4th International Energy Conversion Engineering Conference and Exhibit (IECEC)
26 - 29 June 2006, San Diego, California

AIAA 2006-4003

An Initial Non-Equilibrium Porous-Media Model for CFD Simulation of Stirling Regenerators

Roy Tew*
NASA Glenn Research Center, Cleveland, OH, 44135

Terry Simon†
University of Minnesota, Minneapolis, MN, 55455

David Gedeon‡
Gedeon Associates, Athens, OH, 45701

Mounir Ibrahim§ and Wei Rong**
Cleveland State University, Cleveland, OH, 44115

The objective of this paper is to define empirical parameters (or closure models) for an initial thermal non-equilibrium porous-media model for use in Computational Fluid Dynamics (CFD) codes for simulation of Stirling regenerators. The two CFD codes currently being used at Glenn Research Center (GRC) for Stirling engine modeling are Fluent and CFD-ACE. The porous-media models available in each of these codes are equilibrium models, which assume that the solid matrix and the fluid are in thermal equilibrium at each spatial location within the porous medium. This is believed to be a poor assumption for the oscillating-flow environment within Stirling regenerators; Stirling 1-D regenerator models, used in Stirling design, use non-equilibrium regenerator models and suggest regenerator matrix and gas average temperatures can differ by several degrees at a given axial location and time during the cycle. A NASA regenerator research grant has been providing experimental and computational results to support definition of various empirical coefficients needed in defining a non-equilibrium, macroscopic, porous-media model (i.e., to define "closure" relations). The grant effort is being led by Cleveland State University, with subcontractor assistance from the University of Minnesota, Gedeon Associates, and Sunpower, Inc. Friction-factor and heat-transfer correlations based on data taken with the NASA/Sunpower oscillating-flow test rig also provide experimentally based correlations that are useful in defining parameters for the porous-media model; these correlations are documented in Gedeon Associates' Sage Stirling-Code Manuals. These sources of experimentally based information were used to define the following terms and parameters needed in the non-equilibrium porous-media model: hydrodynamic dispersion, permeability, inertial coefficient, fluid effective thermal conductivity (including thermal dispersion and estimate of tortuosity effects), and fluid-solid heat transfer coefficient. Solid effective thermal conductivity (including the effect of tortuosity) was also estimated. Determination of the porous-media model parameters was based on planned use in a CFD model of Infinia's Stirling Technology Demonstration Convertor (TDC), which uses a random-fiber regenerator matrix. The non-equilibrium porous-media model presented is considered to be an initial, or "draft," model for possible incorporation in commercial CFD codes, with the expectation that the empirical parameters will likely need to be updated once resulting Stirling CFD model regenerator and engine results have been analyzed. The emphasis of the paper is on use of available data to define empirical parameters (and closure models) needed in a thermal non-equilibrium porous-media model for Stirling regenerator simulation. Such a model has not yet been implemented by the authors or their associates. However, it is anticipated that a thermal non-equilibrium model such as that presented here, when incorporated in the CFD codes, will improve our ability to accurately model Stirling regenerators with CFD relative to current thermal-equilibrium porous-media models.

* Research Engineer, Thermal Energy Conversion Branch, 21000 Brookpark Rd./MS 301-2, AIAA Member
† Professor, Department of Mechanical Engineering, 111 Church St., S.E., AIAA Member
‡ Proprietor of Gedeon Associates/Stirling Software Developer, 16922 South Canaan Road, Non-Member AIAA
§ Professor, Mechanical Engineering Department, 1960 E. 24th Street, AIAA Associate Fellow
** Research Associate, Mechanical Engineering Department, 1960 E. 24th Street, AIAA Member

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Roy Tew, NASA GRC
Terry Simon, U. of Minnesota
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Presented by Roy Tew

for
IECEC 2006
June 26-29, 2006
San Diego, CA

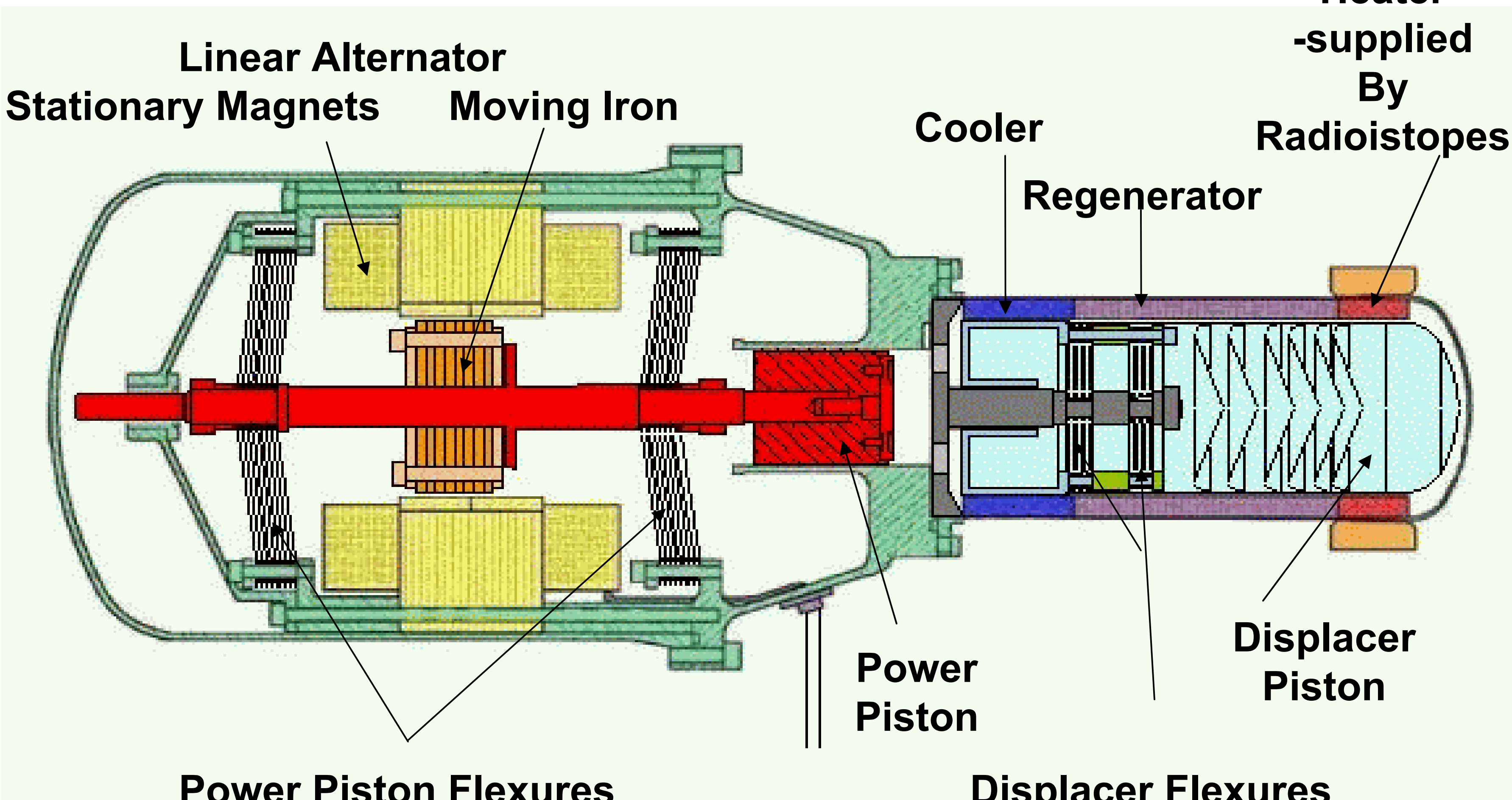
Acknowledgments

The work described in this paper was performed for the NASA Science Mission Directorate (SMD) and the Radioisotope Power System (RPS) Program. Any opinions, findings and conclusions or recommendations expressed in this report, are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration.

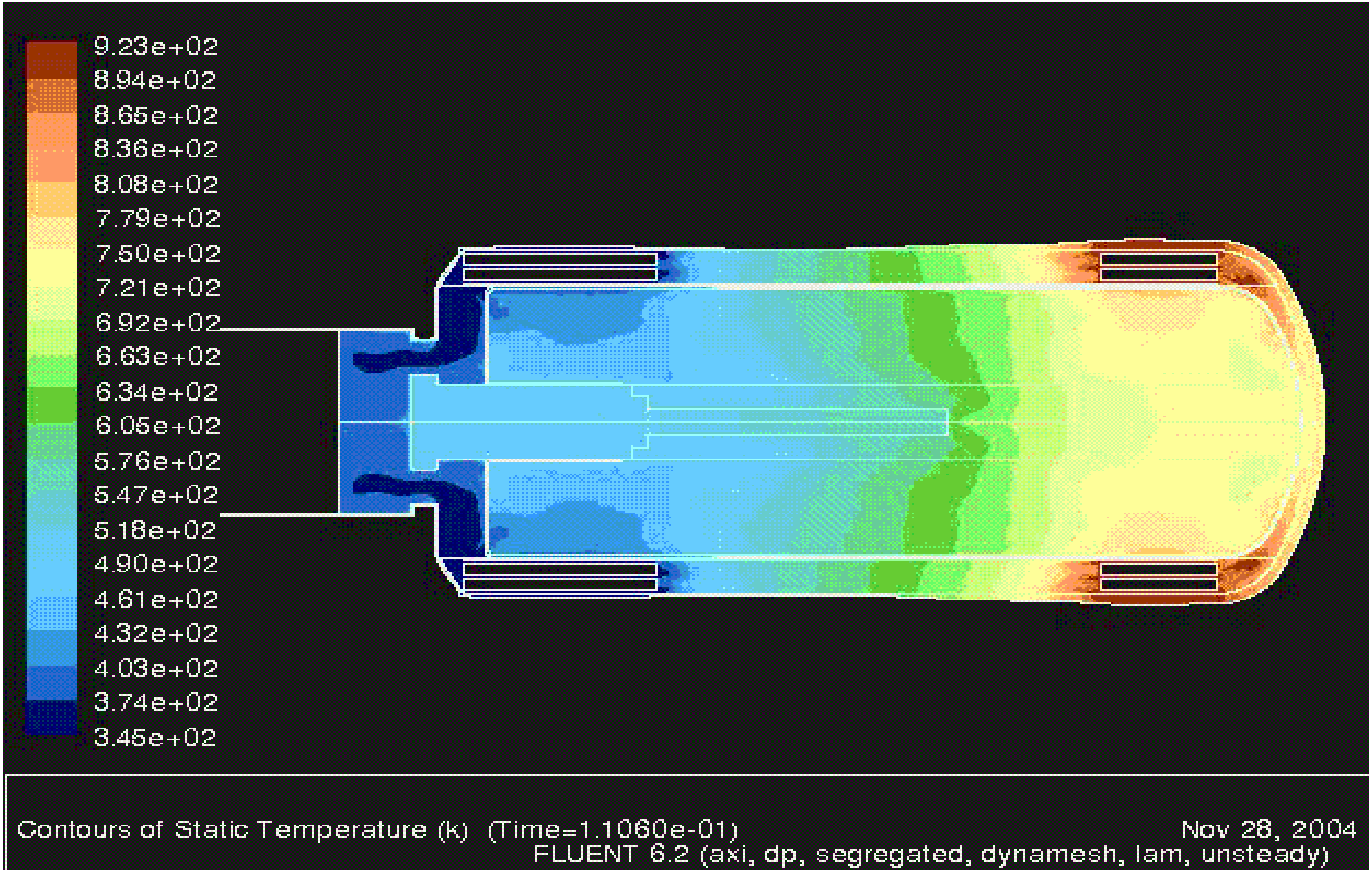
Presentation Outline

- Illustrations of Stirling engine, 2-D simulation results, regen. material
- Evidence that assumption of gas/solid thermal equilibrium in regenerator not valid
- For reference, thermal non-equilibrium porous-media conservation equations
- Porous-media model quantities in these equations needing definition or “closure”
- Summary of available information for defining these quantities
- Concluding remarks

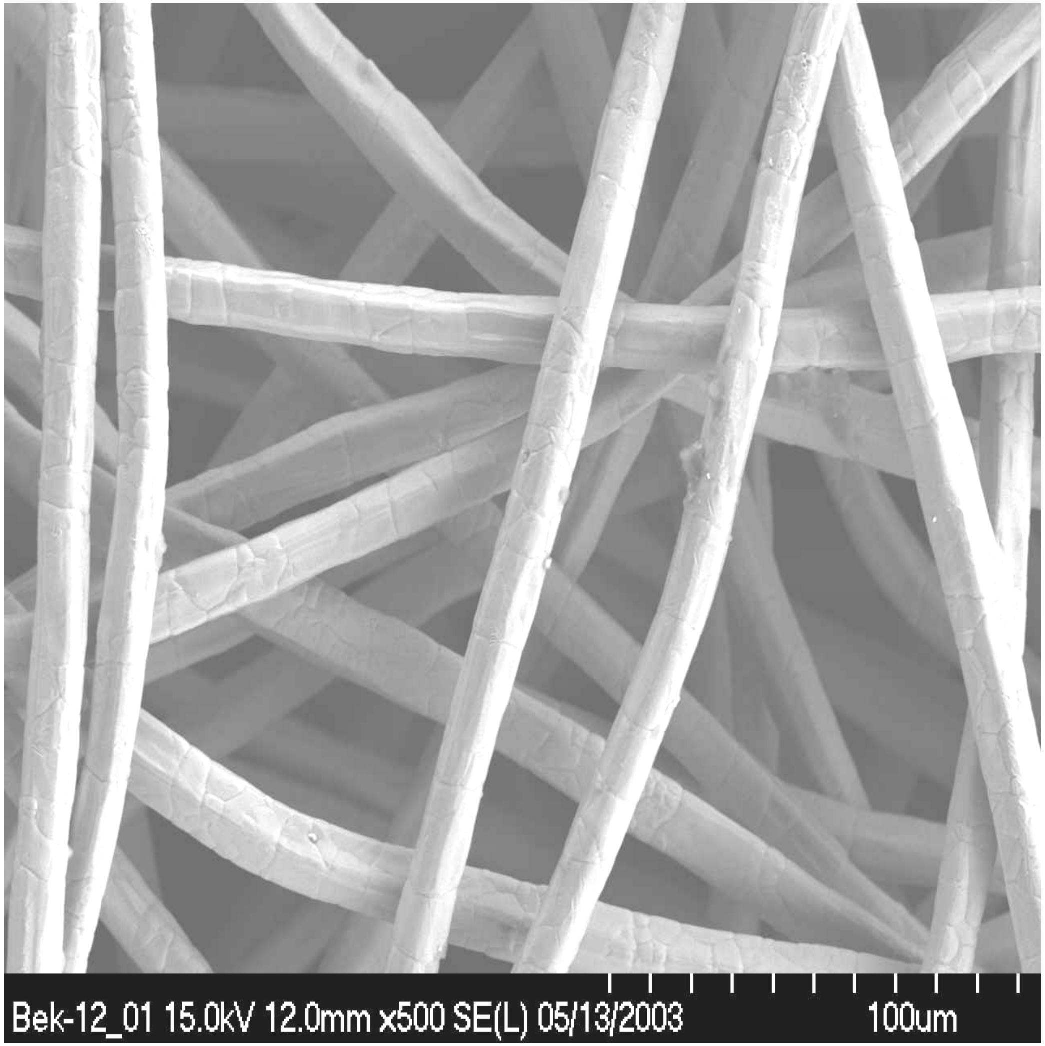
Schematic of Infinia's TDC 55 We Stirling Engine
Pistons have Clearance Seals=>No Contact with Cylinders
Uses Helium "Working" Gas



GRC Temperature Contours: Generated with Dyson's Fluent 2-D Axisymmetric CFD Model of Infinia's TDC Stirling Engine (Using Thermal-Equilibrium Porous-Media Model for Regenerator Simulation)



Currently Used Regenerator Random Fiber Material



Bekaert
316 Stainless Steel
Nominal 12 Micron Round Fibers
Mean Eff. D.=13.4 microns
& Fibers Are Not Round
90 % Porosity

---from David Gedeon memo. reporting on DOE Regenerator Research Contract Work, Photo By GRC.

Figure 1 Bekaert nominal 12 micron round fibers 316 stainless steel. Measured mean effective diameter 13.4 microns. From 90% porosity regenerator matrix made and tested under current DOE regenerator research program. Micrograph courtesy of NASA GRC.

Evidence that Gas/Solid Thermal Equilibrium Assumption is not Valid

- UMN Tests with Engine Values of Dimensionless Variables show Significant Temperature Differences between Gas and Solid
  - To be shown
- Sage 1-D Model of Infinia's TDC 55 We Engine shows significant Gas/Solid Temperature Differences in Regenerator
  - To be shown
- Enthalpy Flux Losses through the Regenerator from the Hot to the Cold calculated by Sage are significant (to be shown) and are expected to be sensitive to thermal equilibrium/non-equilibrium assumption
- Enthalpy Flux at Point along 1-D Flow Axis is:

$$\oint \dot{m}\, c_p\, T_{gas}\, dt$$

Regenerator Research Test Section at UMN

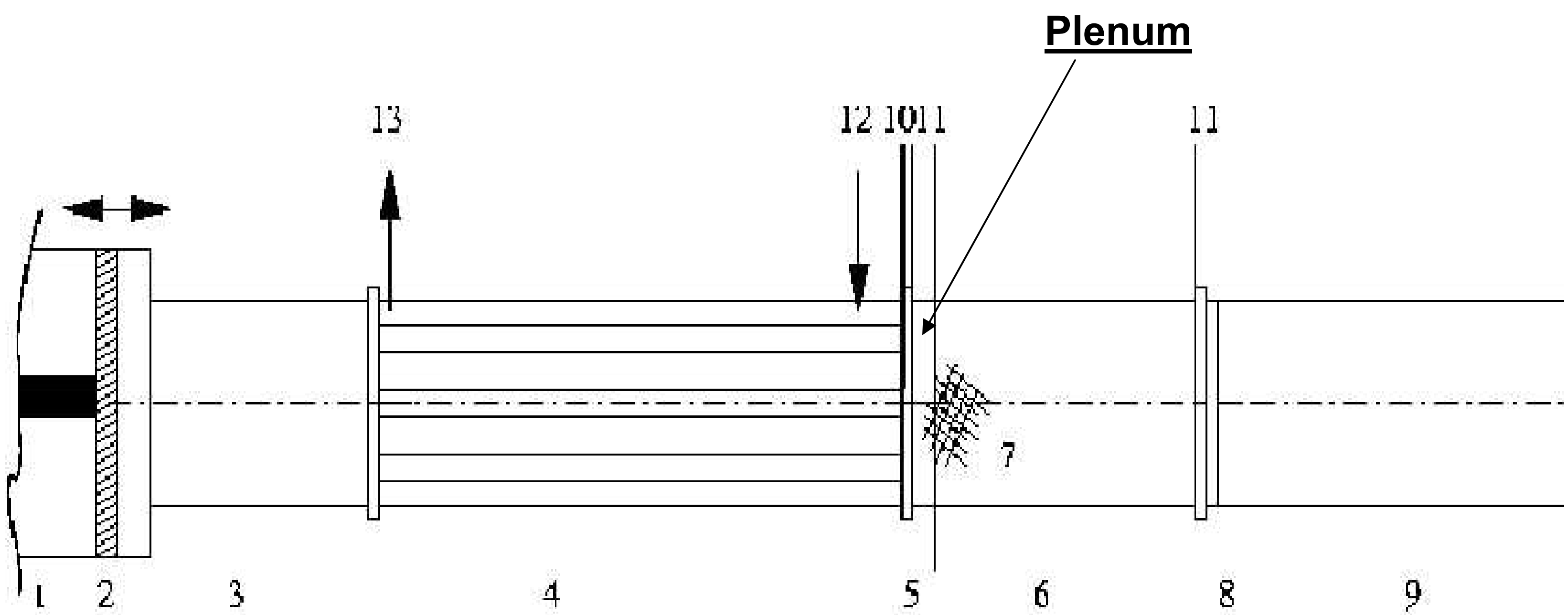
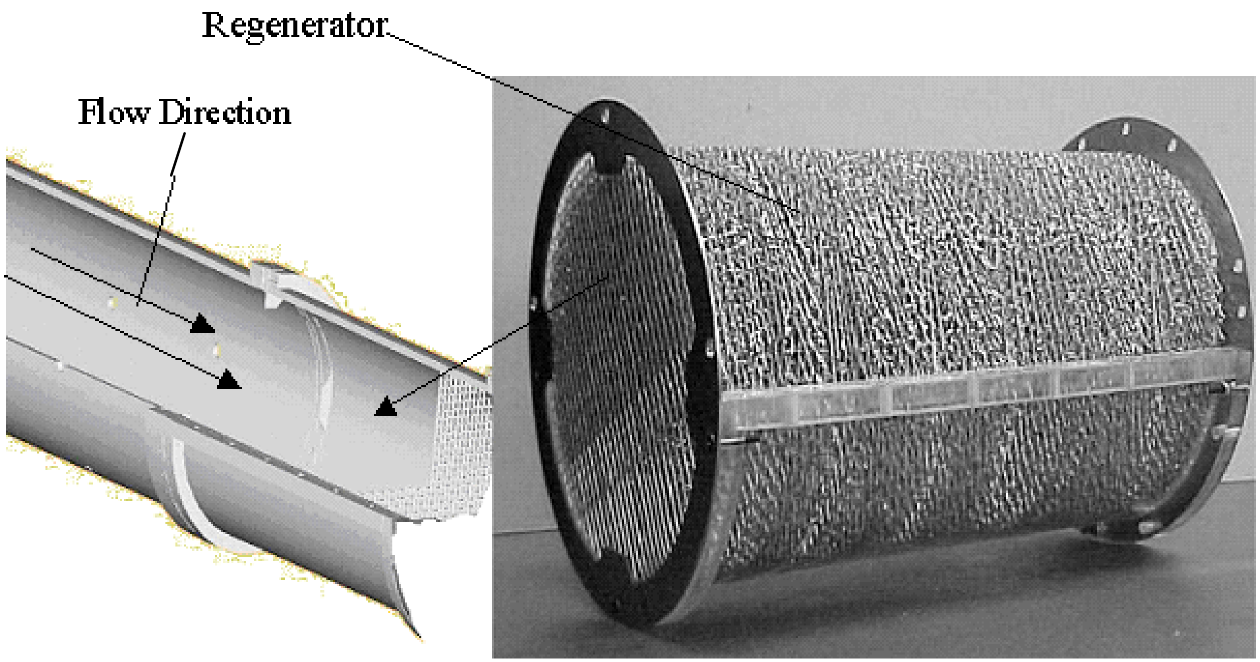


FIGURE 9. The Schematic of the UMN Experimental Facility and the Test Section.
1----oscillatory flow generator, 2----piston, 3----flow distributor, 4----cooler, 5----plenum, 6----regenerator, 7----screen matrix, 8----electrical hating coil, 9----isolation duct, 10----hot-wire, 11----thermocouple, 12----cooling water in, 13----cooling water out

Frequency = 0.4 Hz (24 RPM), Stroke = 356 mm, Piston Diameter = 356 mm
Regen. Porosity = 90%, Dimensionless Parameters Similar to an Infinia Engine

UMN Wire Screen Regenerator

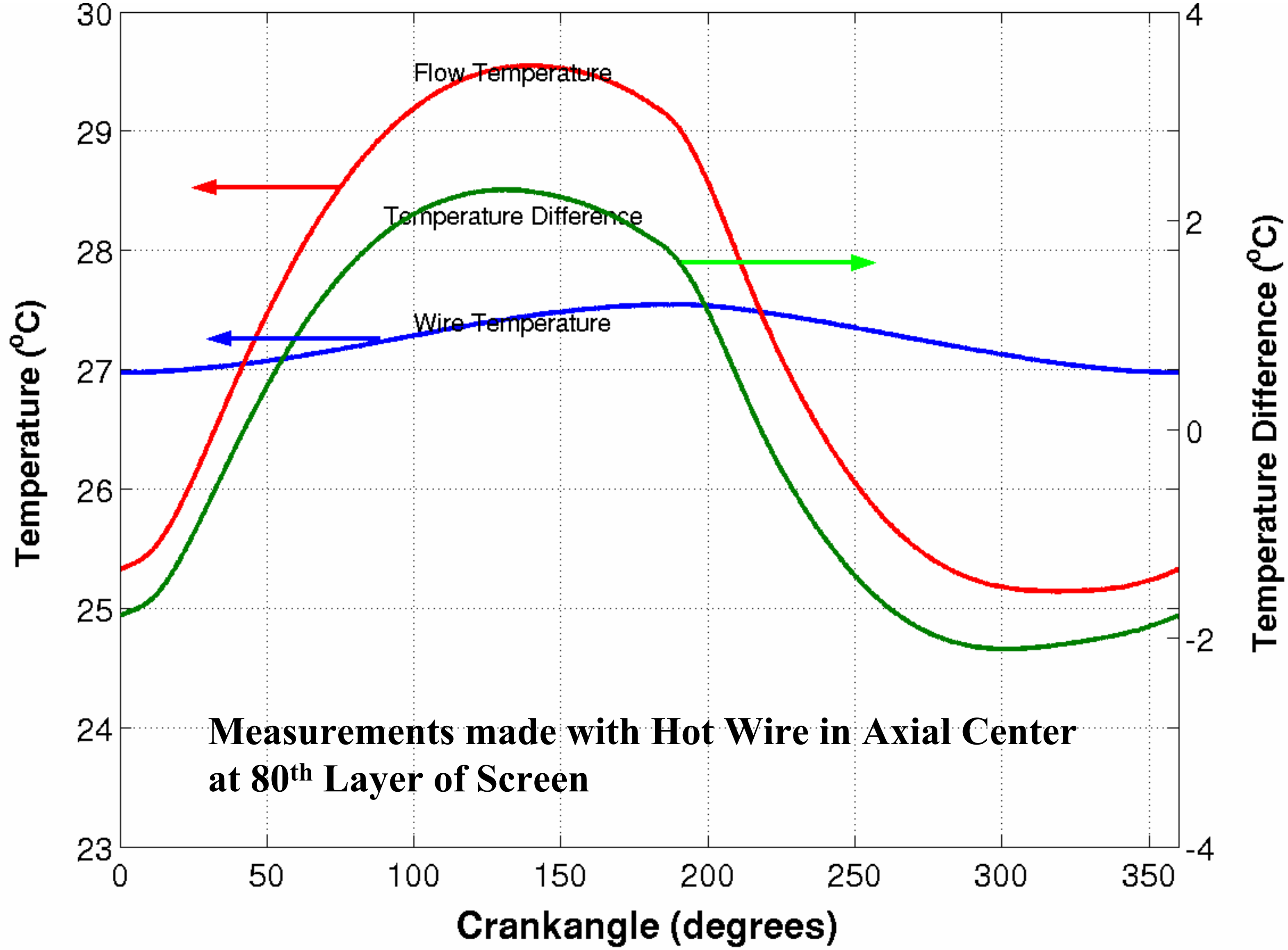
(Considered to Yield an Approximation of Random Fiber)



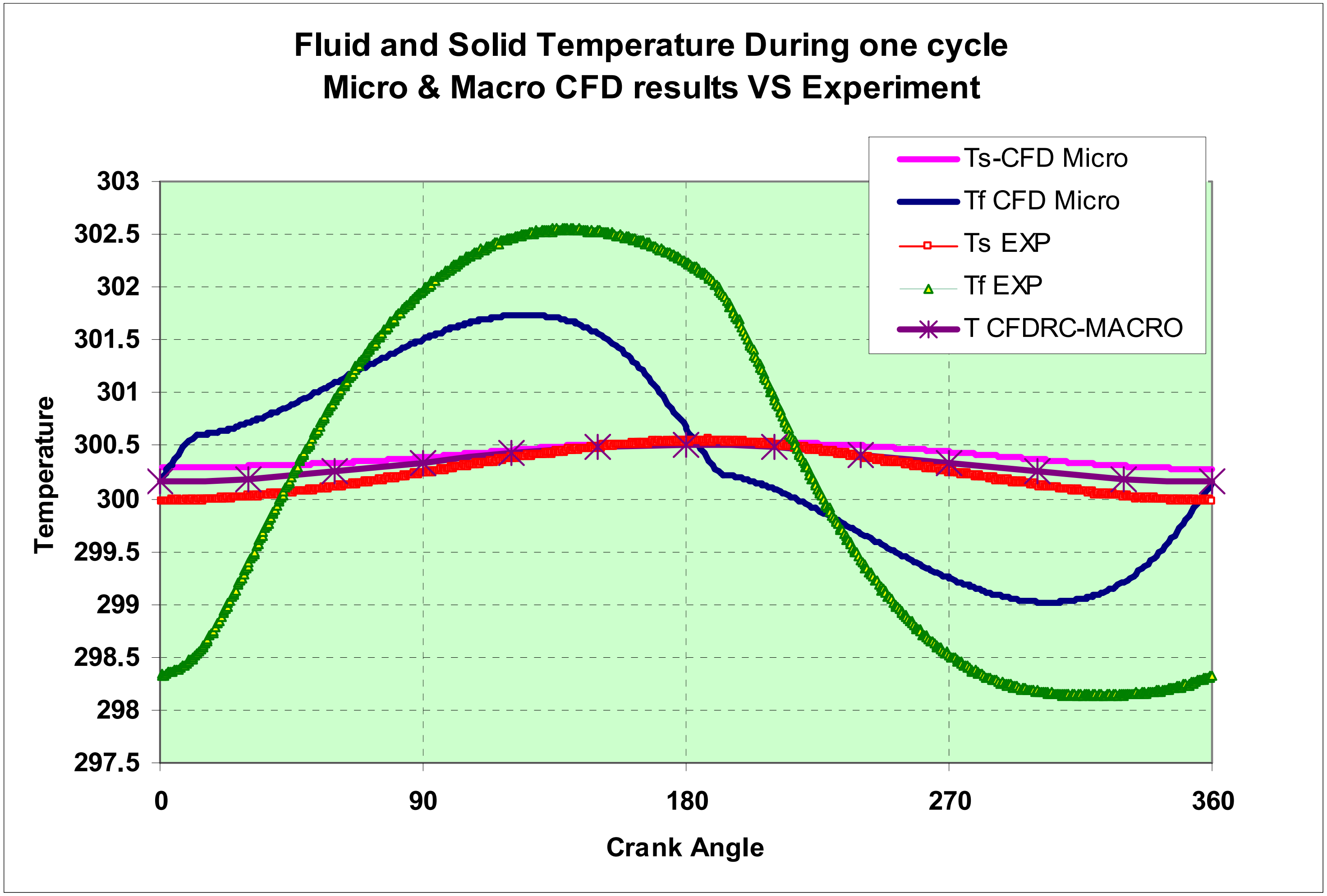
- 90 % Porosity
- Stainless Steel 304 Welded Screens
- 200 Layers of 6.3 mm x 6.3 mm Mesh
- Wire Diameter = 0.81 mm
- Each Screen Rotated 45 Deg. Rel. to Next

- Representative Stirling Engine Chosen to define Test Section Dimensionless Parameters (Max. Reynolds # & Valensi #)
- Hot-Wire Anemometry Measurements

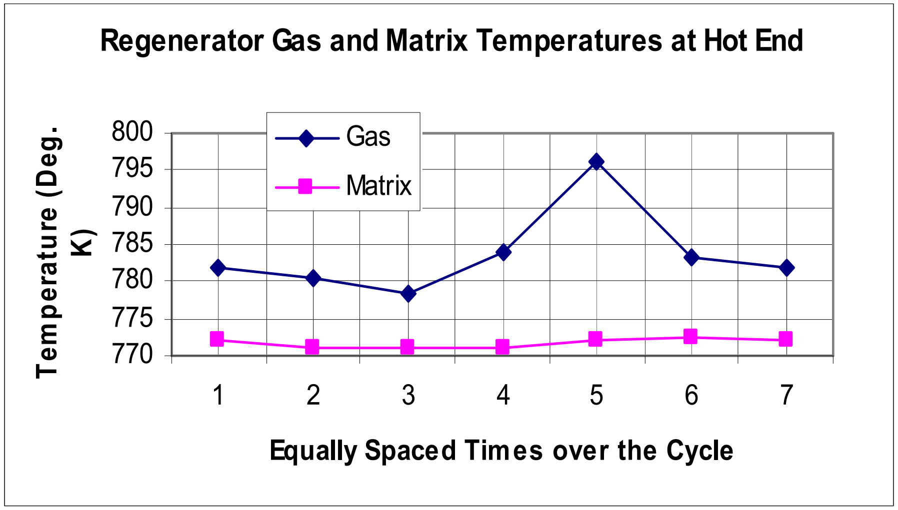
UMN Test Data: Temperature Difference between Gas/Solid in Regenerator

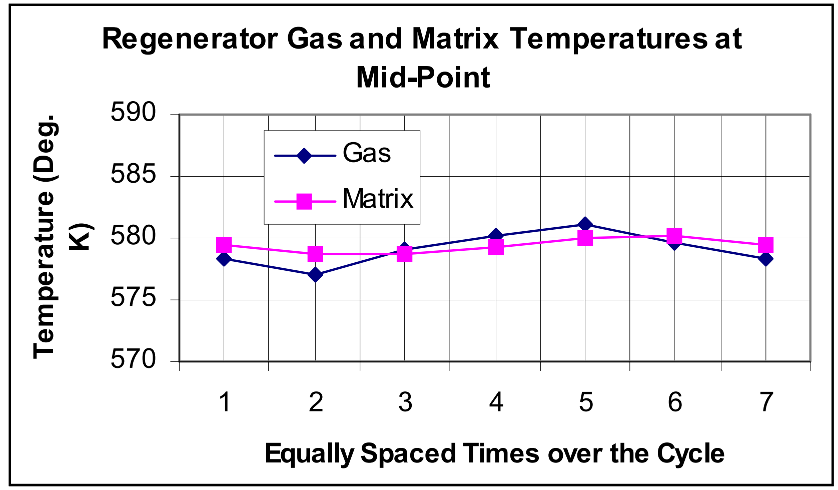


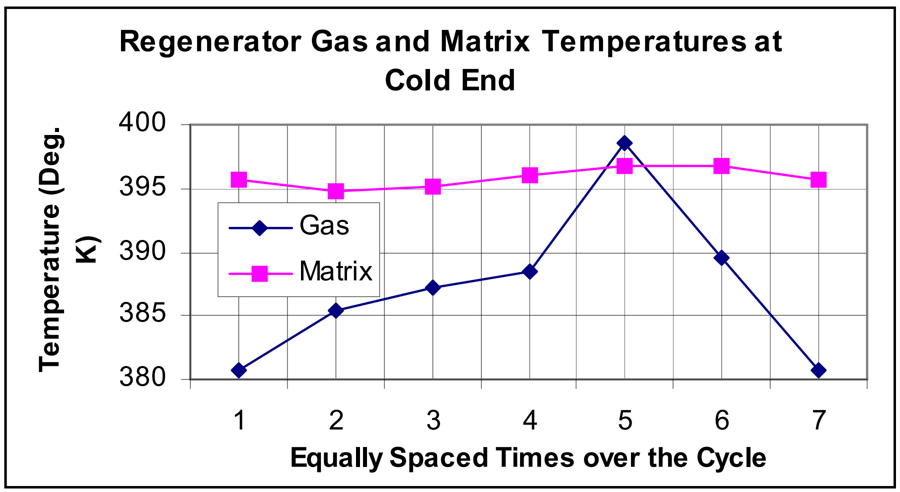
CSU CFD “Micro-” and “Macro-” Calculations & UMN Test Data
(Micro-Calculations made using REV of actual geometry)
(Macro-Calculations made with CFD-ACE Thermal Equilibrium Porous Media Model)



Regenerator Gas and Solid Temperatures at Hot-End, Mid-Point and Cold-End as Predicted by 1-D Sage Code—for Infinia's TDC 55 We Engine (Using 1-D Thermal Non-Equilibrium Regenerator Model)







TDC Operating Conditions & Predictions

- Operating Conditions (Not Design) were
- Hot-End Temperature = 823 K (550 C)
- Cold-End Temperature = 363 K (90 C)
- Frequency of Operation = 82.5 Hz
- Helium Mean Pressure = 2.59 MPa (376 psi)
- Performance Predictions were
- Engine Electrical Power = 55.9 We
- Engine Efficiency = 19.4%
- Heat Into Engine = 288 W
- Regenerator Enthalpy Flux Loss = 22.4 W

Regen. Enthalpy Flux Loss ≡ $\oint \dot{m}\, c_p\, T_{gas}\, dt$

Glenn Research Center

at Lewis Field

Non-Equilibrium Porous-Media Conservation Equations

Continuity:

$$\frac{\partial \langle \rho \rangle^f}{\partial t} + \frac{1}{\beta} \nabla \bullet \left[\langle \rho \rangle^f \langle \mathbf{u} \rangle \right] = 0$$

Momentum:

$$\frac{1}{\beta} \frac{\partial \left(\langle \rho \rangle^f \langle \mathbf{u} \rangle \right)}{\partial t} + \frac{1}{\beta^2} \nabla \bullet \left[\langle \rho \rangle^f \langle \mathbf{u} \rangle \langle \mathbf{u} \rangle \right] = -\nabla \langle p \rangle^f + \nabla \bullet \left(\frac{\langle \nu_{eff} \rangle^f}{\beta} \langle \rho \rangle^f \nabla \langle \mathbf{u} \rangle - \frac{\langle \rho \rangle^f}{\beta} \langle \tilde{\mathbf{u}} \tilde{\mathbf{u}} \rangle \right) - \frac{\langle \mu \rangle^f}{K} \langle \mathbf{u} \rangle - \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} \left| \langle \mathbf{u} \rangle \right| \langle \mathbf{u} \rangle$$

Fluid Energy:

$$\frac{\partial \left(\langle \rho \rangle^f \langle h \rangle^f \right)}{\partial t} + \frac{1}{\beta} \nabla \bullet \left[\langle \rho \rangle^f \langle \mathbf{u} \rangle \langle h \rangle^f \right] = \nabla \bullet \left[\bar{\bar{k}}_{fe} \bullet \nabla \langle T \rangle^f \right] + \left(\frac{\mu}{K} + \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} |\mathbf{u}| \right) \mathbf{u} \cdot \mathbf{u} + h_{sf} \frac{dA_{sf}}{dV_f} \left(\langle T \rangle^s - \langle T \rangle^f \right) + \frac{d \langle p \rangle^f}{dt}$$

Solid Energy:

$$\frac{\partial \left(\rho_s C_s \langle T \rangle^s \right)}{\partial t} = \nabla \bullet \left[\bar{\bar{k}}_{se} \bullet \nabla \langle T \rangle^s \right] - h_{sf} \frac{dA_{sf}}{dV_s} \left(\langle T \rangle^s - \langle T \rangle^f \right)$$

Porous-Media Quantities Needing Definition or "Closure"

Momentum Equation:

Hydrodynamic Dispersion

$$\frac{1}{\beta}\frac{\partial\left(\langle\rho\rangle^f\langle\mathbf{u}\rangle\right)}{\partial t}+\frac{1}{\beta^2}\nabla\bullet\left[\langle\rho\rangle^f\langle\mathbf{u}\rangle\langle\mathbf{u}\rangle\right]=-\nabla\langle p\rangle^f+\nabla\bullet\left(\frac{\langle\nu_{eff}\rangle^f}{\beta}\langle\rho\rangle^f\nabla\langle\mathbf{u}\rangle-\frac{\langle\rho\rangle^f}{\beta}\langle\tilde{\mathbf{u}}\tilde{\mathbf{u}}\rangle\right)$$

$$-\frac{\langle\mu\rangle^f}{K}\langle\mathbf{u}\rangle-\langle\rho\rangle^f\frac{C_f}{\sqrt{K}}\left|\langle\mathbf{u}\rangle\right|\langle\mathbf{u}\rangle$$

Porosity

Permeability

Inertial Coefficient

Fluid Energy:

$$\frac{\partial\left(\langle\rho\rangle^f\langle h\rangle^f\right)}{\partial t}+\frac{1}{\beta}\nabla\bullet\left[\langle\rho\rangle^f\langle\mathbf{u}\rangle\langle h\rangle^f\right]=\nabla\bullet\left[\bar{\bar{k}}_{fe}\bullet\nabla\langle T\rangle^f\right]+\left(\frac{\mu}{K}+\langle\rho\rangle^f\frac{C_f}{\sqrt{K}}|\mathbf{u}|\right)\mathbf{u}\cdot\mathbf{u}+$$

$$h_{sf}\frac{dA_{sf}}{dV_f}\left(\langle T\rangle^s-\langle T\rangle^f\right)+\frac{d\langle p\rangle^f}{dt}$$

Fluid Effective Thermal Conductivity

Fluid-Solid Heat Transfer Coefficient

Solid Energy:

$$\frac{\partial\left(\rho_s C_s\langle T\rangle^s\right)}{\partial t}=\nabla\bullet\left[\bar{\bar{k}}_{se}\bullet\nabla\langle T\rangle^s\right]-h_{sf}\frac{dA_{sf}}{dV_s}\left(\langle T\rangle^s-\langle T\rangle^f\right)$$

Solid Effective Thermal Conductivity

Determination of Permeability and Inertial Coefficient

(1) U. of Minn. Large-Scale Wire Screen Measurements

(2) Assume Quasi-Steady Flow, Equate Darcy-Forchheimer Steady 1-D Momentum Equation with Darcy Friction Factor Momentum Eq. & Use NASA/Sunpower Oscillating-Flow Rig Darcy Friction-Factor Data

$$\frac{\nabla p}{L} = \frac{\mu}{K} u + \frac{C_f}{\sqrt{K}} \langle \rho \rangle^f u^2$$

$$\frac{\nabla p}{L} = \frac{f_D}{d_h} \frac{1}{2} \langle \rho \rangle^f u^2$$

Coefficient	UMN Large-Scale Screens (d_w=8.1E-4 m)				TDC Random Fiber	
	UMN Old, Experimental	UMN New, Experimental	CSU Calcs.	Sage Cor.	Sage Cor.	Unidirectional Flow Tests
K (m^2)	1.07E-7	1.86E-7	8.9E-7	8.24E-7	4.08E-10	3.52E-10
$K/d_W{}^2$	0.163	0.283	1.36	1.26	-	-
C_f	0.049	0.052	0.14	0.13-0.11 Re=25-100	0.19-0.17 Re=25-100	0.154-0.095 Re=25-100

Fluid & Solid Effective Thermal Conductivity Tensors

- **Assume only diagonal elements of tensors are non-zero**
- **Then, in terms of 3-D cylindrical coordinates--**

$$\bar{\bar{k}}_{fe} \equiv \begin{bmatrix} k_{fe,rr} & 0 & 0 \\ 0 & k_{fe,\vartheta\vartheta} & 0 \\ 0 & 0 & k_{fe,xx} \end{bmatrix} = \begin{bmatrix} k_f + k_{f,tor,rr} + k_{dis,rr} & 0 & 0 \\ 0 & k_f + k_{f,tor,\vartheta\vartheta} + k_{dis,\vartheta\vartheta} & 0 \\ 0 & 0 & k_f + k_{f,tor,xx} + k_{dis,xx} \end{bmatrix} =$$

$$\begin{bmatrix} k_{f,stag,rr} + k_{dis,rr} & 0 & 0 \\ 0 & k_{f,stag,\vartheta\vartheta} + k_{dis,\vartheta\vartheta} & 0 \\ 0 & 0 & k_{f,stag,xx} + k_{dis,xx} \end{bmatrix}$$

$$\bar{\bar{k}}_{se} \equiv \begin{bmatrix} k_{se,rr} & 0 & 0 \\ 0 & k_{se,\vartheta\vartheta} & 0 \\ 0 & 0 & k_{se,xx} \end{bmatrix} = \begin{bmatrix} f\left(k_s, k_{s,tor,rr}\right) & 0 & 0 \\ 0 & f\left(k_s, k_{s,tor,\vartheta\vartheta}\right) & 0 \\ 0 & 0 & f\left(k_s, k_{s,tor,xx}\right) \end{bmatrix}$$

Experimental Values of Fluid Thermal Dispersion Conductivity

- **In Table below, $k_{dis,yy}$, $k_{dis,xx}$, respectively, are the radial and axial thermal dispersion conductivities determined by various experimental measurements**

	Estimated Thermal Dispersion	Porous media
Current Direct Measurements at UMN, Niu[7]	$\varepsilon_{M,eddy} = \frac{k_{dis,yy}}{\rho_f c_p} = 0.02 d_h U$ or $\frac{k_{dis,yy}}{k_f} = 0.02 Pe$	Welded Screen
Hunt and Tien[22]	$\frac{k_{dis,yy}}{k_f} = 0.0011 Pe$	Fibrous Media
Metzger, Didierjean, and Maillet[23]	$\frac{k_{dis,yy}}{k_f} = (0.03\text{–}0.05) Pe$ and $\frac{k_{dis,xx}}{k_f} = 0.073 Pe^{1.59}$	Packed Spheres
Gedeon[9]	$\frac{k_{dis,xx}}{k_f} = 0.50 Pe^{0.62} \beta^{-2.91}$ or $\frac{k_{dis,xx}}{k_f} \approx 0.06 Pe$ for $\beta = 0.9, Pe = 560$	Woven Screen

Estimates of Fluid-Stagnant and Solid-Effective Thermal Conductivities --for 90% porosity wire screen or random fiber matrix---#1

- **Base calculations below on air, $k_f =$ 0.026 W/m-K and stainless steel, $k_s =$ 13.4 W/m-K**

- **For radial & azimuthal directions, assume approximately parallel solid & fluid flow paths & thus based on the parallel model:**

$$k_{f,stag} = k_f \beta = (26x10^{-3} W/mK)(0.90) = 0.0234 W/mK \longleftarrow$$

$$k_{se} = k_s (1-\beta) = (13.4\, W/mK)(0.1) = 1.34\, W/mK$$

- **McFadden's (UMN) calculations based on actual welded screen geometry suggest parallel model value of solid effective thermal conductivity should be multiplied by 0.625**

$$\therefore\ k_{se} = k_s (1-\beta) 0.625 = 1.34\, W/mK\ x\ 0.625 = 0.838 \longleftarrow$$

Estimates of Fluid-Stagnant and Solid-Effective Thermal Conductivities --for 90% porosity wire screen or random fiber matrix---#2

- For the axial direction use a modification of the series model
- A lumped effective solid + fluid effective thermal conductivity (not including thermal dispersion), based on the series model is --

$$k_{eff,s+f} = \left(\frac{1}{\frac{\beta}{k_f} + \frac{1-\beta}{k_s}} \right) = \left(\frac{1}{\frac{0.90}{26\ x\ 10^{-3}} + \frac{0.1}{13.4}} \right) = 0.0289\ W\ /\ mK$$

- Only slightly larger than molecular fluid cond. of 0.026 W/m-K & likely too small since series model implies wires not touching in axial direction
- 3-D CFD microscopic simulation of REV of the UMN welded screen by by Rong of CSU suggests above series value should be multiplied by 2.157

$$\therefore\ k_{eff,s+f} = (0.0289\ W\ /\ mK)(2.157) = 0.0623 W\ /\ mK$$

- The above would be appropriate for an equilibrium model. Seeing no obvious way to separate values for a fluid & solid in the axial direction—propose using the same value for fluid-stagnant and solid-effective cond. in the axial direction, hoping that overall effect will be reasonable.

Heat Transfer Coefficients Between Fluid & Solid Matrix

- Good sources of heat transfer coefficient correlations for wire screen and random fiber matrices are Gedeon's Sage manuals and NASA Technical Memorandums containing NASA/Sunpower oscillating-flow rig data.

- The following correlations are in terms of Nusselt No., Peclet Number (Reynolds No. x Prandtl No.), and porosity:

- For wire screen: $Nu = (1. + 0.99\, Pe^{0.66})\, \beta^{1.79}$

- For random fiber: $Nu = (1. + 1.16\, Pe^{0.66})\, \beta^{2.61}$

- where--

$$Nu = \frac{h d_h}{k}, \quad Pe = Re\, Pr = \frac{\rho\, u\, d_h}{\mu} \frac{c_p \mu}{k}$$

Oscillating-Flow Rig

Hydrodynamic Dispersion Term (Possibly Negligible Effect for this Application—This Needs to be Checked)

- Hydrodynamic dispersion term--fluid momentum equation: $\frac{1}{\beta}\langle \tilde{\mathbf{u}}\tilde{\mathbf{u}} \rangle \cong \tilde{\mathbf{u}}\tilde{\mathbf{u}}$

$\beta =$ porosity; $\mathbf{u} =$ average-channel fluid, or local, velocity inside matrix
$\tilde{\mathbf{u}} =$ spatial-variation of average-channel fluid velocity inside matrix;

$$\tilde{\mathbf{u}}\,\tilde{\mathbf{u}} = \mathbf{ii}\,\tilde{u}\tilde{u} + \mathbf{ij}\,\tilde{u}\,\tilde{v} + \mathbf{ik}\,\tilde{u}\,\tilde{w} + \mathbf{ji}\,\tilde{v}\,\tilde{u} + j\mathbf{j}\,\tilde{v}\,\tilde{v} + \mathbf{jk}\,\tilde{v}\,\tilde{w} + \mathbf{ki}\,\tilde{w}\tilde{u} + \mathbf{kj}\,\tilde{w}\tilde{v} + \mathbf{kk}\,\tilde{w}\,\tilde{w}$$

- Most important terms from the above tensor expression are for transport normal to the flow and are: $|\tilde{u}\,\tilde{v}|$ or $|\tilde{v}\,\tilde{u}|$

- Niu of UMN—based on regenerator experiments--showed that:

$$\frac{\overline{\langle \tilde{u}\tilde{v} \rangle}}{\beta^2} = \frac{\overline{\langle \tilde{v}\tilde{u} \rangle}}{\beta^2} \approx \frac{\langle \overline{u'v'} \rangle}{\beta^2} = -\frac{1}{\beta^2}\varepsilon_M \frac{\partial U}{\partial r} = -\frac{1}{\beta^2} 0.02\, d_h U \frac{\partial U}{\partial r} = -\frac{1}{\beta^2}\lambda\, d_h U \frac{\partial U}{\partial r}$$

- This information could be used to check the significance of the term—but has not been done, yet.

Concluding Remarks

- A thermal-non-equilibrium porous-media model is needed to replace the existing thermal equilibrium models in CFD codes for accurate simulation of Stirling regenerators
- Transient, compressible-flow, conservation equations are summarized for reference in discussing porous-media model parameters needing definition for a macroscopic, thermal-non-equilibrium porous-media model
- Available experimental information is discussed for definition of the hydrodynamic-dispersion term and the permeability & inertial coefficients in the momentum equation, and the heat transfer coefficient & thermal dispersion terms in the energy equations
- Methods are outlined for estimating fluid-stagnant and solid-effective thermal conductivities
- Adequate information is given for definition of an initial thermal non-equilibrium porous-media model for use in a CFD model of the regenerator of a TDC Stirling engine. Still need to implement.
- Application of this model in Stirling CFD codes may demonstrate that further refinement of these parameters, or of the model itself may be required