Can the Equivalent Sphere Model Approximate Organ Doses in Space Radiation Environments?

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Abstract - *In space radiation calculations it is often useful to calculate the dose or dose equivalent in blood-forming organs (BFO), the skin or the eye. It has been customary to use a 5cm equivalent sphere to approximate the BFO dose. However, previous studies have shown that a 5cm sphere gives conservative dose values for BFO. In this study we use a deterministic radiation transport with the Computerized Anatomical Man model to investigate whether the equivalent sphere model can approximate organ doses in space radiation environments. We find that for galactic cosmic rays environments the equivalent sphere model with an organ-specific constant radius parameter works well for the BFO dose equivalent and marginally well for the BFO dose and the dose equivalent of the eye or the skin. For solar particle events the radius parameters for the organ dose equivalent increase with the shielding thickness, and the model works marginally for BFO but is unacceptable for the eye or the skin. The ranges of the radius parameters are also shown and the BFO radius parameters are found to be significantly larger than 5 cm in all cases.*

I. INTRODUCTION

Space radiation to astronauts from solar particle events (SPE) and galactic cosmic rays (GCR) is a major hazard for human space explorations, and in space radiation calculations one often needs to calculate the dose or dose equivalent in organs such as BFO, the skin or eyes. The detailed thickness distribution functions of organs can be used for accurate calculations,[1,2] while sometimes the equivalent sphere model (ESM) with an organ-specific constant radius parameter is used to estimate the organ dose in order to save the computation time. It has been customary to use a 5cm equivalent sphere to simulate the BFO dose. However, previous studies[1−4] have demonstrated that a 5cm sphere gives higher (i.e., conservative) dose values compared with the exact BFO dose. One study[3] concluded that a 9cm sphere is a reasonable approximation for the BFO dose or dose equivalent in SPE environments when there is an Aluminum shielding of 2-20 g/cm^2.

In this study we investigate whether the equivalent sphere model with an organ-specific constant radius parameter can approximate dose or dose equivalent values in BFO, the eye or the skin in SPE or GCR environments. We use a deterministic radiation transport[5] with organ geometry from the Computerized Anatomical Man (CAM) model.[6] Six different SPE environments

and seven different GCR environments are used in order to see how much the results depend on the choice of the space radiation environment. Calculations have been performed with and without extra shielding materials surrounding the organs in order to study the dependence of the ESM radius parameters on the thickness of the shielding material. We also give a summary of the ranges of the radius parameters of each organ in SPE and GCR environments so that one can easily see in which cases the equivalent sphere model works.

II. METHODS

For the following cumulative thickness distribution function of an organ:

$$F(t) \equiv \int_0^t f(t')dt', \qquad (1)$$

the thickness distribution function is given by

$$f(t) \equiv \frac{dF(t)}{dt}, \qquad (2)$$

and it is already normalized. In Fig. 1 the curves with circles represent the cumulative thickness distribution function $F(t)$ of BFO (solid), the eye (dashed) and the skin (dot-dashed) from the CAM model.[7] The curves

without circles, obtained numerically from the cumulative thickness distribution functions, represent the corresponding organ thickness distribution functions multiplied by the thickness, i.e., $tf(t)$. In this way, the area under the curves without circles is proportional to the probability of the corresponding thickness range.
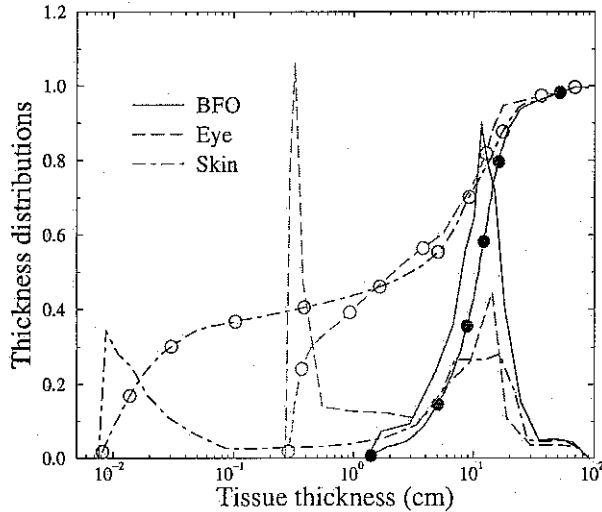

FIG. 1: Thickness distributions for the skin, the eye and BFO from the CAM model. Curves with circles represent the cumulative distribution functions and curves without circles represent the thickness distribution functions multiplied by the thickness.

We use a version of the deterministic radiation transport HZETRN[5]. The six SPE environments used in this study are the Aug. 4, 1972, Aug. 12, 1989, Sept. 29, 1989, Oct. 19, 1989, Mar. 23, 1991 events,[8] and the Jan. 20, 2005 event,[9] respectively. For the Jan. 20, 2005 event a parameterized primary alpha particle flux[10] is also included. The seven GCR environments are taken from those implemented in HZETRN,[5] and they are the solar maximum environments in 1958-1959, 1970-1971, 1981-1982, 1989, and the solar minimum environments in 1965, 1977 and 1986-1987. The dose and dose equivalent values are calculated as a function of the thickness of a water slab that is behind a shielding material of a given thickness, and the ICRP 60[11] quality factor is used for the conversion to the dose equivalent if not specified otherwise. For example, the dose equivalent as a function of water depth when there is no extra shielding material is shown in Fig. 2 for the SPE environments and in Fig. 3 for the GCR environments.

We use water to simulate tissue and calculate the organ dose and dose equivalent according to the following:[1]

$$D_i = \int D(t) f_i(t) dt, \quad H_i = \int H(t) f_i(t) dt, \quad (3)$$

where i labels the different organ, and $D(t)$ and $H(t)$ represents the dose and dose equivalent, respectively, as a function of water depth t in a space radiation environment, which for these 1-dimensional slab calculations is considered to be uni-directional instead of being isotropic. We calculate the dose-depth curves $D(t)$ and $H(t)$ both with and without a slab shielding material of thickness t_s in front of the semi-infinite water slab of thickness t. Because the deterministic radiation transport we use here assumes the straight-ahead approximation, i.e., the nuclear fragments do not change their directions relative to the projectile nucleus, the organ dose calculated using $D(t)$ or $H(t)$ behind a slab shielding material of thickness t_s is equivalent to the dose of the organ that is surrounded by a shell of the shielding material of thickness t_s with the shell having a large inner radius.
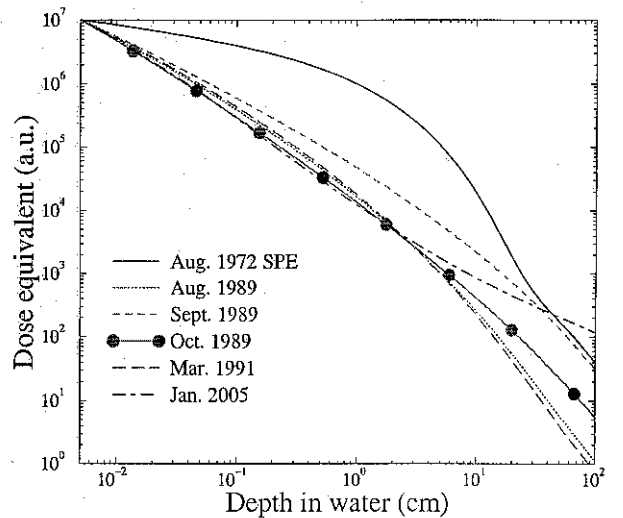

FIG. 2: Dose equivalent as a function of water depth in different SPE environments with no extra shielding.

The ESM radius parameters in the equivalent sphere model, R_i^D for the dose and R_i^H for the dose equivalent of organ i, are calculated by finding the water depth where the dose or dose equivalent is equal to the exact organ values calculated from Eq. (3), i.e.,

$$D(R_i^D) = D_i, \quad H(R_i^H) = H_i. \quad (4)$$

Eqs. (3-4) show that the radius parameters are determined by the shapes of the dose-depth curves but not by their overall magnitudes, and this is the reason that we have renormalized the dose-depth curves to arbitrary units in Fig. 2-3 in order to better show the difference in their shapes.
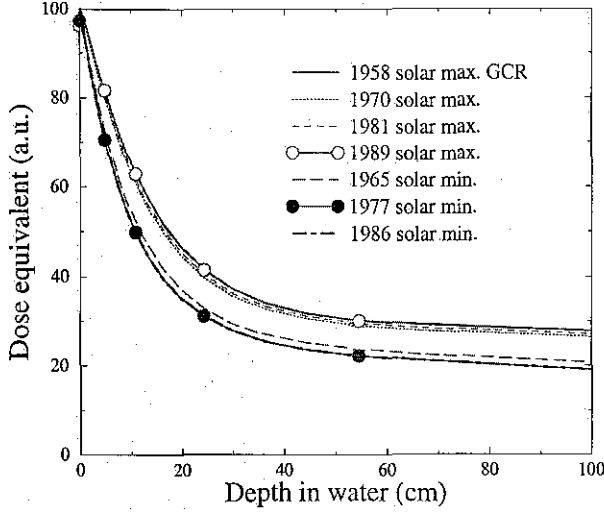
FIG. 3: Dose equivalent as a function of water depth in different GCR environments with no extra shielding.

III. RADIUS PARAMETERS OF ORGANS WITH NO EXTRA SHIELDING

First we have calculated the ESM radius parameters for the organ dose and dose equivalent when there is no extra shielding material. These radius parameters for each SPE environment are shown in Table I. We find that the BFO radius parameters for different SPEs for both the dose equivalent and the dose are within about 25% of each other. The radius parameters of the eye for different SPEs are within a factor of 2 of each other. However, the SKIN radius parameters for the Aug. 1972 SPE are far from those for the other SPEs, as much as larger by a factor of 8. This can be understood qualitatively from the dose-depth curves shown in Fig. 2. As the dose equivalent values of SPEs decrease by orders of magnitude from 0.005 cm to a few cm in water, the total skin dose or dose equivalent mostly comes from that at small thicknesses, say, below 1 g/cm², which make up roughly about half of the thickness distribution of the skin. Therefore the total skin dose is about half of the dose value at a small depth scale that is representative of the skin thickness distribution at small thicknesses. As a result, the skin radius parameter is roughly determined by the water depth where the dose at that representative small depth scale decreases by a factor of 2. We can see that the dose-depth curve of the Aug. 1972 SPE has a much smaller slope at small thicknesses than those of the other SPEs, and this leads to larger radius parameters for the skin in the Aug. 1972 SPE environment.

The ESM radius parameters for each GCR environment are shown in Table II. We find that the radius parameters of each organ are much larger than the corre-

TABLE I: Radius parameters (in cm) for the organ dose equivalent (H) and organ dose (D) in SPE environments when there is no shielding material. The SPE number 1 to 6 represents the Aug. 1972, Aug. 1989, Sept. 1989, Oct. 1989, Mar. 1991, and Jan. 2005 event, respectively.

SPE Event #	1	2	3	4	5	6
R_{BFO}^{H}	6.8	6.6	7.4	7.1	6.4	7.7
R_{Eye}^{H}	1.1	0.68	0.77	0.69	0.67	0.69
R_{Skin}^{H}	0.21	0.038	0.043	0.035	0.039	0.035
R_{BFO}^{D}	6.9	6.8	7.7	7.2	6.6	8.0
R_{Eye}^{D}	1.2	0.74	0.87	0.76	0.74	0.78
R_{Skin}^{D}	0.41	0.055	0.070	0.049	0.057	0.049

sponding value in the SPE environments, especially for the eye and the skin. Furthermore, the radius parameters in solar maximum GCR environments are larger than those in solar minimum GCR environments. For the organ dose equivalent, the radius parameters for different GCR environments are close to each other for each of the three organs, especially for BFO. However, the radius parameters for the organ dose for different GCR environments are close to each other only for BFO.

TABLE II: Radius parameters (in cm) for the organ dose equivalent (H) and organ dose (D) in GCR environments when there is no surrounding material. The GCR number 1 to 4 represents the solar maximum environments in 1958-1959, 1970-1971, 1981-1982, and 1989, respectively, while number 5 to 7 represents the solar minimum environments in 1965, 1977 and 1986-1987, respectively.

GCR Environment #	1	2	3	4	5	6	7
R_{BFO}^{H}	10.7	10.6	10.6	10.7	10.3	10.2	10.2
R_{Eye}^{H}	4.8	4.7	4.7	4.8	4.1	3.9	3.9
R_{Skin}^{H}	5.5	5.3	5.4	5.5	4.5	4.3	4.3
R_{BFO}^{D}	11.9	11.5	11.6	11.8	11.0	11.0	11.0
R_{Eye}^{D}	12.	9.3	10.	11.	5.6	5.1	5.1
R_{Skin}^{D}	18.	14.	15.	17.	7.3	6.6	6.6

Because the dose-depth curves in GCR environments change much more slowly with water depth compared with those in SPE environments, they have much smaller curvatures. Consequently the ESM radius parameters of each organ in GCR environments are significantly larger than those in SPE environments, and they are also on the same order of magnitude as the average thickness value of the organ, which is about 13., 6.8 and 7.5 cm, respectively, for BFO, the eye and the skin from the CAM model. Also, a solar maximum GCR environment has about the same particle flux at high energies, above about 10 GeV per nucleon, but far less lower energy particles than a solar minimum GCR environment. Consequently the dose equivalent-depth curves for solar

maximum GCR environments decrease more slowly than those for solar minimum GCR environments, as shown in Fig. 2, and this leads to somewhat larger values for the ESM radius parameters in solar maximum GCR environments.

IV. EFFECTS OF SHIELDING MATERIALS ON THE RADIUS PARAMETERS OF ORGANS

Organ doses are often calculated with space radiation environments that have been modified by surrounding materials, such as a spacesuit, a spacecraft or the habitat. To investigate how the ESM radius parameters change in the presence of such shielding, we have calculated the dose and dose equivalent as a function of water depth behind a shielding material. The slab shielding material is either water or Aluminum at thickness between 0 and 20 g/cm^2, and it is positioned in front of the water slab that simulates tissue. We then determine the radius parameters using Eqs. (3-4).

The radius parameters for the Aug. 1972 and the Oct. 1989 SPEs are shown in Fig. 4 for BFO, in Fig. 5 for the eye, and in Fig. 6 for the skin, as a function of the areal density of the water shielding. We see that all the radius parameters increase with the thickness of the shielding material. Furthermore, the difference in the radius parameters between the two SPE environments can be significant. The BFO radius parameters increase from about 7 cm when there is no shielding material to up to 10.5 cm when there is a shielding material of 20 g/cm^2. This increase is only up to 50%, therefore one may use the equivalent sphere model with a constant radius parameter to simulate the BFO dose or dose equivalent in SPE environments in situations where an error on the order of a factor of 2 (see Fig. 2) can be tolerated. On the other hand, the increase with the shielding thickness is much stronger for the eye (by a factor of up to 5) and the strongest for the skin radius parameters (by a factor of up to 100). This is mainly due to the large curvatures of the dose-depth curves in SPE environments. As a result, the dose behind a finite shielding material decreases much more slowly with water depth, and the smaller curvature of the dose-depth curve leads to a larger radius parameter. Figs. 4-6 show that the equivalent sphere model with an organ-specific constant radius parameter works marginally for BFO but cannot reliably simulate the dose or dose equivalent of the eye or the skin in SPE environments.

The radius parameters for both the 1977 solar minimum and the 1989 solar maximum GCR environments are shown in Fig. 7 for BFO, in Fig. 8 for the eye, and in Fig. 9 for the skin, as a function of the areal density of the water shielding. For the radius parameters for the organ dose equivalent, we find that the ranges of change with the shielding thickness in GCR environments are much smaller than those in SPE environments.
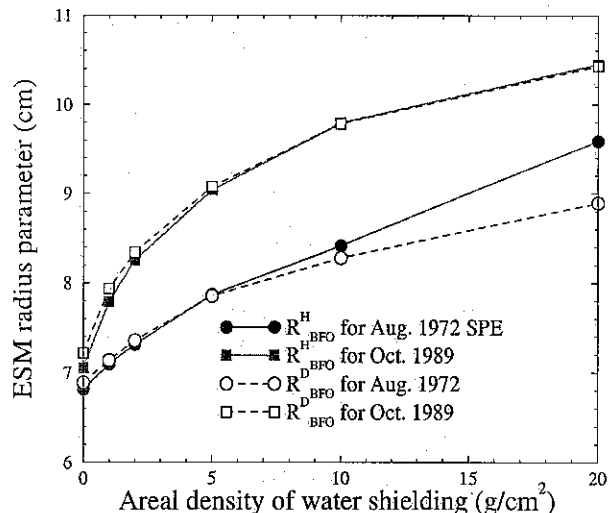

FIG. 4: The radius parameters for BFO in two SPE environments as a function of the areal density of the water shielding.
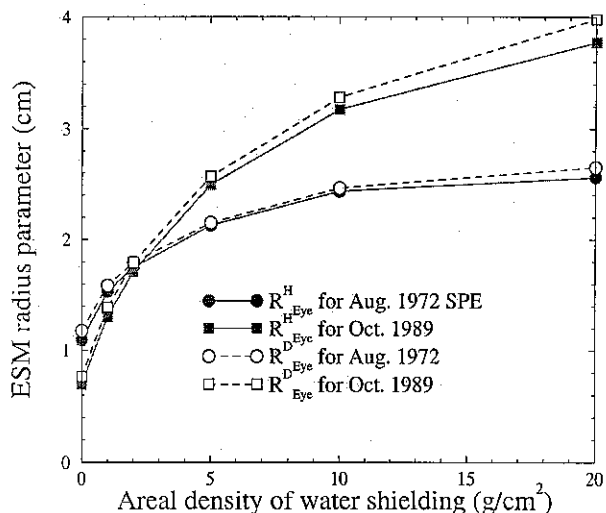

FIG. 5: The radius parameters for the eye in two SPE environments as a function of the areal density of the water shielding.

Furthermore, the radius parameters for the organ dose equivalent in solar minimum GCR environments are essentially the same as those in solar maximum GCR environments when there is a shielding material of about 20 g/cm^2. Fig. 7 also shows the radius parameters for the BFO dose equivalent when the older ICRP26 quality factor[12] is used (dot-dashed curves), and we see that the difference between the ICRP26 and ICRP60 quality factors has little effect on the BFO radius parameters. We
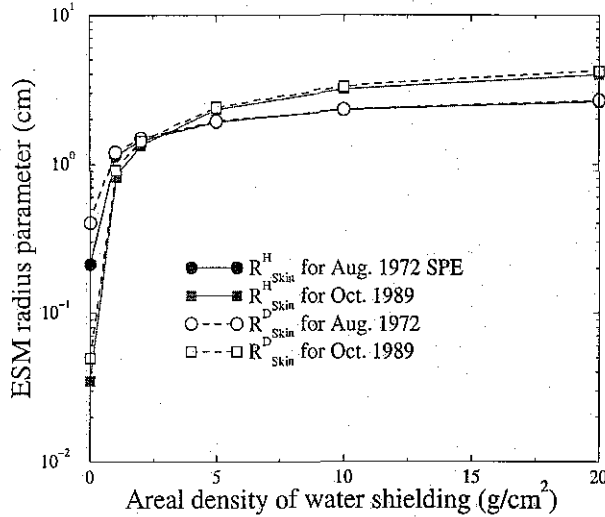
FIG. 6: The radius parameters for the skin (in the logarithmic scale) in two SPE environments as a function of the areal density of the water shielding.
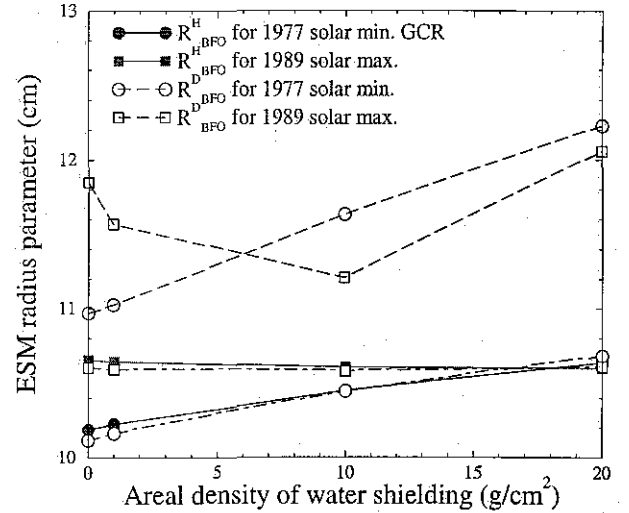

FIG. 7: The radius parameters for BFO in GCR environments as a function of the areal density of the water shielding surrounding the organ. The dot-dashed curves represent the radius parameters for the BFO dose equivalent when the ICRP26 quality factor is used.
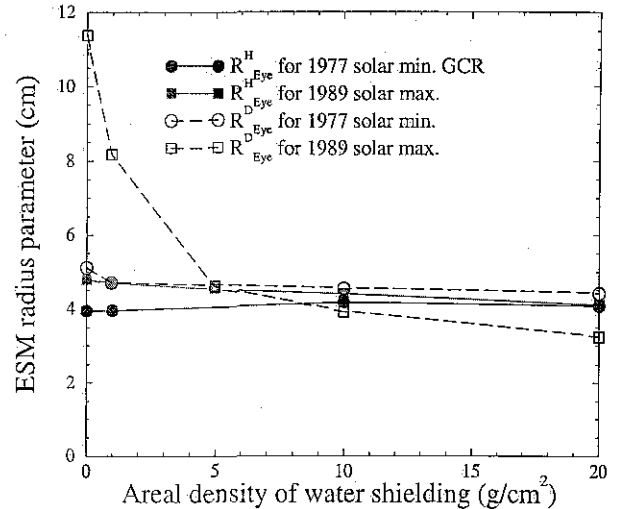

FIG. 8: The radius parameters for the eye in GCR environments as a function of the areal density of the water shielding.

find that this is also true for the radius parameters of the eye and the skin. Overall, the radius parameters for the organ dose equivalent are found to be approximately from 10 to 11 cm for the BFO, 3.7 to 4.8 cm for the eye, and 3.5 to 5.6 cm for the skin in GCR environments. Therefore the equivalent sphere model with a constant radius parameter works well for the BFO dose equivalent and marginally well for the eye or skin dose equivalent in GCR environments.

The ESM radius parameters for the organ dose in GCR environments, as shown in Figs. 7-9 as well as Table II, can be quite different from the corresponding radius parameters for the organ dose equivalent. This difference is much smaller in SPE environments where heavy ion contributions to the organ dose equivalent is negligible. For the organ dose, the range of the ESM radius parameters is approximately from 10 to 13 cm for the BFO, 2.8 to 12 cm for the eye, and 2.3 to 19 cm for the skin. Therefore the equivalent sphere model with a constant radius parameter works marginally for the BFO dose but not for the eye or skin dose in GCR environments.

Fig. 10 shows the BFO radius parameters with water or Aluminum shielding in the Oct. 1989 SPE (curves with symbols) or the 1977 solar minimum GCR environment (curves without symbols). We find that the difference in the radius parameters between water and Aluminum shielding is typically smaller than the range of the radius parameters over the studied shielding thicknesses, especially for the radius parameters for the dose equivalent. Fig. 10 and Fig. 4 are also consistent with an earlier study[3] which concluded that a 9cm sphere is

a reasonable approximation for the BFO dose or dose equivalent in SPE environments when there is an Aluminum shielding of 2-20 g/cm^2.

A summary of the ranges of the ESM radius parameters for two SPE and two GCR environments for each of the three organs is provided in Table III. When the change of the dose or dose equivalent over a given range
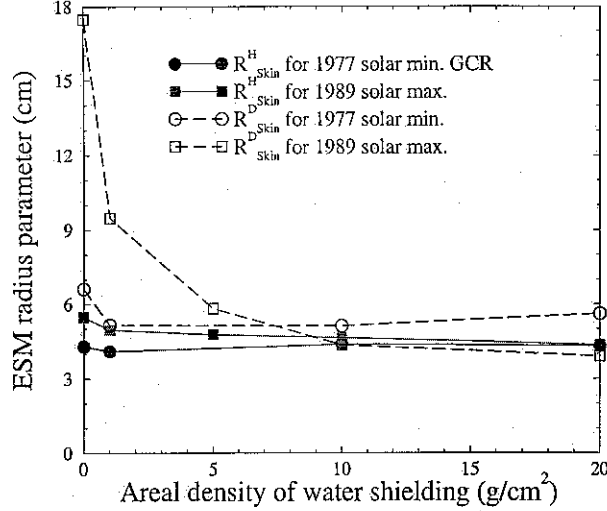
FIG. 9: The radius parameters for the skin in GCR environments as a function of the areal density of the water shielding.
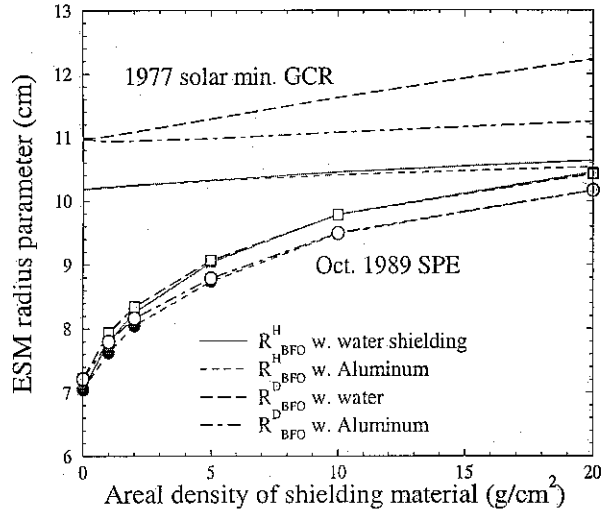

FIG. 10: The BFO radius parameters as a function of the areal density of the water or Aluminum shielding.

is small, the equivalent sphere model with a constant radius parameter within that range can provide a reasonable approximation of the organ dose or dose equivalent.

V. DISCUSSION

We see that all the radius parameters for the organ dose equivalent are smaller than the average thickness value of the organ. This is because the dose equivalent-depth curves in both GCR and SPE environments are

TABLE III: Ranges of the ESM radius parameters (in cm) for the organ dose equivalent (H) and organ dose (D) in two GCR and two SPE environments with a water or Aluminum shielding of thickness between 0 and 20 g/cm^2.

	1977 and 1989 GCRs	Aug.'72 and Oct.'89 SPEs
R_{BFO}^{H}	10-11	6.8-10.5
R_{Eye}^{H}	3.7-4.8	0.69-3.8
R_{Skin}^{H}	3.5-5.6	0.035-4.0
R_{BFO}^{D}	10-13	6.9-10.5
R_{Eye}^{D}	2.8-12	0.76-4.0
R_{Skin}^{D}	2.3-19	0.049-4.2

mostly convex over the range of water depths that is important for the organ dose equivalent, i.e., the curvatures are non-negative. Jansen's inequality states that, for a real convex function $\phi(x)$, one has

$$\phi(E\{x\}) \leq E\{\phi(x)\}, \tag{5}$$

where $E\{Y(x)\}$ represents the expectation value of $Y(x)$ with respect to a non-negative probability distribution function in variable x. Replacing $\phi(x)$ by $H(t)$ leads to

$$H(\bar{t}_i) \leq H_i, \tag{6}$$

where $\bar{t}_i$ is the average thickness value of organ i. As the dose equivalent-depth curves in both GCR and SPE environments mostly decrease with depth, Eq. (6) means

$$R_i^{H} \leq \bar{t}_i. \tag{7}$$

On the other hand, the radius parameters for the organ dose in GCR environments could be higher than the average thickness value of the organ, as seen from Table II, because sometimes a significant part of the dose-depth curves can be concave. Note that the curvature of the dose equivalent-depth curve matters more to the above inequality equations than the slope of the curve. One can easily see from Eqs. (3-4) that, if the dose equivalent changes linearly with depth over the relevant depth range, one has $R_i^{H} = \bar{t}_i$ regardless of the value of the coefficient of the linear term.

We have shown that the equivalent sphere model with an organ-specific constant radius parameter does not work well for either the dose or the dose equivalent of the eye or the skin in SPE environments. In GCR environments it works marginally for the dose equivalent of the eye or the skin but does not work for their doses. One possibility to improve the equivalent sphere model is to use two radius parameters to represent the eye or the skin, because it is clear from Fig. 1 that these two organs are better represented by a superposition of two thickness distribution functions instead of one.

VI. SUMMARY

We have used a deterministic radiation transport with the organ geometry from the Computerized Anatomical Man model to study when the equivalent sphere model with an organ-specific constant radius parameter can approximate the organ dose or dose equivalent in space radiation environments. The organ doses have been evaluated with a water or Aluminum shielding of an areal density from 0 to 20 g/cm^2. For the six different SPE environments, the BFO radius parameters are approximately between 6.4 and 8.0 cm in case of no shielding, and they increase with the thickness of the shielding material (by up to 50% at 20 g/cm^2). On the other hand, the radius parameters for the eye or the skin increase sharply with the shielding thickness. Therefore for SPE environments the equivalent sphere model works marginally for BFO but does not work for the estimation of the dose or dose equivalent of the eye or the skin. For the seven different GCR environments, the radius parameters for the dose equivalent are approximately between 10 and 11 cm for the BFO, 3.7 to 4.8 cm for the eye, and 3.5 to 5.6 cm for the skin. The radius parameters are between 10 and 13 cm for the BFO dose. Therefore for GCR environments the equivalent sphere model works well for the BFO dose equivalent, marginally well for the dose equivalent of the eye or the skin as well as the BFO dose, but not for the dose of the eye or the skin.

ACKNOWLEDGMENTS

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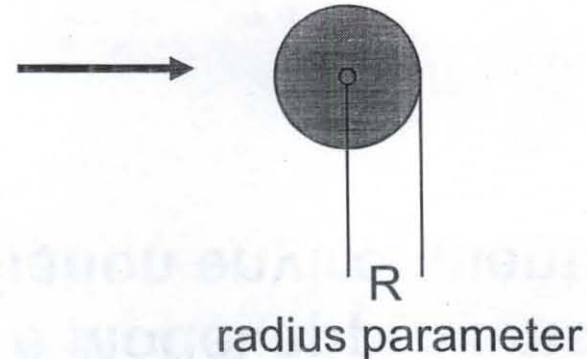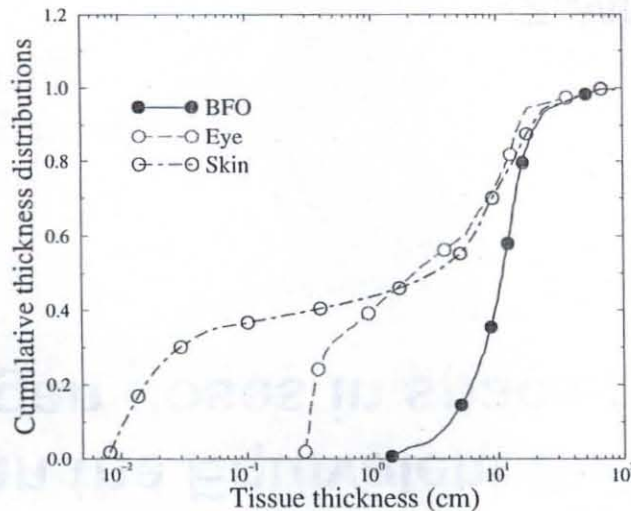# Can the Equivalent Sphere Model approximate organ doses in space radiation environments?

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Overview

Computation is often much easier if one represents an organ with an equivalent sphere. For blood forming organs (BFO), it has been customary to use a 5 cm equivalent sphere to simulate its dose.

It is well known that a 5cm sphere gives conservative dose values.
A 9cm sphere was found to better approximate BFO doses in SPEs.

Bier, Townsend & Maxson, Adv. Space Res. 21 (1998)

Goal

Investigate the Equivalent Sphere Model (ESM)
radius parameters for
•Dose
•Dose equivalent
in
•BFO
•Skin
•Eye
in different space radiation environments
•Solar Particle Events (SPE)
•Galactic Cosmic Ray (GCR)

→Where does ESM work and what are the R (radius) values?
→Where does it not work?

Method

Take 6 different SPEs and 7 GCRs for radiation environments

Use a 1-D deterministic radiation transport (HZETRN)
to get dose-depth curves D(t) & H(t)   *Wilson et al., NASA TP-3495 (1995)*

Take skin, eye and BFO thickness distributions f(t)
from the Computerized Anatomical Man (CAM) model
   Billings & Yucker, NASA CR-134043 (1973)

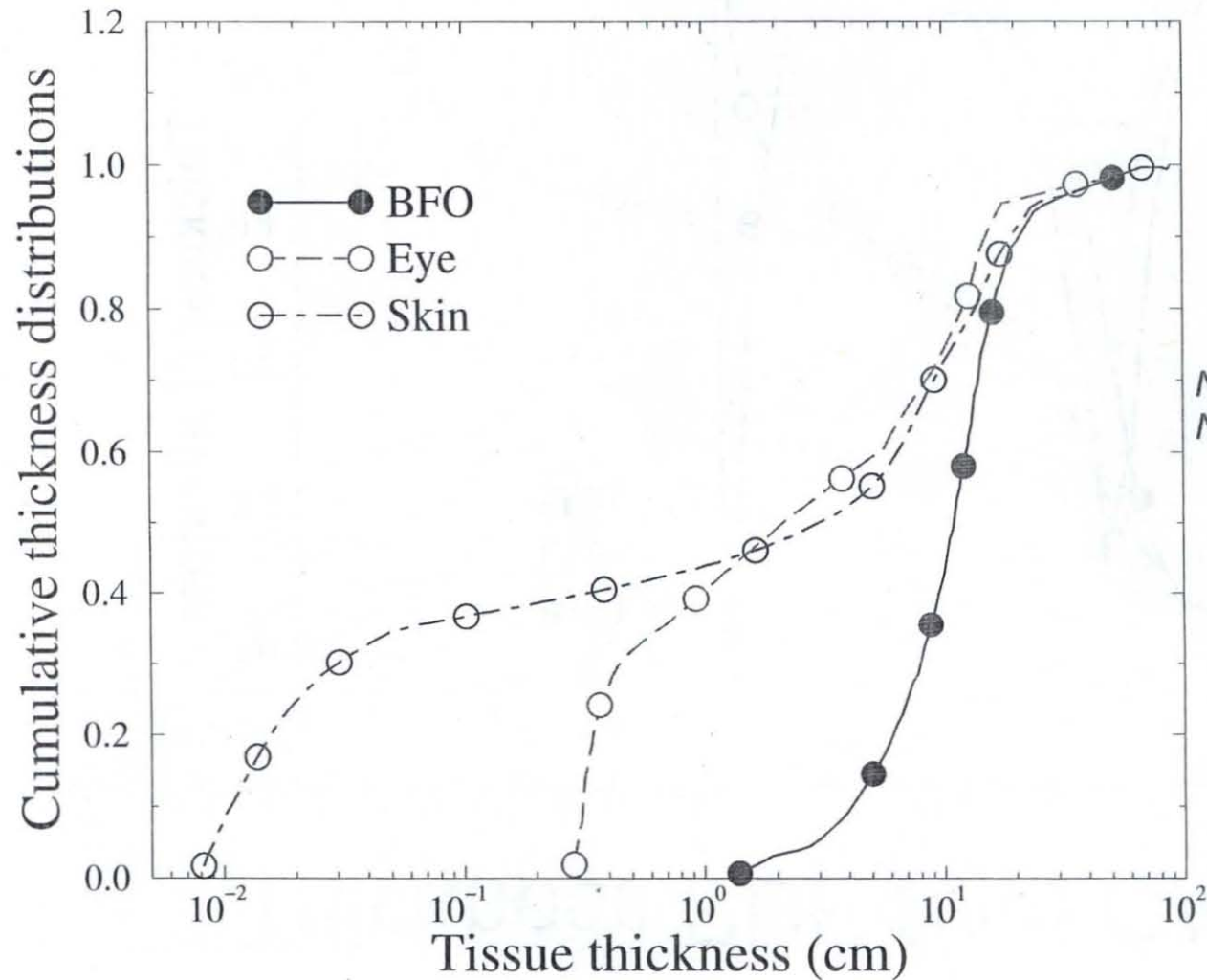Calculate dose(D) and dose equivalent(H) of organ i

$$D_i = \int D(t) f_i(t) dt, \; H_i = \int H(t) f_i(t) dt$$

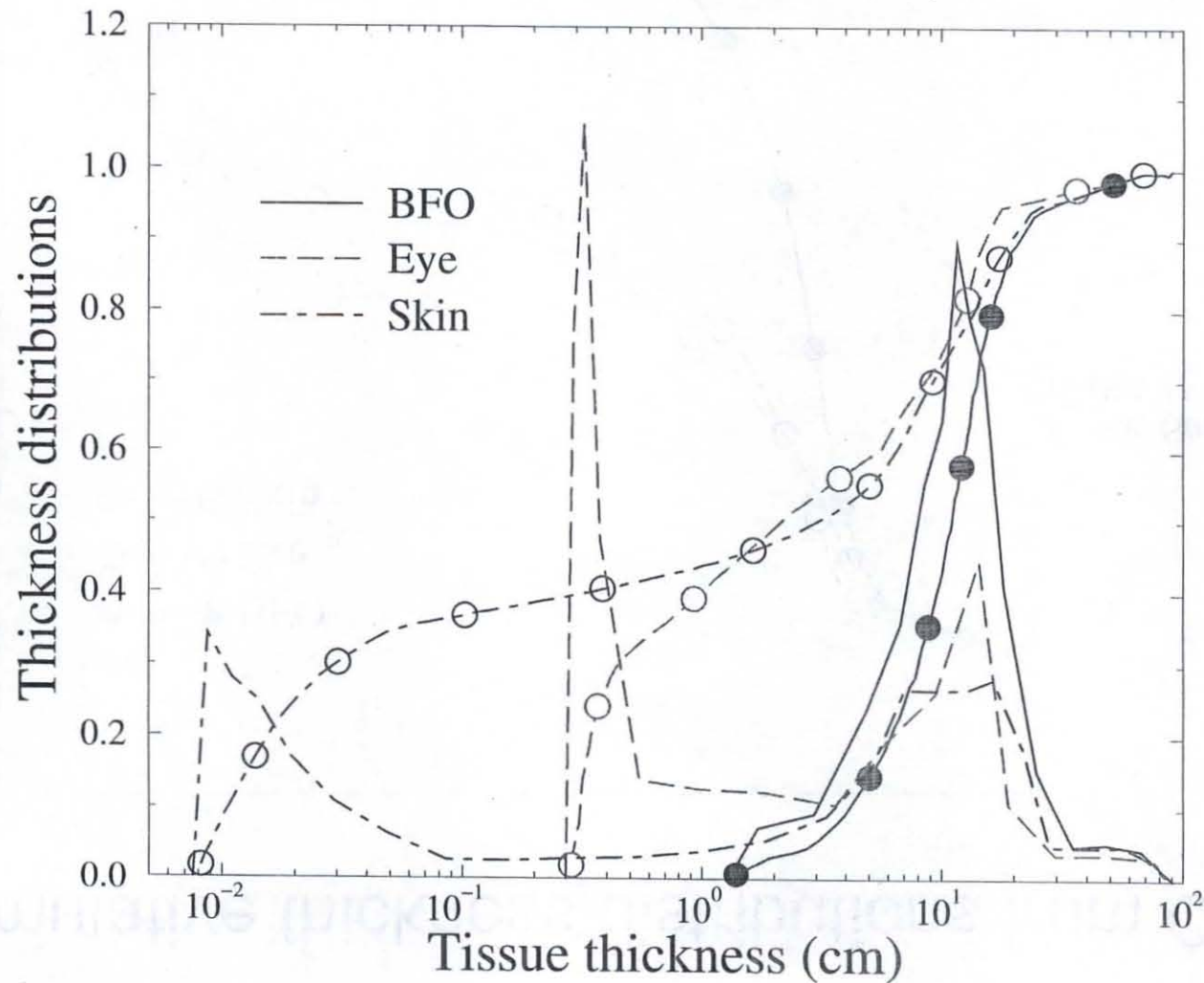Determine the ESM R parameter

$$D(R_i^D) = D_i, \; H(R_i^H) = H_i$$
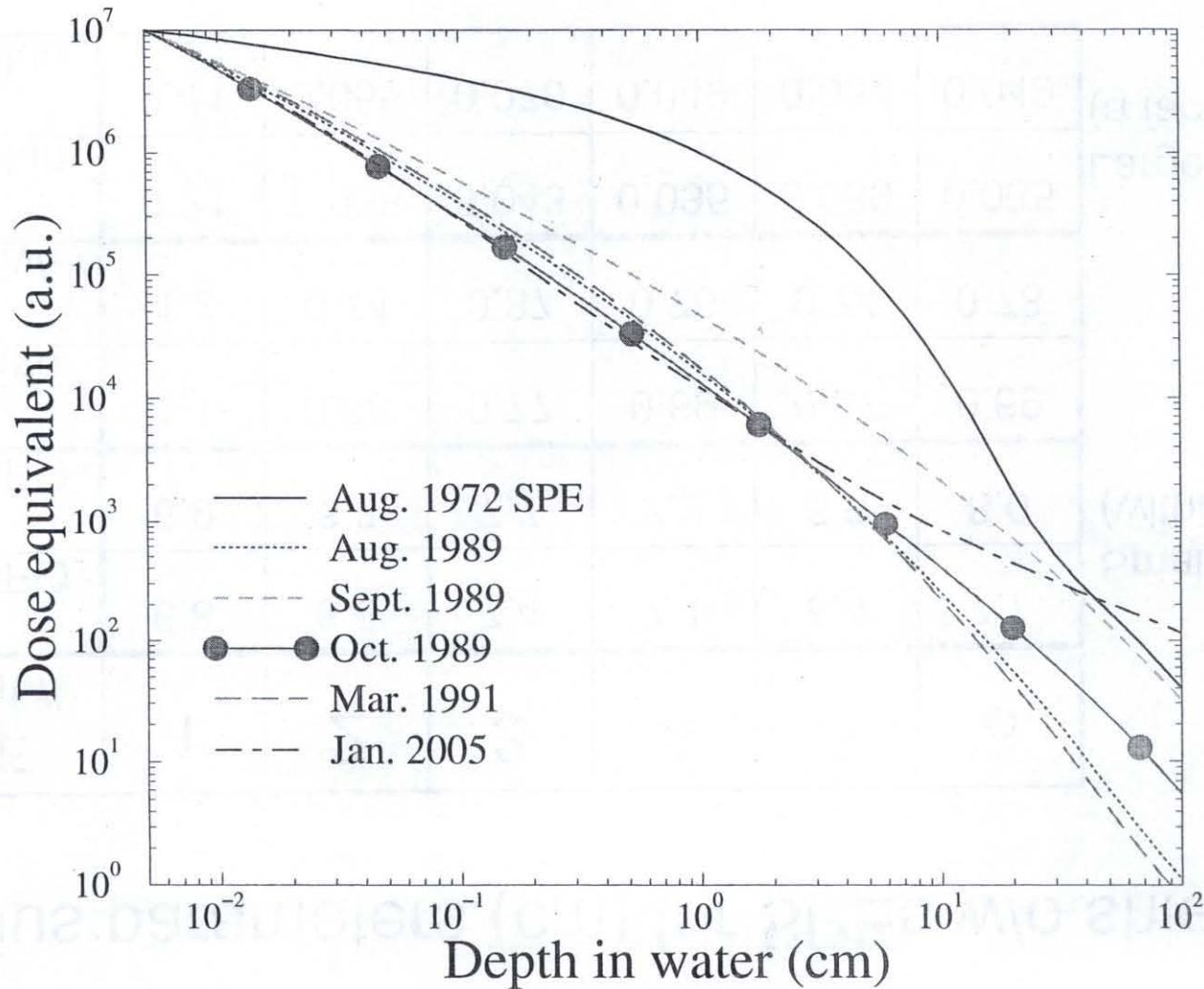
Cumulative thickness distributions from CAM

Nealy, Striepe & Simonsen,
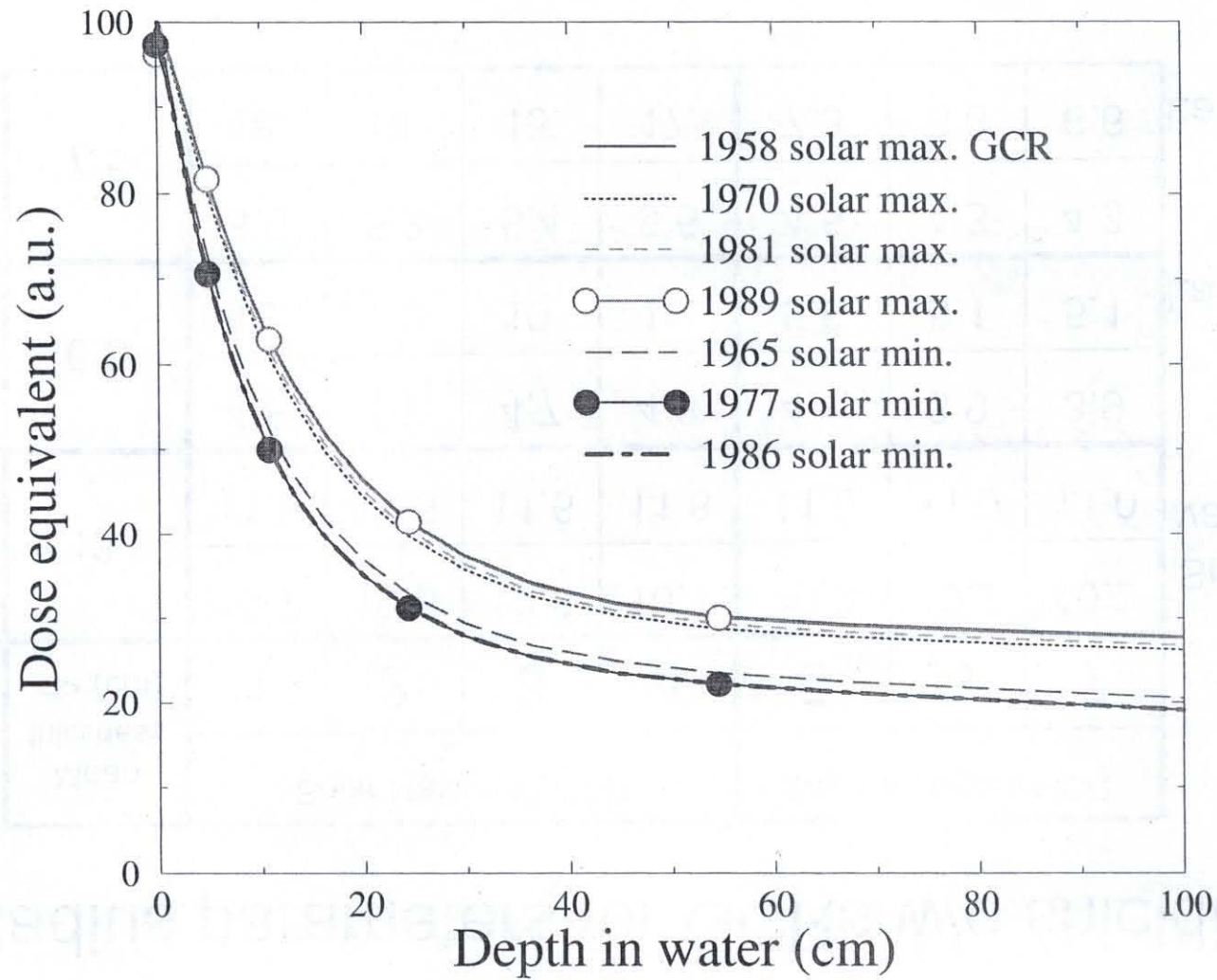NASA TP-3211 (1992)

Thickness PDF from CAM

Dose-depth curves for 6 SPEs

Radius parameters (cm) for SPEs w/o shielding

SPE Event #	1	2	3	4	5	6	
R^H_{BFO}	6.8	6.6	7.4	7.1	6.4	7.7	Small variations (within 25%)
R^D_{BFO}	6.9	6.8	7.7	7.2	6.6	8.0	
R^H_{Eye}	1.1	0.68	0.77	0.69	0.67	0.69	
R^D_{Eye}	1.2	0.74	0.87	0.76	0.74	0.78	
R^H_{Skin}	0.21	0.038	0.043	0.035	0.039	0.035	Large variations (a factor of 8)
R^D_{Skin}	0.41	0.055	0.070	0.049	0.057	0.049	

Dose-depth curves for 7 GCRs

Dose-depth curves for 7 GCRs. Y-axis: Dose equivalent (a.u.) ranging from 0 to 100. X-axis: Depth in water (cm) ranging from 0 to 100. Legend:
— 1958 solar max. GCR
······ 1970 solar max.
– – 1981 solar max.
—○— 1989 solar max.
– – – 1965 solar min.
—●— 1977 solar min.
–·– 1986 solar min.

Radius parameters for GCRs w/o shielding

Radius Parameter (cm)	Mean thickness $<t>$ (cm)	Solar Maximum GCR				Solar Minimum GCR			
		1	2	3	4	5	6	7	
R^H_{BFO}	13.	10.7	10.6	10.6	10.7	10.3	10.2	10.2	Small variations
R^D_{BFO}		11.9	11.5	11.6	11.8	11.0	11.0	11.0	
R^H_{Eye}	6.8	4.8	4.7	4.7	4.8	4.1	3.9	3.9	
R^D_{Eye}		12.	9.3	10.	11.	5.6	5.1	5.1	Large variations
R^H_{Skin}	7.5	5.5	5.3	5.4	5.5	4.5	4.3	4.3	
R^D_{Skin}		18.	14.	15.	17.	7.3	6.6	6.6	Large variations

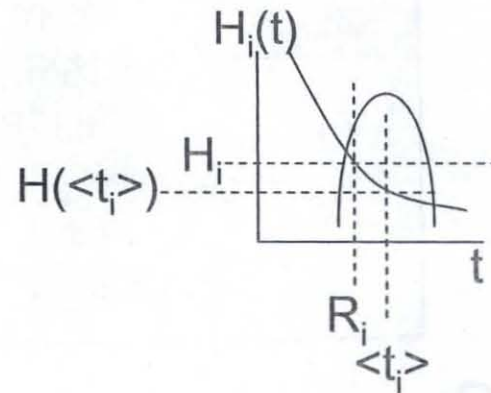# Jansen's Inequality

For a real convex function φ(x),

$$\phi\left(E\{x\}\right) \le E\{\phi(x)\}$$

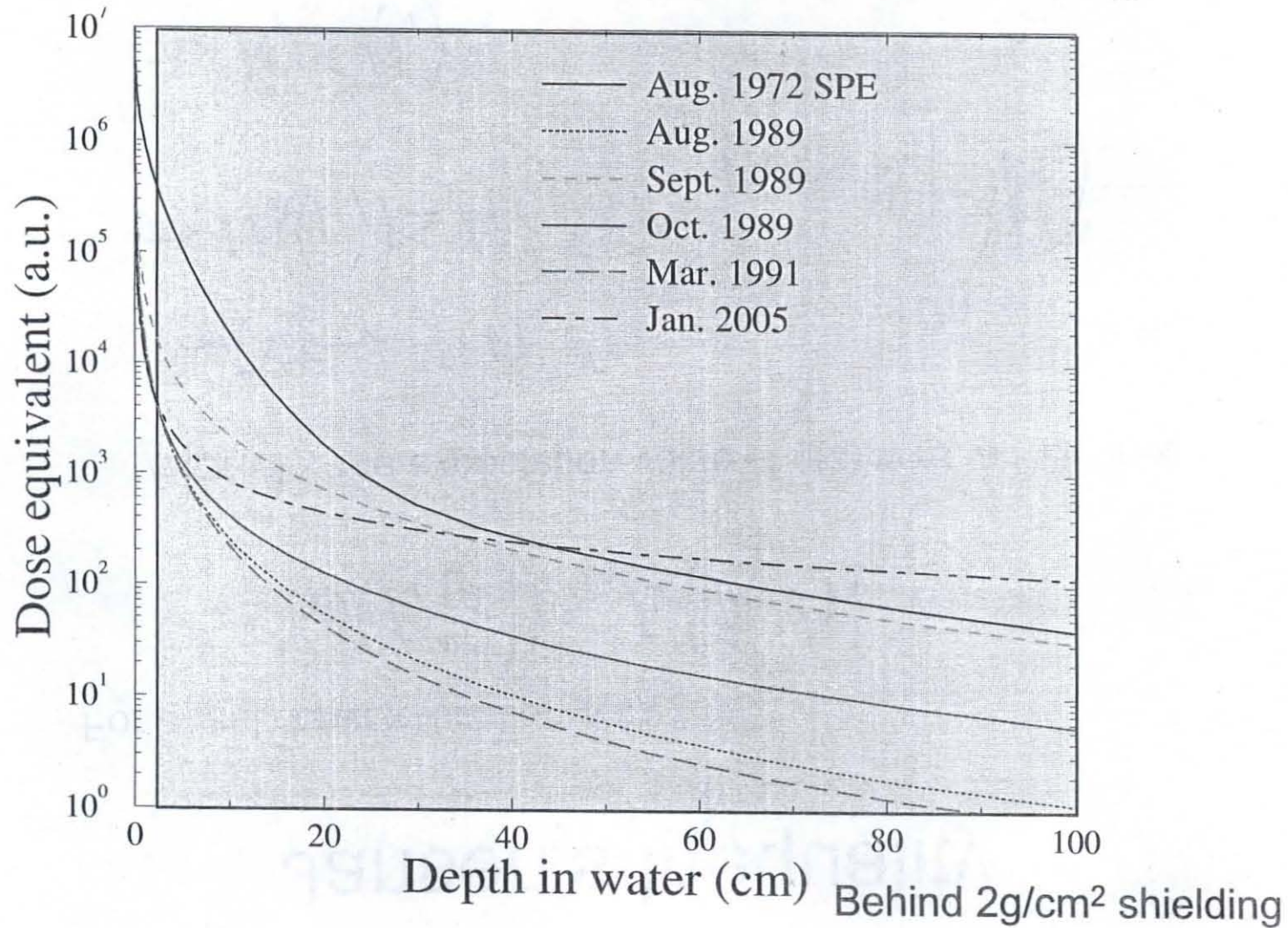$E\{\phi(x)\}$: the expectation value of φ(x) w.r.t. a PDF in x

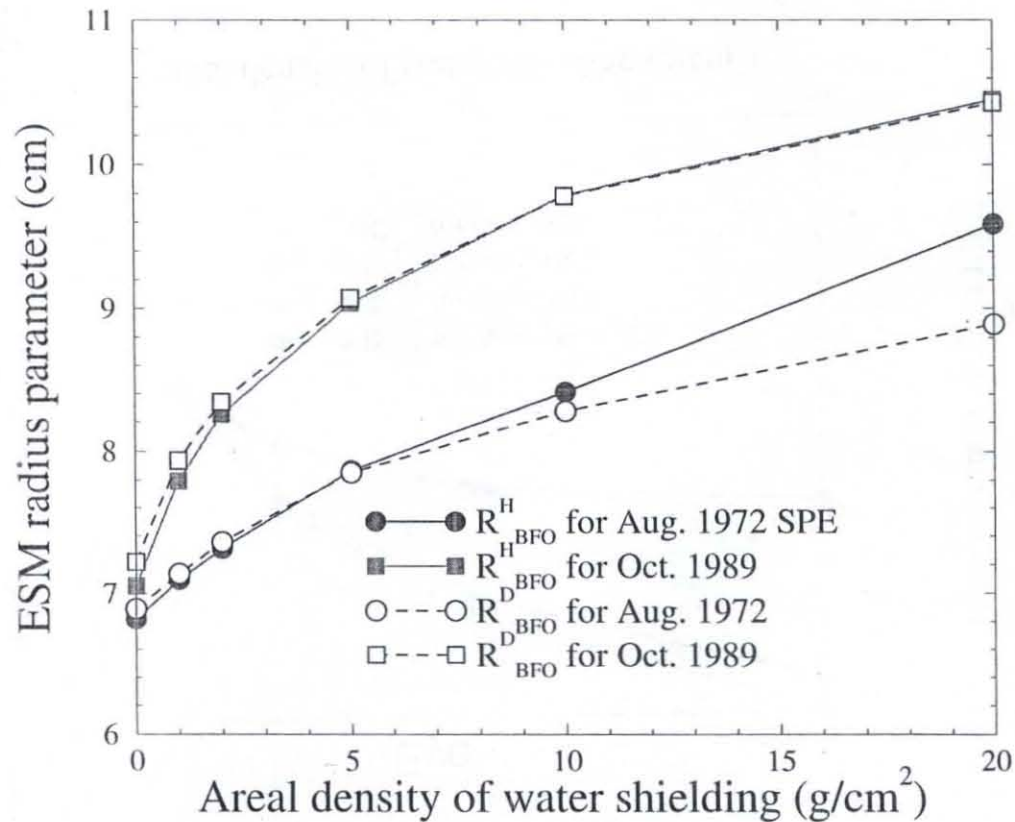$$\phi(x) \rightarrow H(t)$$

$$\Rightarrow H\left(\langle t_i \rangle\right) \le H_i$$

$$\Rightarrow R_i \le \langle t_i \rangle$$

H(t) for SPEs with shielding

Figure: Dose equivalent (a.u.) versus Depth in water (cm), Behind 2g/cm² shielding. Legend: Aug. 1972 SPE, Aug. 1989, Sept. 1989, Oct. 1989, Mar. 1991, Jan. 2005.
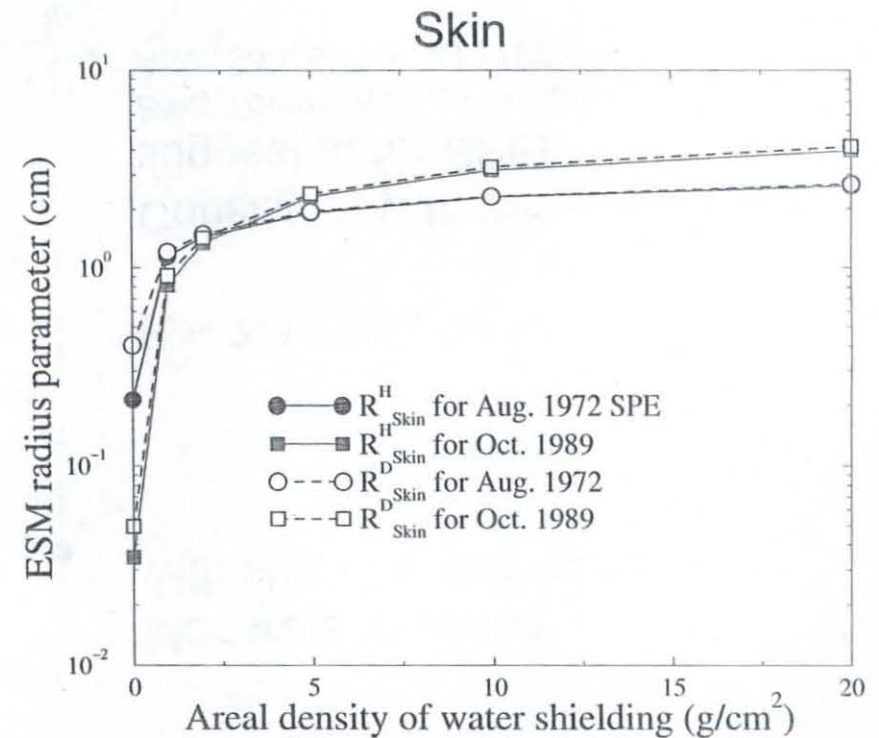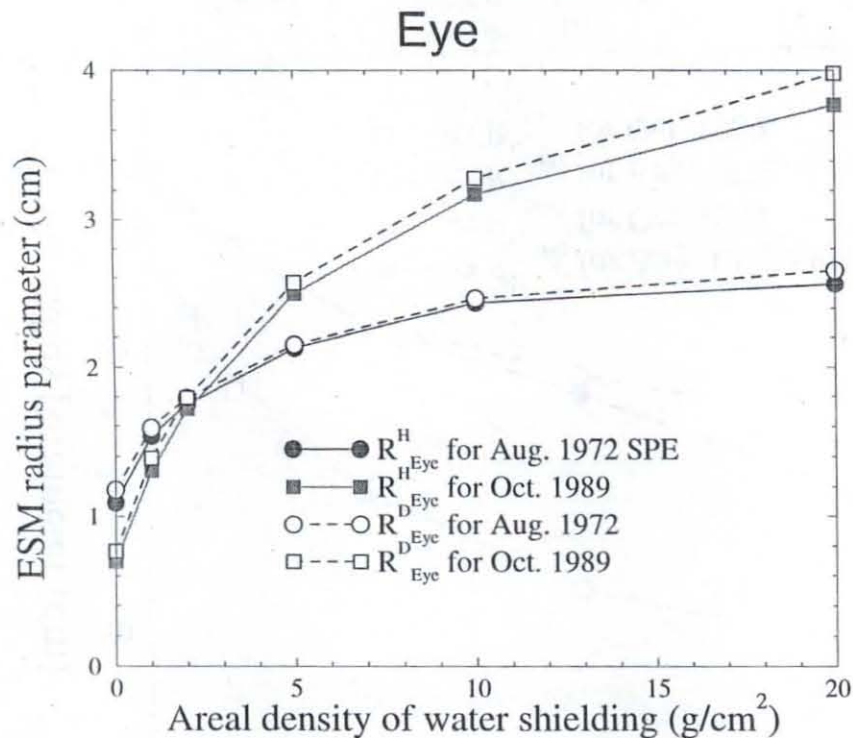
R$_{BFO}$ for SPEs with shielding

Moderate increase
with shielding thickness

R ~ 7-11 fm

Consistent with the 9cm sphere
suggested for SPEs
Bier, Townsend and Maxson,
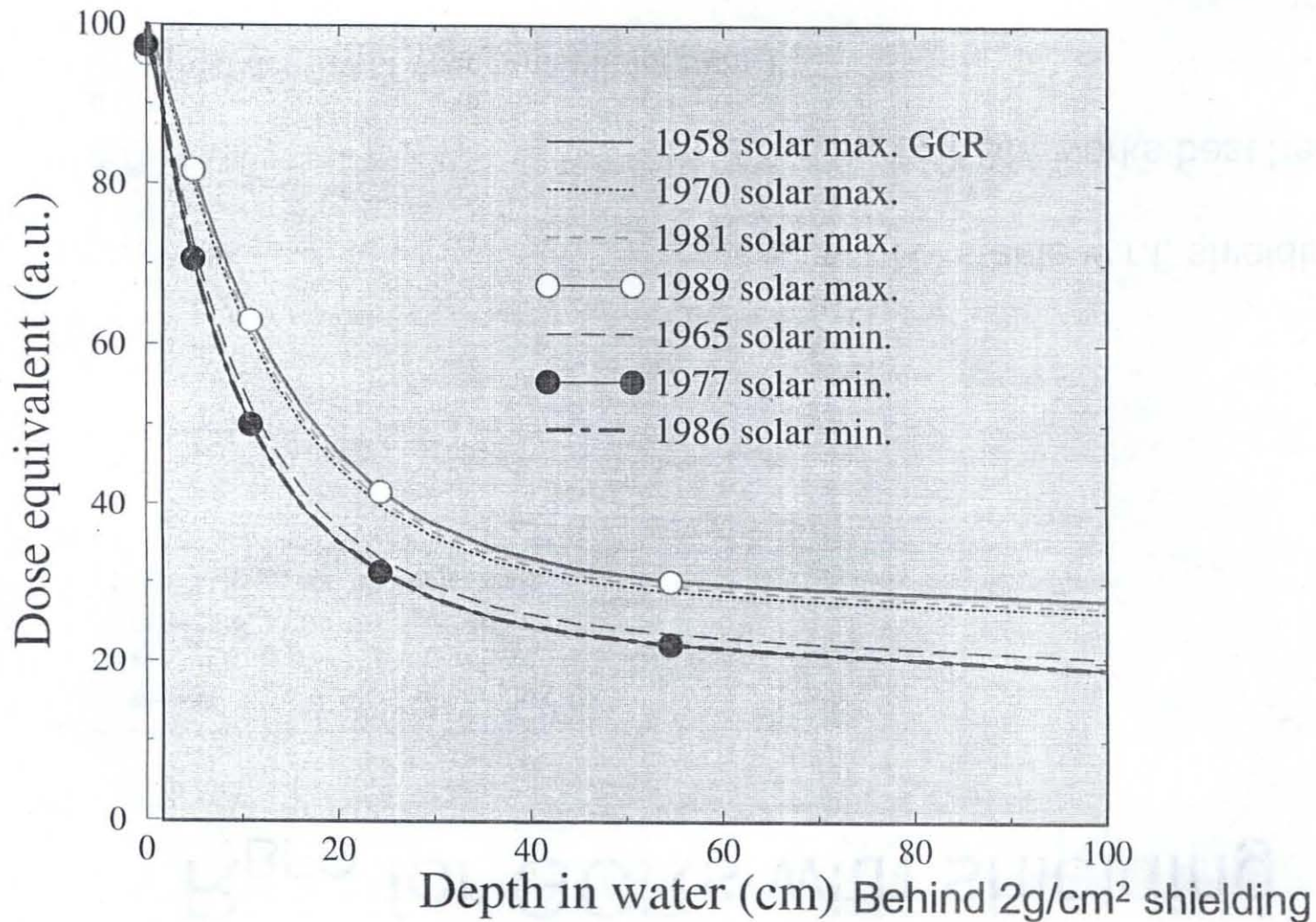*Adv. Space Res. 21 (1998)*

For SPEs with shielding

up to x5   up to x100
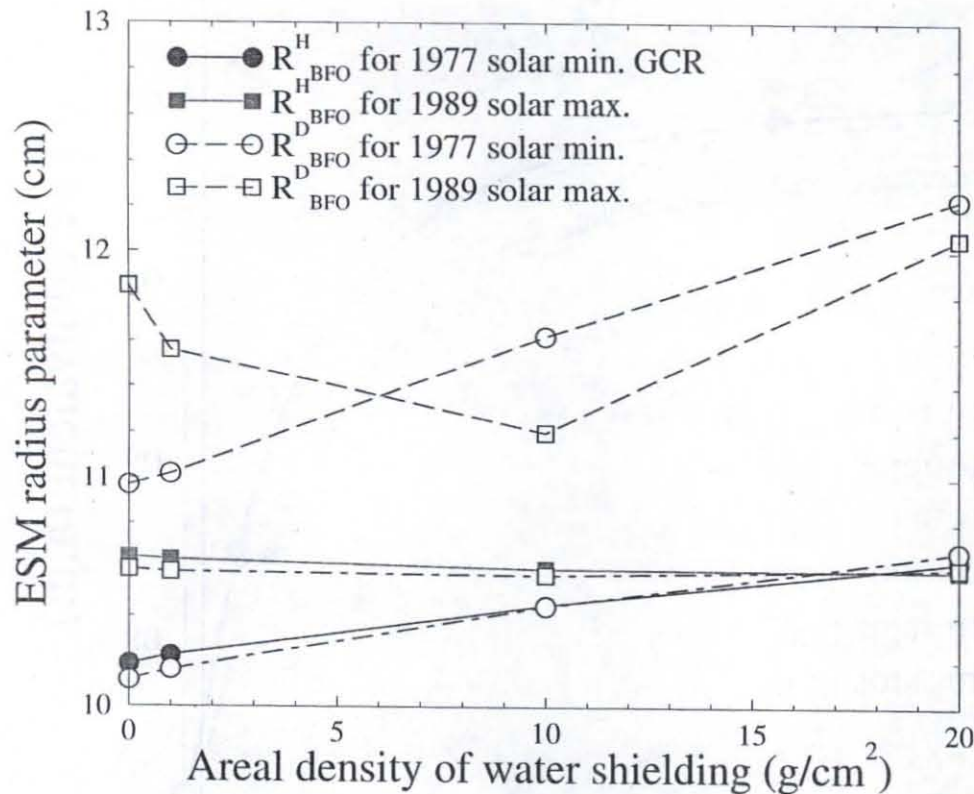
Strong increase with shielding thickness

⇒ ESM does not work here
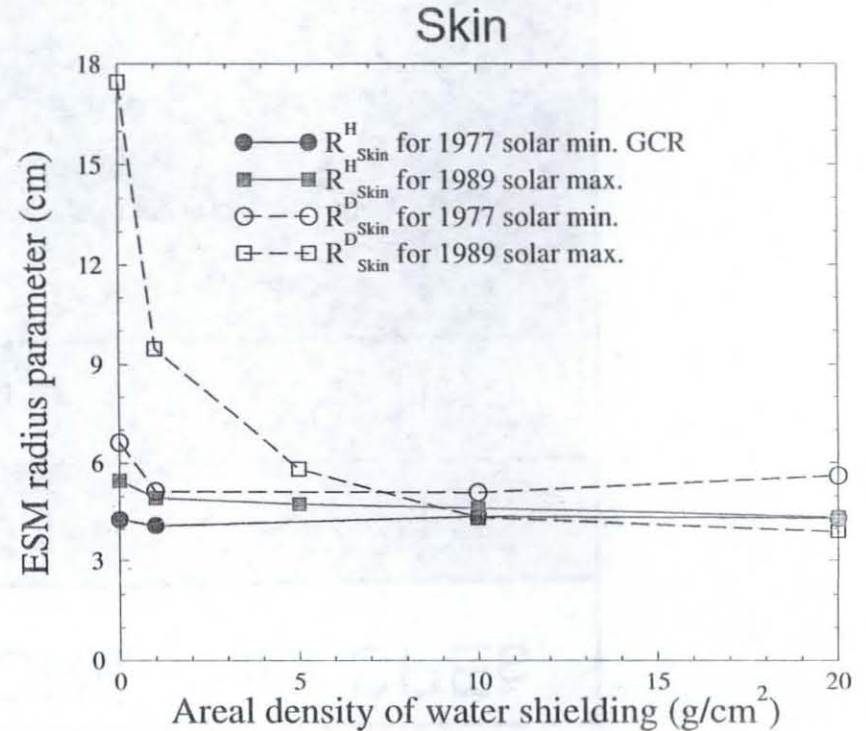
H(t) for GCRs with shielding

Figure plotting Dose equivalent (a.u.) versus Depth in water (cm) Behind 2g/cm² shielding, with legend:
- —— 1958 solar max. GCR
- ······ 1970 solar max.
- – – – 1981 solar max.
- ○——○ 1989 solar max.
- – – – 1965 solar min.
- ●——● 1977 solar min.
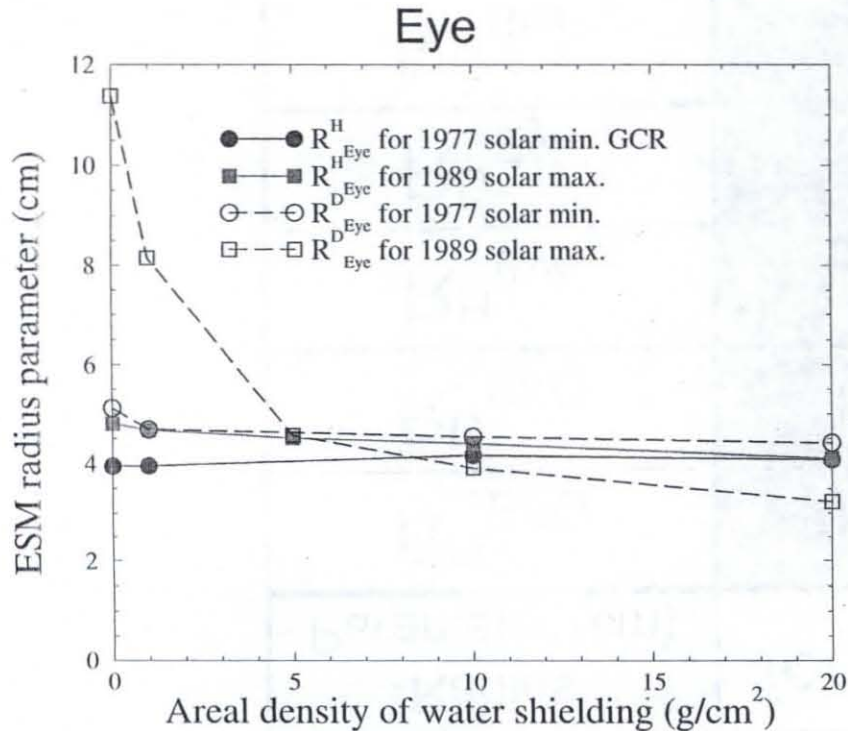- –·–·– 1986 solar min.

R$_{BFO}$ for GCRs with shielding

R^H stable w.r.t. shielding thickness

⇒ ESM works best here

For GCRs with shielding

Eye

Skin

R^H change moderately $\Rightarrow$ ESM is marginal for R^H

R^D changes sharply with shielding thickness $\Rightarrow$ ESM does not work for R^D

ESM Goodness Matrix

Color coding: Good, Marginal, Bad

Radius Parameter (cm)	for GCRs	for SPEs
R^H_{BFO}	10-11	6.8-10.5
R^D_{BFO}	10-13	6.9-10.5
R^H_{Eye}	3.7-4.8	0.69-3.8
R^D_{Eye}	2.8-12	0.76-4.0
R^H_{Skin}	3.5-5.6	0.035-4.0
R^D_{Skin}	2.3-19	0.049-4.2

Summary

Organ doses evaluated for water or Aluminum shielding from $t_S=0$ to 20 g/cm^2; Radius parameters from Equivalent Sphere Model (ESM) calculated.

$\Rightarrow$ For SPEs:

$R_{BFO} \sim 7\text{-}11$ cm, increases with t_S by up to 50% (ESM marginal)

R_{Eye} & R_{Skin} increase sharply with t_S (ESM does not work)

$\Rightarrow$ For GCRs:

$R^H_{BFO} \approx 10\text{-}11$ cm (ESM works)

$R^H_{Eye} \sim 4\text{-}5$ cm, $R^H_{Skin} \sim 3\text{-}6$ cm, $R^D_{BFO} \sim 10\text{-}13$ cm (ESM marginal)

R^D_{Eye} & R^D_{Skin} change sharply with t_S (ESM does not work)