Computational Aeroelastic Modeling of Airframes and TurboMachinery: Progress and Challenges

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Computational analyses such as computational fluid dynamics and computational structural dynamics have made major advances toward maturity as engineering tools. Computational aeroelasticity is the integration of these disciplines. As computational aeroelasticity matures it too finds an increasing role in the design and analysis of aerospace vehicles. This paper presents a survey of the current state of computational aeroelasticity with a discussion of recent research, success and continuing challenges in its progressive integration into multidisciplinary aerospace design. This paper approaches computational aeroelasticity from the perspective of the two main areas of application: airframe and turbomachinery design. An overview will be presented of the different prediction methods used for each field of application. Differing levels of nonlinear modeling will be discussed with insight into accuracy versus complexity and computational requirements. Subjects will include current advanced methods (linear and nonlinear), nonlinear flow models, use of order reduction techniques and future trends in incorporating structural nonlinearity. Examples in which computational aeroelasticity is currently being integrated into the design of airframes and turbomachinery will be presented.

1. Introduction

Aeroelasticity as a science is the investigation of the interplay between fluid dynamics, structural dynamics and structural elasticity. This triad of interactions, first represented in the famous diagram of Collar (1946), is evident in many of the problems encountered in aircraft designs. Examples of such problems are flutter, buffeting, gust response, acoustic resonances, limit cycle oscillations, wing divergence, control surface free play induced oscillation and control reversal, as well as flexibility induced effects on stability and control. Because flexibility induced problems have occurred from the earliest moments of flight aeroelastic analysis has always been an important tool in the hand of the aircraft designer.

For much of the history of flight, aeroelasticians have relied on wind tunnel and flight testing supplimented with empirical and theoretical methods. The advent of digital computers started a new era for this field of engineering. Although early computational methods were an adaptation of the theoretical methods already in use, digital computing has led to the growing sophistication of aeroelastic analyses. It was with the development of the disciplines of computational structural dynamics (CSD) and computational fluid dynamics (CFD) that the possibility of whole new avenues of aeroelastic analysis opened. The coupling of computational fluid dynamic models of various forms with computational structural dynamic models in a simulation of a fluid/structure interaction is the origin of what is now called computational aeroelasticity (CAE). And while early methods employed simplified linear structural and fluid dynamic models, the rapid progress of computer technology in speed and memory has allowed for continuous development of numerical models and opened the way for new methods of simulation, design and analysis. These in turn have led to significant strides in the understanding of aeroelastic phenomena.

The highly competitive nature of the aircraft market with its continuous pressure on lowered cost, increased efficiency and safety, enhanced maneuverability and lowered gust response, lower emissions and tough noise regulations has provided new challenges to the aeroelastician and resulted in the growing use of higher fidelity methods. With expanding capability and successes has come an increasingly important role that computational aeroelastic analyses play in the design of an aircraft. The complexity of aircraft design also drives the need for algorithmic efficiency and robustness as well as the integration of computational aeroelasticity into a larger multidisciplinary aircraft design framework.

Computational aeroelasticity is now one, among many, in a suite of multidisciplinary tools that find use at all stages of airframe or turbomachine design. The use of accurate and efficient multidisciplinary optimization tools at the pre-design stage can indeed be the dividing line between success and failure. This has motivated the slow but steady growth in the use of higher fidelity computational tools in the early stages of a design when much of the aircraft structural and aerodynamic definition takes place. Aeroelastic analysis plays a role in the loads and flutter certification and the stability and control design stages as well. However, because of the enormous cost of the large number of flight conditions and configurations to be analyzed, a significant part of aircraft and engine certification still relies on test data to tune simple linear and theoretical models.

The applications to which aeroelastic analyses are directed have grown. The use of CAE can be found in the design of airframes, turbomachinery, rotor-craft, wind turbines and long span bridges among many others. This paper will be restricted to aerospace applications, discussing the current state of CAE applied to airframes and turbomachinery. As with many subjects, the computational aeroelasticity of airframes and turbomachinery can be viewed from many sides. On one side is its usage in industry while on another side is the research that will eventuate in future production methods. This paper will address as much as possible the current state of both aspects of CAE. There are also many roles that CAE plays in aircraft design, ranging from static aeroelastic parametric sensitivity studies and static aeroelastic optimization, aeroservoelastic design to post design analyses of flight anomolies. This paper will focus on the application of CAE to the analysis of several of the most important dynamic aeroelastic phenomena. Even so it is impossible to cite all of the publications that apply CAE to these phenomena.

Since the subject of computational aeroelasticity has grown rapidly in the past decade, a review of this sort will necessarily be restricted largely to the most recent advances. Other reviews covering the general field of aeroelasticity, with its problems, successes and challenges can be found in Collar (1946, 1978), Garrick (1976), Garrick & Reed (1981), Friedmann (1999), Livne (2003), Yurkovich (2003), Dowell *et al*. (2003). The present paper will be organized in the following way: Numerical methods will be discussed that range from linear to high fidelity, followed by applications of the analysis of airframes and turbomachinery and finally the challenges and goals that appear achievable in the near future.

2. Numerical Approaches to Modeling Fluid/Structure Interaction

In the most general sense CAE can be considered a three field problem requiring solutions of the discrete equations of structural dynamics, fluid dynamics and the motion of the fluid and structure meshes (Farhat *et al*. 2006). The computational modeling of each of these problems is the focus of intense research and deserves separate reviews of each. While the space is insufficient to discuss CFD and CSD development comprehensively, it is worthwhile to summarize the various formulations of the fluid and structure in connection with CAE and recent advances in fluid/structure coupling.

To define a framework for the following discussions, it is noted at the outset that the computational aeroelastic analyses of airframes and turbomachinery are often performed in either the frequency or the time domain. Both of these approaches have relative advantages and both the equations of fluid and structural dynamics may be formulated in either domain. The frequency domain approach assumes harmonic forcing and response. The discrete formulation of the frequency domain fluid dynamics or structural dynamics requires a solution of either a linear or a nonlinear system of equations for the amplitudes of the response at a given frequency. The full dynamics of the problem then becomes an assembly of responses computed over a range of discrete frequencies. On the other hand, the discrete formulation of the time domain fluid or structures equations requires a solution of either a linear or a nonlinear system at discrete points in time to obtain the full transient response.

(a) Structure models

Having constructed a framework for the different formulations of the fluid and structure, additional details on the modeling of the structure will now be given. A semi-discrete time domain representation of a flexible dynamic structure is

$$[M]\ddot{\delta} + [C]\dot{\delta} + [K]\delta = Q \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix and δ is the displacement vector of elements over the structural domain Ω_S. The forcing vector Q is due to fluid tractions acting on the boundary, Γ_S. If the mass, damping and stiffness matrices are constant, the structural system is linear and is amenable to linear analysis methods. If the mass, damping or stiffness matrices are nonlinear in displacement, i.e. $[K] = [K(\delta)]$, as for example occurs with strain hardening or structural free-play, the system must be modeled by numerical means.

The fluid/structure interaction is sometimes solved using the complete set of structural dynamics equations. An example of this is the time accurate computation of a finite element model of an airframe coupled with a computational fluid dynamic solution of the flow field. Solving in this manner requires the computation of the deflections throughout the structural domain, Ω_S, in some instances updating the structures mesh, coupling of the fluid and structure, and updating of the flow field mesh and solution. This approach allows the high fidelity simulation of a complex flexible structure having potentially large nonlinear deflections. It is also very computationally expensive.

The expense of a solution over the entire structure domain has motivated the use of reduced order models of the structural dynamics. One of the most elegant reduced order models is obtained by the method of orthogonal modes. This transformation reduces the complexity of the structure model by producing an independent equation of motion for each mode in terms of a generalized mass and stiffness. The generalized mass and stiffness matrices are obtained by pre and post multiplying the mass and stiffness matrices by the transformation matrix. The generalized mass matrix is diagonal, having the modal masses as its diagonals. The generalized stiffness matrix is also diagonal in which each diagonal entry is the square of the modal frequency times the generalized mass. The mode shapes are obtained from the orthogonal transformation matrix.

The advantage of this method is that the solution over the entire domain Ω_S is replaced with a reduced decoupled modal equation set with which the equations of motion for each mode can be separately solved. Furthermore, the structural displacements are updated very simply as the summation of the contributions of each mode. That is

$$\delta = \sum_{n=1}^{N} \phi_n q_n = [\Phi] q \tag{2}$$

where q is the modal coordinate and $[\Phi]$ is the matrix of modal vectors. The displacement can be approximated using a limited number of modes, say N modes, without significant loss of accuracy. Typically the first few lowest frequency modes to the first hundred or so for a highly complex aircraft structure are necessary to accurately capture the static and dynamic flexibility of the structure.

(b) Fluid models

The fluid domain is the second field that must be modeled computationally. For a time marching computation, the flow equations can be written as

$$\dot{U} = R(U) \tag{3}$$

where U is the solution vector over the flow field domain, Ω_F. The right hand side $R(U)$ contains the residual vector. In its most general form, the residual includes terms due to flow field nonlinearity (Euler fluxes), flow field viscosity (e.g. the Navier-Stokes terms) and other body forces. The tractions on the fluid boundary Γ_F are the forcing applied to the structural domain. Because of the extreme challenges in computing time varying aeroelastic behavior and the wide range of phenomena to be simulated, there are many ways in which the fluid dynamics equations are formulated and solved. The methods of today range from a continued use of the well established lower fidelity linear methods to the growing use of the full Navier-Stokes equations.

To solve the full nonlinear flow field equations in a time marching manner, both spatial and temporal discretizations are required. For airframe and turbomachinery applications, spatial discretizations are most often based on the finite volume and finite element methods. Until the mid 1990's, nearly all nonlinear computational aeroelastic codes used a structured CFD model of the flow field. Examples of CAE methods using structured CFD codes can be found in Erdos *et al.* (1977), Rai (1986,1987), He & Denton (1993) and Ning & He (1998) for turbomachinery and Guruswamy (1990), Robinson *et al.* (1991) and Lee-Rausch & Batina (1995, 1996) for airframes. However, methods based on unstructured grids of mixed cells, emerged and gained momentum by mid-1990's. Turbomachinery examples of those can be found in Vahdati & Imregun(1997), Sayma *et al*. (1998), Sbardella *et al.* (1999), Moinier (1999), Sayma *et al.* (2000a, 2000b) and Contreras *et al.* (2002). Examples of CAE codes for airframes using unstructured CFD are found in Rausch *et al.* (1993), Frink *et al.* (1995) and Farhat *et al.* (2003) and finite element CFD in Tezduyar *et al.* (1992, 2006). These methods allow more efficient generation of grids for complex airframe geometries such as multiple wing store configurations or turbomachinery applications such as fans with intake ducts, tip gaps and compressors with bleed-off takes. For temporal discretization, various methods are used ranging from explicit methods with multi-stage Runga-Kutta schemes to implicit temporal schemes with dual time stepping. Traditional solution acceleration techniques are usually used such as residual smoothing and multigrid acceleration.

The technique of linearization is widely used because it reduces the complexity of either the flow field physics or the geometry. An example is the linearization of the fluid equations for a subsonic, supersonic or hypersonic flow field. A number of widely used aeroelastic methods for airframes analytically solve the linear flow field equations, converting a solution over the full domain into a simplified boundary value problem. Examples using this approach are the doublet lattice method (Albano & Rodden 1969) in the subsonic regime or the harmonic gradient method in the supersonic regime (Garcia-Fogeda & Liu 1987, Liu *et al*. 1991). Besides a reduction in the complexity of the flow physics, these methods also eliminate the necessity to compute the motion of a full flow field mesh. When linear aerodynamics is combined with a linear modal structure the complicated fluid/structure system can be reduced to a matrix to which the classical methods of linear analysis can be applied.

Other moderate fidelity methods solve a reduced or even linearized form of the Navier-Stokes or Euler equations over the complete flow domain. In some instances this may be a linearization of the unsteady part of the solution combined with a nonlinear and non-uniform steady mean flow (Euler or Navier-Stokes). This allows the original nonlinear unsteady equations to be cast as one set of nonlinear steady-state equations plus another set of linear unsteady equations that depend on the underlying steady flow. Chen *et al*. (2000) have developed a method based on this approach for airframes that assumes the unsteadiness is harmonic in time. Whitehead (1982) and Verdon & Caspar (1984) developed a similar method for turbomachines. In the case of turbomachine applications, the linearization of flow variables also allows the computation to be performed, for example, on a single blade passage using the inter-blade phase shifted boundary conditions. Other linear methods for turbomachinery based on the Euler equations have been developed by Giles (1992). Hall & Crawley (1989) also developed a finite-element method for solving the linearized Euler equations. Further work is presented by Hall & Clark (1993), Hall & Lorence (1993), Hall *et al.* (1999) and Montgomery & Verdon (1997a, 1997b).

When flow field nonlinearity is confined in a viscous boundary layer, it is possible to use a linear inviscid flow solver coupled with a nonlinear boundary layer solver. Notable methods based on this feature are those by Cizmas and Hall (1995), Holmes *et al*. (1997), Ning (1998) and Sbardella & Imregun (2000) for turbomachinery applications. A method which takes into account multistage effects in turbomachinery was developed for inviscid flows by Silkowski & Hall (1998) and Hall & Kivanc (2005) and later extended along this vein to viscous flows for rotor stator interaction by He *et al.* (2002) and to multiple blade row viscous flows by Saiz *et al.* (2006).

In other flows nonlinearity cannot be so easily isolated. For instance, in the case of an oscillating shock wave a solution incorporating some level of nonlinearity is required through much of the flow field. Methods have been developed that use the full potential equation or the transonic small disturbance equation for the inviscid region (e.g. Nitzsche & Voss 2003, Batina *et al*. 1987, Batina 2005) which can also be coupled with a viscous boundary layer. For airframe applications Edwards (1993) developed a method that couples a solution of the transonic small disturbance equation with a quasi-steady boundary layer solver, while Henke (2003) and Zhang *et al.* (2006) construct a viscous-inviscid approach that couples with the Euler equations. Likewise, a method using an unsteady boundary layer coupled with the full potential equation has been developed for turbomachinery by Epureanu *et al.* (2000). The harmonic balance method is representative of a frequency domain method that can be formulated to solve a nonlinear system of coupled partial differential equations for the Fourier components of the unsteady flow (Hall *et al*. 2002, Thomas *et al*. 2003, 2004). This method has been applied to both airframe and turbomachinery. Similar methods for turbomachinery have been presented by Ning & He (1998) and Maple *et al.* (1994).

(c) Fluid-Structure Coupling

Before discussing the topic of fluid/structure coupling in a more general sense it is perhaps worth mentioning what is sometimes referred to as the classical method in turbomachinery analyses, namely the uncoupled approach. This method is still used for the prediction of forced response in turbomachine components. In this approach the objective is to obtain the effect of an unsteady flow field, on for instance turbine blade rows, in the most efficient manner possible within the constraints of the needed accuracy. In order to obtain the unsteady flow field forcing, a rigid geometry model is used. The unsteady forcing is then imposed on a flexible model of the structure.

By far the most common approach to the fluid/structure problem in airframes and turbomachinery is an interacting or coupled solution of the flexible structure and flow field. In order to model the full interaction in a time accurate way, a method by which the fluid and structure domains can be coupled at each instant in time (or frequency) is required. This coupling, or compatibility condition, expresses the requirement that the fluid displacements and tractions on Γ_F remain in equilibrium with the structure displacements and tractions on Γ_S at the interface for all time. This coupling involves the exchange of displacement and pressure information between the fluid and structure media. The discrete modeling of this interface coupling can be accomplished in several ways. The most widely used method is called the partitioned approach, (Farhat *et al*. 2006) also called a loose fluid/structure coupling (Cebral & Lohner 1997). In the partitioned procedure the discrete solution is advanced over separate fluid and structure partitions having different mesh distributions and non-point matching interfaces. While this complicates the coupling by requiring additional interface steps, the advantage of partitioning is that it also readily facilitates the use of a variety of interchangeable off-the-shelf software packages to model the fluid and structure.

There are two related issues with the partitioned fluid/structure coupling, namely the time advancement and the procedure used to project quantities from one side of the interface to the other. The first issue applies only to dynamic aeroelastic simulations performed in the time domain. It has to do with the way the fluid and structure are cycled as the solution is advanced in time. Time marching fluid/structure procedures in use can be broadly classed as ranging from fully explicit to fully implicit. These also generally range from the least to the most accurate and robust methods of coupling respectively. Several widely used methods of time advancement are the explicit lagged procedure, the implicit iterative method, the conventional serial staggered approach, the central staggered differencing and the backward second order differencing of the fluid and structure with predictor/corrector steps. (e.g. Edwards *et al.* 1983) Farhat *et al*. (2006), Yee *et al*. (1997), Morton *et al*. (1997) provide useful analyses of various coupling methods. With the application of the appropriate constraints, these schemes can be strongly-coupled and conserve the overall second order accuracy of the solution (Geuzaine *et al*. 2003, Farhat *et al*. 1998).

The transfer of information from one side of the fluid/structure interface to the other when using the partitioned approach presents the second challenge. The challenge is coupling interface node points that do not coincide. This requires an interpolation step that transfers information from nodes on one side to non-coincident nodes on the other. An essential requirement is a projection of loads and displacements that enforces the conservation of forces, momentum and energy. By the principle of reciprocity, conservation will be maintained when the same geometric representation of the fluid and structure surface model is used in the transfer of information going both ways (Farhat *et al*. 1998, Samareh & Bhatia 2000). Additional features such as smoothness influence the accuracy of the result. Several studies have evaluated methods of multidisciplinary coupling, among those evaluated are the interpolation-based algorithm (IBA) (Maman & Farhat 1995), the non-uniform rational B-spline (NURBS) method (Samareh & Bhatia 2000). Smith *et al*. (1995) evaluated the infinite-plate spline (Harder & Desmarais 1972), the multiquadratic biharmonic, the nonuniform B-spline, the thin-plate spline, finite-plate spline and the inverse isoparametric mapping. Each of these coupling methods have relative advantages and disadvantages for the overall accuracy and behavior of the aeroelastic solution.

3. Computational Aeroelasticity of Airframes

While linear aero and structural dynamics models have been and remain the first line of aeroelastic analysis tools, there has been a steady rise, since the mid 1990's in the use of computational aeroelastic tools using the Euler and Navier-Stokes equations. Several reviews have outlined progress on the development of computational aeroelasticity. For instance Schuster *et al*. (2003) present a broad overview of the development and application of computational aeroelasticity tools to many unique and challenging problems. Kamakoti (2004) reviews progress in fluid/structure coupling with emphasis on the coupling of Navier-Stokes solvers and linear structural models. Yurkovich (2003) presented the state of the art in the use of computational aeroelasticity in the design of high-performance aircraft. While reviewing the status of CFD solvers in European aircraft design, Voss *et al*. (2002) survey the extent to which fluid/structure coupling had found its way into CFD analysis. Today most Navier-Stokes codes have multidisciplinary analysis capability embedded within them.

Voss *et al*. (2002) identifies six European Navier-Stokes codes used for airframe CFD analysis having aeroelastic capability, namely ENSOLV, EURANUS, FLOWer, AETHER, FASTFLO and TAU. Other codes can be found that present computational aeroelastic analysis. For example, simulations of a geometrically complex flexible transport have been reported by Prananta *et al.* (2005) using the Euler code ENFLOW. The development of aeroelastic versions of the UES3D/UNS3D code (Girodroux-Lavigne *et al.* 2003) and the elsA code (Girodroux-Lavigne 2005) are reported as well. A few widely used CFD codes in the US that also have published aeroelastic capability are CFL3D (Robinson 1991, Bartels *et al*. 2006, McNamara *et al.* 2005) , USM3D (Frink *et al*. 1995, Bounajem 2000, Allison & Cavallo 2003) , ENSAERO (Guruswamy 1990), ENS3DAE (Schuster *et al*. 1990), and AERO-F (Farhat *et al*. 2003). There are many other research codes as well as several commercial codes that have aeroelastic and multidisciplinary capability.

The ultimate goal of the aeroelastician is to produce an airframe free of performance degrading or structurally damaging effects due to flexibility. The need to address several key aeroelastic problems in airframe design has motivated advances in computational aeroelastic tools. The most catastrophic problem to be addressed is the onset of flutter. Other phenomena have important design ramifications as well, such as buffeting, limit cycle oscillations (LCO) and nonlinear effects due to large deflections. These subjects will now be addressed respectively.

(a) Flutter analysis

Flutter is a self induced instability which occurs when the flow field feeds energy into the structure. If unattenuated flutter results in an explosive structural failure. Because it was the earliest aeroelastic problem to be recognized, flutter analyses have a long history of development. Flutter analyses that use a linear model have been the mainstay for many decades, and to this day flutter clearance of an aircraft begins with a linear flutter analysis. The value of a linear flutter model is that it allows a rapid assessment of the flutter onset of a vehicle over a broad sweep of the flight envelope. Several commercial software packages such as NASTRAN and ASTROS now have a wide range of aeroelastic and aeroservoelastic analysis options available including linear flutter analysis.

As higher fidelity computational aeroelastic tools evolved they have also found their way into flutter analyses. In fact, the earliest successes in high fidelity CAE were in the prediction of flutter onset. These early analyses focused on critical conditions in the flight envelope at which flutter onset is known to be sensitive to flow field nonlinearity. Several flight regimes in which nonlinearity must be taken into account are the transonic range and at high angle of attack. The growing use of higher fidelity tools at such critical points has allowed designers to gain greater confidence earlier in a design cycle by providing insight into the sensitivity of a design to nonlinear effects. This has resulted in cost savings by reducing the changes that would otherwise be required as a result of flight test failures.

As computational tools have matured and computing power has increased, CAE methods are finding a greater role in the preliminary aircraft design stage. Future aircraft designs can be envisioned that will be more aggressive and in which structural sizing becomes less conservative. Such designs will require the use of higher fidelity multidisciplinary methods at an earlier stage in the design. In anticipation of such aggressive designs, extensive evaluation of high level CAE methods has taken place in the last decade by a broad base of industry and academic investigators. Recent collaborative Boeing-NASA efforts have been aimed at validating, verifying and calibrating computational aeroelastic tools for use in the design of the next generation of aircraft (Hong *et al*. 2003). The report by Hong *et al*. (2003) was the culmination of a multi-year experimental and computational effort.

The analysis was performed on a twin-engine transport model that had been tested multiple times in the NASA Langley Transonic Dynamics Tunnel (TDT) during the 1980's and 1990's. Previous analyses using the full potential plus boundary layer code TRANAIR considered the influence of tunnel walls on flutter onset (SenGupta *et al*. 2001). The results of that work were that both the static deformations of a flutter model and the tunnel walls can have a significant influence on flutter characteristics. As a follow on, the work reported in Hong *et al*. (2003) compared the flutter onset computed by a high level CAE code modeling tunnel walls and in free air with the experimental flutter onset. This study also further investigated the influence of static aeroelastic shape on flutter onset. The CAE code used in that study was CFL3D v6.0. This code solves the Reynolds averaged

Navier-Stokes equations for the flow field and the modal equations of motion for the model dynamics. The flow field, mesh movement and structures time advancement use second order backward differencing. The fluid/structure coupling also uses a second order predictor/corrector scheme to provide close coupling. For details regarding the coupling procedure see Edwards *et al.* (1983).

The half-span model of a twin-engine transport mounted to the tunnel wall is shown in Figure 1. Because of the structural and aerodynamic interactions of the flexible wing, engine nacelle and strut, this configuration can display complex flutter onset behavior. As seen in Figure 2, the flutter characteristics of this model reveal the appearance of a hump mode followed by a hard wing tip flutter onset at a higher dynamic pressure. A hump mode is a mode that becomes lightly undamped for a limited range of dynamic pressure and then becomes damped again, giving it the characteristic hump shape. The computations match the hump mode flutter onset experimental data well for both configurations. Furthermore, the trend of the hump mode with the variation in Mach number is reproduced. On the other hand, the tip mode was not reproduced well by any CAE method. Tip mode flutter onset was over predicted by 25 percent in dynamic pressure by the RANS code CFL3D using the jig shape, but it was over predicted by nearly 50 percent using the cruise shape with every CAE method. All of the lower fidelity CAE codes over-predicted the tip mode flutter for any wing shape by about 50 percent. These results indicate the superiority of the RANS code over lower fidelity CAE codes, but also indicate the sensitivity of the tip mode flutter onset to the static wing shape, nacelle aerodynamics and angle of attack. One of the interesting results of these studies is the influence of wind tunnel walls on flutter onset. The tunnel walls were found to reduce the undamping of the hump mode by 50 percent and increased the dynamic pressure of the tip mode flutter onset by 4 percent.

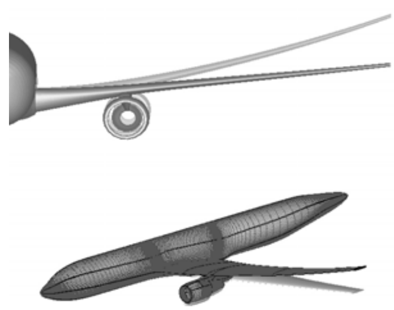

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Figure 1. Twin-engine transport geometry

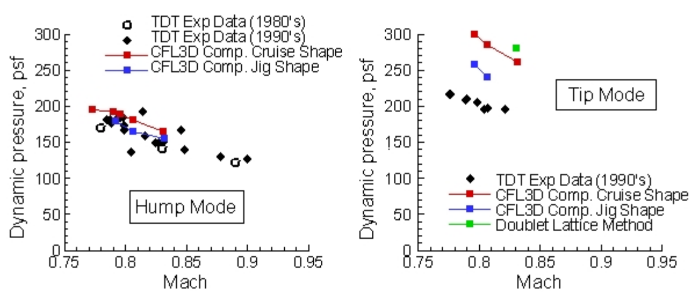

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Figure 2. Twin-engine transport transonic flutter onset. Hong *et al.* (2003)

(b) CAE modeling of buffeting

Some aeroelastic problems do not result in immediate catastrophic failure, but instead produce structural degradation, loss of controllability or flying qualities. These design issues need to be addressed and means found to mitigate ill effects. For some aeroelastic problems of this nature, such as gust response, linear analysis may be sufficient. In other instances the nonlinear nature of the problem precludes analysis by linear methods. Buffeting induced by nonlinear aerodynamics is of this type. Buffeting is a structural response to forcing by an unsteady flow field. Examples of nonlinear aerodynamic phenomena that induce buffeting are vortex bursting or transonic shock induced flow oscillation.

The interaction of a leading edge strake or leading edge extension (LEX) vortex with the vertical tail of a fighter is an example of flow induced aeroelastic interaction. The purpose of the LEX is to entrain flow over the wing and vertical tails to enhance lift and aircraft stability. However, at high angle of attack conditions the vortex bursts upstream from the vertical tail. Conditions at which buffet occurs are characterized by strong vortical flow that bursts combined with massive three dimensional flow separation. The result of this highly unsteady flow is to produce severe structural vibrations that lead to the eventual failure of the vertical tail.

The F/A-18 aircraft has been a subject of numerous research efforts aimed at identifying the buffet characteristics of the vertical tail. At high angle of attack conditions the first bending mode of the F/A-18 vertical tail strongly couples with the characteristic frequency of the buffet flow. At low angles of attack the buffeting is in the first torsional mode of the vertical tail. Several computational aeroelastic studies have been conducted in an attempt to increase understanding of the phenomena, such as that by Sheta (2004). That analysis used a Navier-Stokes solver for the flow field coupled with a finite element structural solver. The coupling was accomplished with the software MDICE (Multidisciplinary Computing Environment). See Kingsley *et al*. (1998). Comparisons of the aeroelastic computations using this method correlate well with wind tunnel and flight test rms pressures with respect to magnitudes and onset angles of attack (Sheta 2004). Further analysis using a LEX fence has shown a reduction in the rms pressures of the vertical tail in full agreement with flight test. Additional insights have been gained from the computational investigation. The computations have demonstrated that the LEX fence spreads the energy of the vortex and delays onset of vortex bursting. The computations show that the influence on the vertical tail is a reduction in rms pressures predominantly on the inboard side (Sheta 2004).

(c) Limit cycle oscillations (LCO)

As the scope of problems addressed by computational aeroelastic methods has expanded, structural nonlinearity has also received increasing attention. Structural nonlinearities can be subdivided into either concentrated or distributed nonlinearities. Examples of concentrated nonlinearities are control surface free-play, friction and hysteresis due to worn or loose hinges or linkages. Distributed nonlinearities such as material or geometric nonlinearity or buckling are spread over an entire structure. One class of aeroelastic response that can be due either to nonlinear aerodynamics or nonlinear structures is what has been termed limit cycle oscillations (LCO). An LCO initially grows in a manner similar to flutter instability but reaches a sustained limit amplitude periodic oscillation. LCO behavior has been classified by whether LCO onset occurs exclusively beyond a linear flutter onset (e.g. LCO due to amplitude dependent hardening of a structure), whether it exhibits hysteresis when the flow speed passes below flutter onset (e.g. flow field induced LCO) and whether it occurs entirely separate from linear flutter onset (e.g. control surface or linkage free-play). Examples of aircraft related LCO are found in fighters with wing/store configurations, such as the F-16 (Bunton & Denegri 2000, Denegri *et al*. 2005) and the F/A-18 C/D (Goodman *et al.* 2003). Other examples of LCO are a result of control surface oscillation induced by structural free-play, (Croft 2001) or are a result of nonlinear strain hardening as illustrated in static measurements of the F/A-18 wing (Thompson & Strganac 2000).

Many LCO phenomena have been attributed to nonlinear aerodynamic effects due to transonic shock waves and shock induced trailing edge separation. Examples of aircraft exhibiting flow field induced LCO are the F/A-18, B-1 and the B-2. Many LCO's can be attributed to a self excited flow field, but in some cases LCO is induced by an interaction with a flexible structure. An example is the B-2 residual pitch oscillation (RPO) (Driem *et al*. 1999). RPO is an uncommanded low amplitude periodic aircraft plunging motion that occurs within a narrowly defined Mach number and altitude range. Simulations using an aeroelastic transonic small disturbance code coupled with a boundary layer solver and a closed loop control system showed that the RPO was produced by an oscillating shock induced separation interacting with the aircraft short period mode and the wing first symmetric bending mode. Additional contributors to the RPO identified by the simulations were nonlinearity and hysteresis in the control system actuators.

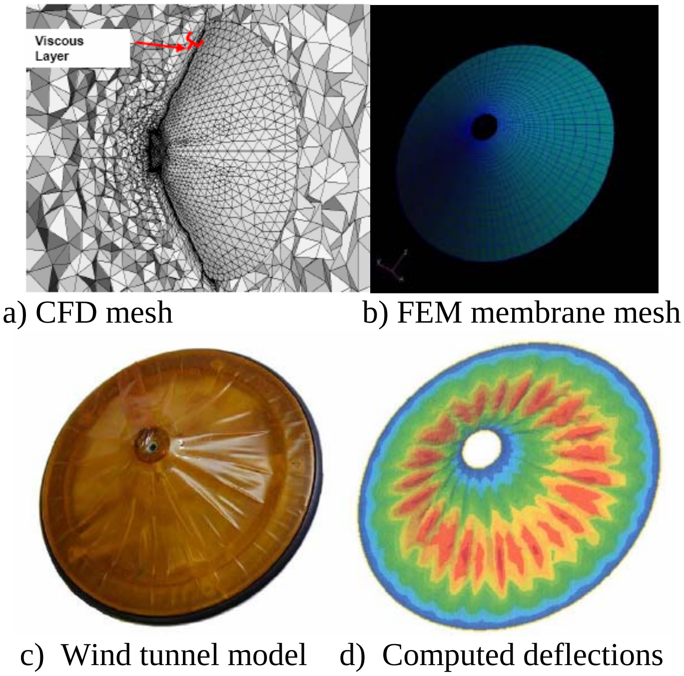
To further illustrate the role that computational aeroelasticity has had we consider recent investigations of the LCO of an F-16 fighter with under-wing stores. This fighter configuration has shown a tendency for transonic LCO behavior at moderate to high angles of attack at a range of altitudes. This LCO typically involves the wings and stores in an anti-symmetric oscillation. It has been suggested that this LCO has its origins in classical flutter onset, supported by the fact that linear flutter analysis predicts reasonably well the instability onset speed and frequency. Melville performed a series of studies using Euler and Navier-Stokes aerodynamics investigating the behavior of a flexible F-16 including the influence of under-wing store masses but with only the tip launcher modeled aerodynamically (Melville 2001, 2002a, 2002b). The structure was linear and modeled in a modal sense. The results using Euler aerodynamics showed the onset of flutter, but did not reveal LCO at any angle of attack. The Navier-Stokes results showed flutter at cruise angle of attack, but LCO at a higher angle of attack indicative of a high g maneuver. The onset of LCO at high angle of attack was coincident with the onset of extensive shock induced trailing edge separation. These studies suggest that at least at higher angles of attack, flow field nonlinearity has a role in producing limit cycle oscillation.

Another suggested mechanism for wing/store LCO in fighters is the nonlinear structural damping resulting from mechanical linkages (Chen *et al.* 1998). If the structural damping increases with amplitude and the negative aerodynamic damping beyond flutter onset is sufficiently small, a limit cycle oscillation can be produced. Further analysis of the F-16 LCO and flutter onset has been performed with the ZTAIC code which uses linear unsteady aerodynamics combined with a nonlinear steady transonic flow field (Chen *et al*. 2000). The nonlinear steady flow field data for the F-16 simulation was supplied by Navier-Stokes CFD. This analysis predicts aeroelastic instability at speeds, frequencies and negative damping levels that are consistent with the F-16 flight test data. It is suggested that with the addition of nonlinear structural damping this analysis may well predict the LCO observed in flight (Chen *et al*. 2001).

(d) Modeling structural nonlinearity in airframes

Recent interest in the use of wing flexibility for control augmentation and very high aspect ratio wings for long endurance flight has motivated the use of structural models that include at the very least geometric nonlinearities if not also elastic nonlinearities. Although it is possible in some instances to include structural nonlinearity modally, (Chen & Sulaeman 2003, Bae *et al*. 2002) most analysis has used a nonlinear finite element model of the structure. Strganac *et al*. (2005) address the development of a nonlinear structural model for highly deforming high aspect ratio wings that accounts for in-plane, out-of-plane, and torsional couplings. Garcia (2005) recently has found that the coupling of nonlinear CFD code with a nonlinear finite element structural code for the analysis of a high aspect ratio wing can significantly alter the lift distribution of such a wing compared with a linearly flexible structure. The influence of structural buckling on LCO onset for a transport wing has been investigated using a state space formulation with aerodynamics produced by the frequency domain unsteady aerodynamic method ZONA6 (Chen & Sulaeman 2003). Attar and Gordnier have modeled the interaction of a vortical flow with a structurally nonlinear delta wing (Attar & Gordnier 2006). Bae *et al*. (2002) have used modal analysis with the concept of fictitious mass to investigate the flutter and LCO onset of a fighter wing having control surface free-play.

To extend the idea of an 'airframe' somewhat, we consider the interesting and novel aeroelastic problem presented by a highly flexible inflatable thin-membrane decelerator designed for aero-capture of an atmospheric re-entry vehicle. Because of their flexibility, thin-membrane decelerators can have large and potentially catastrophic aeroelastic responses, even at low dynamic pressures. Membrane flexibility also complicates analysis by requiring the modeling of geometric and possibly material nonlinearities. Furthermore, the high temperature of re-entry also requires coupling of the flow and structure thermal fields. The ability to compute the nonlinear fluid/structure and thermal interaction to predict the membrane deflections and temperatures is a requirement for a successful design. A recent study has been directed at developing the computational aero-thermo-elastic methods necessary to model all these effects and validating that model with wind tunnel data (Armand *et al.* 2005). The wind tunnel article was a 4 inch diameter 1 mil thick Kapton film model shown in Figure 3 tested at Mach 6 in the NASA Langley Research Center Mach 10 Wind Tunnel. The CFD code used in that study was USM3D, while the structure was modeled with MSC Marc using follower loads and geometrically nonlinear analysis. Although the test article experienced secondary and tertiary creep due to thermal loads the analysis used linear material properties. Despite these simplifications the computed maximum film displacement was within 6.7 percent of the displacement observed in the wind tunnel test. The wind tunnel article with displacements after testing is shown in Figure 3c. The wrinkle patterns and maximum displacements of the computed solution in Figure 3d are in good agreement with experiment.



a) CFD mesh b) FEM membrane mesh

c) Wind tunnel model d) Computed deflections

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Figure 3. Comparison of wind tunnel thin-film ballute test article and computed results. Armand *et al.* (2005)

(f) CAE using reduced order models (ROM)

Because a prohibitive amount of computing can be required to determine flutter onset for a complex aircraft configuration, various ways of reducing the computing have been explored that still retain the important physics. An exciting area of recent research is in the development of reduced order models (ROM) of nonlinear aerodynamics and structures. These methods apply linear and nonlinear system identification and order reduction to the problem of computational aeroelasticity. Several methods under exploration for use in computing linear and nonlinear dynamic aeroelastic phenomena are proper orthogonal decomposition, Karhunen-Loeve decomposition, harmonic balance and Volterra series. Consider the method of proper orthogonal decomposition applied to a nonlinear flow field. In a fashion similar to that used for a modal transformation of a linear structure, this method transforms the discrete flow model from a physical coordinate to a modal coordinate system. Since the analysis orders the modes by the size of the eigenvalue, the lowest order modes can then be retained. In this way, the high-dimensional discrete CFD model is reduced to a lower dimensional system in modal space. This method has been applied successfully to the analysis of flutter onset and to the rapid computation of nonlinear flow induced limit cycle oscillations (Carlson & Feng 2005). Another promising method is the harmonic balance method. In this method the nonlinear flow field equations are transformed into frequency domain and solved in a coupled manner with the frequency domain structure equations. The advantage of this approach is that the time marching solutions of certain unsteady dynamic problems can be reduced to a limited set of steady nonlinear solutions. Hall *et al*. (2002) have developed the harmonic balance method for application to aeroelastic problems and have applied the method to the flutter and LCO onset of an F-16 wing (Thomas *et al*. 2004, see also Thomas *et al*. 2001, 2003).

An illustration of the successful use of reduced order modeling methods for the computation of flutter onset for a complex aircraft configuration is found in the work by Hong *et al*. (2003) later extended by Kim *et al.* (2005). The configuration analyzed is shown in Figure 1. This effort was the first to utilize ROM methods to evaluate the flutter onset of a realistic aircraft configuration. The use of a ROM method allows the creation of an aerodynamic model of the flexible aircraft response due to prescribed modal impulse excitations. The model thus created can be considered the aerodynamic response due to a unit dynamic pressure. The aerodynamic model is then used in a separate simulation of the fully coupled flexible dynamic modal response. The simulation of flutter onset can be quickly performed over a wide range of dynamic pressures by using the reduced order aerodynamic model. The original ROM method was developed for use with CFL3D CAE analysis by Silva & Bartels (2004). For system identification and extraction of the ROM, this method uses the Eigensystem Realization Algorithm (ERA) (Juang 1994). Additional enhancements used in the analysis of Hong *et al.* (2003) include a frequency-domain Karhunen-Loeve method to further reduce the aerodynamic model size. The time histories of a full CAE computation and the ROM simulation near flutter onset were shown to be in excellent agreement. The ROM based method was shown to require 4 to 6 times less computing than did the full CAE simulation (Kim *et al*. 2005). This method was used to compute the twin-engine transport flutter onset boundary shown in Figure 2.

4. Computational Aeroelasticity of Turbomachinery

The combination of high-speed transonic flows with the structural flexibility of turbomachinery gives rise to a range of complex vibration problems. Virtually all turbomachine blade rows are susceptible to vibration induced problems, caused either by inherent self-induced motion (flutter), or by response to flow distortions and blade wakes (forced response). Although significant progress has been made in understanding both flutter and forced response, the ability to predict reliably the vibration amplitude within acceptable error bounds is still some time away. This is true not only because of the difficulties of simulating the unsteady flow at off-design conditions but also due to the complex structural behavior that arises from blade mistuning and the uncertain knowledge of structural damping. The most complex and least understood aeroelasticity phenomena occur in multi-stage core compressors because of their wide operating envelope. During engine development programs, very costly structural failures are known to occur because of a mixture of instabilities such as acoustic resonances, rotating stall, surge and buffeting. Bendiksen (1993) presents a review of some of these problems related to turbomachinery. Because of the importance of these problems to the design of turbomachinery, this section will explain in some detail the above mentioned aeroelastic phenomena as well as the computational aeroelasticity methods that have been applied to their analysis.

Computational aeroelasticity methods that use linear aerodynamic and structural models are still in wide use in the design of turbomachinery. New methods that incorporate nonlinear aerodynamic and even nonlinear structural models are emerging and gaining ground in industrial level analyses. As discussed earlier many developments and contributions have made higher fidelity fluid and structural dynamics modeling possible in turbomachine aeroelasticity. Marshall & Imregun (1996) present a review of aeroelasticity methods applied to turbomachinery. Some of the key contributions that have made nonlinear turbomachine aeroelastic analysis economically feasible will now be outlined.

Nonlinear turbomachinery flow computations were made possible by the nonlinear 2D inviscid method of Erdos *et al.* (1977) for unsteady flow in a fan stage that uses a specific algorithm to deal with the unequal pitches. This work introduced the new concept of phase-shifted periodic boundary condition for a sector of the annulus, which allows for stage computations to be performed using a single passage or a sector, reducing the computational requirements significantly. This work generated great interest because of the introduced concept of temporal periodicity. Later, this work was extended to 3D by several researchers. A 3D Navier-Stokes algorithm was used to compute a stator/rotor interaction problem by Rai (1986, 1987). This algorithm relied on selecting a number of blades from the stator and rotor such that the resulting sectors have equal pitch angle, thus automatically producing a periodic sector. Giles (1988) introduced the notion of “time-inclined” computational planes used to handle arbitrary stator/rotor pitch ratios as an alternative to the above methods. Giles (1992) gives a fairly comprehensive list of the various combinations of the above methods found in the literature.

The techniques of Giles (1992) and Erdos *et al.* (1977) can not be applied to more than two blade row interaction problems when the blade-numbers in a third blade row are different. An example in which this analysis is not possible is when the forcing on a blade row arises from interaction with wakes or potential waves of a blade row which is not an immediate neighbour. Other problems in which this analysis is not applicable is when the excitation is due to low engine order or inlet distortion passing through several blade rows. The obvious approach to model these problems is to perform simulations with whole annulus multi-blade rows. This is a computationally expensive problem to simulate, however with increases in computing power, these calculations are becoming possible in recent years at least for after-design analyses. Examples of the extent to which the complexity of these analyses have grown are found in Sayma *et al.* (2000a, 2000b), which recently performed whole annulus forced response predictions for a combined turbine stage and multi-stage fan, or Wu *et al.* (2005), which performed forced response predictions for a 17 blade row compressor using a 3D viscous solver.

(a) Forced response analysis

When the rotating blades pass through flow defects created by the interaction of the flow with upstream and downstream blade rows, the ensuing large unsteady aerodynamic forces can cause excessive vibration levels even in otherwise stiff components. This gives rise to the term forced response (Sayma *et al.* 2000b). Stator blades are also subject to similar so called forced response problems caused by rotating flow defects arising from the interaction of the flow with rotating rows. Intake distortions and fluctuating back pressures can also be significant excitation sources.

One of the first design steps for minimizing the response levels is the Campbell diagram which indicates the possibility of an assembly mode being excited at a particular rotational speed or its multiples. This is the so-called engine-order excitation. What is of real interest to the designer, which cannot be inferred from the Campbell diagram, is the prediction of the vibration amplitude under unsteady aerodynamic loading. Because of the numerous unknown factors such as structural damping, nonlinear damping in the blade roots, and the forcing itself, forced response analyses often aim at ranking potential designs rather than predicting absolute vibration levels. Such analyses are also used to explain failure mechanisms and to evaluate the effects of possible remedial modifications.

While it is always possible to anticipate the likelihood of blade passing (BP) forced response at design stage, and consequently, try to either eliminate it or keep the response levels to tolerable amplitudes, a more subtle forced response regime is likely to occur at much lower frequencies. This is the so called Low Engine-Order (LEO) forced response (Sayma *et al.* 2000a, Vahdati *et al.* 2000). The unsteady aerodynamic forcing that causes LEO forced response is called LEO excitation, the creation mechanism of which is much more complex than that for blade passing forced response. In some cases, the causes of the LEO forced response are obvious, such as inlet distortions due to non-symmetric intake ducts, cross wind, or other forms of non-stream line entry of the flow into fan rotors. Other cases of LEO in multistage compressors and turbines result from upstream or downstream stator blade numbering. But in most cases, the causes are less obvious and can be due to many sources such as blade to blade variations either in rotor or stator blade rows, combustion variations, general flow unsteadiness or any other mechanism causing loss of cyclic symmetry. Both BP and LEO excitation co-exist in modern engines and the potential cost of undetected in-service forced response problems is enormous because of expensive re-designs and retrofits.

The practical way to obtain response levels for turbomachinery blades under synchronous aerodynamic loading is to use a forced response analysis. The unsteady aerodynamic forcing thus obtained can be used to compute the limit cycle modal displacement and hence the actual displacement if the total damping is known. The total damping, composed of both aerodynamic and mechanical damping, is a critical component of this analysis but is one which also carries the most uncertainty. Aerodynamic damping can be estimated computationally but it is difficult to compute the mechanical damping. For that reason values are usually obtained either with test data, or from approximate values estimated using similar configurations. This limitation in the accuracy of structural damping estimates can represent a serious limitation to force response computations. For blisks, in which the blades and disk are forged from a single piece of metal, mechanical damping is negligible. The use of the blisk construction technique for blade and disk construction thus eliminates a major source of uncertainty in a forced response analysis.

To illustrate the use of forced response analysis consider an entire Low Pressure Compression (LPC) system shown in Figure 4. This includes the flight intake, bypass duct outflow guide vanes (OGVs) and pylon. The aim of this case study is to evaluate the fan assembly forcing levels at different engine speeds for a given LPC configuration and to check these against available measured vibration levels for the blade 1st flap vibration mode.

The analysis domain, shown in Figure 4, was discretized using a mixture of semi-structured grids for the blades and unstructured tetrahedral grids for the intake and pylon. The final mesh contained over 10 million grid points. The forced response computations were performed for sea-level-static boundary conditions. Atmospheric stagnation pressure and temperature were applied upstream while a static pressure, adjusted to yield the correct bypass ratio, was applied downstream of the pylon. The computations were performed at three speeds, namely 90%, 94% and 100%. The modal force due to the 1EO mode was then used to determine the stresses. In this case the 1EO mode is not resonant with the 1F mode, so the elastic stiffness terms are dominant in determining the blade response and no estimate of the damping is required. Figure 5 shows a comparison of the predicted and measured blade root stresses at a given gauge location for the three speeds, the measured values representing an averaged reading from a number of gauged blades. It can be seen that both the stress level and its variation with speed are captured with good accuracy. This is true even at peak stresses occurring at around 94% speed. The shock instability was found to be linked to the steady-state shock position which can be seen from Figure 6, where the shock is most unstable when the steady state solution has a shock that is just swallowed. An animation of the unsteady flow variables also confirmed the previously-identified forcing mechanism.

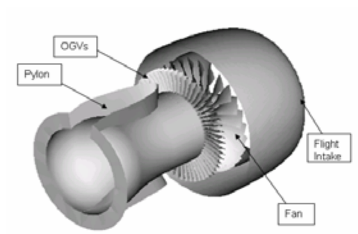


Figure 4. View of the analysis domain

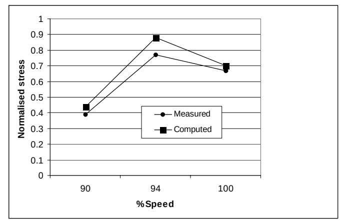


Figure 5. Comparison between computed and measured response levels

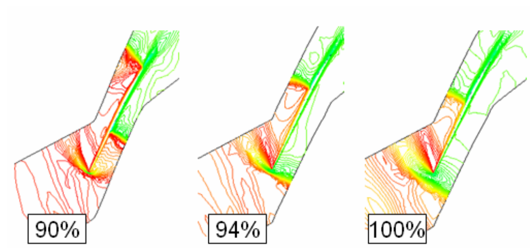


Figure 6. Comparison of steady state shock position near tip for the three analysis speeds

(b) Flutter analysis

As pointed out earlier, flutter is a self-excited phenomenon. This implies that a flutter analysis does not need to take into account the excitation from neighbouring blade rows. Hence, most flutter analyses are performed on a single blade row. At a given operating condition, it is more likely that flutter occurs at a single inter-blade phase angle. Thus, methods relying on single passage analysis with phase shifted boundary conditions are sufficient to predict the onset of flutter. In these methods, the blade is oscillated with fixed small amplitude with the given frequency and mode shape. The periodic boundaries are treated using time lagged boundary conditions. When the unsteady solution reaches a periodic state, the work done by the fluid on the structure is integrated over a vibration cycle. The magnitude and sign of the work done on the structure determines stability. Vahdati *et al.* (2001) presented a detailed flutter analysis of a fan rig using different levels of modelling. That study found that this method is capable of predicting the onset of flutter provided that unsteady viscous effects are taken into account.

Single passage flutter computations are efficient if the mode of interest and the inter-blade phase angle are known before hand. But during the design stage, the unstable mode or inter-blade phase angle are not known. Single passage analyses will not only require large amounts of computer time for all possible modes and inter-blade phase angles, but also will require large amounts of human effort. An alternative is to perform the calculations using whole annulus analysis. Thus, in principle any number of modes can be included in one computation for all possible inter-blade phase angles.

Whole annulus analysis is usually performed in the time domain by exciting all modes simultaneously using an initial impulse excitation of the system.

Flutter occurs at frequencies which are not multiples of the engine order at different places of the operating map. In turbomachinery applications, flutter is usually associated with fan blades, though other compressor and low-pressure turbine blades may also suffer from such instabilities. As the speed is increased towards conditions of flutter, the vibration character alters gradually with amplitudes controlled by nonlinear effects. Flutter sometimes occurs on only a few blades in a row, with widely different amplitudes, but as the amplitude rises, the flutter tends to become more coherent and to involve all the blades at a common frequency with a fixed interblade phase angle.

In axial compressor stages, or fans, the nature of flutter is complicated. Figure 7 shows a compressor map with five regions. This diagram was originally presented by Mikolajczak *et al.* (1975). Region I is the most common and is usually called subsonic stall flutter. One of the main problems in this region is the difficulty in deciding whether vibration problems are a result of flutter or forced vibrations due to rotating stall. In this region, flow separation from the blades may be an essential part of the flutter onset mechanism. However, Vahdati *et al.* (2002) postulated that other mechanisms could exist in this region originating from a match between the mode shape and cut-off acoustic modes trapped in the intake region. This phenomenon gives rise to a sharp drop in the flutter stability boundary known as flutter bite. Region II occurs at negative incidence at speeds below the design speed. This is what is called choke flutter. This usually occurs when the passage is terminated by a strong shock causing boundary layer separation. Torsion modes are most commonly affected by flutter in this region. Region III is called supersonic unstalled flutter or low backpressure supersonic flutter. Flutter in this region is mostly associated with high aspect ratio blades with part-span shrouds. Region IV is called high back-pressure supersonic flutter and is mostly driven by the in-passage shock, and mostly occurs for highly loaded blades. Region V is called supersonic stall flutter. This type of flutter is most common in fans or front-stage compressor blades without part-span shrouds (Halliwell 1975). Additional discussion of stall flutter is found in Sisto (1953).

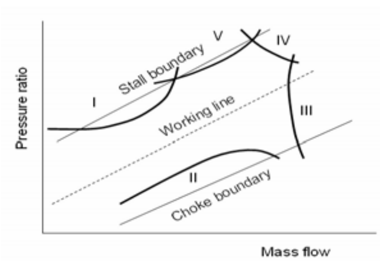
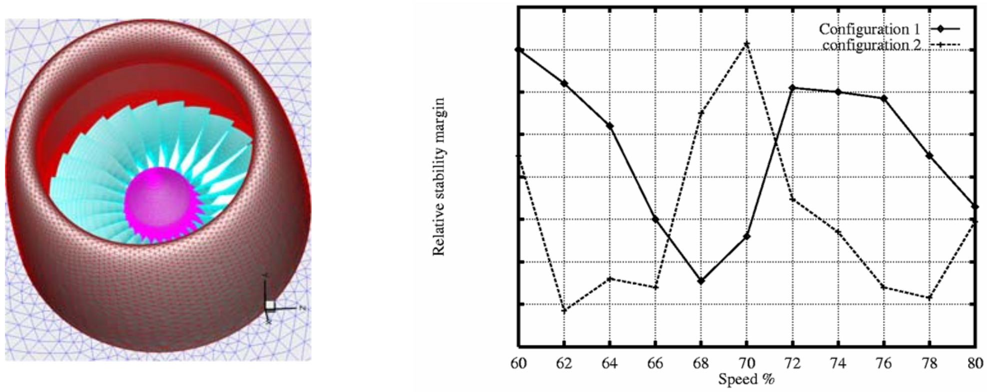


Figure 7. The various flutter regions on the operating map of a transonic compressor. Mikolajczak *et al.* (1975)

Stall flutter is still somewhat difficult to define because of the many ways in which it can occur. A variant, known as the flutter bite, manifests itself as a series of very sharp stability drops for very narrow part-speed ranges. Such behavior is due to an impedance match between the intake acoustics and the upstream pressure perturbations created by the vibration of the fan assembly in a given nodal diameter mode. Figure 8 presents results from Vahdati *et al.* (2002) of a stall flutter computation performed on a complete fan assembly plus an intake duct. A series of flutter analyses was conducted for the 60-80% speed range in 2% speed increments by considering two different intake duct geometries. An important requirement in an analysis of this type is the inclusion of the intake duct and the whole annulus of the fan. The key result shown in Figure 8b is that the relative flutter margin, defined with respect to some arbitrary damping value, is markedly different for each intake. Since the numerical model of the fan assembly and the aerodynamic conditions were kept the same, the difference in the flutter behavior was clearly linked to the intake duct properties. The exact mechanism was identified by examining the ratio of the unsteady pressures and axial velocities in the duct domain.



(a) Flight intake and fan grid (b) Flutter margin
(Configuration 1: test-rig intake, Configuration 2: flight intake)
Figure 8. Fan + intake grid and flutter margin. Vahdati *et al*. (2002)

(c) Rotating stall and surge analysis

This phenomenon occurs most often in core compressors, but can occur also in fans. Rotating stall is instability local to the compressor, in which a circumferentially uniform flow is disturbed. A local region or regions appear where the flow is stagnant. The regions propagate in the same direction as the blades, rotating around the annular path at a fraction of the rotational speed. For fully developed rotating stall this speed is between a fifth and half of the shaft speed. Rotating stall may occur in some parts of the machine only, e.g. in some stages. It is regarded as the precursor of a more severe and potentially dangerous flow instability known as surge. At least two types of aeroelasticity problems can result from rotating stall. The first is the force excitation of the blades in the blade row undergoing rotating stall. This excitation results from a match in the rotating stall cell number and relative blade row frequency compared with frequency the blade row natural modes. The second is the forced response excitation of upstream or downstream blade rows. This excitation also occurs if the relative frequency and shape of the rotating stall cells matches any of the blade row natural modes.

Surge is a self-excited cycle phenomenon, affecting the compression system as a whole. It is characterized by large rises in the annulus unsteady pressure amplitude and average mass flow fluctuation. Flow reversal is also possible. Surge also can cause at least two other aeroelasticity related problems. The first is known as surge deflection, where blade rows, particularly cantilevered ones undergo axial deflection under surge loads leading to the possibility of rotors rubbing into or hitting stator blades, or the casing. The other form of instability is indirect, in which the axial deflection of a blade row can lead to the initiation of flutter in one of the blade rows.

(d) Acoustic resonance

Acoustic resonance occurs when an unsteady aerodynamic mode locks onto a structure mode. In turbomachinery cavities or ducts, this results from the formation of standing waves that can take place only at discrete frequencies. These are known as resonant modes. At resonance conditions, an initial disturbance is either sustained or amplified. According to Cargill (1987), acoustic resonance occurs when acoustic waves change their character from propagating (cut-on) to non-propagating (cut-off) waves. Acoustic resonance can occur in intake ducts, bleed chambers (Yamaguchi *et al.* 1991) and combustion chambers (Caraeni *et al.* 2003). Acoustic resonance can also be triggered within blade rows by circumferential non-uniformity of the flow or by blade vibration. It can also occur in seal cavities where the seal gap acts as a reed and the downstream cavity acts as the body of an instrument, amplifying the excitation.

(e) Blade aerodynamic untwist

As an assembly of high aspect ratio, thin, flexible airfoils rotates, the centrifugal and aerodynamic loads deform the blades relative to their stationary shape. Such assemblies are typically found in the fan system of gas turbine engines powering large civil airframes. The deformation induced by the running forces can reduce the tip stagger of the blades by as much as five degrees, producing the well known phenomenon of fan blade untwist. Typically, fan blades open up under aerodynamic and centrifugal loading, resulting in a positive value of untwist.

The untwist phenomena can be differentiated from other aeroelasticity phenomena mentioned above because it is essentially a static aeroelasticity problem similar to that in airframes. Examples of the analysis of this problem can be found in Sayma *et al*. (1998) and Vahdati *et al.* (2001). Wilson *et al.* (2005, 2006) used nonlinear models to study the aerodynamic untwist of large fan blades. Their study focused on two aspects, the first is the change in performance of the fan when the flexibility of the blades is taken into account along a given speed characteristic. The other objective of their investigation is to assess the effect of blade’s geometric variability on the running shape of a fan assembly.

(f) Modeling structural nonlinearity in turbomachinery

Although most turbomachinery problems can be modeled accurately with a linear structural model, there are a number of applications where nonlinear structural effects must be taken into account. One example is the nonlinear behavior of friction dampers. Friction dampers are small dampers placed under the platforms or in the shrouds of turbomachinery rotors to reduce the amplitude of either flutter or forced response vibrations. One of the main effects of a friction damper is to introduce nonlinearity into the structure. A hallmark of structural nonlinearity is the variation of natural frequencies and damping levels with vibration amplitude. Iterative approaches have been used to track the resonance frequency in those applications (see for example Bréard *et al.* 2000). Other nonlinear effects can arise from friction at contact points and turbine shroud interlocks. The increasing use of blisks also introduces a new class of problems, namely the lack of structural damping. Because of the lack of mechanical damping in blisks, recent research efforts have been aimed at adding damping by using viscoelastic material within the blades. The introduction of such materials adds beneficial nonlinear damping into the structure, but at the expense of requiring nonlinear structural analysis. An important contribution toward coupling nonlinear structural and aerodynamic models can be found in the work of Doi & Alonso (2002). That work coupled an existing CFD solver to a commercial structural analysis code via transfer of boundary conditions between the two solvers at the interface. Doi and Alonso demonstrated their approach by using it for flutter computations of a single blade passage. Although demonstrating the technique for critical applications, they found the expected result that the use of both nonlinear aerodynamic and structural models is still unduely expensive. This is particularly true when modeling whole annulus multiple blade rows. A similar coupling of nonlinear aerodynamic and structure codes for flutter analysis has also been presented by Carstens *et al.* (2003).

5. Future Trends in Computational Aeroelasticity

Attempting to predict the future always risks failing on some points. None the less, it seems that a paper such as this must make the attempt to predict future trends in CAE. It is likely that there will be several trends in the computational modeling of the aeroelasticity of airframes and turbomachinery. One trend will be the increasing use of CAE models at a variety of levels, from low to high fidelity, at all stages of the design and certification of aircraft and engines. As discussed in this paper, many recent research efforts have focused on assessing the accuracy of higher fidelity methods and the feasibility of their expanded use. There have been instances cited in this paper of the focused application of higher fidelity CAE tools to aeroelastic problems that have yielded real insights into the nature of complex interactions. And although currently too expensive for routine aeroelastic analysis for the foreseeable future, the focused application of higher fidelity CAE methods on flow field conditions and configurations not amenable to test or known to exhibit complex nonlinear interactions will continue to expand. The drive toward structural designs that have tighter margins and reduced failure rates also drives the use of higher fidelity methods. These methods will provide new and unique physical insights into complex aeroelastic phenomena.

The design of aircraft will also see the integration of CAE into a larger multidisciplinary design framework. Although the aeroelastic and multidisciplinary optimization of designs and aeroservoelastic analyses are already making use of higher level methods of CAE, these methods are still too expensive for routine application. In certain applications discussed in this paper, nonlinear aeroservoelastic interactions have been clearly shown to cause anomolous flight behavior. This will continue to motivate the integration of higher fidelity multidisciplinary analyses in the design of aircraft. The incorporation of thermal, catalytic surface and radiation effects on structural properties will be included in CAE analyses. With increases in computing power and the massive parallelization of CAE codes, it can be expected that the complexity of the modeled flight dynamics, flow field, structure or the control system will increase. At the same time the growing use of linear and nonlinear system identification techniques will help to reduce the computational expense and allow a larger region of the flight envelope to be simulated. The useability and robustness of higher fidelity methods is a major thrust in development that will need to continue.

It can be expected that aircraft and engine designs will continue to rely on the synergistic use of simulation in conjunction with testing. As computational models have become more sophisticated, they have indeed replaced some testing with what can be called virtual testing. There are isolated examples of virtual testing in turbomachines. Examples are the emerging application of forced response analysis to entire components such as a whole intermediate pressure compressor, an entire low pressure compression system, or a whole turbine. Examples in airframes include the nonlinear CAE analysis of transonic flutter or LCO onset of a full aircraft or the analysis of flight conditions unattainable in a wind tunnel. There are other phenomena that are difficult to simulate because of the time scale over which they develop or perhaps the complexity of the flow field physics or structural geometry. That complexity at least at the present time makes it unlikely that simulations will in the near future wholly replace testing.

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