EUROMECH Colloquium 483
Geometrically Non-linear Vibrations of Structures
9 – 11 July 2007, FEUP, Porto, Portugal

POD/MAC-BASED MODAL BASIS SELECTION FOR A REDUCED ORDER NONLINEAR RESPONSE ANALYSIS

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Keywords: Geometric nonlinear dynamics, reduced order modeling, proper orthogonal decomposition, modal assurance criterion.

1 INTRODUCTION

Reduced order nonlinear simulation is often times the only computationally efficient means of calculating the extended time response of large and complex structures under severe dynamic loading. This is because the structure may respond in a geometrically nonlinear manner, making direct numerical integration in physical degrees of freedom (DoF) prohibitive. As for any type of modal reduction scheme, the quality of the reduced order solution is dictated by the modal basis selection. The techniques for modal basis selection currently employed for nonlinear simulation are ad hoc and are strongly influenced by the analyst's subjective judgment. The authors have shown that the common approach for basis selection, consisting of only modes in the excitation bandwidth, is insufficient because it does not capture the nonlinear coupling between low and high frequency modes [1-3]. For all but the simplest structures, the choice of which high frequency modes to include is itself problematic, as was recently demonstrated for a large aircraft fuselage sidewall structure [4]. This work is aimed at developing a reliable and rigorous procedure through which an efficient modal basis can be chosen.

2 FORMULATION

The proposed procedure requires a short, but representative time history of the full-field nonlinear displacement response. This may be obtained by a finite element (FE) method simulation in physical DoF or from an experiment. A Proper Orthogonal Decomposition (POD) [5, 6] analysis is first employed to determine the Proper Orthogonal Values (POVs) and Proper Orthogonal Modes (POMs). The POVs are then used to determine the most contributing POMs. Since POMs change as the loading condition changes, they do not themselves form the preferred basis as the nonlinear modal transformation would need to be repeated for each loading condition. Instead, a set of normal modes (NMs) which resembles a desired set of POMs is identified by employing the Modal Assurance Criterion (MAC) [7]. Such an approach permits determination of a reduced order system that remains applicable over a relatively large nonlinear response regime.

2.1 Finite Element Model and Physical DoF Analysis

An aluminum beam structure previously considered [1-3] served as the basis for the current investigation. The beam measured 0.4572 m x 25.4 mm x 2.286 mm (l x w x h), and had clamped boundary conditions at both ends. Material properties used were: Young's modulus (73.11 GPa), shear modulus (27.59 GPa), mass density (2763 kg/m^3), and mass proportional damping (14.52 1/s).

The beam response was analyzed with the FE code ABAQUS [8]. The FE model consisted of 144 B21 beam elements (145 nodes), each 3.175 mm in length. The B21 element allows single-plane bending and has two translational and one rotational DoFs at each node. Therefore, the total number of DoFs ($3M$) in the model was 435. The clamped boundary conditions were modeled by constraining both translational (transverse and in-plane) and the rotational DOFs at both ends of the beam, reducing the active number of DoFs ($3m$) to 429. The ABAQUS/Explicit solution was used with an automatic time step adjustment, referred in ABAQUS as 'element-by-element'. This approach is known to yield a conservative time step increment.

The beam was subjected to a uniformly distributed random pressure loading. A flat, band-limited random pressure time history was generated by summing equal amplitude sine waves having random phase in the frequency range 0 – 1500 Hz [9]. An overall sound pressure level of 170 dB was used, resulting in a strongly nonlinear response regime. The simulation time was 2.1384 s. The initial transient of 0.5 s was removed to provide 1.6384 s of developed response for the subsequent analysis. A total of 32,768 data points (n) at an output sampling of 50 μs were utilized in the analysis.

2.2 Proper Orthogonal Decomposition

For the planar structures considered in this paper, the NMs are uncoupled between the transverse and in-plane DoFs. Since the POMs most closely matching the NMs are sought, the POD analysis was conducted separately for the transverse and in-plane displacement DOFs. Furthermore, since NM rotational DoFs are not independent from transverse displacement DoFs, no attempt was made to conduct a third POD analysis for the rotational DoFs.

Nonlinear displacement time histories are stored in two matrices $[X_i]$, where $i = t$ (transverse) or m (in-plane membrane). Both matrices are of size n x M, where n is the number of data points in a numerical simulation (or experimental acquisition), and M is the number of active and constrained DoFs for the selected component (or the number experimentally captured response DoFs). The correlation matrix of size M x M can be formed

$$[R_i] = \frac{1}{n}[X_i]^T [X_i] \tag{1}$$

Eigenanalyses are next performed on the correlation matrices $[R_i]$ to yield vectors of M POVs and the corresponding POMs in a form of M x M matrices $[V_i]$.

2.3 Linear Eigenanalysis

The NMs of the system under investigation, $[E]$, are obtained as part of either a normal modes analysis of the numerical model, or via an experimental modal analysis. For the clamped beam, there are $3m$ NMs of size $3M$ each, therefore the matrix $[E]$ is of size $3M$ x $3m$. Two new matrices, $[E_t]$ and $[E_m]$, can now be formed through partitioning of $[E]$. The matrix $[E_t]$ has rows corresponding only to the transverse displacements, and $[E_m]$ has rows corresponding to the in-plane displacements. Consequently their dimensions are M x $3m$.

2.4 Modal Assurance Criterion

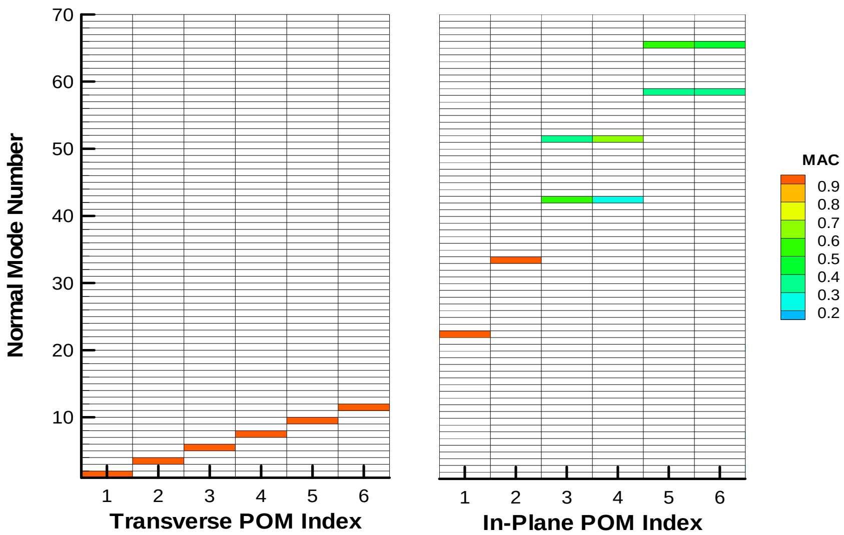
The MAC value sought is a measure of similarity between the k-th column of POM matrix $[V_i]$ and the l-th column of NM $[E_i]$ matrix, and is computed as

$$MAC\left(\{V_i\}_k, \{E_i\}_l\right) = \frac{\left|\{V_i\}_k^T \{E_i\}_l\right|^2}{\left(\{V_i\}_k^T \{V_i\}_k\right)\left(\{E_i\}_l^T \{E_i\}_l\right)} \tag{2}$$

Two MAC matrices of size M x $3m$ can be formed containing the MAC values for the transverse and in-plane pairs of $[V_i]$ and $[E_i]$. In practice, the process of computing MAC values can be accelerated by computing only the values corresponding to the most contributing POMs of interest. This can be achieved by ranking the POMs by their corresponding POVs, which are known to be indicative of the signal power associated with a certain POM [5, 6].

3 RESULTS

A MSC.NASTRAN implementation of a computer code RANSTEP [2, 4] was used to perform the nonlinear reduced order analysis. The formulation is omitted here for the sake of brevity. Two sets of modal basis were considered. The first basis, referred to as the Low Frequency Basis, was chosen on the assumption that inclusion of all of the symmetric modes present in the excitation bandwidth is sufficient. The second set of basis functions was guided by the POD/MAC-based technique, and is subsequently referred to by that name. The Low Frequency Basis approach resulted in a selection of six eigenvectors. For consistency, the six most contributing transverse modes, in terms of their POVs, were selected for the POD/MAC basis. Not surprisingly, the POD/MAC-based technique identified the same transverse modes as the Low Frequency Basis. Their ordering numbers and the corresponding frequencies and MAC values are indicated in Figure 1a and 1c. For the POD/MAC basis, the six most contributing in-plane modes were also selected giving a basis consisting of twelve modes. The in-plane modes ordering numbers and their corresponding frequencies and MAC values are provided in Figure 1b and 1c. Figure 1b confirms earlier results which indicated the need for both low and high frequency modes in the basis [1-3]. The transverse modes selected all had MAC values exceeding 0.9. The correlation between the in-plane NMs and the in-plane POMs however is generally weaker than for the transverse modes. Four in-plane modes (42, 51, 58 and 65) are correlated with more than one POM each, and with generally smaller MAC values relative to the transverse modes. Note that the approach used for POD/MAC basis selection is one of a few alternatives, and is based on fixing the number of included basis functions, since the computational benefit of the reduced order analysis is strongly influenced by this. An alternative approach could be for selection of POMs corresponding to all POVs above some threshold, and then inclusion of all NMs that correlate with those selected POMs by a certain minimum value of MAC.



Mode #	Frequency
Transverse	(Hz)
1	57.78
3	312.1
5	770.1
7	1431
9	2293
11	3354
In-plane	(Hz)
22	11,240
33	22,480
42	33,710
51	44,920
58	56,110
65	67,270

(a) (b) (c)

Figure 1. Correlation matrix between (a) transverse and (b) in-plane NMs and POMs, and (c) NM frequencies. Note the correlation of high frequency modes with in-plane POMs in Figure 1(b).

Figure 2 shows the improvement of reduced order simulation results obtained with the POD/MAC-based modal basis selection over those obtained using the Low Frequency Basis. It is seen that even though the in-plane modes do not contribute directly to the transverse response, their presence is important for accurate modeling of the overall system dynamics.

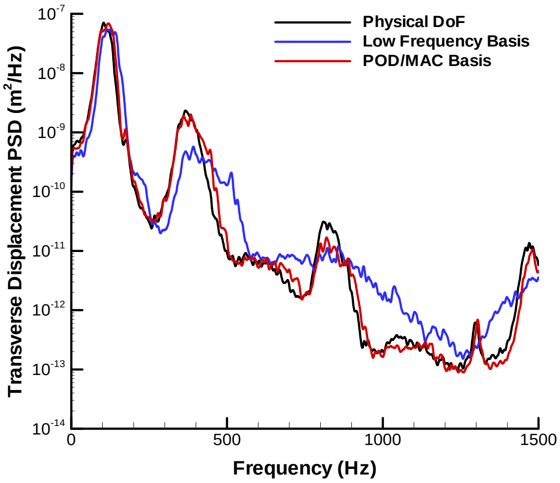


Figure 2. Comparison of reduced order simulations using POD/MAC basis and a Low Frequency Basis with a physical DoF solution.

4 CONCLUSIONS

A feasibility study was conducted to explore the applicability of a POD/MAC basis selection technique to a nonlinear structural response analysis. For the case studied the application of the POD/MAC technique resulted in a substantial improvement of the reduced order simulation when compared to a classic approach utilizing only low frequency modes present in the excitation bandwidth. Further studies are aimed to expand application of the presented technique to more complex structures including non-planar and two-dimensional configurations. For non-planar structures the separation of different displacement components may not be necessary or desirable.

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