in the relative attitude quaternion. The measurement of the relative attitude quaternion from the vision and feature based sensor was...
the largest source of error for HRV. When the true quaternion is unknown, equations 6 and 7 cannot be implemented. Instead, the erroneous measured attitude \( q_{t,m} \) is used in place of \( q_t \), resulting in

\[
\dot{q}_t = \frac{1}{2} Q(\tilde{q}_t) R(\tilde{q}_{t,m})^T [I_v^{-1} R_{t,m} \dot{h}_{t,t} + k \tilde{e}_{t,m} \text{sign}(\tilde{h}_{t,m})] 
\]

\[
\dot{h}_{t,t} = \hat{T}_{t,t} + \beta \frac{1}{2} R_{t,m}^T I_t^{-1} \tilde{e}_{t,m} \text{sign}(\tilde{h}_{t,m})
\]

\( \hat{T}_{t,t} \) is the estimated external torque. The estimator, however, remains robust to disturbances in the relative attitude measurement. The addition of a leakage term ensures that the system remains bounded in the event of unusually large disturbances.

### IV. Chase Vehicle Control Algorithm

Prior to docking with the target vehicle, the chase vehicle control system must force the chase vehicle to match the attitude and attitude rates of the target vehicle to within some mission specific tolerance. In the non-cooperative scenario considered here, the target vehicle attitude and rates are provided by the nonlinear estimator of the previous section. The chase vehicle is equipped with star trackers and calibrated gyros to provide the necessary feedback signals to the control algorithm. In this work we consider the nonlinear adaptive controller of reference 8.

The attitude dynamics of the chase vehicle, modelled as a rigid spacecraft, are given as (the time dependence is omitted for clarity)

\[
I_c \omega_c - S(I_c \omega_c + h_c) \omega_c = u + \dot{h}_c
\]

where \( I_c \) is the inertia matrix, \( u \) is the applied external torque, \( \omega_c \) is the angular velocity, and \( h_c \) is the wheel momentum in body coordinates. The goal of the control law is to force the attitude of the chase vehicle, \( q_c \), to asymptotically track the target vehicle attitude, \( q_t \), and the target vehicle rate, \( \omega_t \). The attitude tracking error is computed with equation 2 as

\[
\tilde{q}_{tc} = \begin{bmatrix} \tilde{e}_{tc} \\ \tilde{h}_{tc} \end{bmatrix} = q_c \otimes q_t^{-1}
\]

The rate tracking error is given as

\[
\omega_{tc} = \omega_c - R(\tilde{q}_{tc}) \omega_t
\]

where \( R(\tilde{q}_{tc}) \) transforms the angular velocity from the target vehicle body frame to the chase vehicle body frame.

The control law is given as

\[
u + \dot{h}_c = -K_D s(t) + I_c \alpha_r - S(I_c \omega_c + h_c) \omega_r
\]

\( K_D \) is any symmetric, positive definite matrix and \( s \) is an error defined as

\[
s = \omega_{tc} + \lambda \tilde{e}_{tc} = \omega_c - \omega_r
\]

where \( \lambda \) is any positive constant. The reference angular velocity \( \omega_r \) is computed as

\[
\omega_r = R(\tilde{q}_{tc}) \omega_t - \lambda \tilde{e}_{tc}
\]

The derivative of \( \omega_r \) is given as

\[
\alpha_r = \omega_r - R(\tilde{q}_{tc}) \dot{\omega}_t - S(\dot{\omega}_{tc}) R(\tilde{q}_{tc}) \omega_t - \lambda Q_1(\tilde{q}_{tc}) \tilde{\omega}_{tc}
\]

Asymptotically perfect tracking is obtained with the above control scheme, given noise free measurements of the states \( \omega_c \) and \( q_c \). Reference 11 also shows that the control scheme is robust to gyro bias errors and bounded noise disturbances. (The gyros are assumed to be calibrated for scale factor and misalignment errors prior to the approach and capture phase. Gyro bias errors can be estimated a priori as well.)
In a typical control application the desired states are well defined. In this work, however, the desired states are estimated with the nonlinear estimator outlined in the previous section. The nonlinear estimator provides continuous estimates of the desired attitude, desired rate, and derivative of the desired rate. The desired attitude is provided by the estimator as $\dot{q}_t$. The control error is then computed as

$$\tilde{q}_{tc} = \left[ \begin{array}{c} \tilde{\varepsilon}_{tc} \\ \tilde{\eta}_{tc} \end{array} \right] = q_c \otimes \dot{q}_t^{-1}$$

The desired rate, $\omega_t$ in the target body coordinates, is computed from the estimated angular momentum as

$$\omega_t = I_t^{-1} R(\ddot{q}_t) \dot{h}_{i,t}$$

The chase vehicle desired angular acceleration is then

$$\ddot{\omega}_t = I_t^{-1} \left[ R(\ddot{q}_t) \dot{h}_{i,t} + R(\dot{q}_t) \dot{h}_{i,f} \right]$$

Substituting for $R(\ddot{q}_t)$ from equation 1 and $\dot{h}_{i,f}$ from equation 11, $\ddot{\omega}_t$ is written as

$$\ddot{\omega}_t = I_t^{-1} \left[ S(\omega_t) R(\ddot{q}_t) \dot{h}_{i,t} + R(\dot{q}_t) (\ddot{h}_{i,t} + \frac{3}{2} R_{t,m} I_t^{-1} \tilde{\varepsilon}_{t,m} \text{sign}(\tilde{\eta}_{t,m})) \right]$$

where $\tilde{\varepsilon}_{t,m}$, $\tilde{\eta}_{t,m}$, and $R_{t,m}$ are all calculated using the measured attitude. The control algorithm will asymptotically track the estimated states. Since the estimator provides a bounded estimate of the true states in the presence of measurement errors, the chase vehicle will track the target vehicle states within the bounds of the estimator.

V. Simulation Results

The algorithms outlined in the previous sections are tested in two simulation environments. Both simulations are based on an HRV-HST rendezvous scenario. The first simulation is developed in Matlab. The Matlab simulation gives a high level indication of the performance of the combined estimator and control algorithm. Then the simulation is developed in the NASA Goddard Spaceflight Center simulation environment known as Freespace (FSP). FSP is a C-based high performance, simulation environment. FSP utilizes shared memory with a modular environment which allows for simultaneous processing and visualization. The results from the Matlab simulation are presented first, followed by results from FSP.

The HST (target) inertia is

$$I_t = \begin{bmatrix} 36046 & -706 & 1491 \\ -706 & 86868 & 449 \\ 1491 & 449 & 93848 \end{bmatrix} \text{kg} \cdot \text{m}^2$$

The HRV (chase) inertia matrix is

$$I_c = \begin{bmatrix} 18748 & 525 & -2197 \\ 525 & 55903 & 1366 \\ -2197 & 1366 & 53025 \end{bmatrix} \text{kg} \cdot \text{m}^2$$

Both algorithms are initially tested without any errors. In both cases, the initial attitude quaternions are identity, $q_t = \dot{q}_t = [0, 0, 0, 1]$. The initial HST angular velocity estimate is zero, $\omega_t = [0, 0, 0]$, and the true initial angular velocity is $\omega_t = [-0.04, -0.01, 0.14]$ deg/sec.

Figures 1 and 2 show the attitude control error and the angular velocity control error with perfect measurements. The estimator runs for 5000 seconds before the control algorithm is started. The final attitude control error and rate control error (magnitude) are 0.01 deg and 1.6e-5 deg/sec, respectively, and both are still converging.
Figure 1. True Attitude Control Errors, No Measurement Errors
Next, the measured attitude is chosen randomly with a 10 degree uncertainty. The estimator is initialized with the first attitude measurement. The HRV attitude is initialized at $q_e = [0, 1, 0, 0]$. The initial HRV angular velocity is $\omega_e = [0, 0, 0]$ deg/sec. Figures 3 and 4 are samples of the attitude control error and the angular velocity control error. Here the final attitude control error is approximately 1 deg and the final rate control error is 0.001 deg/sec, both are within the long range requirements for the HRV capture. Reducing the attitude measurement error to 1 degree results in final true attitude and rate control errors of 0.04 deg and 0.00017 deg/sec, respectively, which meet the close range requirements. (The measured attitude error is expected to improve as the approach distance decreases.)
Figure 3. True Attitude Control Errors, 10 Degree Attitude Measurement Errors
The simulation was then repeated 100 times, each with a different random measured attitude sample, again with an uncertainty of 10 degrees. Figure 5 shows the true attitude control error for 100 different test cases. In all cases the controller is converging. The final average attitude control error for all 100 cases is 1.3 deg. Figure 6 shows the true angular rate control error. Again, the controller is converging. The final average angular velocity control error for all 100 cases is

\[
\hat{\omega}_{tc}(ave \ in \ deg/sec) = \begin{bmatrix} -0.00032 \\ 0.00021 \\ -0.00036 \end{bmatrix}
\]
Figure 5. True Attitude Control Errors, 100 Test Cases with Random Attitude Measurement Errors
Next, the algorithms are tested in the FSP simulator. The Freespace truth model contains gravity gradient and aerodynamic external torques acting on both HST and HRV. The HRV gravity gradient torques are input to the control algorithm as a feed forward torque. The net control torque, after accounting for gravity gradient, is distributed to four reaction wheels. The Freespace simulation is run first with no additional error sources. Then, errors in the measured attitude are included, similarly to the errors added to the Matlab simulation. Next inertia errors and gyro errors are introduced. (Results to be supplied in the final paper.)

**VI. Conclusions**

An approach for estimating the attitude and rates of a non-cooperative target vehicle and then controlling the chase vehicle to match the estimated attitude and rates is presented. The attitude and rates are estimated with a nonlinear estimation algorithm that is robust to measured attitude errors. The nonlinear control algorithm is asymptotically stable in the absence of errors, and remains robust given gyro measurement errors. The algorithms are applied to the HST robotic servicing mission and are tested first with a Matlab simulation, introducing a random attitude measurement error of 10 degrees. A Monte Carlo simulation demonstrates that the combined algorithms are stable and converge to less than 1.3 deg and 0.00052 deg/sec in attitude and rate control errors (magnitude), respectively. The algorithms are then tested in a high performance simulation environment known as Freespace. Again, the combined algorithms are stable and converge to final error less than TBD deg and TBD deg/sec.

Future work will expand the simulation scenario in Freespace. The vision sensors will be modelled and used to provide the relative attitude measurement. Additional fidelity will be added to the wheel models and the chase vehicle gyros, along with a star tracker based attitude estimation algorithm for the chase vehicle.

**References**


