

# Fast Whole-Engine Stirling Analysis

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An experimentally validated approach is described for fast axisymmetric Stirling engine simulations. These simulations include the entire displacer interior and demonstrate it is possible to model a complete engine cycle in less than an hour. The focus of this effort was to demonstrate it is possible to produce useful Stirling engine performance results in a time-frame short enough to impact design decisions. The combination of utilizing the latest 64-bit Opteron computer processors, fiber-optical Myrinet communications, dynamic meshing, and across zone partitioning has enabled solution times at least 240 times faster than previous attempts at simulating the axisymmetric Stirling engine. A comparison of the multidimensional results, calibrated one-dimensional results, and known experimental results is shown.

This preliminary comparison demonstrates that axisymmetric simulations can be very accurate, but more work remains to improve the simulations through such means as modifying the thermal equilibrium regenerator models, adding fluid-structure interactions, including radiation effects, and incorporating mechanodynamics.

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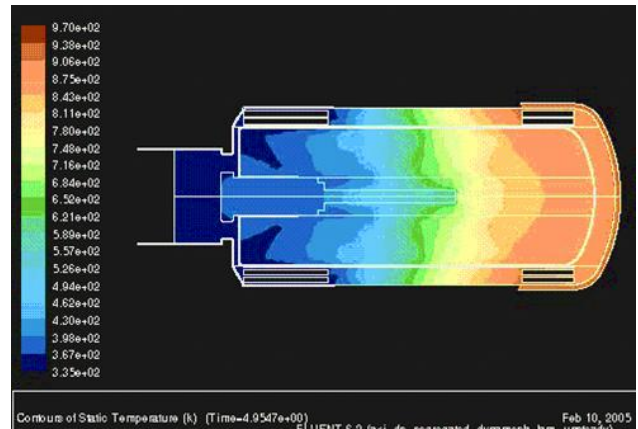
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1. Oscillating flow which changes the effective conduction, flow friction coefficients and heat diffusion.<sup>68</sup>
2. Low mach number flow (no shocks),
3. Compressible flow due to enclosed varying volumes and heat transfer effects,
4. Laminar, Transitional, and Turbulent flow with Reynolds numbers from 100 to 10000 (based on various length scales pertinent to the flow region),
5. Conjugate heat transfer, thermal dispersion and local thermal nonequilibrium in the porous media regenerator,
6. Micron to Millimeter scale geometry<sup>6</sup>
7. Sliding Interfaces in the appendix gap
8. Deforming Flow Regions Due to Compression and Expansion

### III. Current Multi-Dimensional Stirling Cycle Analysis Tools

At the level of multidimensional analysis, relatively little has been completed because the third order analysis, including Sage,<sup>5</sup> and to some extent, DeltaE,<sup>13</sup> and MS\*2 Stirling Cycle Code<sup>11</sup> are faster and for the most part have been adequate engineering design tools. However, to improve efficiency, reliability, and supplement experimental testing (to understand and reduce losses, identify component temperatures, assist in sensor calibration, correlation, and placement) it will likely require a better understanding of the actual flow features and heat transfer throughout the engine. Some of the primary multidimensional analysis tools are shown below.



**Figure 3. Example Transient Simulation of Stirling Converter-Colored by Temperature**

#### 1. Modified Computer Aided Simulation of Turbulence (CAST)

The modified CAST code<sup>70</sup> is based upon the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE)<sup>71</sup> method but is restricted to two dimensions.<sup>72-74</sup> It was modified to

# Nomenclature

$\gamma$	Porosity of the Medium
$\bar{\bar{\tau}}_{\text{eff}}$	Stoke's Tensor
$\rho_f$	Fluid Density
$\rho_s$	Solid Density
$\vec{v}$	Fluid Velocity Vector
$E_f$	Fluid Energy
$E_s$	Solid Energy
$h$	Sensible Enthalpy
$J$	Diffusion Flux of Helium
$k_{\text{eff}}$	Effective Thermoconductivity
$p$	Pressure
$S_f^h$	Fluid Enthalpy Source Term
$T$	Temperature

## I. Introduction

Great strides have been made in the last decade in expanding the designer's analysis options for building improved free-piston Stirling engines<sup>1</sup> ranging from thermal and structural dynamic analysis,<sup>2,3</sup> computer-assisted-design (CAD) software for complete three-dimensional part design,<sup>4</sup> one-dimensional thermodynamic cycle analysis<sup>5–18</sup> and optimization, and one-dimensional system mechanodynamic simulations.<sup>19–25</sup> A review of the progress in Stirling analysis shows a steady relaxation of assumptions/limitations imposed on the analysis over the past century.<sup>26</sup>

Initial two-dimensional computational fluid dynamic (CFD) simulation work began over twenty years ago and focused on specific regions of the engine but were not time-efficient and the projects were not continued.<sup>6,27–30</sup> And other researchers attempted to build specific in-house research codes with mixed success at modeling specific components.<sup>31–36</sup>

More recently, multidimensional multiphysics simulations that allow for realistic Stirling engine system solutions now exist and include the following capabilities that are potentially useful for Stirling convertor design:<sup>37–40</sup>

- Structural
  - linear or nonlinear static or transient deformation
  - modal, spectrum, harmonic, and random vibration dynamics
  - linear or nonlinear buckling

- Thermal
  - steady-state, transient, conduction, convection, and radiation
- CFD
  - Steady-state/transient, compressible/incompressible, laminar/turbulent, forced/natural convection, conjugate and radiation heat transfer, laminar to turbulent transition modeling, scalable parallel performance
- Electromagnetics
  - electrostatics, magnetostatics, low-frequency electromagnetics, current conduction, circuit analysis, harmonic, transient, modal, scattering, perfect electric and magnetic conductors, impedance boundaries, near and far electromagnetic field extension, frequency selective surface, antenna radiation patterns, and specific absorption rate.
- Coupled Physics
  - acoustics-structural, electric-magnetic, fluid-structural, fluid-thermal, electromagnetic-thermal, piezoelectric, piezoresistive, thermal-electric, thermal-structural, electromagnetic-thermal-structural, and electrostatic-structural

Recently, attempts at utilizing these commercially available options have focused on only the thermal dynamic cycle simulation because it is difficult to achieve a useful solution in a reasonable time, let alone attempt a more comprehensive simulation. For example, previous attempts at whole engine simulation with these multiphysics commercial codes ended in unconverged solutions.<sup>31,41</sup> Although a recent simplified axisymmetric solution<sup>76</sup> shows reasonable energy balance convergence, the results are not validated yet by experiment.

The first reported successful three-dimensional whole Stirling engine simulation was reported by Mahkamov<sup>42–45</sup> in the United Kingdom. Two gamma-type Stirling engines were simulated using commercially available software with good reported experimental agreement of 12%-18% error in the indicated power. Seal leakage, appendix gap flow, and other loss mechanisms were not included in this work.

The second reported successful three-dimensional Stirling engine simulation was by Zhang<sup>46</sup> in China in which it took approximately 3 months to reach a steady-harmonic converged solution on a simplified, academic geometry that did not include the appendix gap, solid walls, or internal displacer.

A multi-dimensional analysis has an important place in Stirling engine design as is discussed at length in Ref.<sup>47</sup> and outlined below:

- verify the one-dimensional results,
- properly simulate inherently three-dimensional turbulence—including transition,
- provide empirical heat transfer and friction factor coefficients for complex geometries before the hardware is built
- integrate all the parts into a CAD system and test for structural and relative motion issues in the overall design
- assist experiments with determining hard to reach data
- provide a fluid-structure interaction capability
- generate linear reduced order models for controller development
- solve high-power Stirling applications in which the one-dimensional flow assumption breaks down
- identify areas of excessive flow losses due to unintended dead zones, recirculation zones, dissipative turbulence and other losses such as:<sup>48–50</sup>
  1. Inefficient heat exchange and pressure loss in the regenerator, heater and cooler,
  2. Gas spring and working space loss due to hysteresis and turbulence,
  3. Appendix gap losses due to pumping and shuttle effects,
  4. Mixing gas losses from nonuniform temperature and flow distributions perpendicular to primary engine flow axis,
  5. Conduction losses from the hot to cold regions
  6. Losses due to combined radiation, conduction and convection in hollow spaces
  7. And in general, inaccurate loss representations due to use of 1-D flow design codes to account for flow and heat transfer through area changes (between components) where phenomena such as flow separation and jetting from tube into regenerator may occur.

The current multiphysics simulation tools are not optimized for Stirling analysis as discussed in Ref.<sup>51</sup> For example, recently developed oscillating time advance strategies<sup>52–58</sup> and high order spatial differencing could make modeling oscillating turbulent transition a reality. But given the cost of developing new codes, it is desirable to fully utilize current technology first.

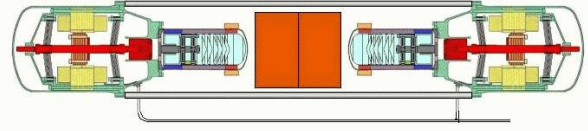
This paper will present the first successful U.S. axisymmetric Stirling engine simulation incorporating an actual engine's geometry including appendix gap, displacer seals (inner and outer), solid walls, and displacer interior. Only the flexures, piston seals, bounce space, radiation shields, and heat exchanger fins were not included for this initial work, but reasonable agreement with experiment has been achieved despite these omissions.

## II. Description of the Problem

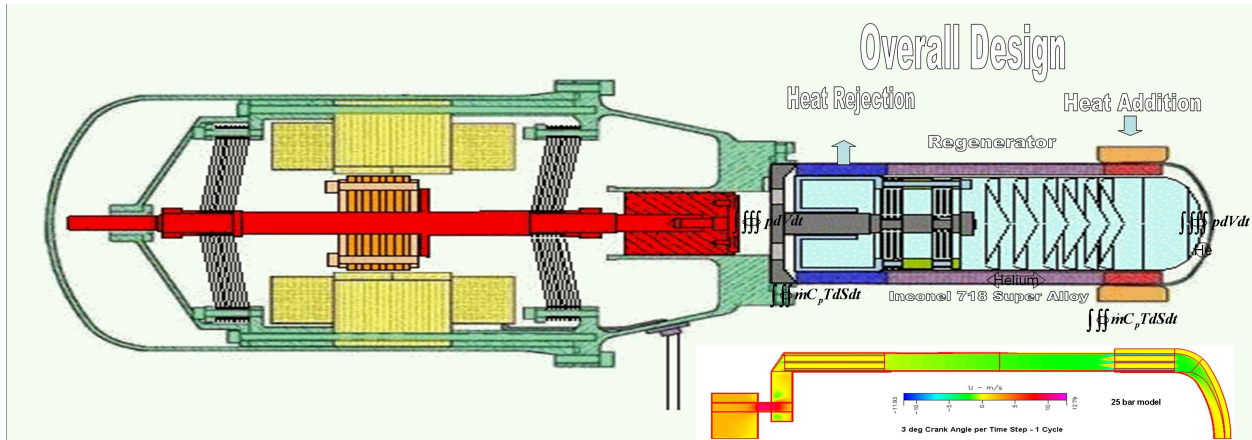
The dual opposed configuration shown in figure 1<sup>59–62</sup> is being developed for multimission use (i.e., for use in atmospheres and space), including providing electric power for potential missions such as unmanned Mars rovers and deep space missions.<sup>63,64</sup>

Only the Stirling engine part of the convertor (Fig. 2) is simulated multi-dimensionally although one could anticipate the entire convertor may one day be simulated using modified multiphysics software.

An example of the desired full Stirling engine simulation is shown in Fig. 3. It is anticipated that full 3D simulations will provide a level of geometric and flow detail necessary for further design improvements. For example, hardware experiments have shown that large performance gains can be made by varying manifolds and heat exchanger designs to improve flow distributions in the heat exchangers.<sup>60,65–67</sup>



**Figure 1. Dual Opposed Stirling Convertors Reduce Vibration (Schreiber)**



**Figure 2. Stirling Engine Part of Convertor Simulated Only**

The kinds of flow and geometry that occur in a Stirling engine for which a modeling technique must address are as follows:<sup>49,50</sup>

include oscillatory boundary conditions and conjugate heat transfer but does not allow for complicated geometry nor sliding interfaces. It has been used to model Stirling components but has not been extended to a whole engine simulation tool.

A pressure-splitting technique was added<sup>75</sup> to reduce the computational requirements. It was based on separating the thermodynamic and hydrodynamic pressures so that these widely varying scales could be solved with less round-off error and better efficiency. And the non-acoustic form of the equations was later incorporated into the code since the fast acoustic waves and small dimensions of the engine essentially resulted in "incompressible" like flow behaviour and removing the acoustical pressure disturbance could enhance computational speed and accuracy.<sup>32</sup>

## 2. *CFD-ACE*

This commercial code has been used to model a two-dimensional representative Stirling engine<sup>31,33,46,76</sup> and it has been used to model an actual engine.<sup>41</sup> Full validation of these simulations has not been completed.

It is also based upon the SIMPLE technique. The regenerator is not currently modeled correctly since thermal equilibrium is assumed between the gas and solid, but as shown later, this error may not be large compared to the overall engine performance. The code does support sliding interfaces on parallel computers in three-dimensions and the company is introducing sliding, overlapping Chimera grids which should enable better appendix gap modeling. Moreover, the highly compressed volumes occurring at both ends of the displacer may be more easily modeled with Chimera grids. This finite volume code can utilize both structured and unstructured grids.

## 3. *Fluent*

This commercial code is based upon the SIMPLE and PISO methods. It currently has similar regenerator modeling limitations as CFD-ACE. It does however have a sliding interface for two-dimensional axisymmetric flows that can be used for appendix gap modeling on parallel computers and will be discussed in more detail later. It is also finite volume based and can utilize both structured and unstructured grids.

It has been used with mixed success by industry for Stirling engine simulations<sup>77</sup> by several commercial manufacturers. It was also used by Mahkamov<sup>43</sup> in what appears to be the first full three-dimensional simulation, but details are embargoed. Also, in the related business of cryocooler modeling it has been used with reasonable success on simplified problems<sup>78–80,82–89</sup> And this report has utilized it to provide a detailed axisymmetric simulation.

#### 4. *STAR-CD*

The Simulation of Turbulent Flow in Arbitrary Regions (STAR)<sup>38</sup> code also uses SIMPLE and PISO methods. Its companion product (STAR-HPC) is the parallel computer version. It also has sliding interfaces and deforming mesh capability. It has been used in the related field of internal combustion engine piston modeling, and some Stirling engines and regenerators have been modeled with it.<sup>90,91</sup>

#### 5. *CFX*

This code also uses SIMPLE and PISO methods on unstructured grids. It also has sliding interfaces implemented, but no stirling engine modeling with this software has been publicly published.<sup>37</sup> It currently seems to have the greatest potential for full multiphysics Stirling convertor simulations because of it's fluid-structure interaction, magnetic flux analysis, and structural/thermal stress analysis capabilities. It's dynamic meshing capabilities are not ideal for highly compressed volumes as occur in the Stirling engine since only the spring analogy mesh adaption is currently available.

#### 6. *Others*

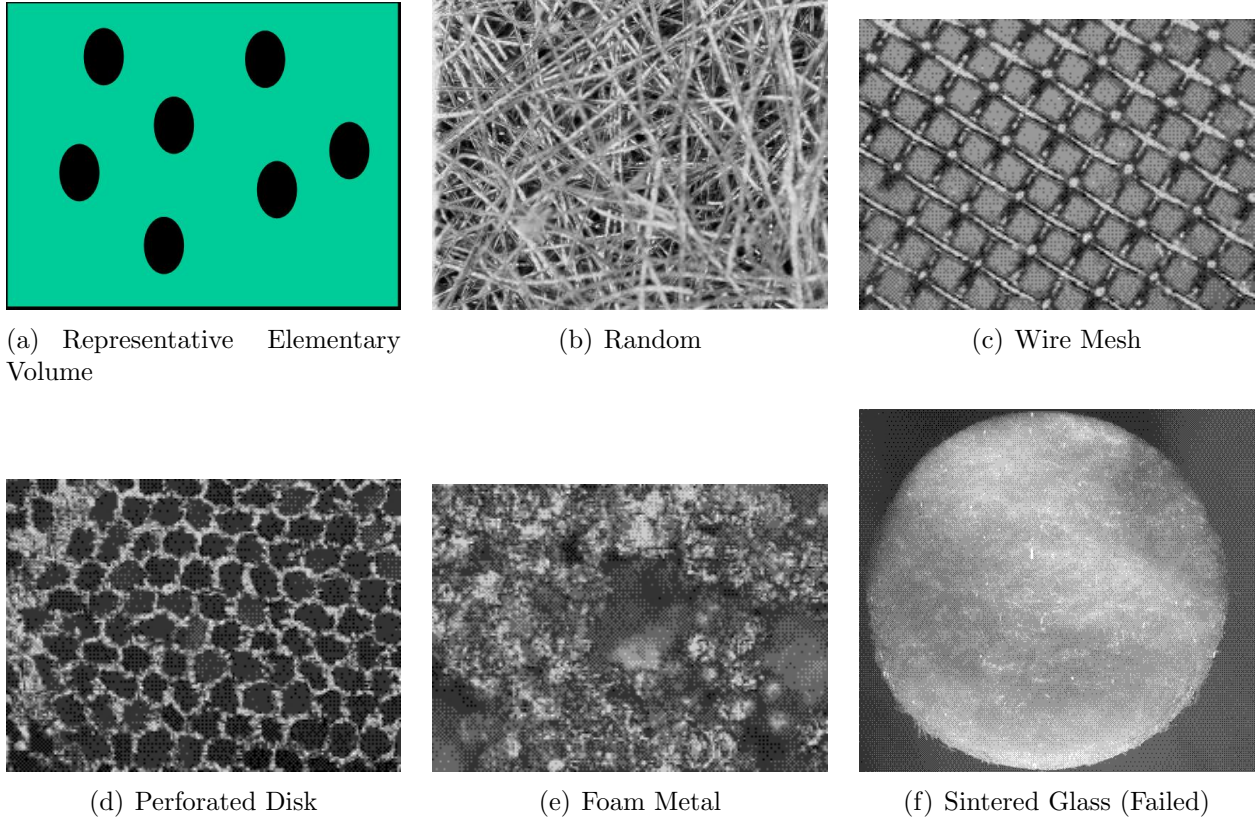
While there are other research codes, they are usually limited to modeling only specific regions of the Stirling engine such as the regenerator, or the displacer.<sup>15,81,82,92,93</sup>

## IV. Regenerator Modeling

A very important specific area of modeling difficulty is the regenerator. Since the regenerator (depending upon one's definition of effective) has roughly 3 to 40 times more effective heat transfer than the heater,<sup>94</sup> any inefficiency of the regenerator represents a significant loss for the entire Stirling engine. Hence, any numerical losses or inaccuracies in this region will disproportionately influence the entire Stirling simulation.

Also, the geometrical shape of the matrix elements in the regenerator is important and as shown in Figs. 4(b-f), many shapes are possible. For practical calculations a Representative Elementary Volume (REV) is often used as shown in Fig. 4(a). An REV requires some closure assumptions that can be partially derived from experiment and potentially from microscale (pore level) numerical simulations.

Since the size and shape of the regenerator fibers do affect Stirling engine performance, numerically simulating a regenerator has been pursued at both microscopic (direct) and macroscopic (REV) levels of fidelity.<sup>6,95–101</sup> An example simulation utilizing Fluent by Harvey from Georgia Tech is shown in Fig. 5.



**Figure 4. Regenerator Geometry**

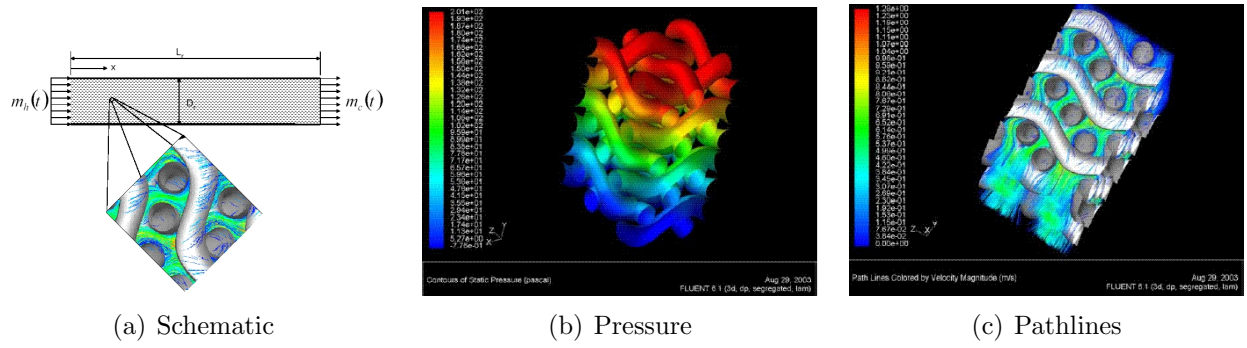
However, our initial approach at whole engine simulation simply adds source terms to the governing equations and does not take into account the different temperatures of the solid matrix and the fluid. The source terms serve to enforce a pressure drop according to the Darcy-Forcheimer<sup>68</sup> equation. And only a single energy equation is used, although it does include an effective solid/fluid average conductivity<sup>69</sup> and an effective solid/fluid average energy.

The continuity equation is not modified, but the following source term is added to the momentum equations (for  $i = x, y, \text{ or } z$ ):

$$S_i = - \left( \frac{\mu}{K} v_i + C_2 \frac{1}{2} \rho v_{mag} v_i \right) \quad (1)$$

where permeability  $K = 4.08e^{-10} m^2$ ,  $C_2 = \frac{2C_f}{\sqrt{K}}$  and inertial coefficient,  $C_f = 0.178$ . These coefficients were determined experimentally<sup>102</sup> and used without modification here. And the fluid/solid averaged energy equation can be written as:

$$\frac{\partial}{\partial t} (\gamma \rho_f E_f + (1 - \gamma) \rho_s E_s) + \nabla \cdot (\vec{v} (\rho_f E_f + p)) = \nabla \cdot \left[ k_{eff} \nabla T - \left( \sum_i h_i J_i \right) + (\hbar \tau \cdot \vec{v}) \right] + S_f^h \quad (2)$$



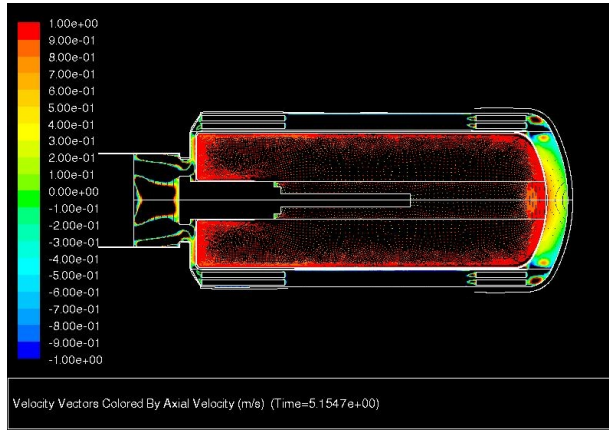
**Figure 5. Flow Through Wire Mesh Screen Regenerator**

## V. Dangers of Component Modeling

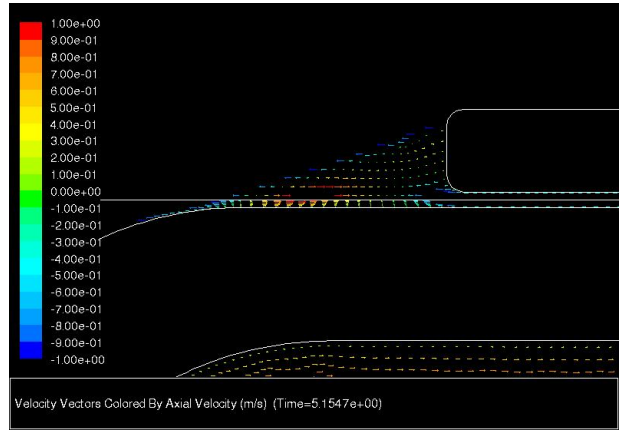
Modeling the entire engine, as demonstrated herein, is preferable to modeling one component at a time due to the oscillatory nature of the flow. A component requires known unsteady boundary conditions for an entire cycle. Since a given boundary may have flow going in both directions, the direction of the characteristics at these boundaries are difficult to define. Incorrect specifications of boundary conditions result in solution instability, nonconvergence of solutions, and/or convergence to inaccurate results. It is well known that even for steady harmonic flows, non-physical reflections can occur at inflow and outflow locations.<sup>103</sup> In essence, by artificially truncating the domain to model a single engine component an artificial boundary condition is required. However, the exact specification of artificial/non-reflecting boundary conditions in nonlinear turbulent flow is an important and as yet unresolved area of research.<sup>104</sup>

For example, a subsonic inflow has four characteristics entering and hence four physical boundary conditions need to be specified, typically total pressure, total temperature, and two flow angles, allowing only one of the velocity components to be free (which is extrapolated from the interior flow solution). A subsonic outflow has only one characteristic entering and hence only one physical boundary condition should be specified, for example, static pressure.

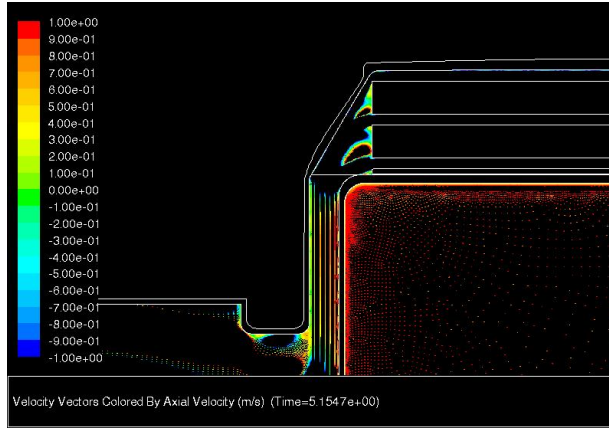
Simultaneous inflow/outflow along each engine component's interface is clearly demonstrated by examining flow velocities near zero (from -1 m/s to 1 m/s). Regions of blue and red in Fig. 6 show borders of mixed inflow/outflow in the axial direction. For example, the piston compression space in Fig. 6(a), the seal exit in Fig. 6(b), the cooler exit in Fig. 6(c), and the appendix gap entrance in Fig. reffig:oscillating(d) all show mixed inflow/outflow borders at this moment in time. Moreover, these mixed boundaries moving during the cycle, further complicating the numerical boundary treatment. If a component boundary crosses any of these indicated areas, the boundary conditions will not be properly defined. Of course in the case of commercial software users the code will attempt to stabilize the solution with essentially guessed boundary conditions, but the solutions will be inherently non-physical.



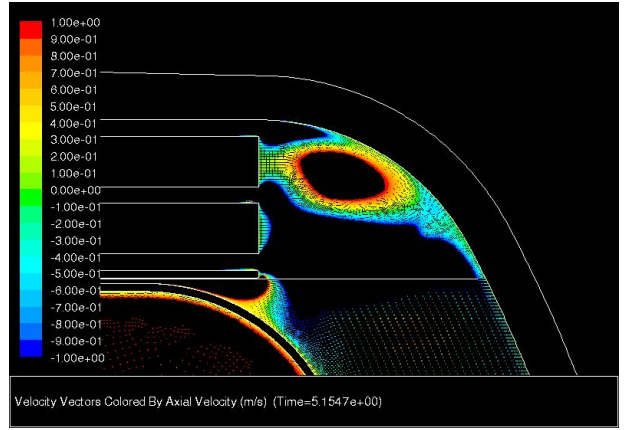
(a) Entire Engine



(b) Seal Region



(c) Compression Space



(d) Heater Manifold and Appendix Gap

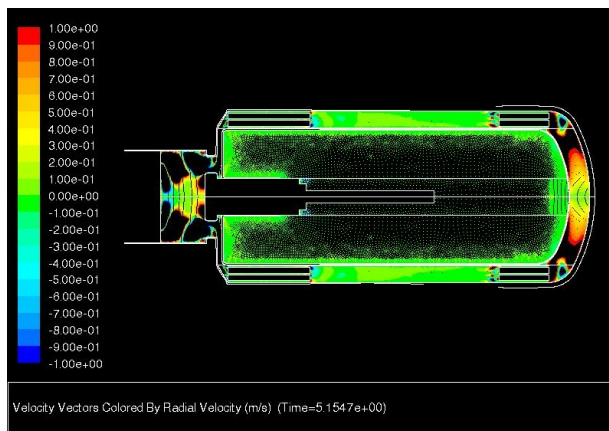
**Figure 6. Oscillating Axial-Direction**

Repeating the same velocity test, but this time examining velocity close to zero in the y-direction, we see a new set of boundary lines that will pose difficulties resulting in non-physical component solutions in Fig. 7.

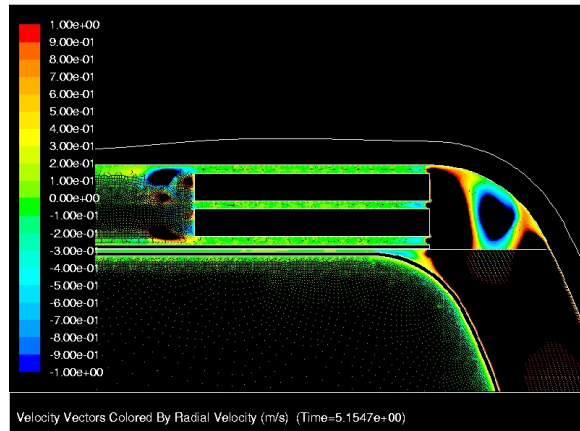
### A. Adjust Components within a Full Engine Simulation

Instead of pursuing the mathematically ill-posed problem of component modeling in complicated oscillating flow, it is easier, more efficient, and more reliable to determine a whole engine steady-harmonic solution with the procedures described herein. The engine components can be easily examined and it only takes a few hours to test the effect of component changes since the overall engine has the correct steady-harmonic temperature distribution.

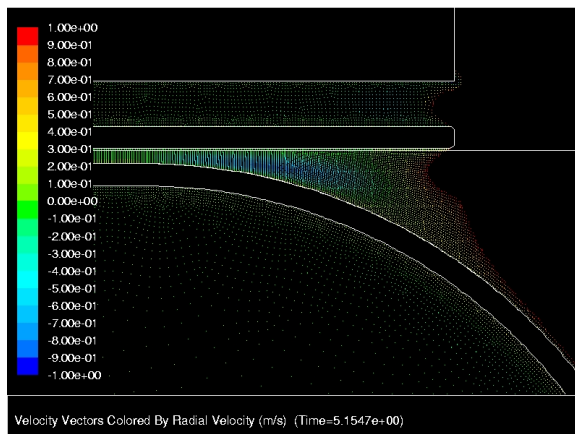
At the same time, it is possible to numerically determine the one-dimensional equivalent friction factors and heat transfer terms in the new components required by one-dimensional analysis tools (e.g. Sage). In this way, a one-dimensional analysis be made more accurate while not requiring the ill-posed boundary conditions that multidimensional component



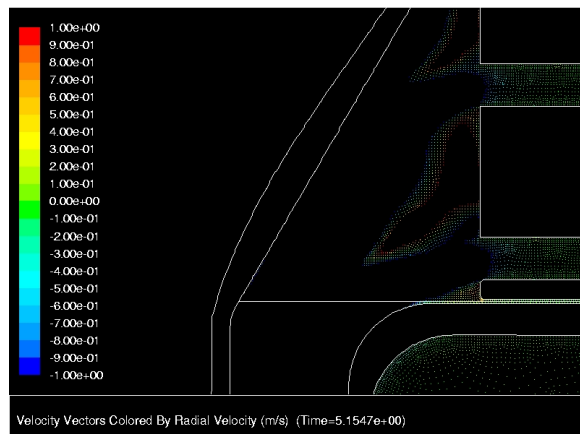
(a) Entire Engine



(b) Heater Region



(c) Appendix Gap Region



(d) Seal Region

**Figure 7. Oscillating Radial-Direction**

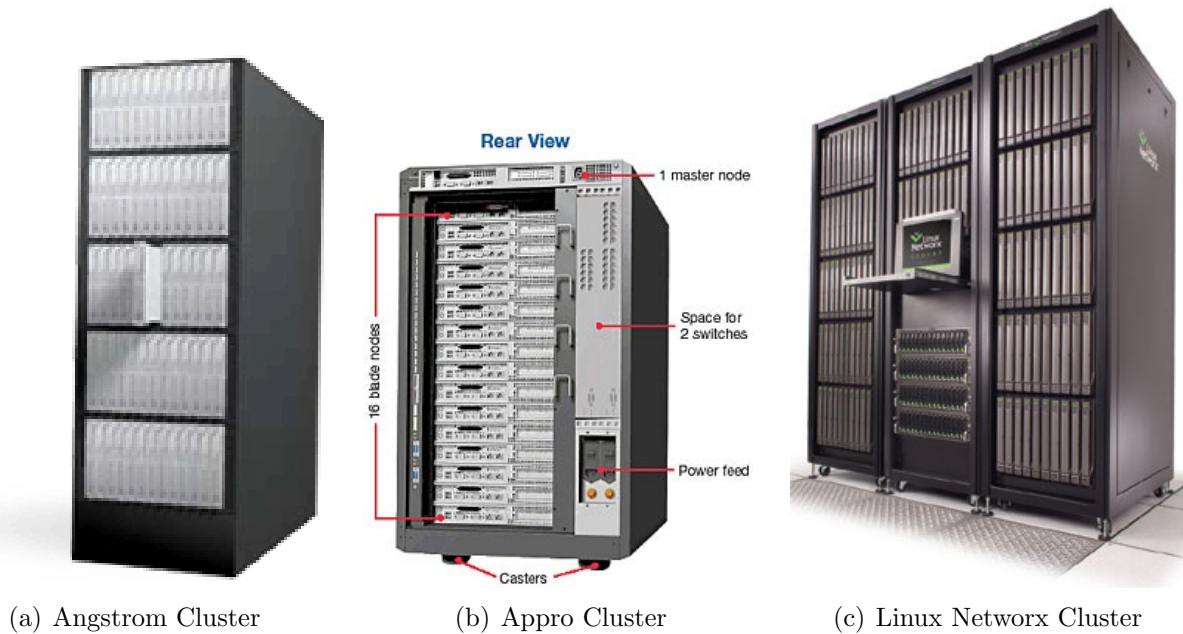
modeling entails.<sup>47</sup> Finally, it is worth noting that one-dimensional analysis does not have difficulty with component modeling because a boundary is always either an inflow or an outflow, never both.

## VI. Approach for Fast Simulations

Achieving practical computer simulations of entire Stirling engines depends upon judicious use of computer hardware, software engineering, grid generation, and algorithm design. A combination of all four areas is presented next that enables axisymmetric whole engine simulations to be accurately completed in less than a week.

### A. Computer Design

An important area new to Stirling analysis is the use of parallel computers for faster simulations. NASA Glenn was one of the first organizations to pioneer computer clusters back in 1992. Many universities have assembled their own low cost computers,<sup>105</sup> and many states have supercomputer consortiums<sup>106</sup> to assist in designing and operating small computer clusters that utilize commercial off-the-shelf (COTS) hardware and software for low cost systems. These systems are referred to as Linux Beowulf clusters<sup>107,108</sup> and now represent the majority of the top 500 supercomputer systems in the world.<sup>109</sup> Recently, many commercial organizations have begun assembling "Beowulf"-like clusters including Dell,<sup>110</sup> Angstrom,<sup>111</sup> LinuxNetworkx,<sup>112</sup> Appro,<sup>113</sup> and Microway<sup>114</sup> as shown in Fig. 8. They cost a bit more, but provide comprehensive health monitoring and controls that significantly reduce the administration and integration costs. The latest systems can house 260 processors in a single 7 foot tall tower.



**Figure 8. Appro, Angstrom, and Linux Networkx Clusters**

Our sixteen dual-Opteron (32) processor Microway cluster is shown in Fig. 9. This was

selected because last year it represented the best value for performance and it integrated well with the commercial software, Fluent. The Opteron chip, at the time, was the only chip that could run 32-bit and 64-bit applications with an exceptional price/performance ratio. All the whole Stirling engine results shown here were calculated using it.



(a) Front View



(b) Dual CPU Board

**Figure 9. Microway Cluster**

A very important consideration when designing your computer cluster is the communication bandwidth and latency since commercial fluid simulation software utilizes older numerical approaches that require a large number of frequent communications between the compute nodes. In Fig. 10, two of the best communications network interface card (NIC) and network switches are shown. The Infiniband system can perform 10 Gb/s bi-directional communication rates or about 20 times faster than the Myrinet in our cluster. Our simulation utilized Myrinet communications because last year Infiniband was not a proven technology.



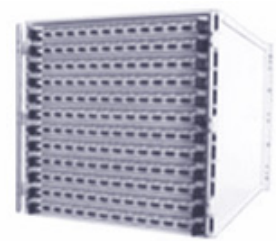
(a) Myrinet Card



(b) Myrinet Switch



(c) Infiniband Card



(d) Infiniband 144 port Switch

**Figure 10. Communications**

## B. Software Decision

We chose Fluent as our computational platform because, at the time, it was the only commercial vendor supplying an axisymmetric sliding interface for parallel computers. Presently, as mentioned in Sec. III, the other vendors are now also providing similar capability. It is important to always have at least two commercial solvers available because of the invariably large number of bugs in their software. Generally, a bug is more easily found if the same simulation is repeated with two different codes. For example, Fluent version 6.1.22 properly handles heated moving solids, but the new release (version 6.2.16) did not as identified by Ibrahim.<sup>115</sup> The latest version 6.2.19 has been corrected.

Another reason for utilizing at least two commercial codes is the lack of control the user has in the solvers utilized. Generally, the commercial codes all utilize slight variants of older, well established lower order numerical techniques. The slight variants are the only independent way to examine the effect of a given solver and to look for possible defects. Of course developing one's own software offers more freedom, but this can be risky in the short-term.

Our second platform utilizes CFD-ACE and reasonable solutions have been achieved with it. It's primary limitation has been the need to work in three-dimensions when solving cases including axisymmetry sliding interfaces. Another strength of this software is it includes a wide range of multiphysics capabilities.

A third very interesting platform is CFX because of it's integrated multiphysics capability enhanced by the merger between ANSYS and CFX. The only apparent limitation of the software is grid layering is not yet available. Grid layering is defined by the automatic addition or removal of a row of cells as the mesh domain shrinks or expands and will be discussed later.

Other packages such as STAR-HPC are very capable as well. However, the commercial cost of these packages are very expensive and it is not practical to test all of them.

Finally, full three-dimensional simulations are still quite challenging and the parallal scalability of the commercial software is somewhat limited due to the implicit nature of their steady flow solvers. Better approaches are currently available<sup>51</sup> but have not yet been applied to the Stirling cycle analysis.

## C. Grid Design

When using commercial codes, it is primarily the grid quality that determines the speed and accuracy of the solution. Ideally, one could simply read in a three-dimensional CAD geometry and a grid would automatically be created. That is in fact the implied goal of computational fluid dynamics code developers. But for complicated geometry as occurs

inside a Stirling engine with all its moving parts, the technology currently available still requires a great deal of human intervention. Taking several weeks to develop a grid is not unusual and is often required to get a reasonable solution. For example, the rate at which conjugate heat transfer can be solved is directly related to the smoothness and quality of the grid at the interface. This is particularly true in Stirling engines when the heat capacity of the solid is vastly different than the Helium. If short-cuts are taken in developing the grid, the price is often vastly longer solution times, if a solution is produced at all.

All the grids produced in this work were done with Gambit, which is the Fluent, Inc. proprietary grid generator. There are other grid generators available with their own distinct advantages, but despite the bugs encountered in Gambit, we found it to be a useful tool.

First, the domain is scaled up by 1000 to compensate for the 32-bit precision available in the grid generator. The Stirling engine, when entirely modeled, has a fairly wide range of length scales ranging from a thousandth of an inch in the narrow seal and appendix gap region to an inch or more in the heat exchanger regions. This scaling has the effect of making very small numbers larger so that smoother grids can be created. Once the mesh is created satisfactorily, the entire grid domain is scaled back down by a 1000 before utilizing it.

Second, try to use structured or paver unstructured grids wherever possible to reduce the number of bad cells and to reduce the number of cells required. Avoid unstructured tetrahedrals because of the considerably reduced rate of solution convergence.

Third, be sure a sliding interface has essentially matching cell sizes on both sides of the interface to prevent interpolation errors and avoid instabilities.

Fourth, be sure to place grid points in the boundary layer to properly model friction losses.

Fifth, use sizing functions to smoothly expand the cell size from the boundaries at a rate of 10 percent.

Sixth, use layering wherever possible, but avoid grid adaption and remeshing because this slows down the solution time considerably. Choose a layering size large enough to permit a reasonable 160 time steps per cycle, but small enough to allow for smooth transitioning to boundary layers.

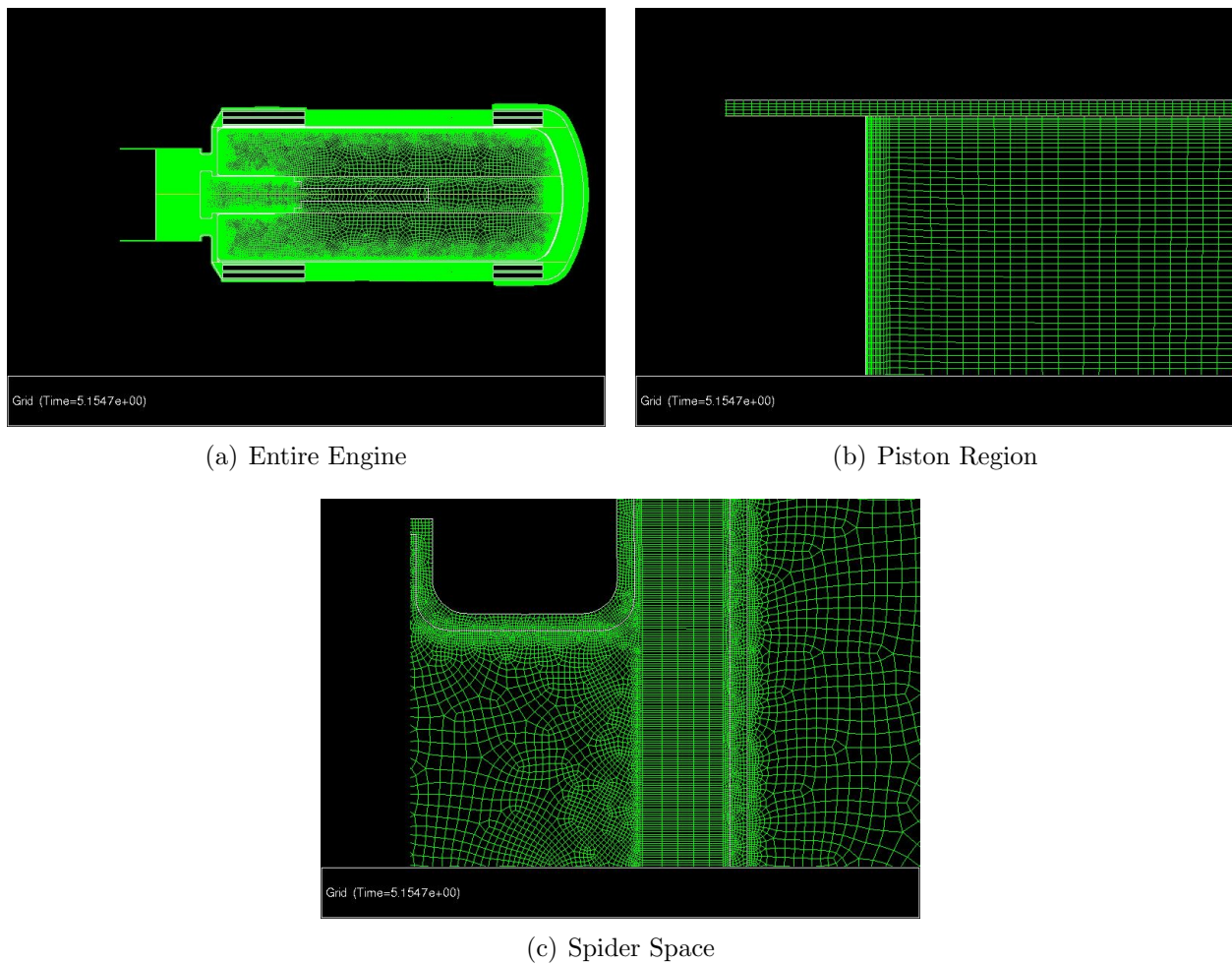
Seventh, utilize moving boundary layers on all walls that are moving to enhance conjugate heat transfer. And place a boundary layer on both sides of the interface (on the solid side and the fluid side).

Eighth, include as much geometry as is practical to avoid having to redo the grid later when simplifying assumptions are found to matter.

Ninth, clean up the geometry first in a CAD package to avoid creating virtual geometry that is not as robust.

Tenth, test your grid by forcing the piston and displacer to be stationary and simulate the heat transfer in the engine. Once a converged solution is observed, continue running the solver and look for grid induced heat sources. It should be noted that Cleveland State University found producing a steady-state result first provided a better starting condition for the transient solution. But this last suggestion goes a bit further in providing a means for improving the grid by "over-converging" the steady solution.

For the successful case presented here, we used about 700,000 grid points and all grid boundary layers start at a distance of .001 in from the fluid/solid interface.



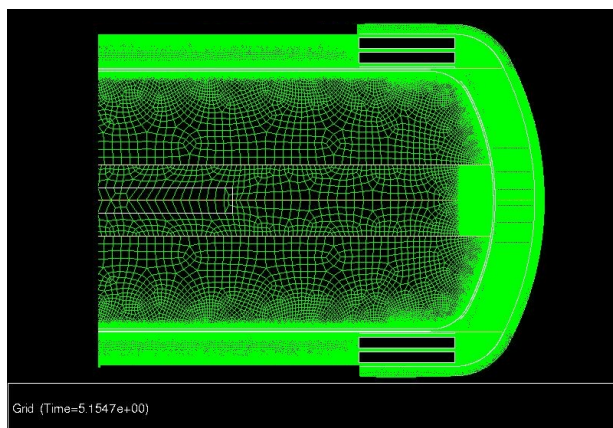
**Figure 11. Grid Strategy**

In Fig. 11, we show the entire engine, piston region and spider region grid utilized in this simulation. Notice the piston has a moving boundary layer and grid layering is used to avoid small cells when the piston fully compresses. Also, notice in the spider region that all helium/wall interfaces have boundary layers on both sides to enhance conjugate heat transfer. The overall engine can be seen to not include any grid inside the heater or cooler since the heat exchanger wall temperature is specified as constant. Also notice there is some extra wasted grid in the spider region near the compression space. This is necessary to maintain grid quality in the other nearby boundary layer regions. Grid quality is more important than grid count for faster conjugate heat transfer calculations.

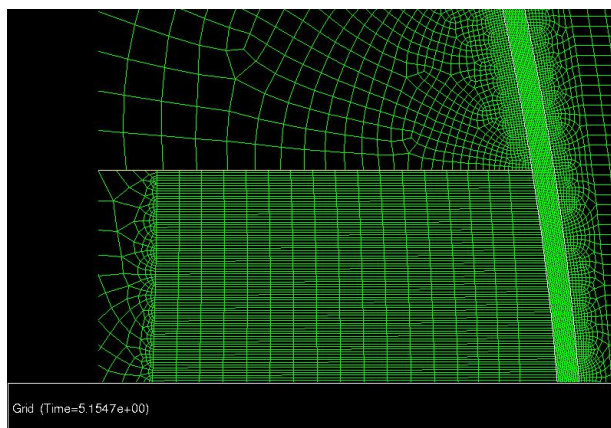
In Fig. 12, the hot half of the engine is shown, including the gridding in the displacer and appendix gap. Notice that despite the complexity, tetrahedral grids were not required. The layering inside the displacer was done to avoid moving all the grid points inside the displacer for faster run-times. For example, without grid layering (the addition or subtraction of rows of cells), all the grid cells inside the displacer would need to move. Instead, only the single layer of cells located at the top of the displacer (right side in picture) moves at a time in Fig. 12(b). Also, the expansion space uses layering to avoid skewed cells when the displacer is at top dead center (TDC).

In Fig. 13, the gridding utilized in the seals and compression space are shown. The challenge in the seal region is to avoid overly skewed cells, overly mismatched cells across the interface, and reasonable cell size variation near the boundary layer. It is quite difficult to balance all these constraints. Layering was used in the compression region to avoid crushed cells and the axial direction spacing was chosen to maximize the time-step.

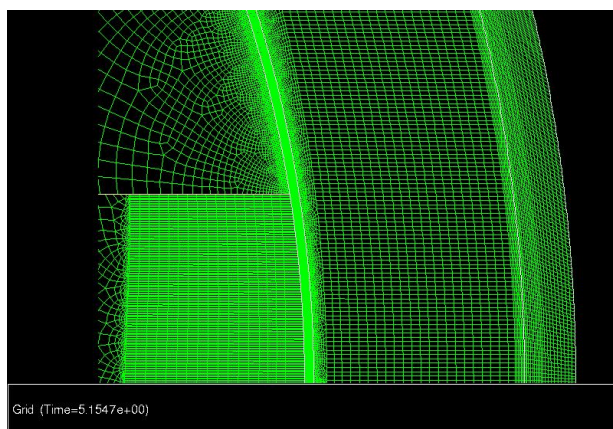
In Fig. 14 the seal inside the displacer is shown. Notice the use of unstructured quadrilaterals throughout to avoid the inefficient tetrahedral gridding. Also notice just how small the seal gap is even compared to the thickness of the displacer wall. This region was very difficult to grid and required utilizing suggestion 10 in Section C. Velocities are high in this region and this grid may not be adequate here, but since this seal region is not being studied this was left as seen.



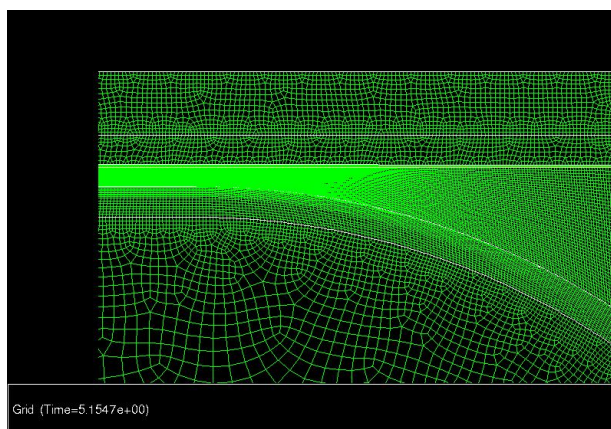
(a) Right Half



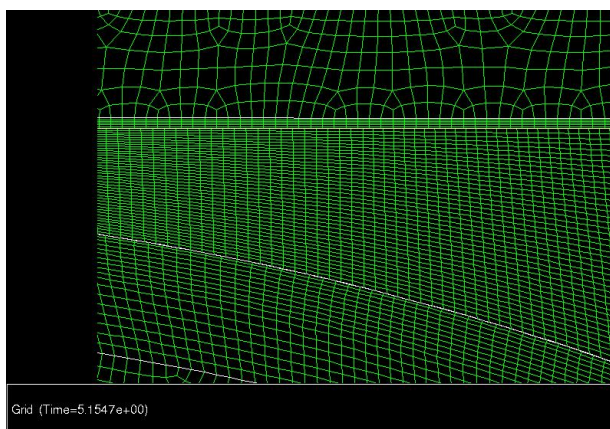
(b) Layering Inside Dome



(c) Layering in Expansion Space

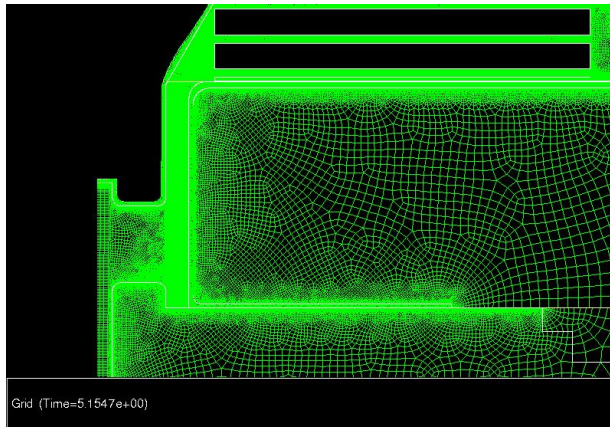


(d) Appendix Gap

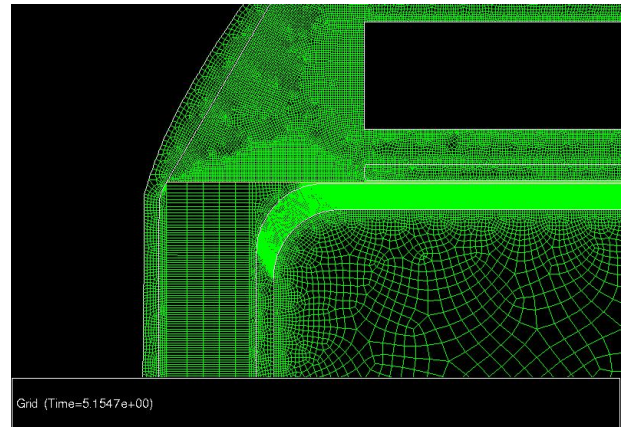


(e) Appendix Gap Close

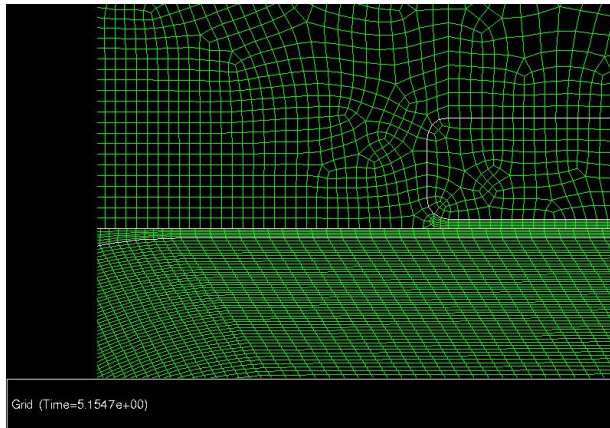
**Figure 12. Hot End Grid Strategy**



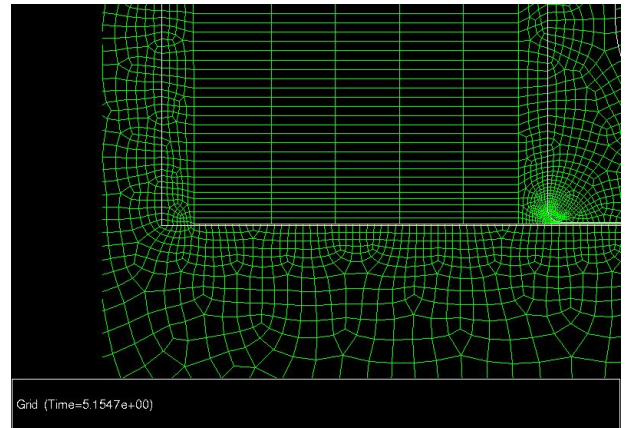
(a) Top and Bottom Seal, Rod, Compression Space



(b) Top Seal and Compression

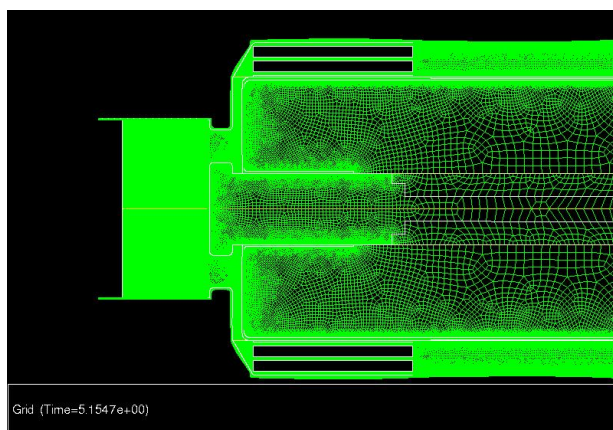


(c) Top Seal Close-up

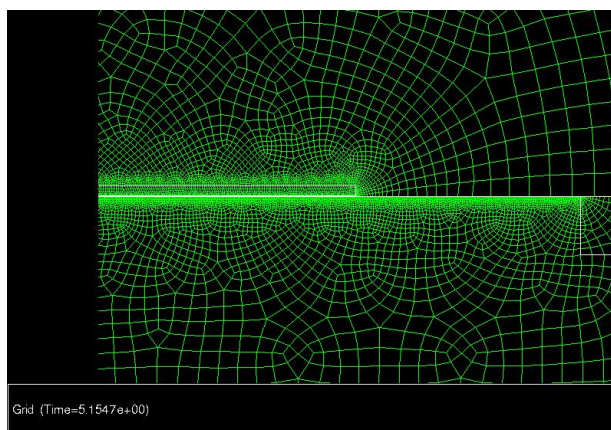


(d) Bottom Seal and Compression

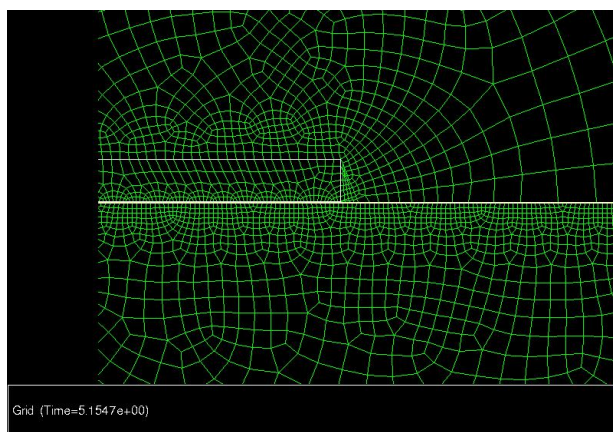
**Figure 13. Seal Grid Strategy**



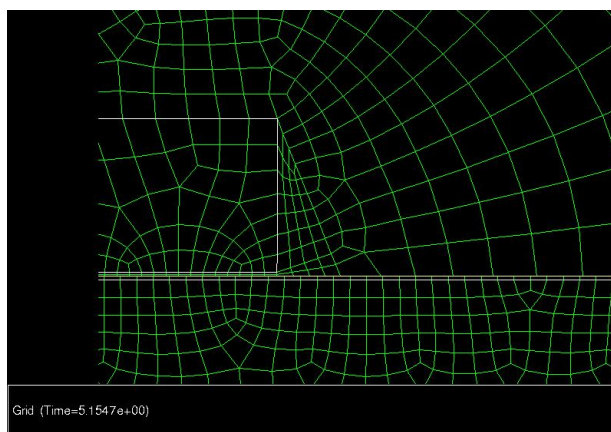
(a) Left Half



(b) Inside Seal



(c) Inside Seal Close-up



(d) Inside Seal Close-up2

**Figure 14. Inner Seal Grid Strategy**

## D. Solver Optimization

While one's choices are limited with commercial codes, they do offer a variety of low-order solvers to choose from, including low Mach number variable density flows.<sup>116</sup> A comparison of two commercial codes with the latest high-order numerical techniques demonstrates some of the differences a solver decision can make.<sup>51</sup> One of the limitations currently in Fluent is only first order accuracy is available in time when the mesh moves. CFD-ACE permits a higher accuracy in time with moving meshes, but does not have a coupled solver. The results shown in the next section utilize the segregated implicit solver in Fluent, although it was later discovered the coupled implicit solver is potentially significantly better. Of course, the grid used for the segregated solver may need further improvement if a coupled solver is used. It was found that the best method in Fluent and the best method in CFD-ACE performed comparably on simpler heat transfer test problems.<sup>51</sup> And with sufficient grid quality and density, the answers are comparable to the latest numerical methods.

One of the challenges to utilizing actual engine geometry is the difficulty of creating a high quality mesh as discussed previously. However, certain switches in the solvers may be adjusted to attempt to compensate for poor grid quality in regions of conjugate heat transfer.<sup>117</sup> In what follows are specific recommendations for improving conjugate heat transfer and they contain Fluent specific switches. The other commercial packages should be modified with their own specific format.

First, skewed meshes require secondary temperature gradients to be turned off to improve stability and to reduce undershoot/overshoot of local cell temperatures. In Fluent, this is accomplished by the command, "(rpsetvar 'temperature/secondary-gradient? #f)"

Second, in conjugate heat transfer problems with vastly different thermal conductivities between the fluid and solid zones, cells near the interface can experience round-off errors. To mitigate this effect, we want to ensure that the coarse levels of the AMG are visited. It is recommended that the default AMG cycle for energy be changed from Flexible to either F, V, or W-cycle. The F and V-cycles are more efficient in parallel simulations. The termination criteria should also be reduced to either 0.01 or 0.001 in order to tighten up convergence.

Third, although a reasonable temperature distribution can be achieved using single precision, in order to ensure that the total heat transfer is balanced the use of double precision is recommended.

Fourth, if convergence still remains problematic with secondary gradients turned off, it is recommended that explicit under-relaxation of temperature be used to stabilize the solution. When this is activated, the energy equation is solved with a URF of 1, but

the solution is explicitly relaxed with respect to the previous iteration. This allows for the equations to be efficiently solved (URF equal 1) and instabilities due to poor quality mesh and non-linear material properties to be mitigated (relaxation factor). The GUI entry of URF for energy will specify the explicit relaxation factor to be used. To active this, the scheme command is: (rpsetvar 'temperature/explicit-relax? #t)

## VII. Comparison with Analytical and Experimental Data

The approach just described was applied to simulating a Stirling engine built by Infinia Corp.<sup>118</sup> (formerly Stirling Technology Company) being considered for use in space missions. The engine is currently being tested for long-duration missions and hence a good deal of experimental performance information is available for comparison. The geometry is export controlled for these engines and so limited information may be shared with the public. The entire simulation was performed first, with no knowledge of the "correct" answer either from experiment or calibrated one-dimensional analysis. The operating conditions used in the simulation are shown below (Table 1):.

Hot-End Temperature, K	923
Cold-End Temperature, K	353
Ambient Temperature, K	293
Frequency, Hz	80
Mean Pressure, Pa	2.429E+6
Power Piston Amplitude, mm	6.0e-3

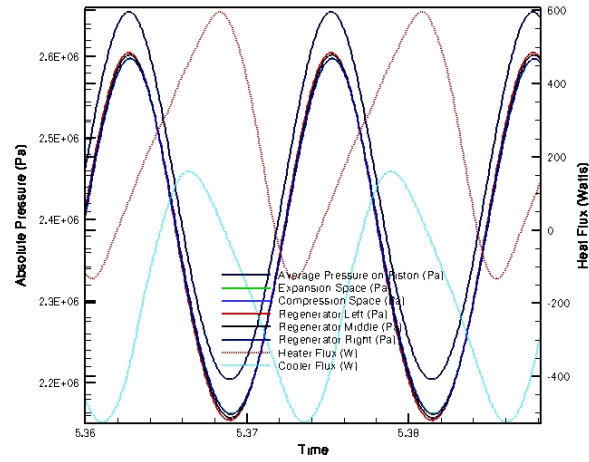
**Table 1. Operating Conditions**

The pressures and heat transfer occurring in various parts of the engine over two cycles are shown in Fig. 15(a). The pressure in the left (cooler end), middle, and right (heater end) of the regenerator is shown in Fig. 15(b). The average pressure in the expansion and compression spaces are shown to be about equal in Fig. 15(c). And, the temperature after 412.376 cycles of simulation are shown in Fig. 16 for an instant in time (135.36 degree Crank angle with reference to 0 degree displacer located at mean position) in the cold end of the machine and for the hot end are shown in Fig. 17. Notice the effects of radial heat transfer on the regenerator.

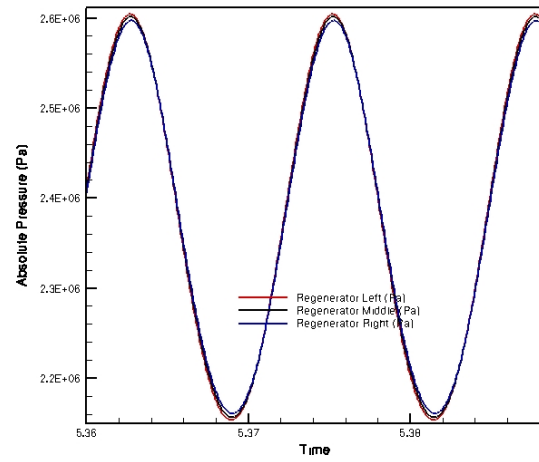
An initial comparison with Sage suggested the axisymmetric simulation was over-predicting the indicated power as shown in Table 2. And the overall heat transfer in and out fell within a band predicted by Sage in which heat loss to the environment was included and then assumed excluded (adiabatic).

But then we gathered the actual performance of two identical engines identified as Technology Demonstrator Converter (TDC) #13 and #14 from over many thousands of hours of operation since August 2003. As shown in Table 3, the axisymmetric solution which assumed no ambient heat loss to the environment, predicted not only the higher indicated power more accurately, but the efficiency was essentially within the variability of each built machine.

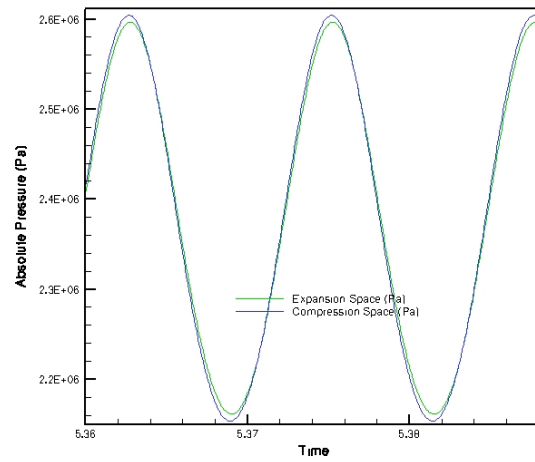
A few words of caution about this encouraging result are in order. First, due to experi-



(a) Pressure and Heat Transfer over Two Cycles



(b) Pressure Along Regenerator over Two Cycles



(c) Comparison of Expansion and Compression Pressure

**Figure 15. Pressure and Heat Transfer over Two Cycles**

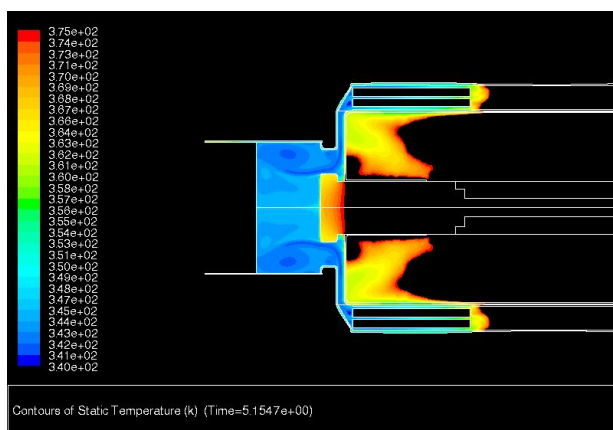
	Axisymmetric Simulation	Sage 1D Ambient Heat Loss	Sage 1D No Heat Loss
PV Power, W	79.65	68.77	70.14
Heat In, W	247.315	260.94	193.8
Heat Out, W	168.313	191.9	123.7
PV Efficiency	.322	.264	0.362
Pressure Ratio	1.209	1.187	1.187
Regenerator Delta Press., Pa	10466.3	15,888	15790
Heater Delta Press., Pa	682	194	192
Cooler Delta Press., Pa	896	316	315
Pressure Amplitude., Pa	225269	203100	203200
Mean Pressure, Pa	2429130	2378000	2378000

**Table 2. Comparison With Sage 1D Results**

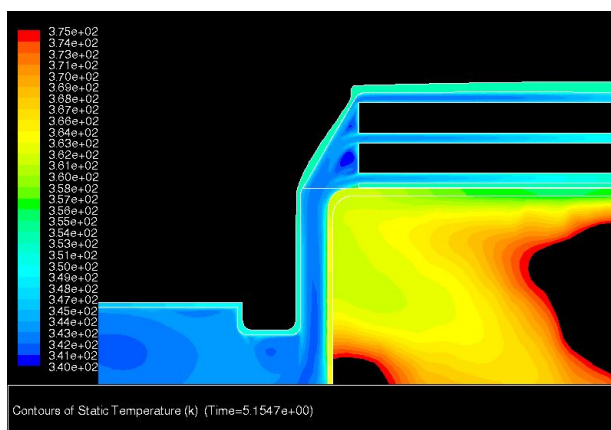
mental limitations the piston amplitude is not known for certain. The original designs call for a piston amplitude of 6+/- .08 mm. And the mean operating pressure is 2.517 Mpa compared to the simulated mean pressure of 2.42913 Mpa. If the piston amplitude in the experiment is exactly 6.00mm, then this mean pressure difference implies the simulated power would need to be corrected up to 82.5874W. But since the engines are tuned for maximum performance and the energy measurements are considered more reliable than the piston amplitudes, it seems appropriate to conclude this simulation did in fact predict the performance and efficiency within the bounds of variability of the two engines tested. Since a slightly lower piston amplitude would counter-balance the corrected higher power just shown, it is important to improve the experimentally derived piston amplitude for future comparisons.

	$T_h$	$T_c$	freq	net Qin	PV Pout	net Qout
TDC #13	646.0	92.4	81.4	242.1	78.2	163.9
TDC #14	646.5	94.4	81.4	250.4	79.6	170.8
Simulation	650.0	80.0	80.0	247.3	79.7	168.3
%Err-AVE	0.6%	0.3%	-1.8%	0.4%	0.9%	0.6%
%Err-TDC#13	0.5%	0.2%	-1.8%	2.1%	1.9%	2.7%
%Err-TDC#14	0.6%	0.3%	-1.8%	-1.2%	0.0%	-1.5%

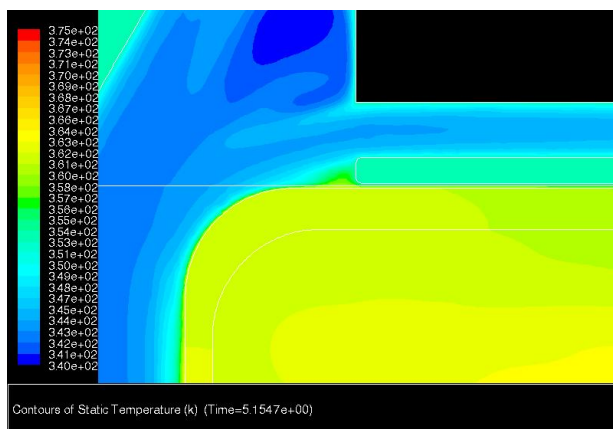
**Table 3. Comparison With Experimental Results**



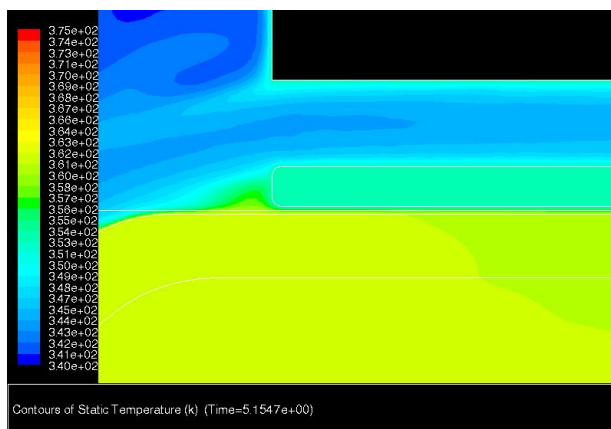
(a) Left Half



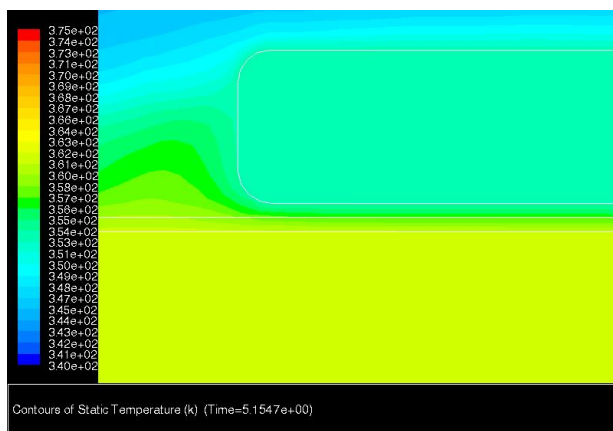
(b) Compression and Cooler



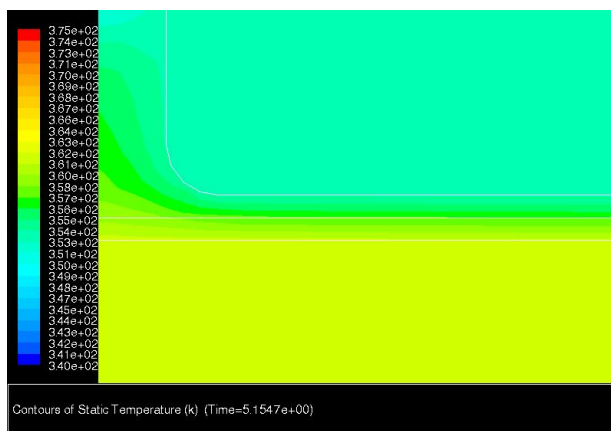
(c) Cooler and Seal



(d) Seal Region

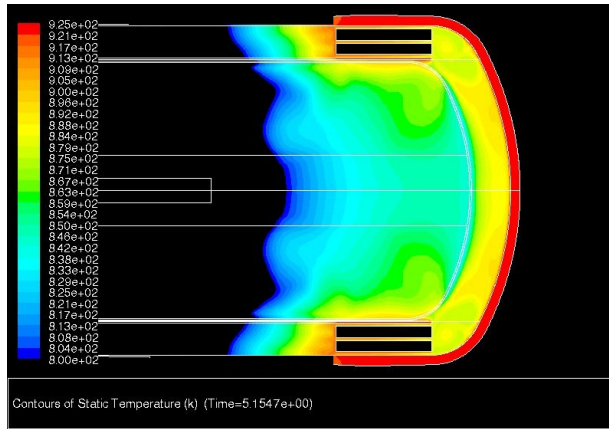


(e) Seal Pretty Close

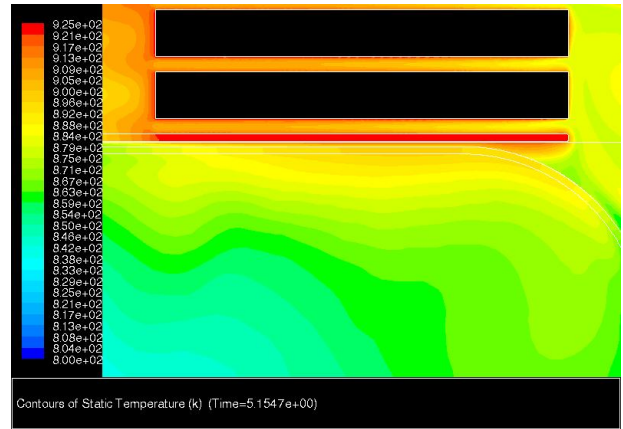


(f) Seal Very Close

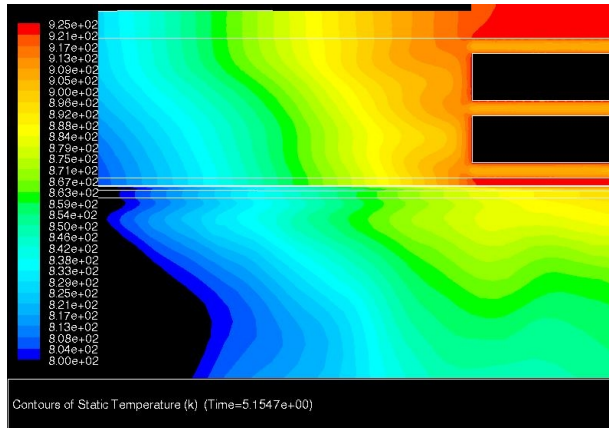
Figure 16. Cold End Temperature



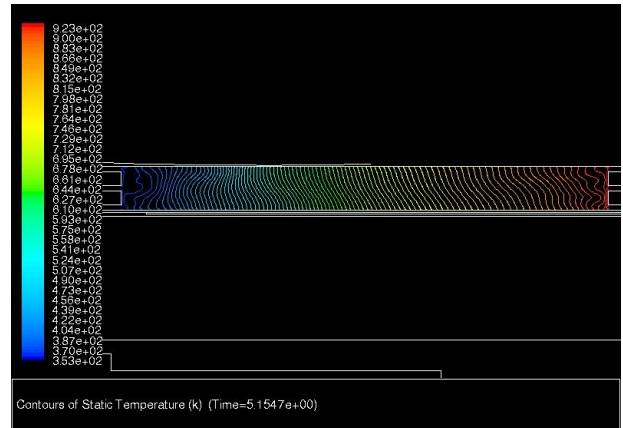
(a) Right Half



(b) Below Heater



(c) Around Heater



(d) Regenerator

Figure 17. Hot End and Regenerator Temperature

## VIII. Conclusion

For the first time in the U.S., a fully converged axisymmetric simulation of an actual Stirling engine was successfully performed. And moreover, by utilizing the latest advances in computer processors, communication hardware, parallel decomposition strategy, dynamic meshing, solution algorithms, and careful grid design, it is possible to compute one complete Stirling cycle in less than an hour.

The full axisymmetric simulation predicted the engine performance within the bounds of both engine test's data. And predicted the overall efficiency and indicated power considerably better than our initial efforts with using a leading one-dimensional Stirling analysis prediction on the first attempt. The one-dimensional analysis can certainly be improved by modifying various friction and heat transfer factors, either from experimental data or from this axisymmetric solution. But the real value of the axisymmetric solution is no adjustments were required because the engine is simulated from first principles. Of course the one-dimensional analysis is valuable for other reasons such as fast optimizations and performance mapping.

These results and observations lead to the conclusion that one, two, and three-dimensional modeling should all be employed and all three paradigms provide important capabilities that when combined provide a potent combination of initial design, empirical coefficient adjustment, optimization, and final prototype demonstration before the first metal is cut.

## IX. Acknowledgments

We'd like to thank Dr. Kumud Ajmani, Dr. Jim Heidman, Dr. Vijay Garg (late), and Dr. Isaac Lopez for their suggestions and guidance with high performance computing and simulation. The work described in this paper was performed for the Science Mission Directorate (SMD) and the Radioisotope Power System (RPS) Program, which provided funding for this project. Any opinions, findings, and conclusions or recommendations expressed in this report, are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration.

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