CASE 3: COMPUTATIONS OF FLOW OVER THE HUMP MODEL USING HIGHER ODER METHOD WITH TURBULENCE MODELING

P.Balakumar
Flow Physics and Control Branch, NASA Langley Research Center, Hampton, VA 23681

Introduction

The flow over the two-dimensional hump model is computed by solving the RANS equations with k-ω (SST) model.

Solution Methodology

The governing equations, the flow equations and the turbulent equations, are solved using the 5th order accurate weighted essentially non-oscillatory (WENO) scheme for space discretization and using explicit third order total-variation-diminishing (TVD) Runge-Kutta scheme for time integration. The WENO and the TVD methods and the formulas are explained in [1] and the application of ENO method to N-S equations is given in [2]. The solution method implemented in this computation is described in detail in [3].

Model used

Standard k-ω (SST) model is used and the equations and the model coefficients are described in [4, 5, 6].

Implementation and Details

The computational domain extends from x/c=-10. to 4. in the streamwise direction and extends from the splitter plate to the upper tunnel wall in the normal direction. The leading edge of the splitter plate is modeled as a super ellipse with 0.25 in. half thickness and an aspect ratio of 2. The leading edge is located at x/c=-5.9. C-Type grid is used around the splitter plate and a rectangular grid is added upstream of the leading edge as shown in Fig. 1. This grid overlaps the C grid and 5^{th} order central interpolation is used to transfer the flow variables from one grid to the other at the boundaries. (651*151) grid size is used around the splitter plate and the hump and (101*51) grid size is used in the rectangular region.

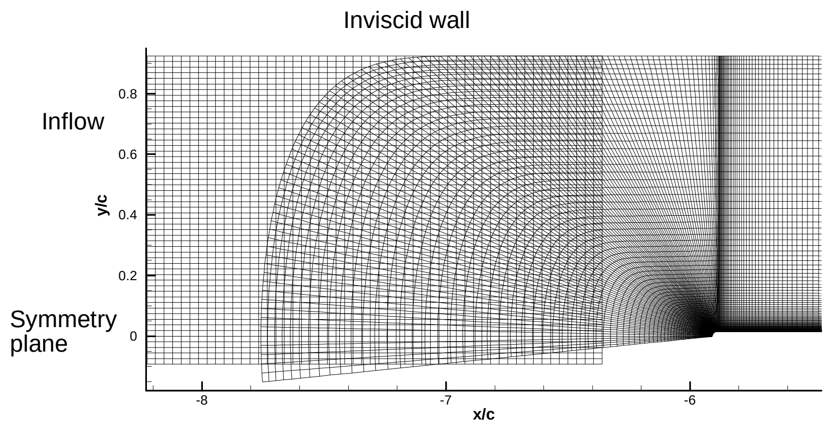

Figure 1: Overlapping grid near the leading edge of the splitter plate.

Following boundary conditions are implemented at different boundaries:

1. At the upper wall inviscid conditions are applied.
$$\frac{\partial \rho}{\partial y} = \frac{\partial u}{\partial y} = \frac{\partial E}{\partial y} = v = 0. \tag{1}$$
2. At the lower wall viscous conditions are used.
$$u = v = 0, \quad T = T_w = T_{free\,stream}, \tag{2}$$
and ρ is computed from the continuity equations.
3. From the leading edge of the splitter plate to the inflow boundary symmetric conditions are used.
4. At the inflow boundary stagnation pressure, one Riemann variable and normal velocity v=0 are prescribed and the second Riemann variable is solved for to obtain the other flow quantity.
5. At the outflow boundary the pressure is specified to obtain the required Mach number and characteristic-type boundary conditions are implemented similar to as described in [7] to obtain other flow variables.
6. In the suction case, boundary conditions are applied on the surface of the hump across the suction slot. The suction slot extends from x/c=.6541 to .6584 and across the slot normal mass flow rate is specified to the experimental value. A suction distribution of the form
$$(\rho v)_n = f_{\max} \sin^2\left(\frac{\pi(x - x_{start})}{(x_{end} - x_{start})}\right) \tag{3}$$
is used. Other forms have been tried and all of them yield the same results for a fixed total suction rate. The other flow quantities are obtained using the characteristic type boundary conditions [7].

Following boundary conditions are implemented for the turbulent quantities at different boundaries.

1. At the inflow boundary small values are prescribed for k and μ_T.

$$\frac{k}{U_\infty^2} = 10^{-7},$$
$$\frac{\mu_T}{\mu_\infty} = .009. \tag{4}$$

2. At the outflow boundary k and ω are solved for from the governing equations. Higher order extrapolation condition is also tried and it gives the same results.
3. At the lower viscous wall k=0 condition is used and following exact boundary condition is derived for ω.
4. In the suction case, across the suction slot linear extrapolation is used to obtain the turbulent quantities on the surface.

Since the variable ω becomes singular near a viscous wall, in practice a large approximate value is prescribed at the wall.

$$\omega_{wall} = \frac{60\mu_w}{\rho_w \beta d_w^2}, \tag{5}$$

where d_w is the distance to the first grid point from the wall. This is an approximate boundary condition and when it is implemented in the higher order scheme, oscillations and convergence problems are encountered and the following exact condition is derived for ω_{wall}. By realizing that $\left(1/y^2 \right)$ type singularity for the variable ω arises because of the balance between the dissipation term and the viscous diffusion term in the ω equation, the singularity is removed by rewriting the variable ω as

$$\omega = \frac{C}{y_n^2}\omega_1, \tag{6}$$

where y_n is the normal distance to the wall, C is a constant and ω_1 is the new variable which is now regular near the wall. When this is substituted into the ω equation the following equation is obtained for ω_1, which is similar to the ω equation except for the source term.

$$\begin{aligned}
&\frac{\partial}{\partial t}(\rho\omega_1) + \frac{\partial}{\partial x_j}(\rho u_j \omega_1) = \gamma\rho\Omega^2 y_n^2 \mathrm{Re} + \frac{1}{\mathrm{Re}}\frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_w}\right)\frac{\partial \omega_1}{\partial x_j}\right] \\
&-\frac{\omega_1}{\mathrm{Re}}\frac{1}{y_n^2}\left[\beta\rho\omega_1 - 6\left(\mu + \frac{\mu_T}{\sigma_w}\right)\left(\frac{\partial y_n}{\partial x_j}\right)^2\right] \\
&\left(-\frac{2}{y_n \mathrm{Re}}\right)\left[\left(\mu + \frac{\mu_T}{\sigma_w}\right)\frac{\partial \omega_1}{\partial x_j}\frac{\partial y_n}{\partial x_j} + \frac{\partial}{\partial x_j}\left\{\left(\mu + \frac{\mu_T}{\sigma_w}\right)\omega_1 \frac{\partial y_n}{\partial x_j}\right\}\right] \\
&+\frac{2}{y_n} u_j \rho\omega_1 \frac{\partial y_n}{\partial x_j} + 2(1 - F_1)\sigma_{\omega 2}\frac{\rho}{\omega_1}\frac{\partial k}{\partial x_j}\mathrm{Re}\, y_n\left(\frac{\partial \omega_1}{\partial x_j} y_n - 2\frac{\partial y_n}{\partial x_j}\omega_1\right).
\end{aligned} \tag{7}$$

The value of ω_1 at the wall becomes

$$\omega_{1wall} = \frac{6\mu}{\beta\rho}\left(\frac{\partial y_n}{\partial x_j}\right)^2. \tag{8}$$

Here the variable ω is nondimensionalised by

$$\omega = \frac{\omega^*}{\left(\frac{U_0^2}{\nu_0}\right)} \text{ and } C = \frac{1}{\mathrm{Re}^2}. \tag{9}$$

Hence the procedure is to use the ω_1 equation for the first few points near the wall and switch to the ω equation away from the wall. In these computations ω_1 equation is solved for the first ten points near the wall. Figure 2 shows the distribution of k, ω and ω_1 near the wall and it is seen that this technique resolves the viscous layer smoothly even though ω is infinite at the wall.

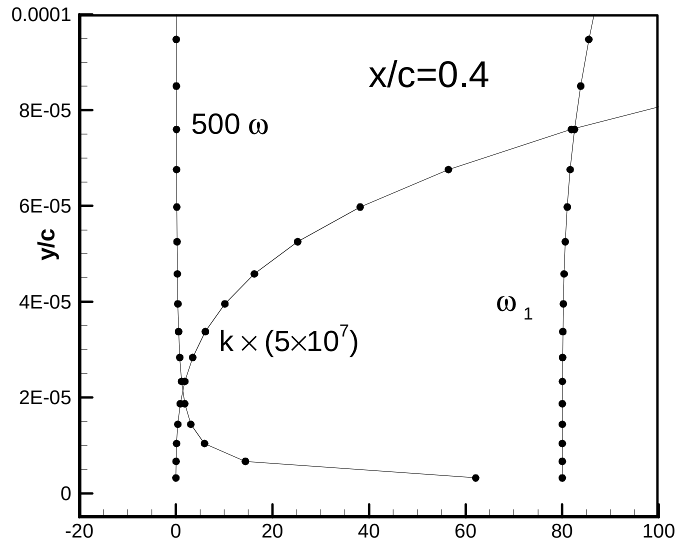

Figure 2: Variation of k, ω and ω_1 near the wall at x/c=0.4.

Figure 3 shows the contours of the U velocity near the leading edge region and for the entire computational domain. Near the leading edge region the flow separates and forms a small separation bubble. In the no flow case, the solution converged for the flow equations and the turbulent equations very well. In the suction case, the maximum residual in the ω equation near the suction slot edges converged only by three orders. This may be due to the boundary conditions used for the turbulent quantities near the suction slot.

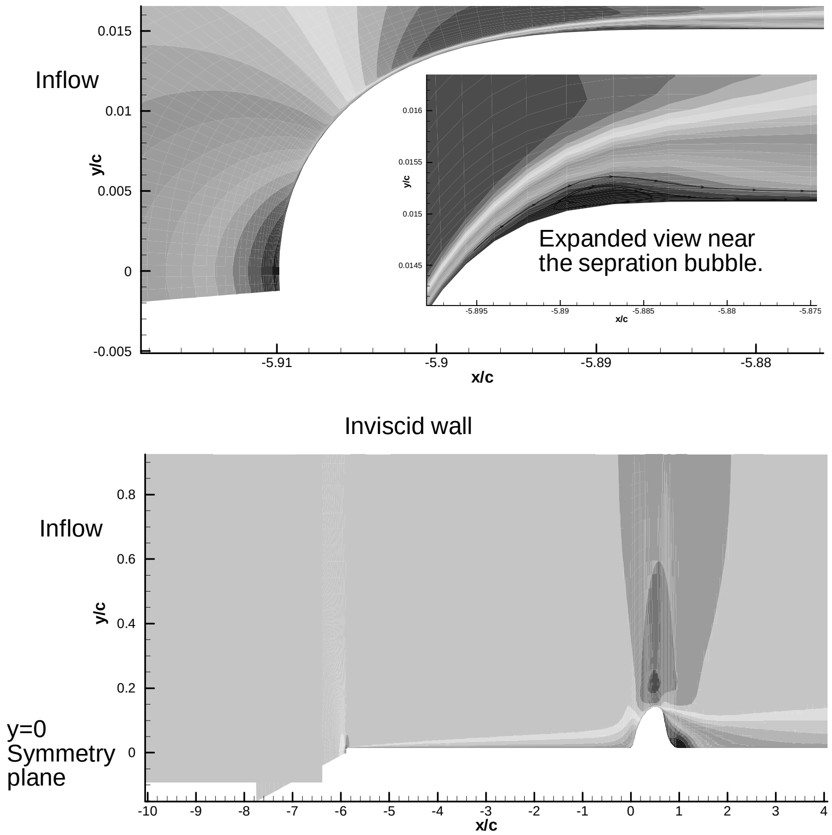

Figure 3: Contours of U velocity near the leading edge and over the hump.

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