Transition Zone Modeling

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This lecture reviews current practice as well as new modeling ideas for the calculation of at least skin friction and heat transfer between the onset and end of transition.

The principle means of transition zone modeling dating from the 1950's is through the intermittency models developed by Narasimha and colleagues. In these methods the length of the transition zone is closely correlated to mean boundary layer properties at the onset location and so the procedure is critically dependent on a correct determination of the onset location. The procedure is independent of the level or character of the free-stream disturbances implying independence of the spot generation mechanism or other detailed physics. Determination of onset location is principally by correlation methods of by e^N for quiescent streams. Recent development of PSE methods by Herbert and others provides a powerful method for calculating onset at all turbulence levels. Additional data for modeling of bypass transition over heated flat plates have been obtained by Sohu and Reshotko in the NASA-Lewis boundary layer channel. With a series or grids placed upstream of the contraction, free stream turbulence levels ranging from 0.4% to 6% can be produced. For Tu=0.4%, heating the plate advances transition as would be expected fro T-S considerations. For Tu>1%, there is negligible effect of heating on transition.

Rai and Moin (1990) have performed a direct Navier-Stokes (DNS) simulation of the grid 2 case of Sohu and Reshotko. This calculation which used about 8000 hrs. of CRAY-YMP time reproduced the onset of transition and most profile features through transition but missed the turbulent skin friction level. Such DNS simulation have to do better before they can be used as validation "experiment" for simpler modeling methods.

Sohu's intermittency conditioned mean profiles show that the nonturbulent part is Blasius near the beginning of transition but is fuller than Blasius as transition progresses. The turbulent part is initially less full than the standard turbulent boundary layer but approaches the Musker profile as transition is completed. Sohu's boundary layer spectra for Grids 2 and above are unambiguously turbulent. The spectra for Grid 0.5 and Grid 1 are somewhat ambiguous but our sense is that the bypass made is dominant. Sohu's spectra have some t-s features. Suder's spectra in the same facility for Grid 0 without plate heating fairly clearly document a T-S path to transition. The turbulent spectra observed in bypass situations suggest that bypass transition might be modeled using turbulent flow methods. Additional experiments are under way for boundary layers with pressure gradients.

The balance of the presentation shows the experience to date of the Wu and Reshotko with a multiple-time-scale k-ε method. The skin friction results for the ERCOFTAC T3A case (Tu=2.8%) are reasonable but subject to improvement. The distributions of the k and ε components through transition are shown. More specifically, the low frequency component of k (kp) through transition is comparable to what is expected from such experiments including law-of-the wall similarity in the near-wall region. For turbulence levels above 12%, the law-of-the-wall similarity breaks down indicating that the outside turbulence is strong enough to change the near wall behavior thus altering the characters of the turbulent boundary layer at sub-elevated turbulence levels.

Work is continuing toward improving the MTS modeling and making the procedure more robust. Also it is important that the receptivity issue be examined toward identifying the physics of bypass initiation. Of particular interest in this regard are the forced algebraically growing subcritical "modes" identified recently by Henningson, Trefethen and colleagues.

INTERMITTENCY MODELS

$\gamma = \gamma(x)$ { Narasimha, Arnal, Chen and Thyson }

Linear combination
$( ) = (1-\gamma)( )_L + \gamma( )_T$

Algebraic
$\nu = \nu_m + \gamma(x)\nu_T$

<u>Narasimha</u>

$$\gamma = 1 - \exp\left[-\frac{(x-x_t)^2 n\sigma}{U}\right]$$

x_t is onset location

n is spot formation rate

σ is a spot propagation parameter

$$\frac{n\sigma}{U} = \frac{0.41}{\lambda^2} = \frac{N}{\theta_t^2}\left(\frac{\nu}{U\theta_t}\right) \quad \left\{ \lambda = \sqrt{\frac{0.41}{N}}\,\theta_t R_{\theta_t}^{1/2} \right.$$

where λ is distance between $\gamma = 0.25$ and $\gamma = 0.75$

N is dimensionless parameter thought to be nearly constant

θ_t is momentum thickness at x_t

DETERMINATION OF ONSET

Quiescent Streams $0 < Tu \lesssim 0.4\%$	Correlation, PSE, e^N, *
Weak Bypass ("Dangerous") $0.4\% \lesssim Tu \lesssim 2\%$	Correlation, PSE, *
Strong Bypass $Tu \gtrsim 2\%$	Correlation, PSE, * k-ϵ (DNS)

* Methods based on Henningson, Trefethen inspired techniques involving non-orthogonal forced modes, principally 3D!!

WHAT ARE THE ALTERNATIVES?

Quiescent Flows – Wilcox: e^4 followed by $g-\tau$

Bypass Flows – Many investigators:

Turbulent Flow methods

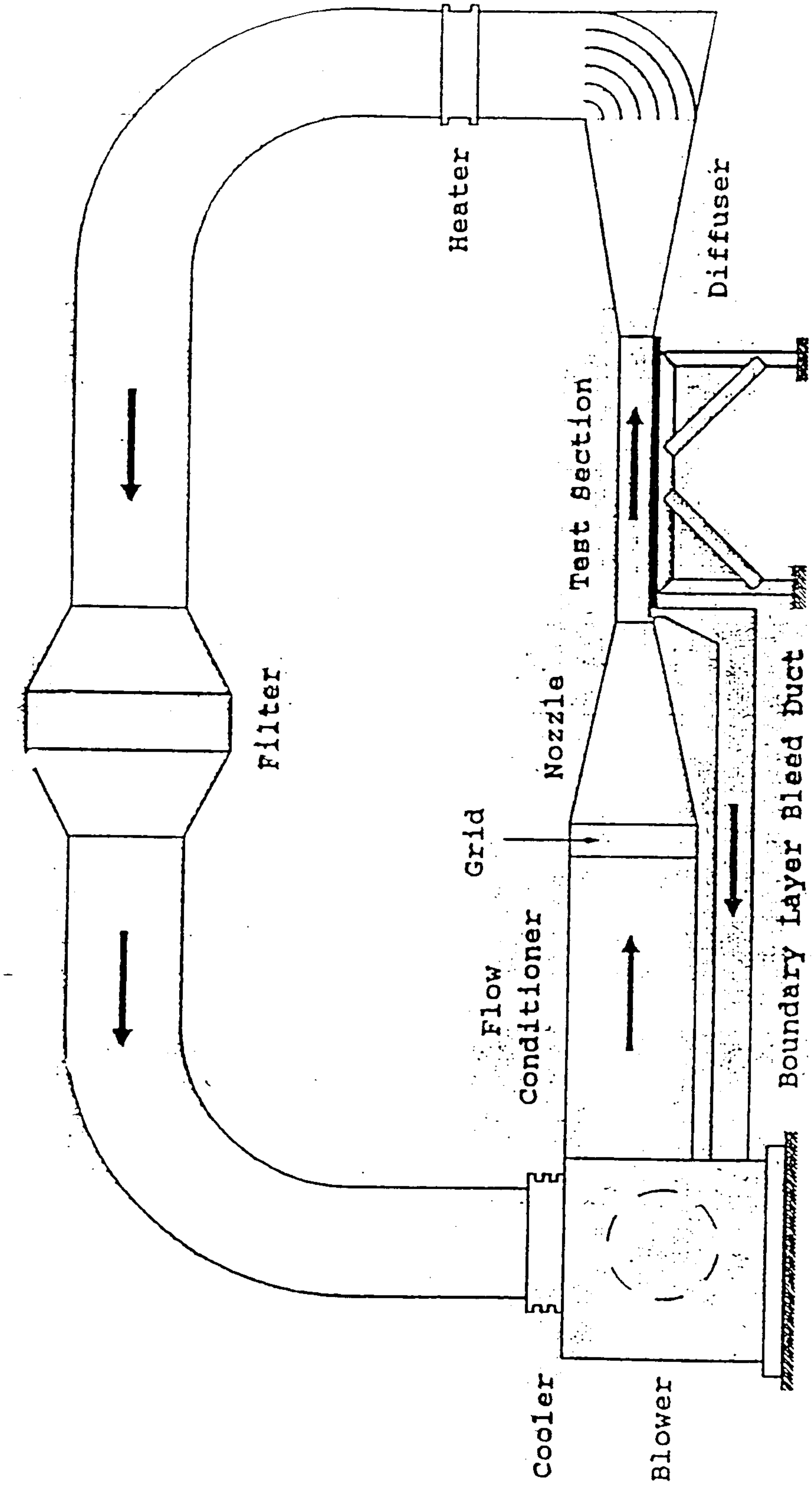


Fig. 1 Schematic diagram of the wind tunnel

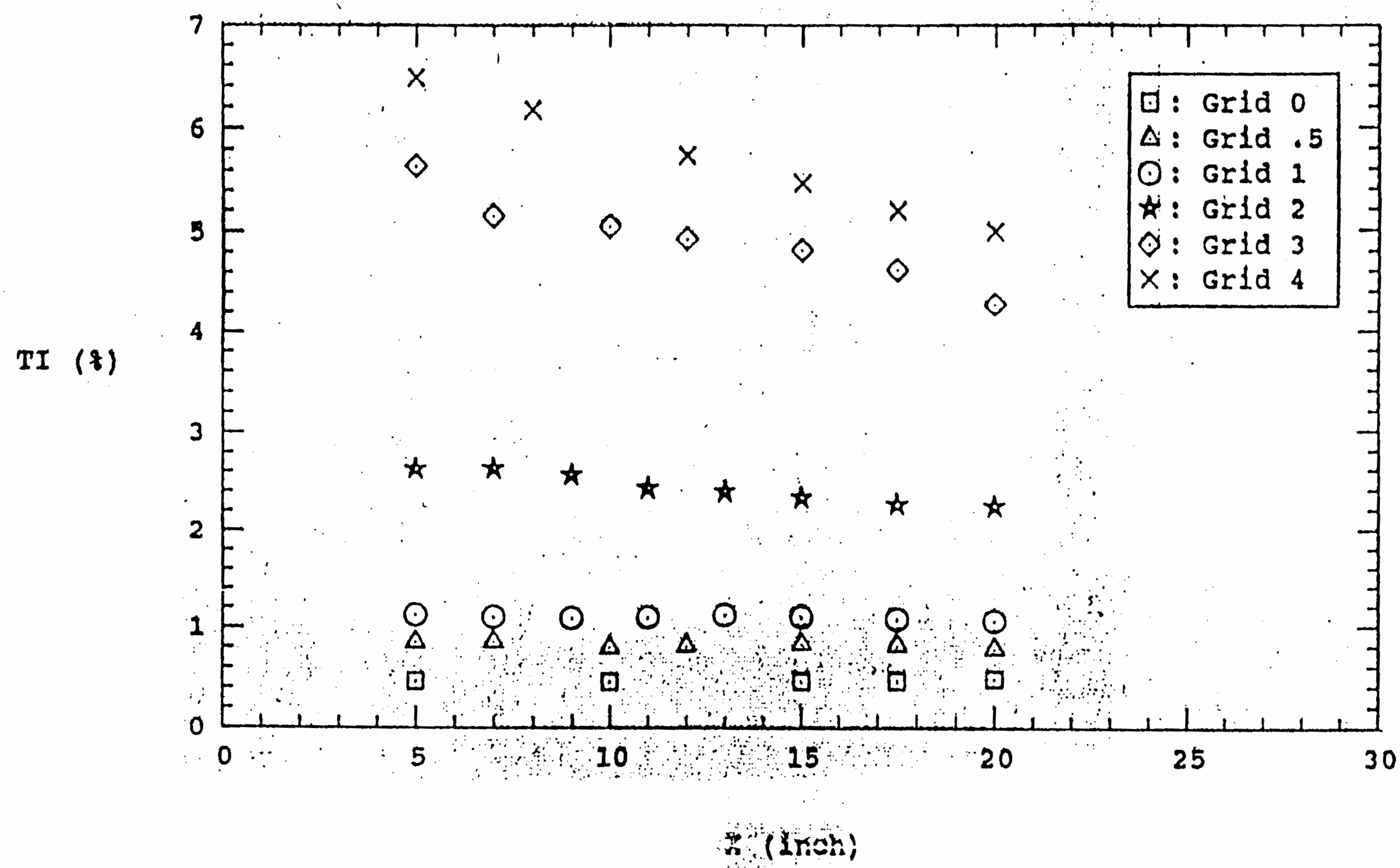
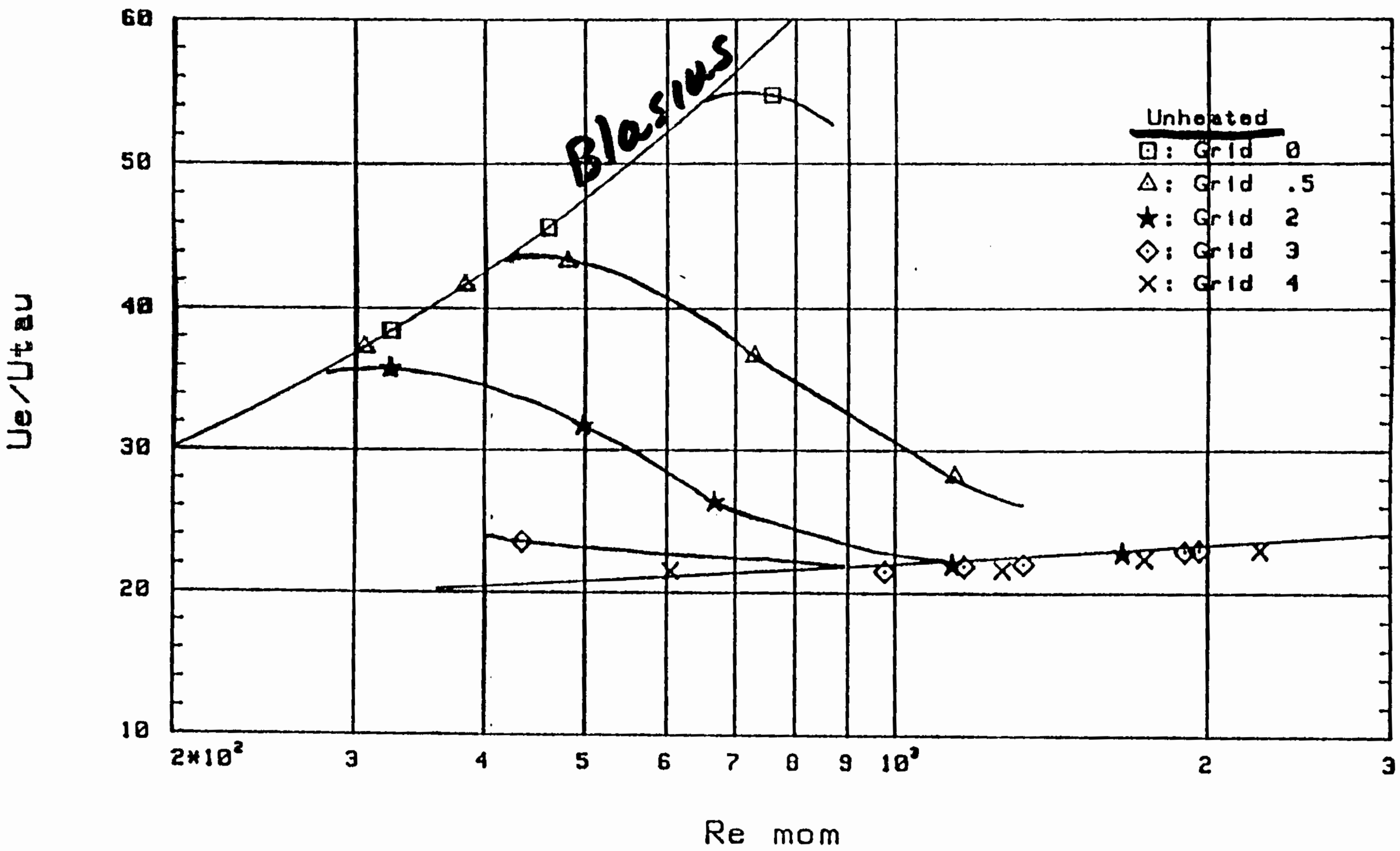


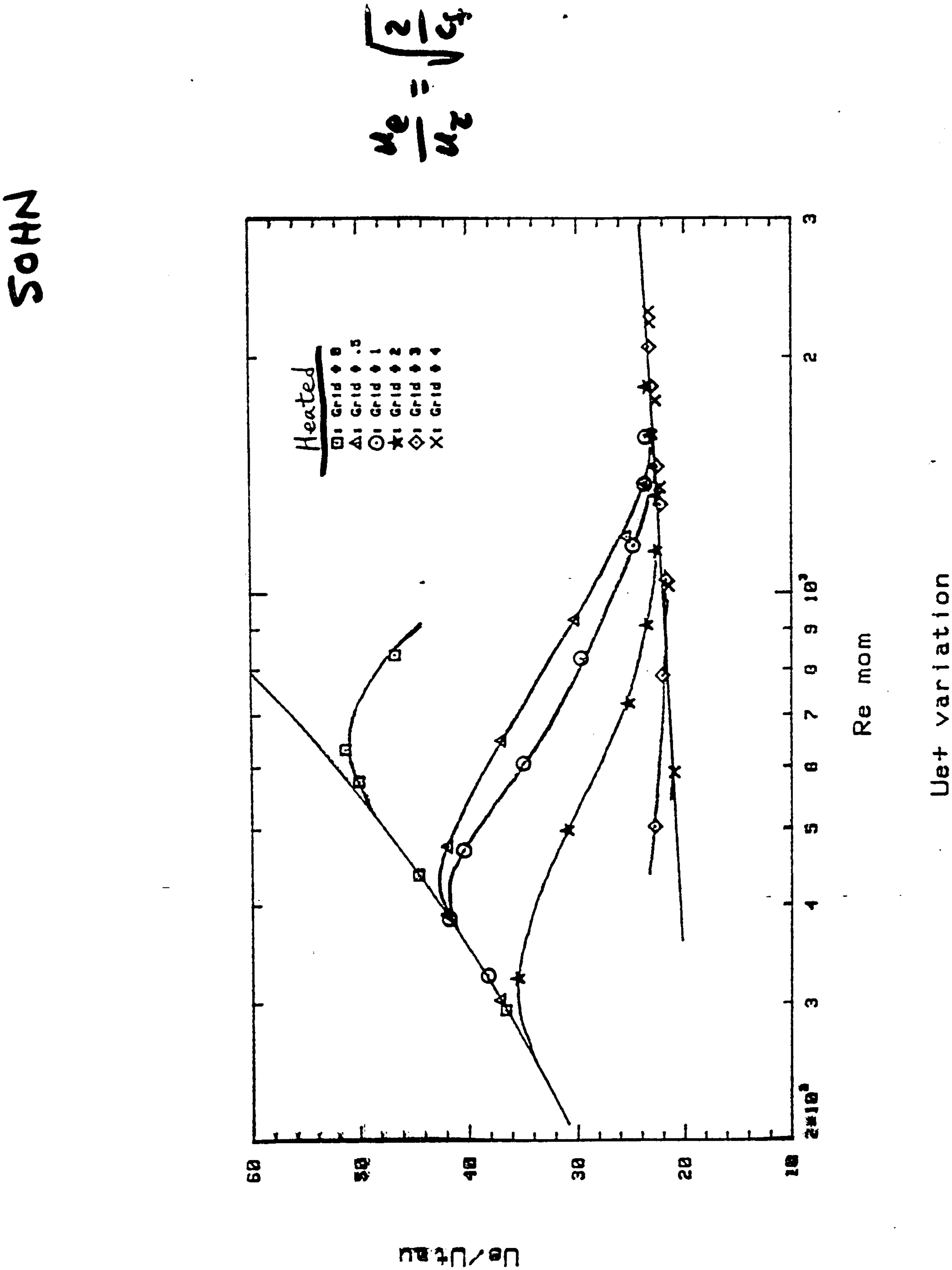
Fig. 19 Streamwise freestream turbulence intensity through the test section

SOHN

$$\frac{u_e}{u_\tau} = \sqrt{\frac{2}{C_f}}$$



Ue+ variation


SOHN
Heated
Grid # 0
Grid # .5
Grid # 1
Grid # 2
Grid # 3
Grid # 4
Re mom
Ue+ variation
Ue/Ut.au

($\Delta T \sim 15^{\circ}F$)
Heating reduces $Re_{x_{tr}}$ by about 20% for Grid 0 (T-S mode)

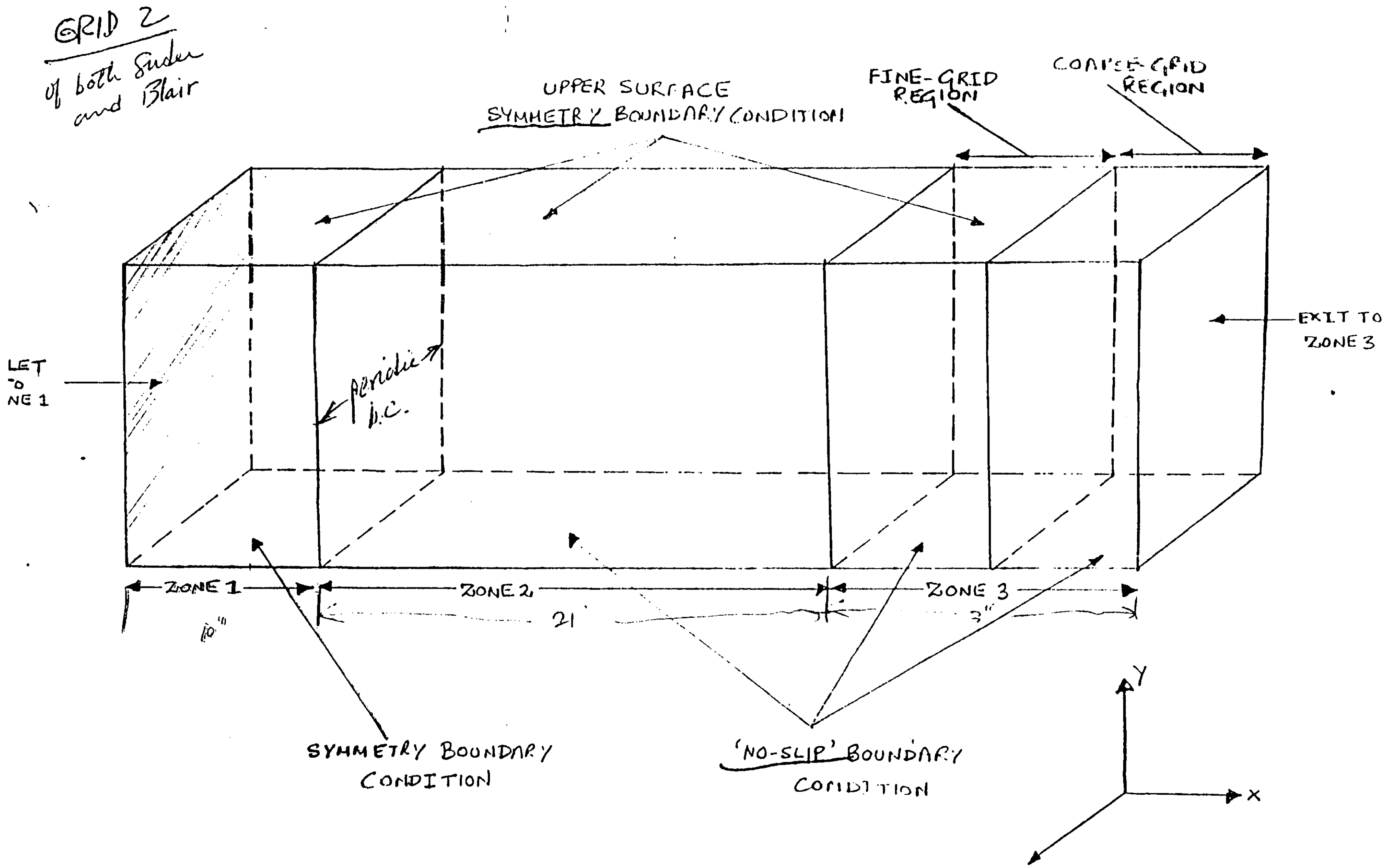
BUT HAS NEGLIGIBLE EFFECT ON Re_{tr} FOR THE OTHER GRIDS (Bypass mode dominant)

Note: For distibuted roughness sufficiently large, stability modifiers are inoperative or may even have reverse effect. THIS IS ALSO A BYPASS.

FIG. 1.

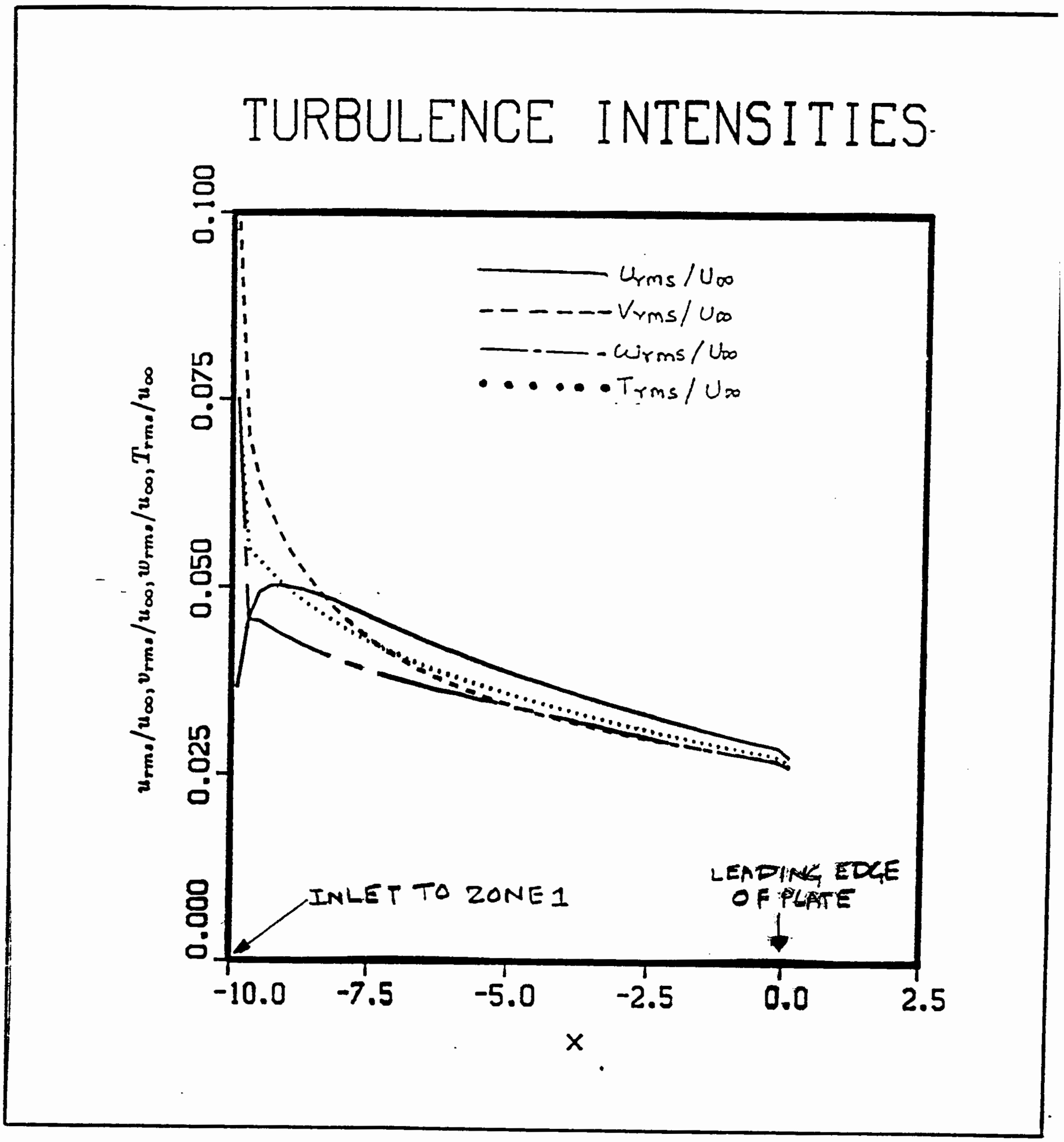
Man Rai, NASA-Ames 1990

SCHEMATIC OF COMPUTATIONAL REGION
(NOT TO SCALE)



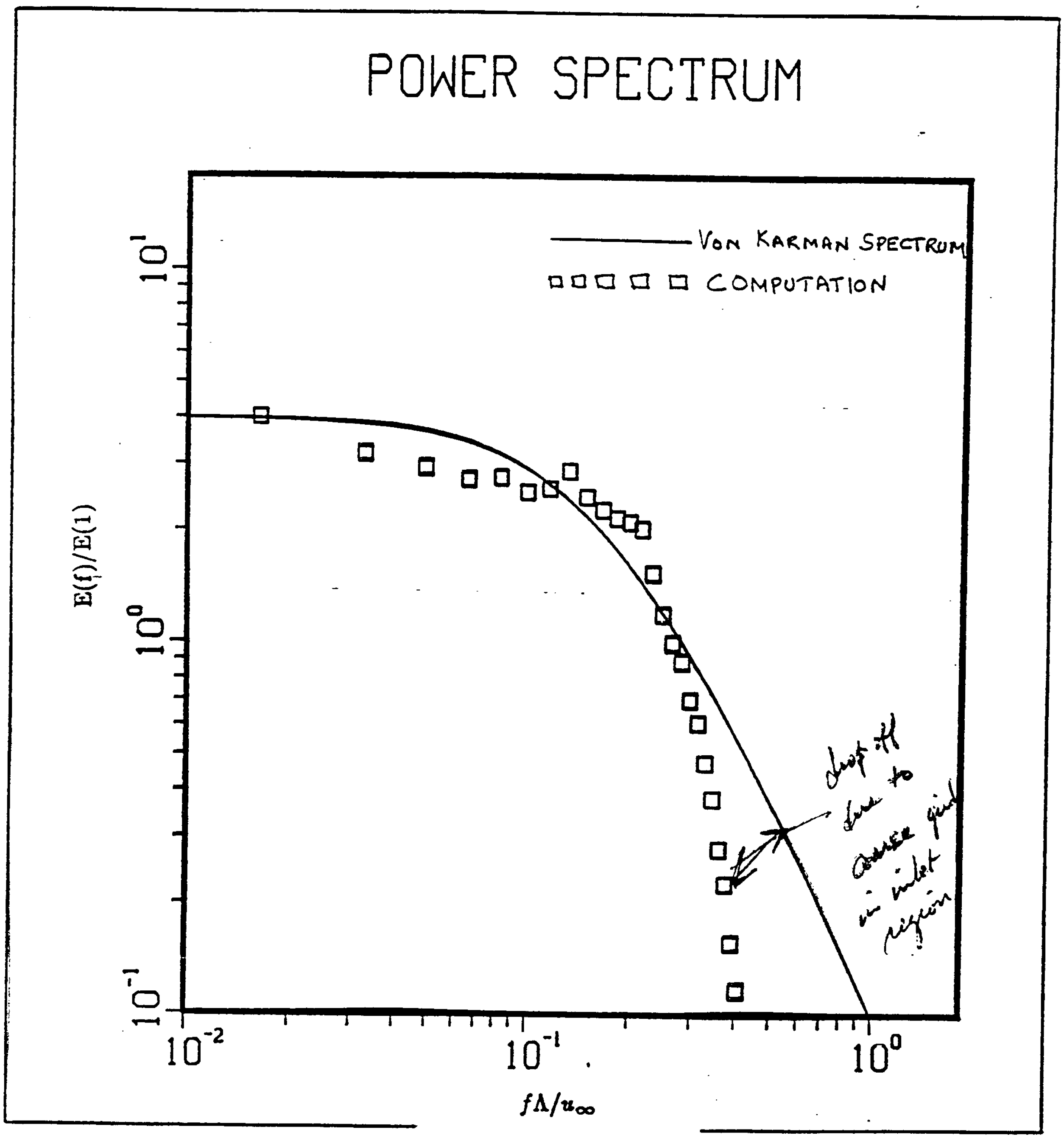
Rai, 1990

FIG. 2



Rai, 1990

FIG. 3

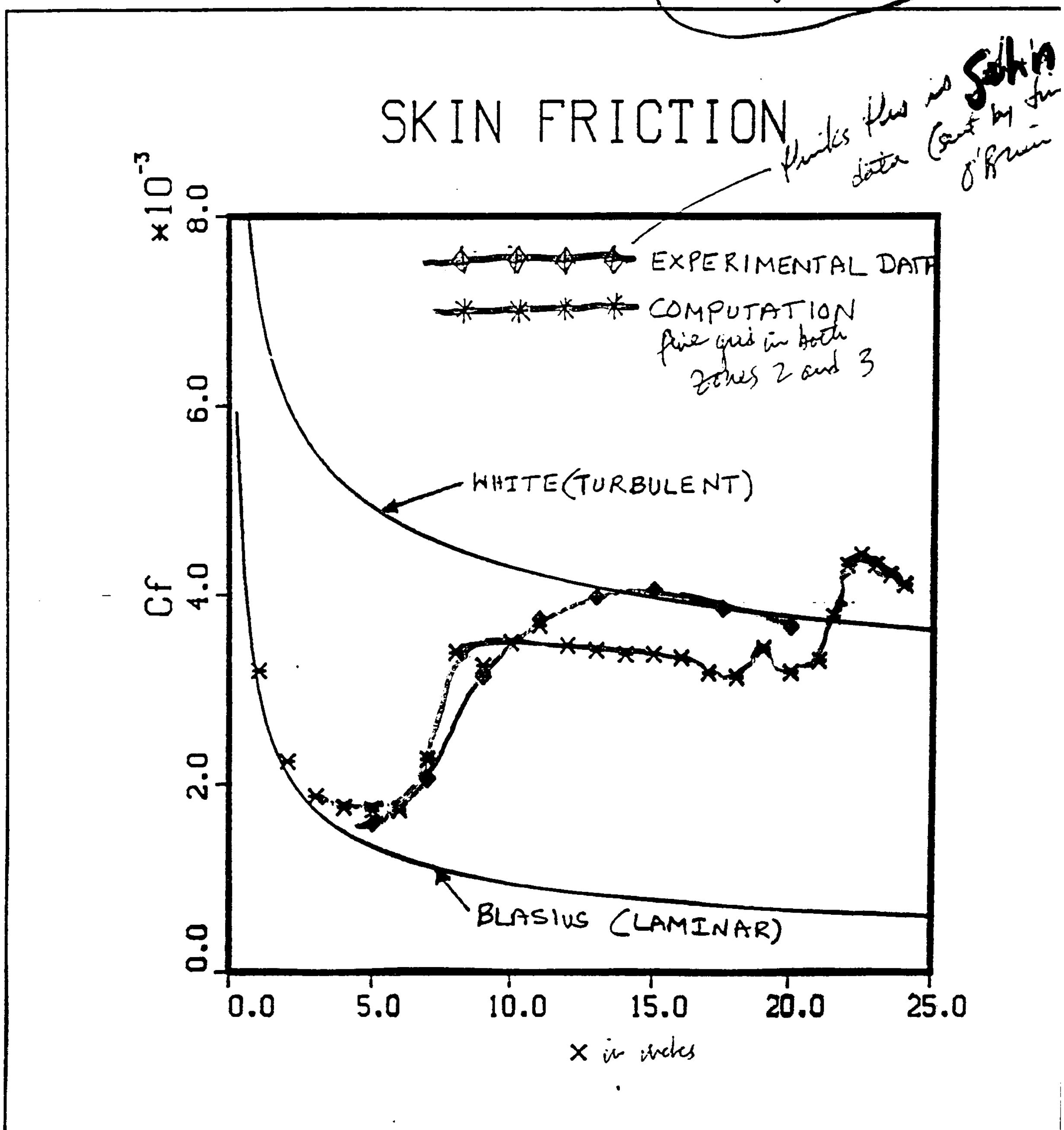


Rai, 1990

Would go a lot faster for M = 0.4 rather than M = 0.1

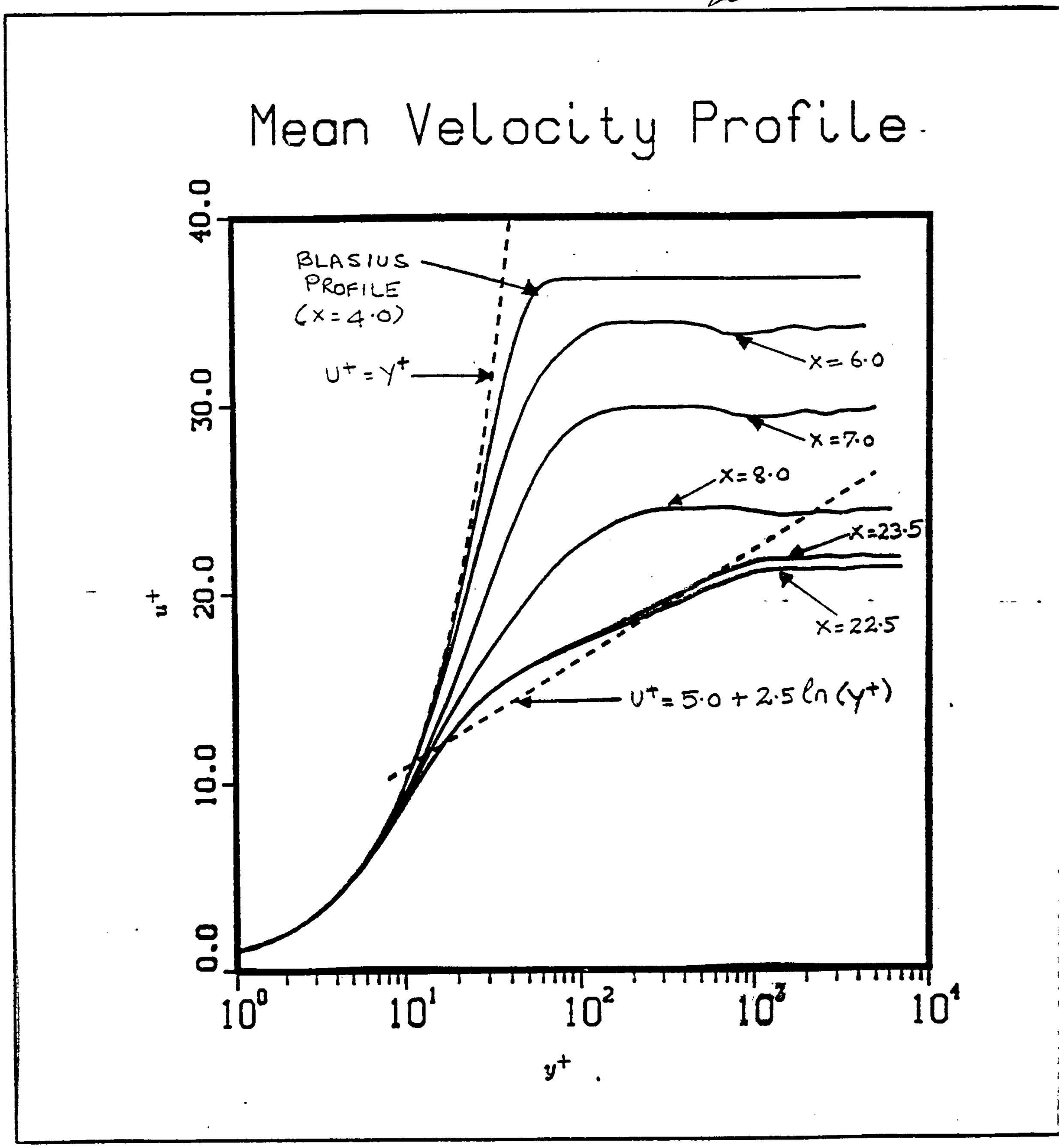
16.8x10^6 grid points 600-800 hrs on CRAY-YMP

FIG 5



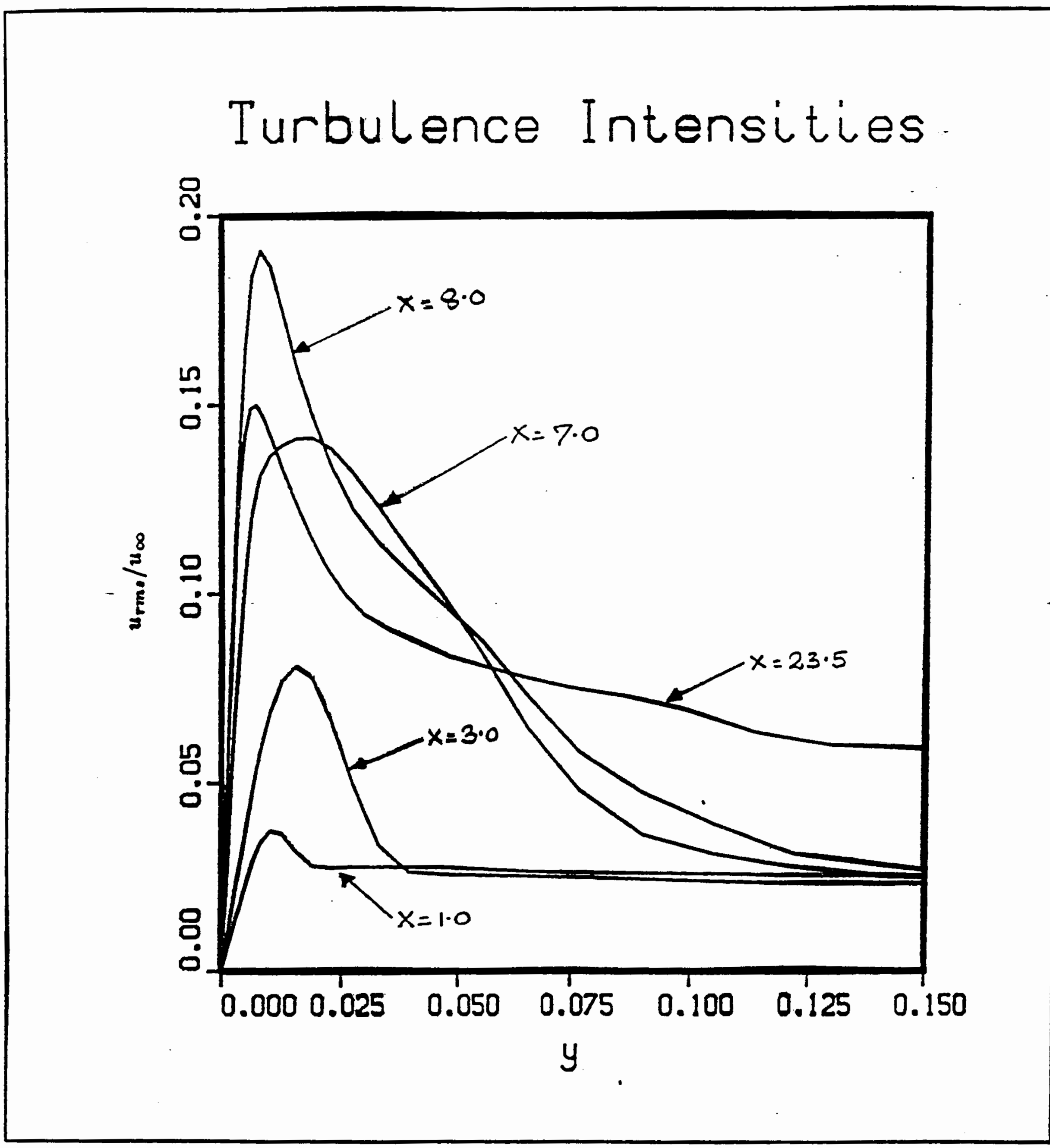
Rai, 1990

FIG. 6 $\Delta x^+ \approx 29$ $\Delta z^+ \approx 10$



Rai, 1990

FIG. 7



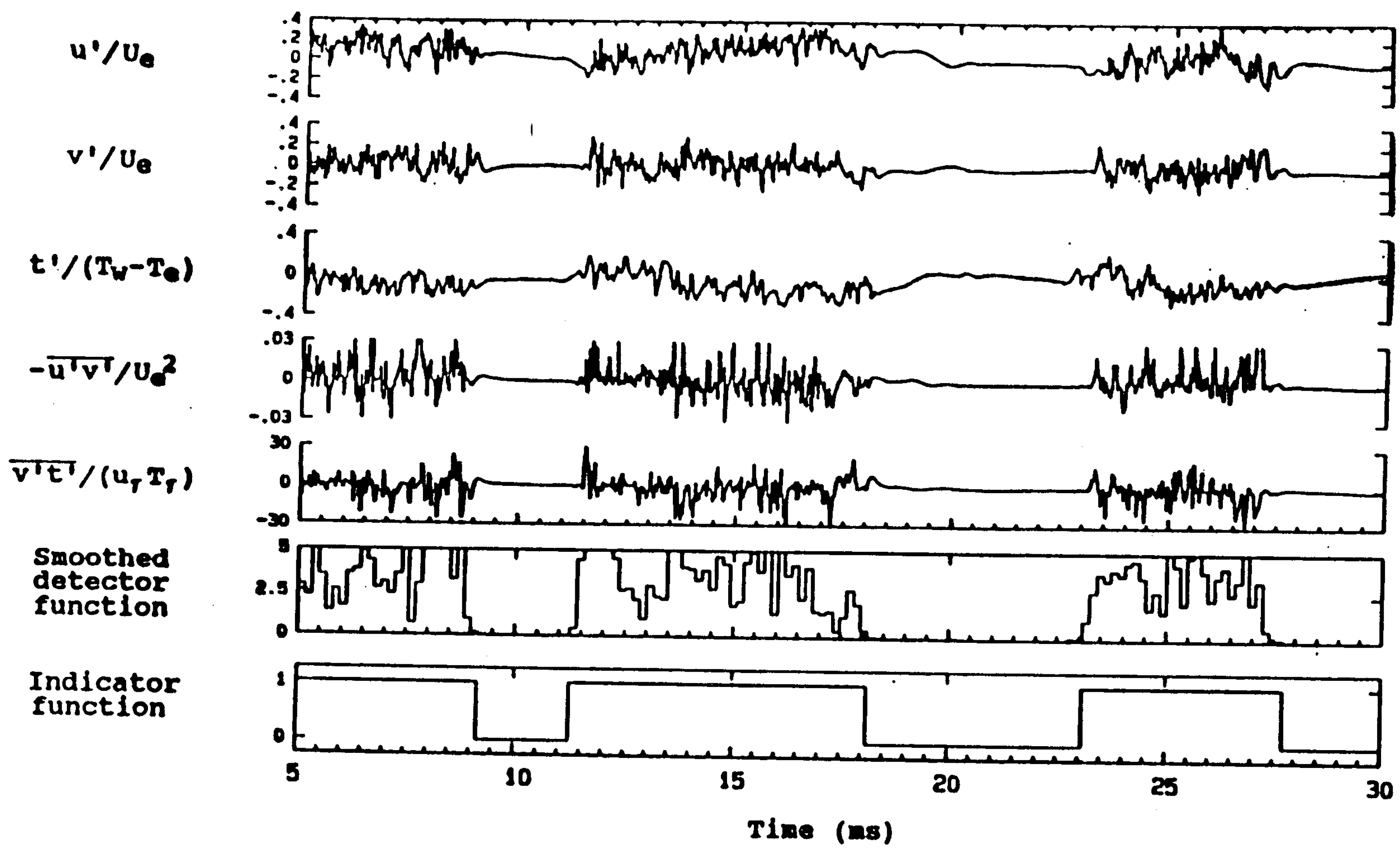


Fig. 18(b) Illustration of indicator function determination technique for 3-wire probe following Hedley and Keffer (1974)

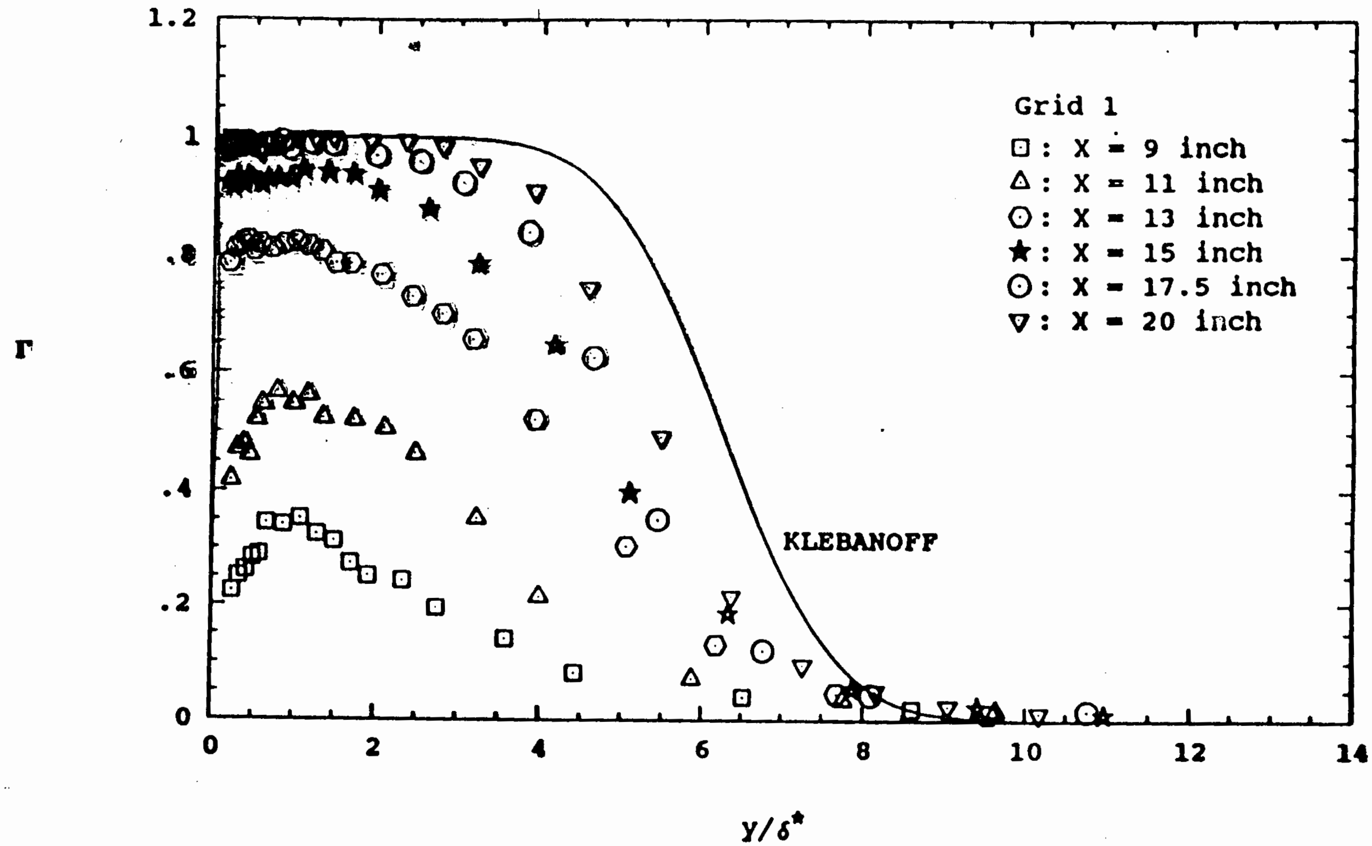


Fig. 28(a) Intermittency profiles across the boundary layer for grid 1

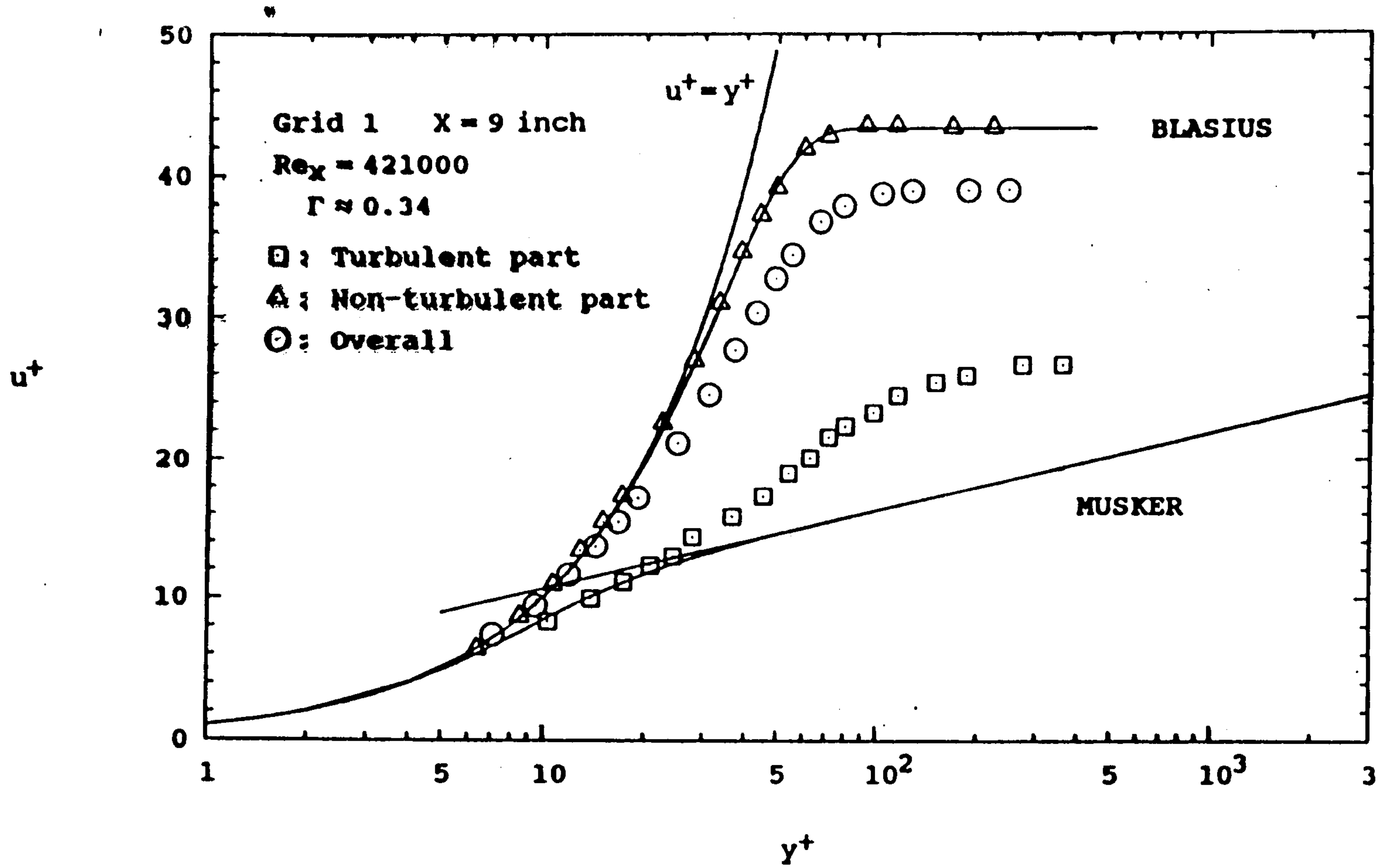


Fig. 29(a) Conditionally sampled mean velocity profiles in wall units at X=9 inches for grid 1

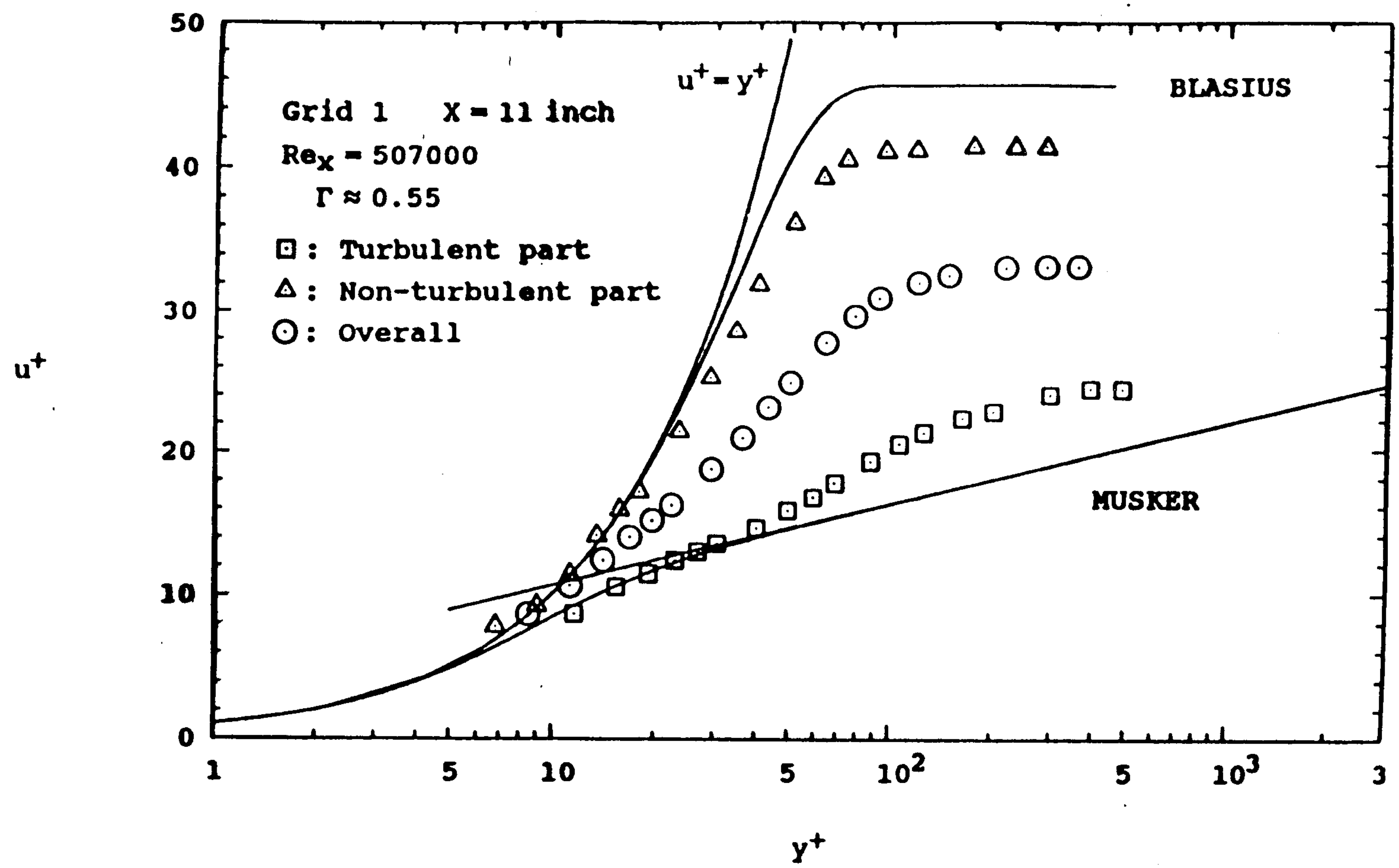


Fig. 29(b) Conditionally sampled mean velocity profiles in wall units at X=11 inches for grid 1

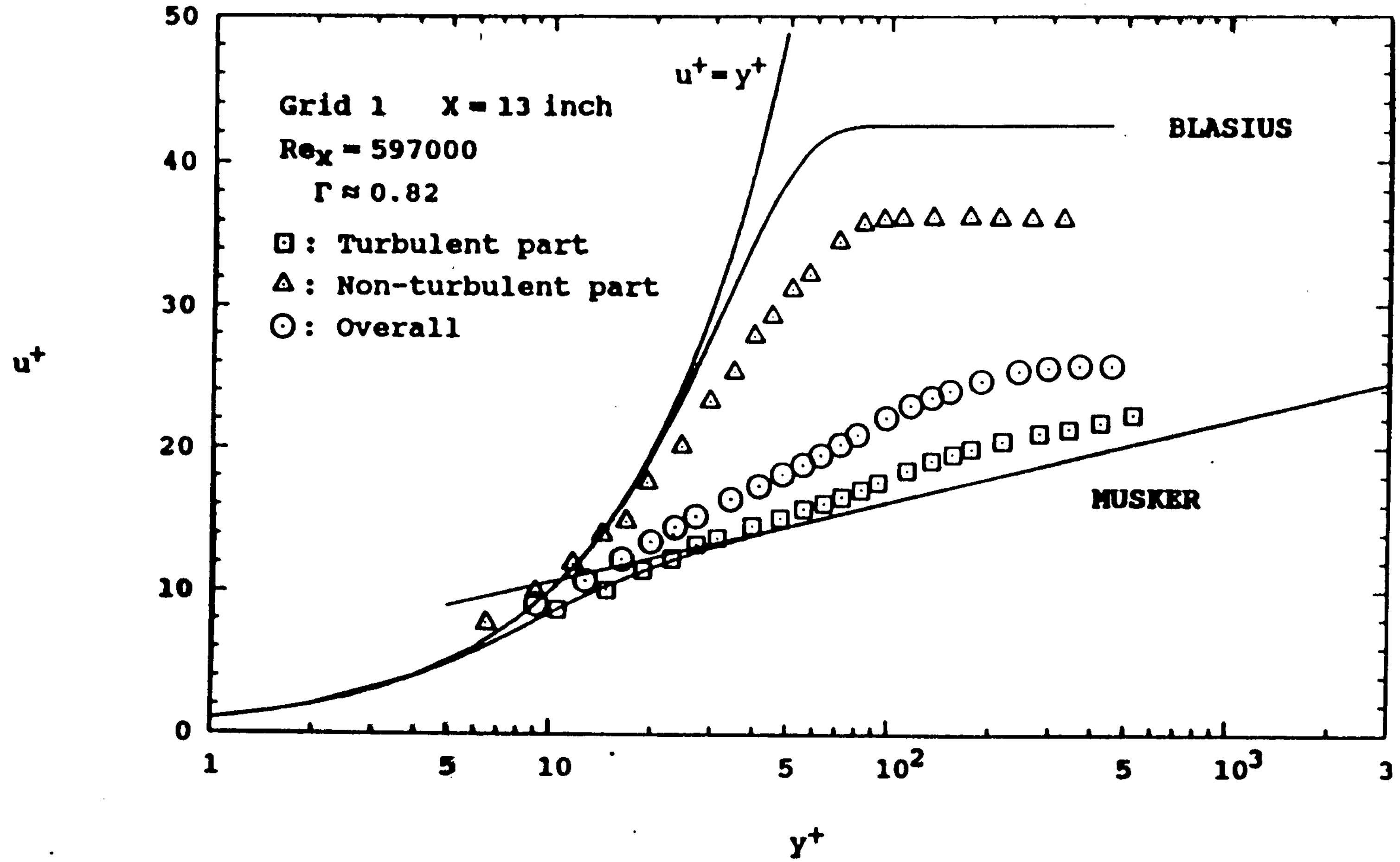


Fig. 29(c) Conditionally sampled mean velocity profiles in wall units at X=13 inches for grid 1

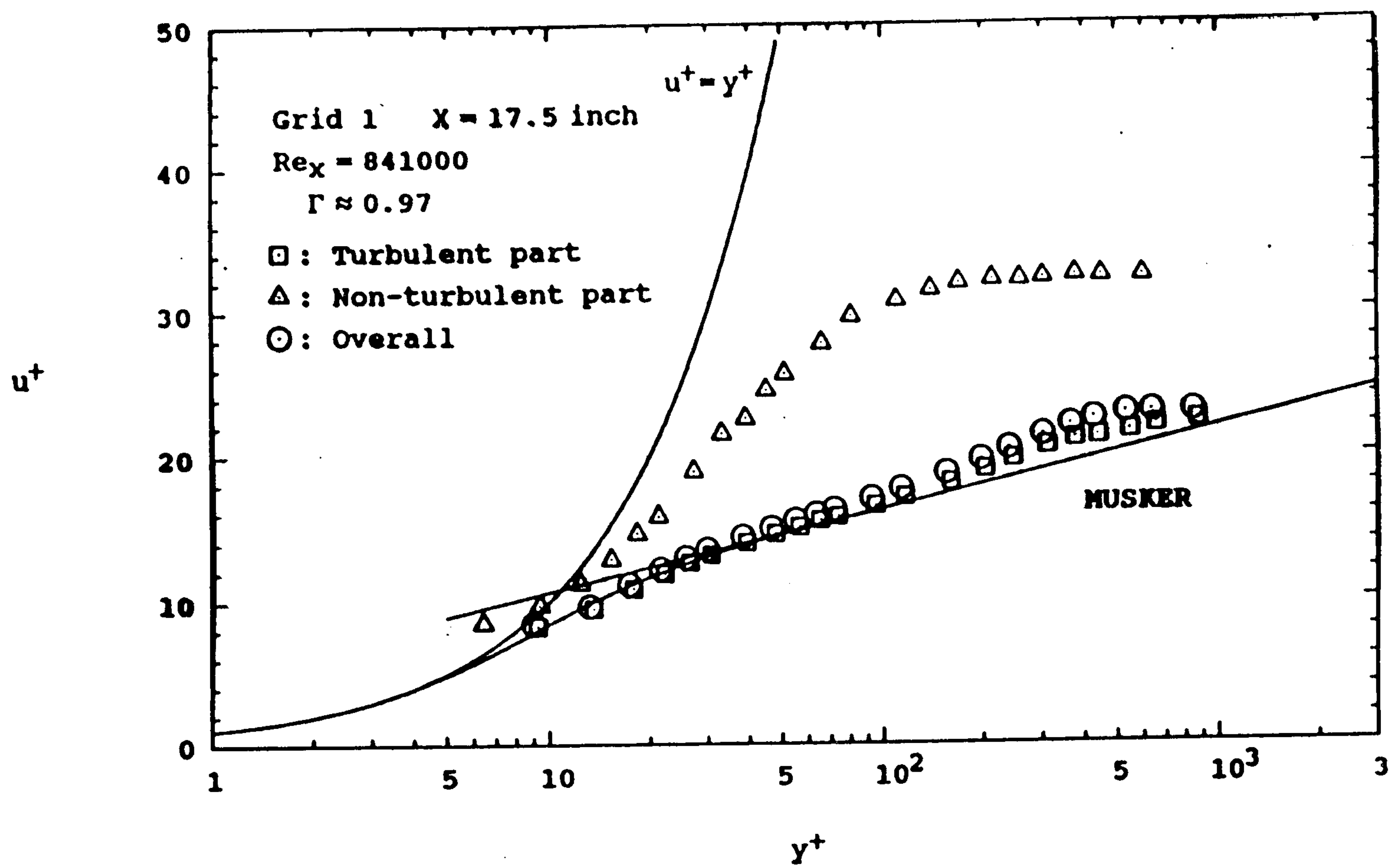


Fig. 29(e) **Conditionally sampled mean velocity profiles in wall units at X=17.5 inches for grid 1**

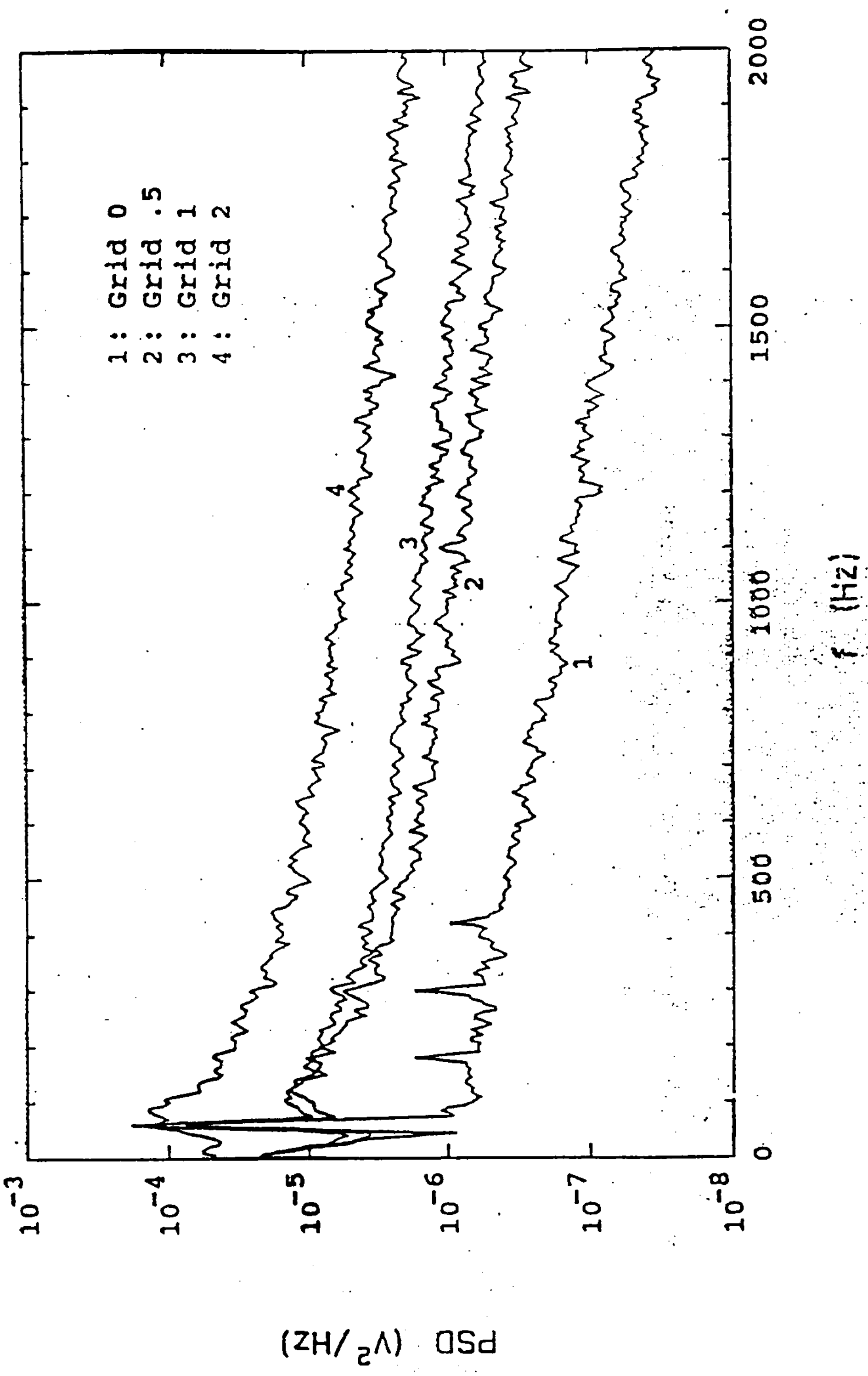


Fig. 54 Freestream spectra for 4 grids

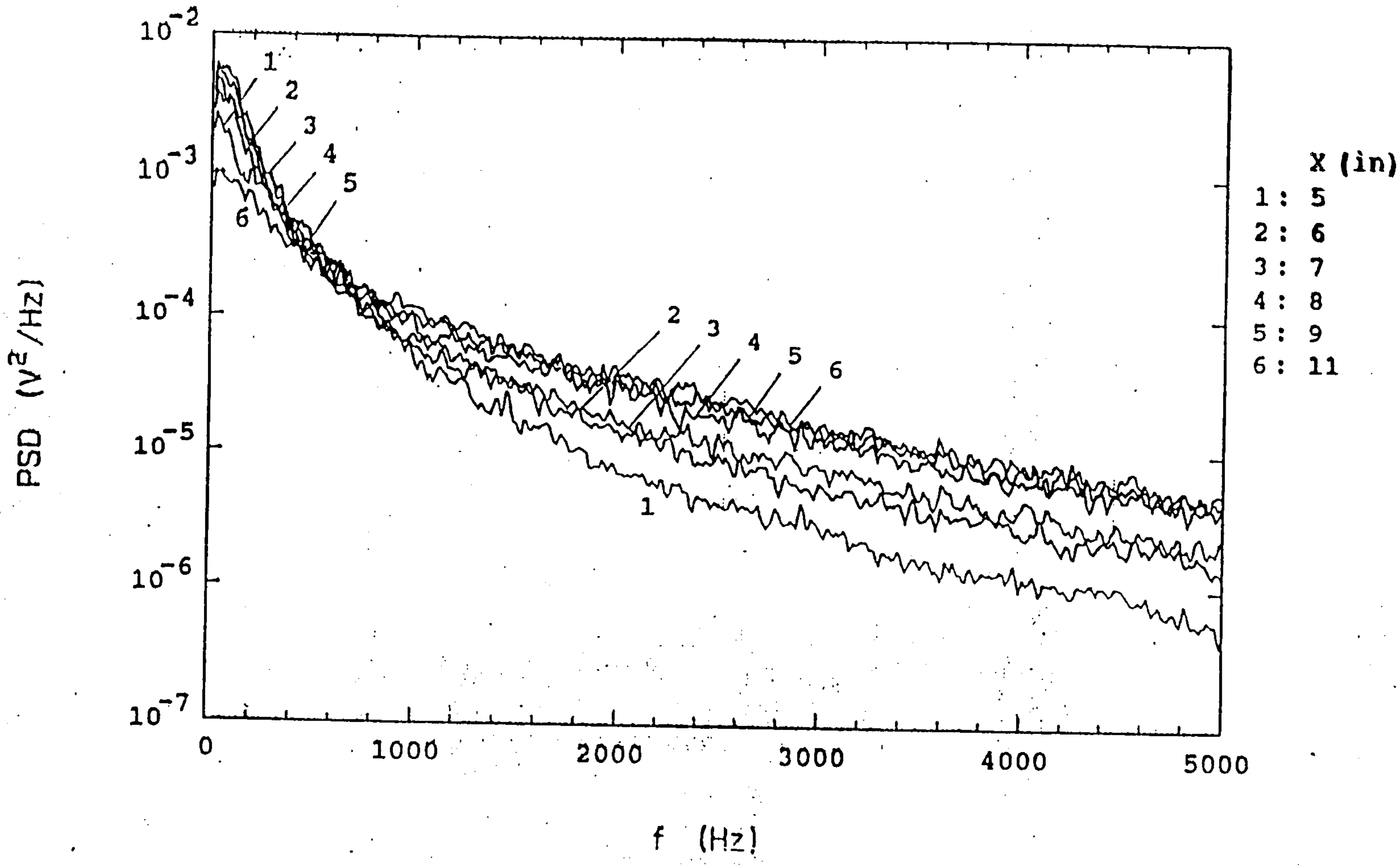


Fig. 53(d) Boundary layer spectra measured at locations where rms velocity reached maximum for grid 2

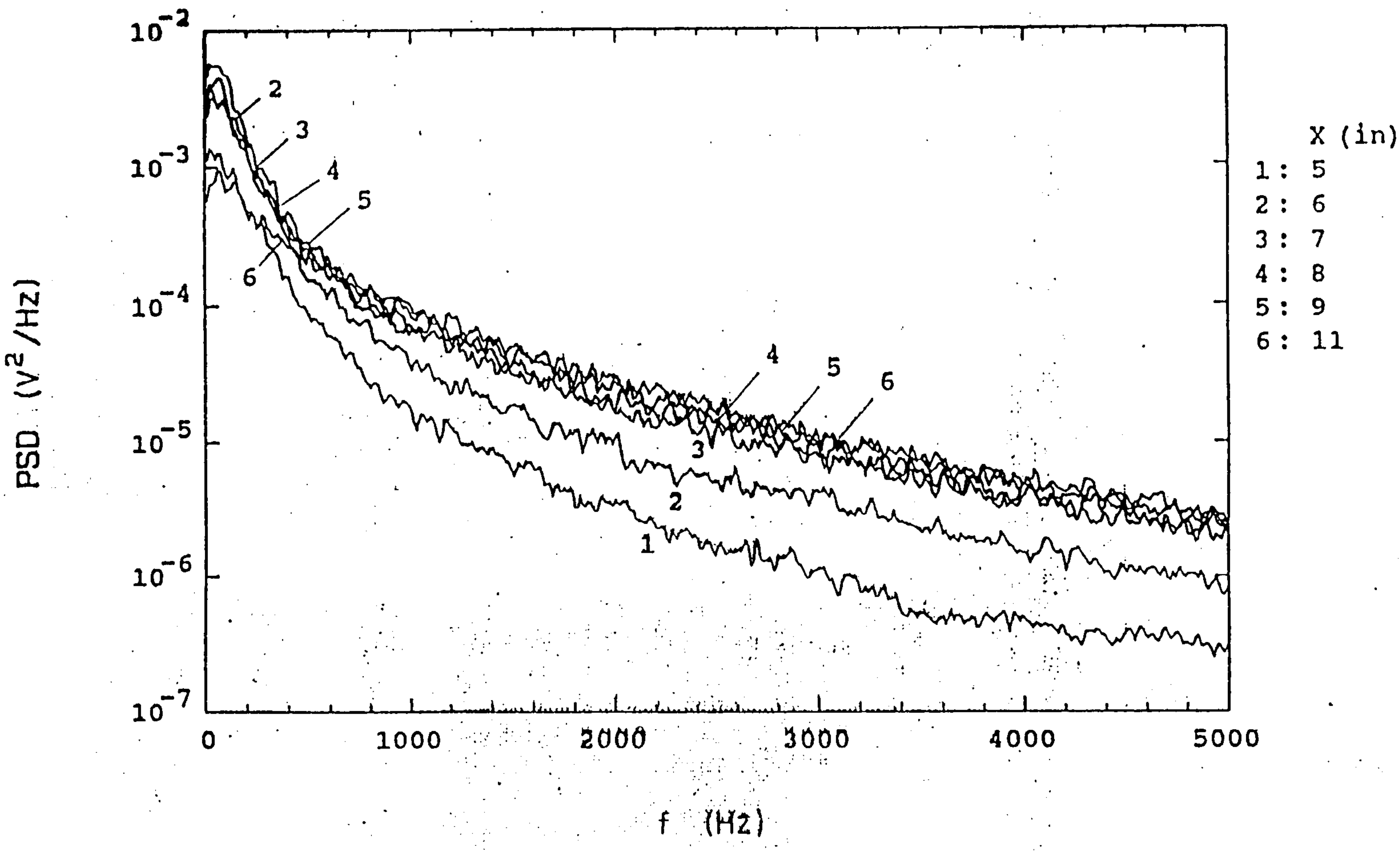


Fig. 52(d) Boundary layer spectra obtained at near-wall locations (Y_0) for grid 2

THIS SUGGESTS THAT BYPASS TRANSITION IN THE CONTEXT OF GAS TURBINE TECHNOLOGY (Tu large) MIGHT BE MODELED USING <u>TURBULENT FLOW METHODS</u>

<u>THESE MUST ALL BE VALID IN THE LAMINAR LIMIT</u>

- one-equation models (k)
- two-equation models ($k-\epsilon$)
- Reynolds stress models
- DNS / LES

<u>BUT</u> - HOW DO YOU START THE CALCULATION ?

<u>ISSUE</u> : RECEPTIVITY

K-ϵ APPROACHES

- Equations should have proper low-Reynolds-number form.
- Turbulence models should have correct near wall behavior for k , $-\overline{u'v'}$, ϵ
- Calculation algorithm should be compatible with equations and turbulence model

STARTING PROFILES FOR k, ϵ ?

FEATURES OF BYPASS TRANSITION TO BE MODELED

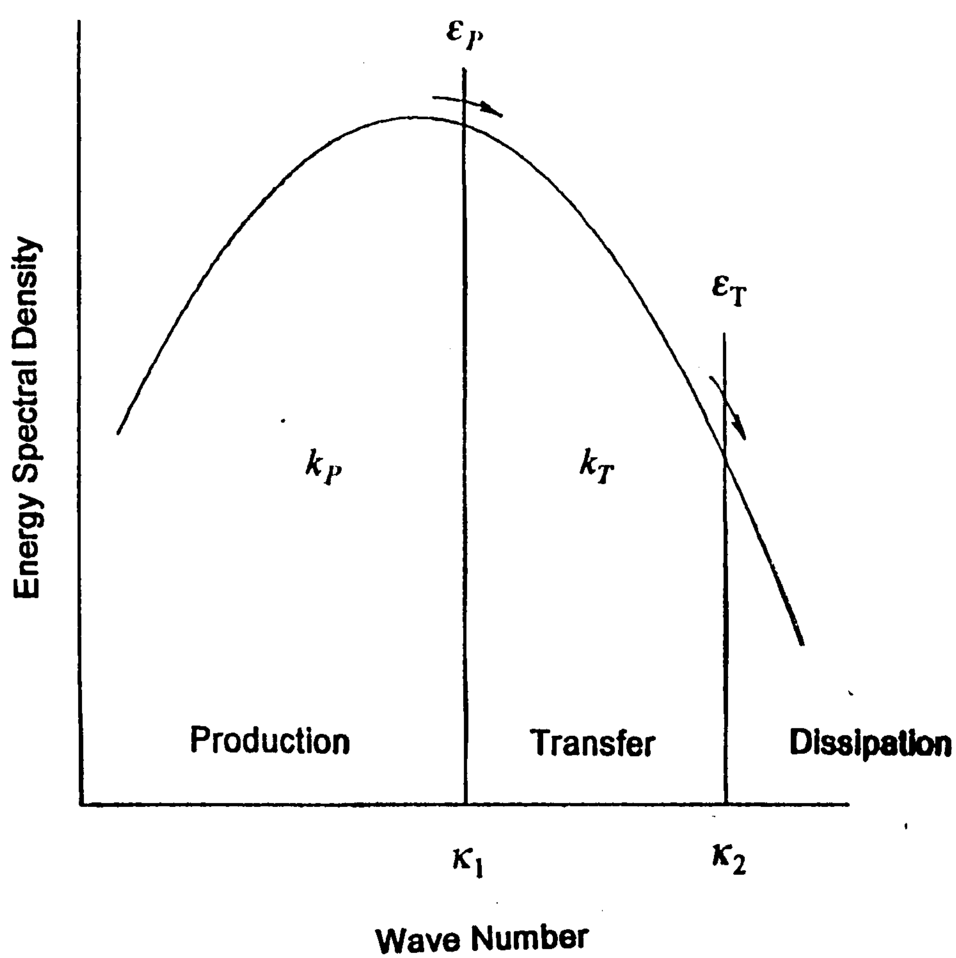
- Turbulent-like spectra throughout
- Intermittency $\gamma = \gamma(x,y)$
- With conditional sampling
  - Turbulent part approaches fully-turbulent characteristic as transition progresses
  - Non-turbulent part departs from laminar after transition onset

Intermittency:
MTS:

	Non-turbulent part	Turbulent part
	Low frequency	High frequency

MODEL EACH PART SEPARATELY
DEVELOP INTERMITTENCY pde (following Kollmann)
COMBINE We gave up on this!

Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model



Hanjalić, Launder & Schiestel, 1980

Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model

S-Y WU

$$U\frac{\partial k_P}{\partial x}+V\frac{\partial k_P}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k_P}{\partial y}\right]+P_k-\varepsilon_P$$

$$U\frac{\partial k_T}{\partial x}+V\frac{\partial k_T}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k_T}{\partial y}\right]+\varepsilon_P-\varepsilon_T$$

$$U\frac{\partial \varepsilon_P}{\partial x}+V\frac{\partial \varepsilon_P}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon_P}{\partial y}\right]+C_{P1}\left[\left(1+C'_{P1}\frac{P_k}{\varepsilon_P}\right)\right]P_k\frac{\varepsilon_P}{k_P}-C_{P2}f_{P2}\frac{\varepsilon_P^2}{k_P}$$

$$U\frac{\partial \varepsilon_T}{\partial x}+V\frac{\partial \varepsilon_T}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon_T}{\partial y}\right]+C_{T1}\left[\left(1+C'_{T1}\frac{\varepsilon_P}{\varepsilon_T}\right)\right]\varepsilon_P\frac{\varepsilon_T}{k_T}-C_{T2}f_{T2}\frac{\varepsilon_T^2}{k_T}$$

where $P_k=\nu_t(\frac{\partial U}{\partial y})^2$ $\quad \nu_t=c_\mu f_\mu\frac{k^2}{\varepsilon_P}$ $\quad k=k_P+k_T$

at wall $k_P=k_T=0$ $\quad \varepsilon_P=2\nu\left(\frac{\partial\sqrt{k_P}}{\partial y}\right)^2$ $\quad \varepsilon_T=2\nu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$

Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model

- Patankar-Spalding method:

  Grid point N=102

  Step size $\Delta x=0.1\delta_2$

  Non-uniform grid $\Delta\omega_n=1.02\Delta\omega_{n-1}$

- Inlet condition:

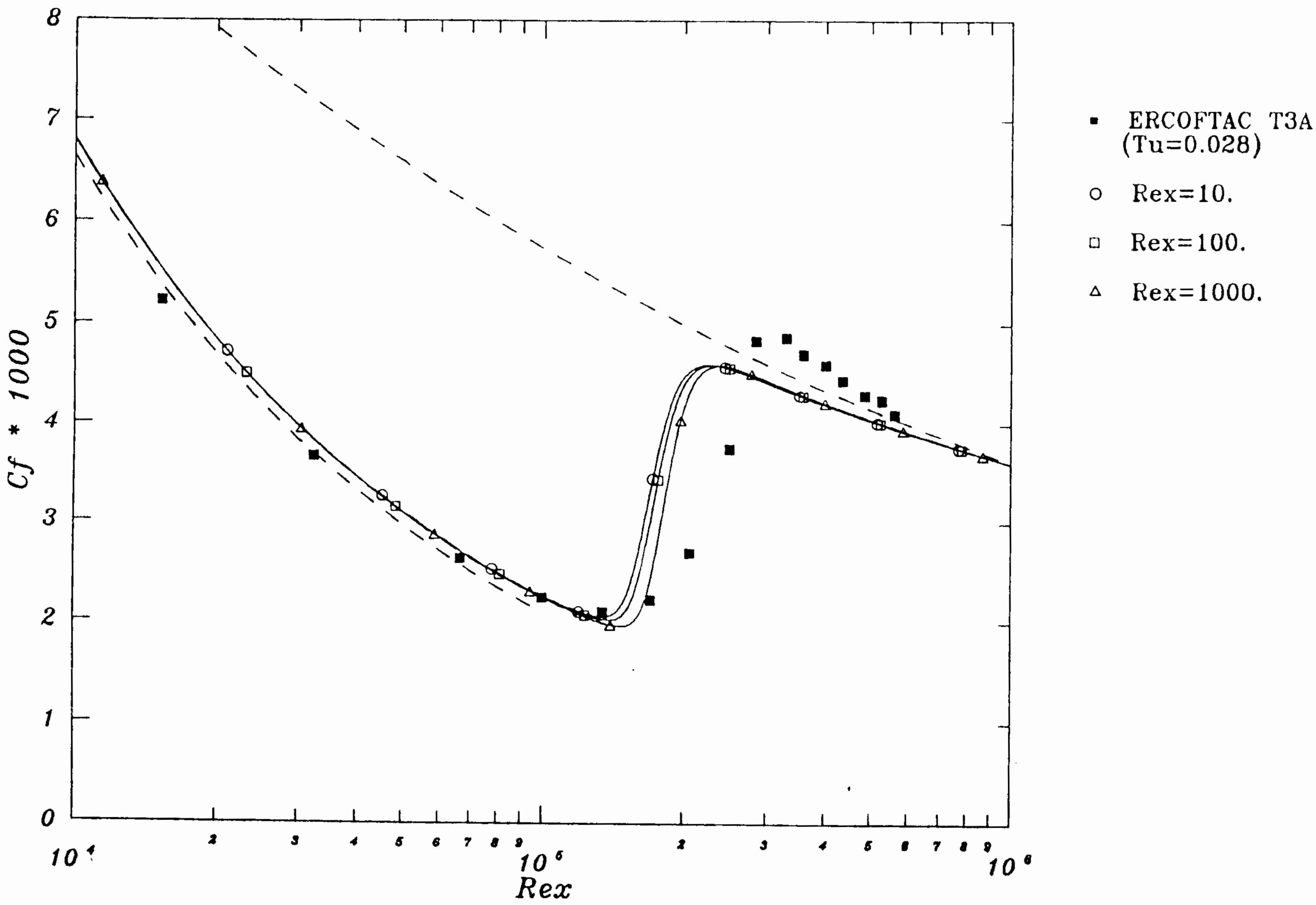
  $Re_x=100$

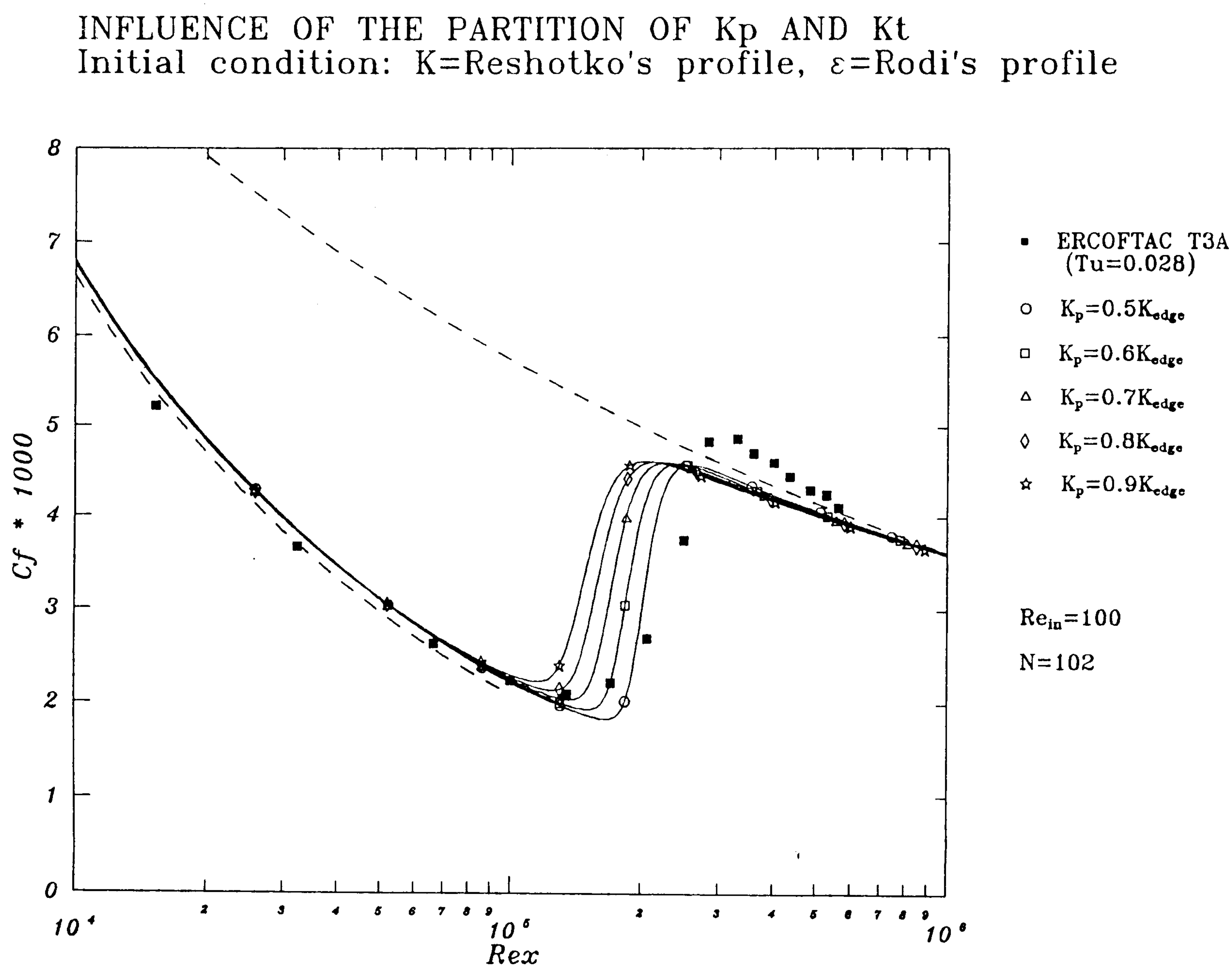
  U=Blasius solution

  k=Reshotko's profile $k=\frac{1}{2}U^2Tu^2\left(f'+\frac{\eta}{2}f''\right)^2$

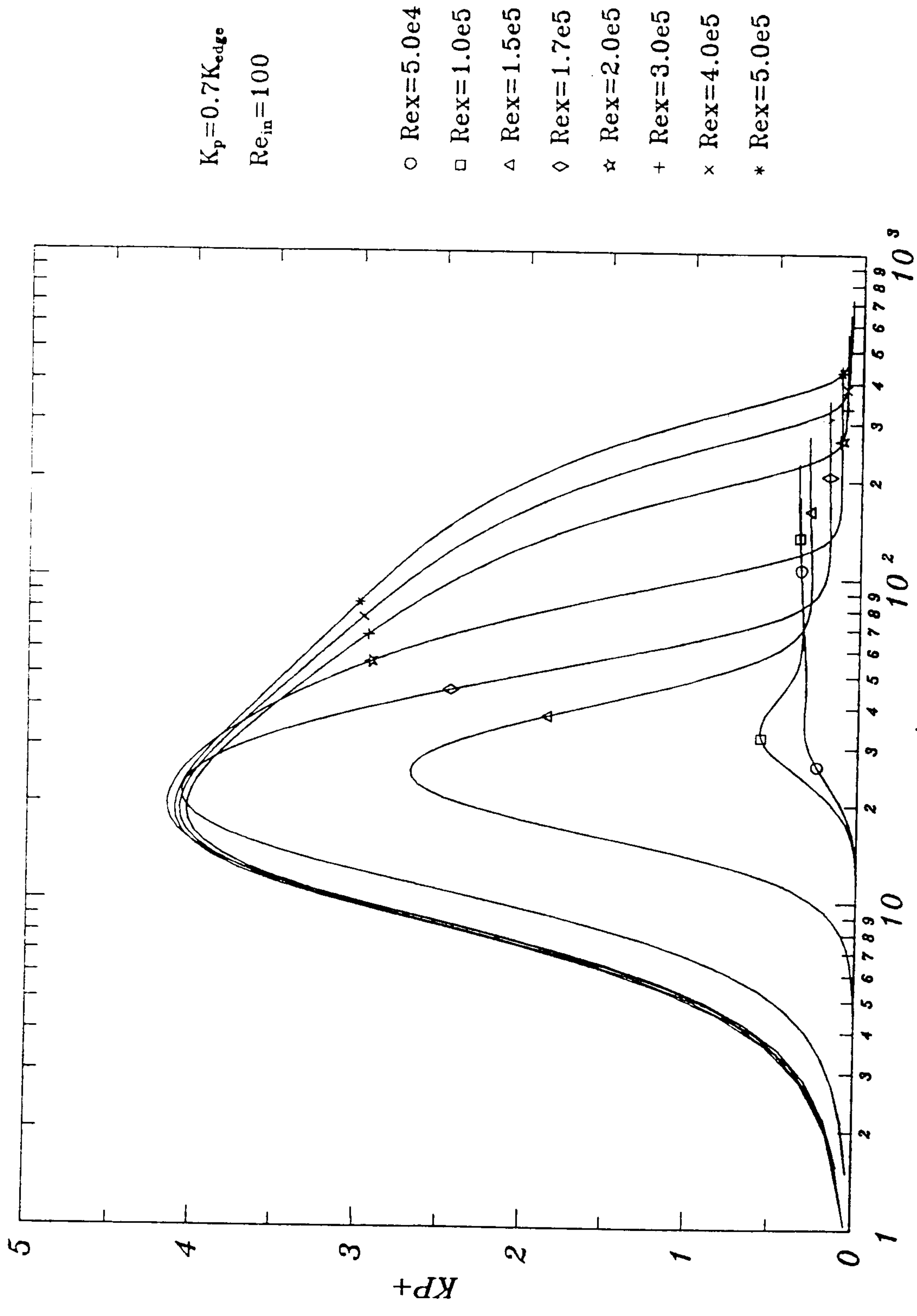
  ε=Rodi's profile $\varepsilon_P=a_1k_P\frac{\partial U}{\partial y}$ $\varepsilon_T=a_1k_T\frac{\partial U}{\partial y}$

INFLUENCE OF THE STARTING LOCATION
Initial profiles: K and ε are Rodi's profiles
Freestream: $K_p=0.7K_e$

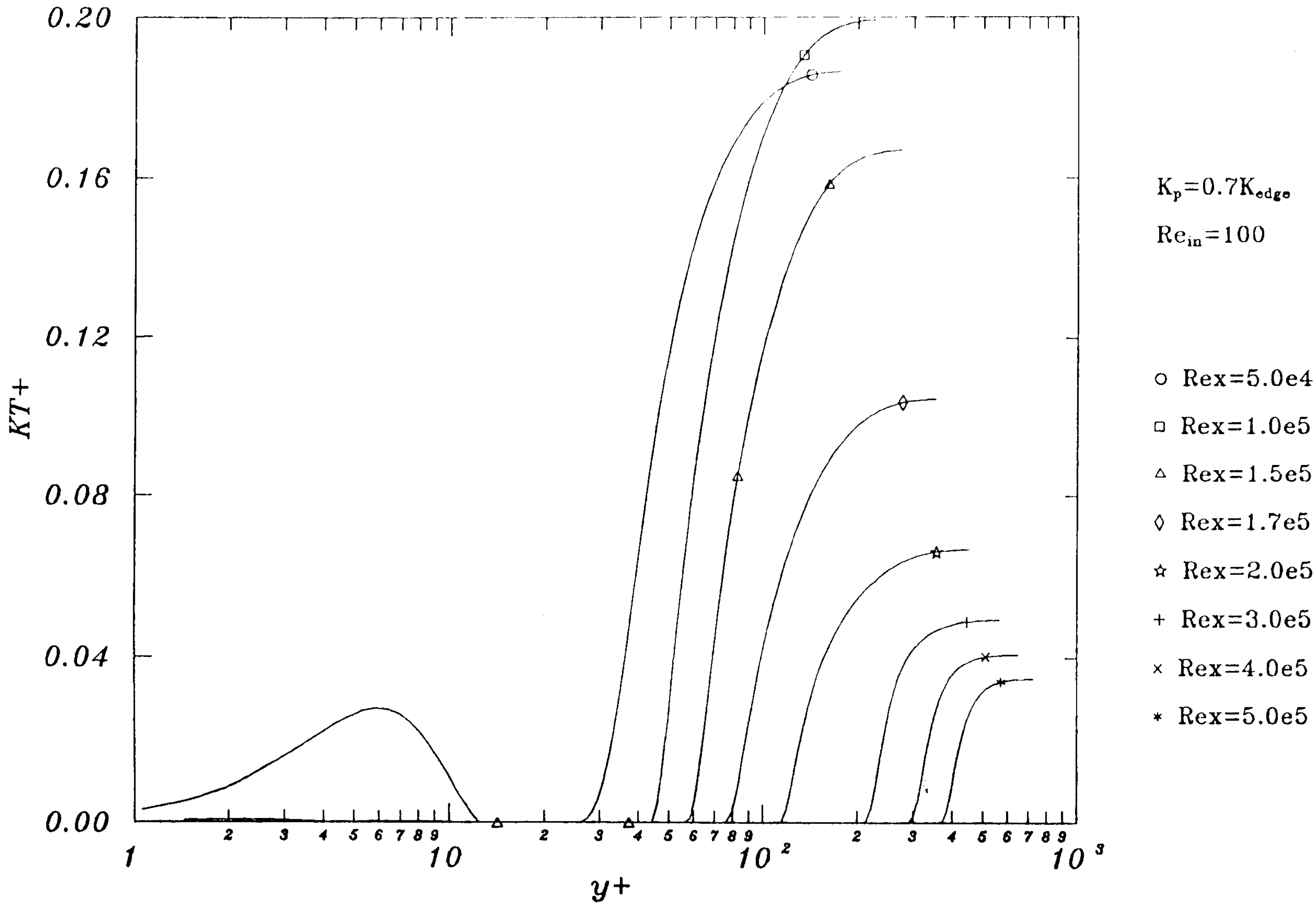


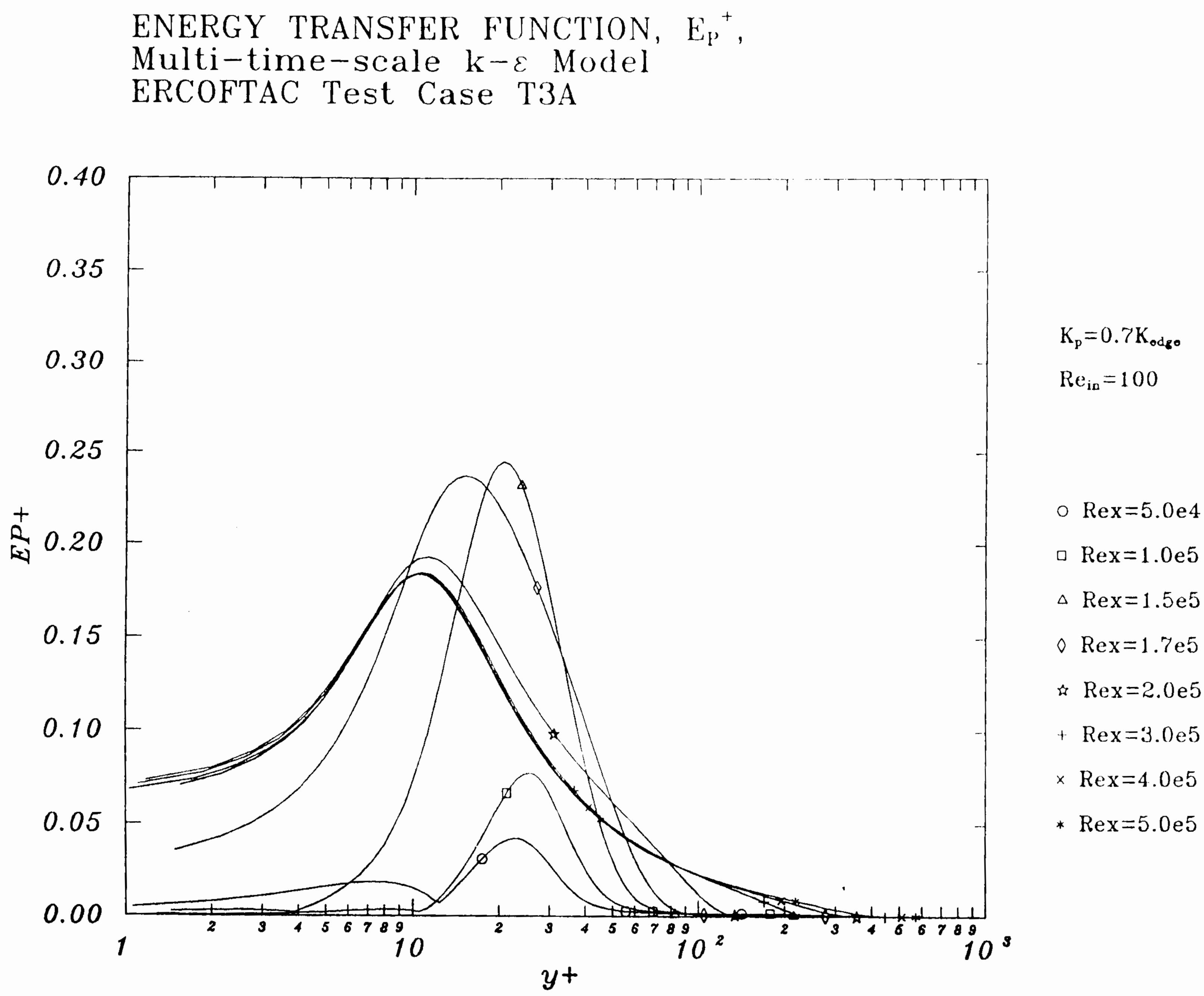

INFLUENCE OF THE PARTITION OF Kp AND Kt
Initial condition: K=Reshotko's profile, ε=Rodi's profile
Cf * 1000
Rex
ERCOFTAC T3A (Tu=0.028)
Kp=0.5Kedge
Kp=0.6Kedge
Kp=0.7Kedge
Kp=0.8Kedge
Kp=0.9Kedge
Rein=100
N=102

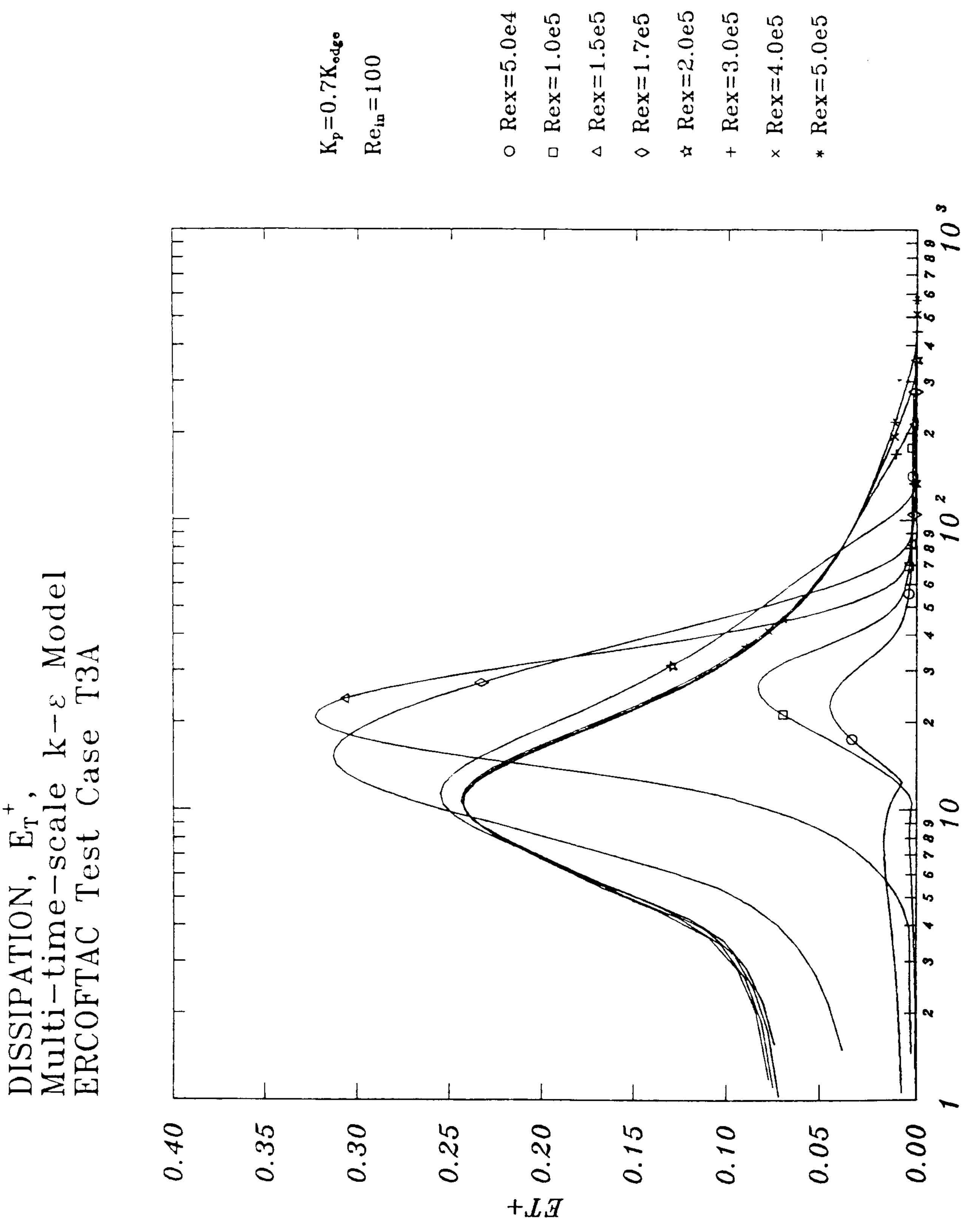
PRODUCTION ZONE TURBULENT KINETIC ENERGY, K_P^+,
Multi-time-scale k−ε Model
ERCOFTAC Test Case T3A



TRANSFER ZONE TURBULENT KINETIC ENERGY, K_T^+,
Multi-time-scale k−ε Model
ERCOFTAC Test Case T3A

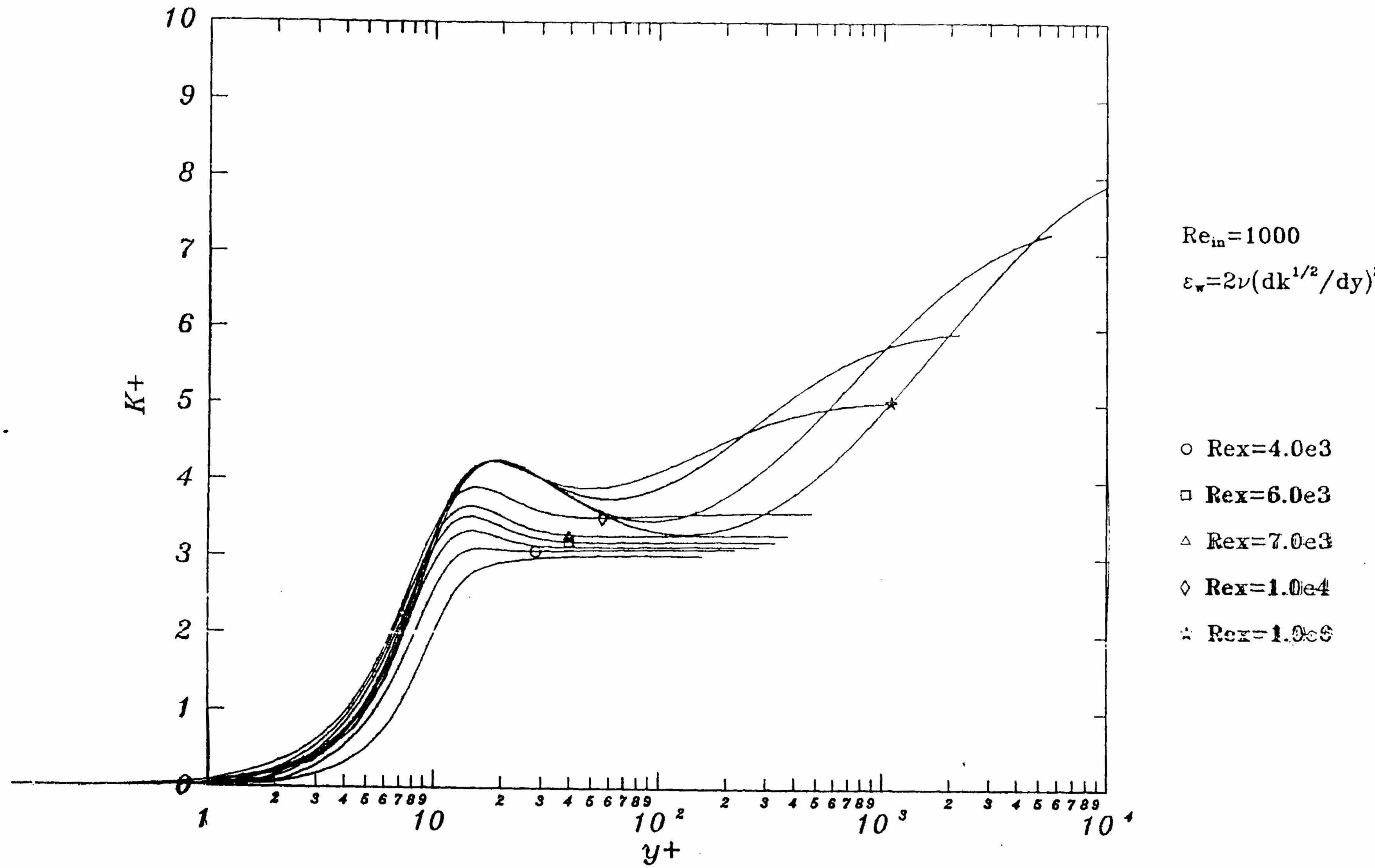



ENERGY TRANSFER FUNCTION, E_p^+,
Multi-time-scale k-ε Model
ERCOFTAC Test Case T3A
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
EP+
1
10
10 2
10 3
y+
$K_p=0.7K_{edge}$
$Re_{in}=100$
○ Rex=5.0e4
□ Rex=1.0e5
△ Rex=1.5e5
◇ Rex=1.7e5
☆ Rex=2.0e5
+ Rex=3.0e5
× Rex=4.0e5
* Rex=5.0e5

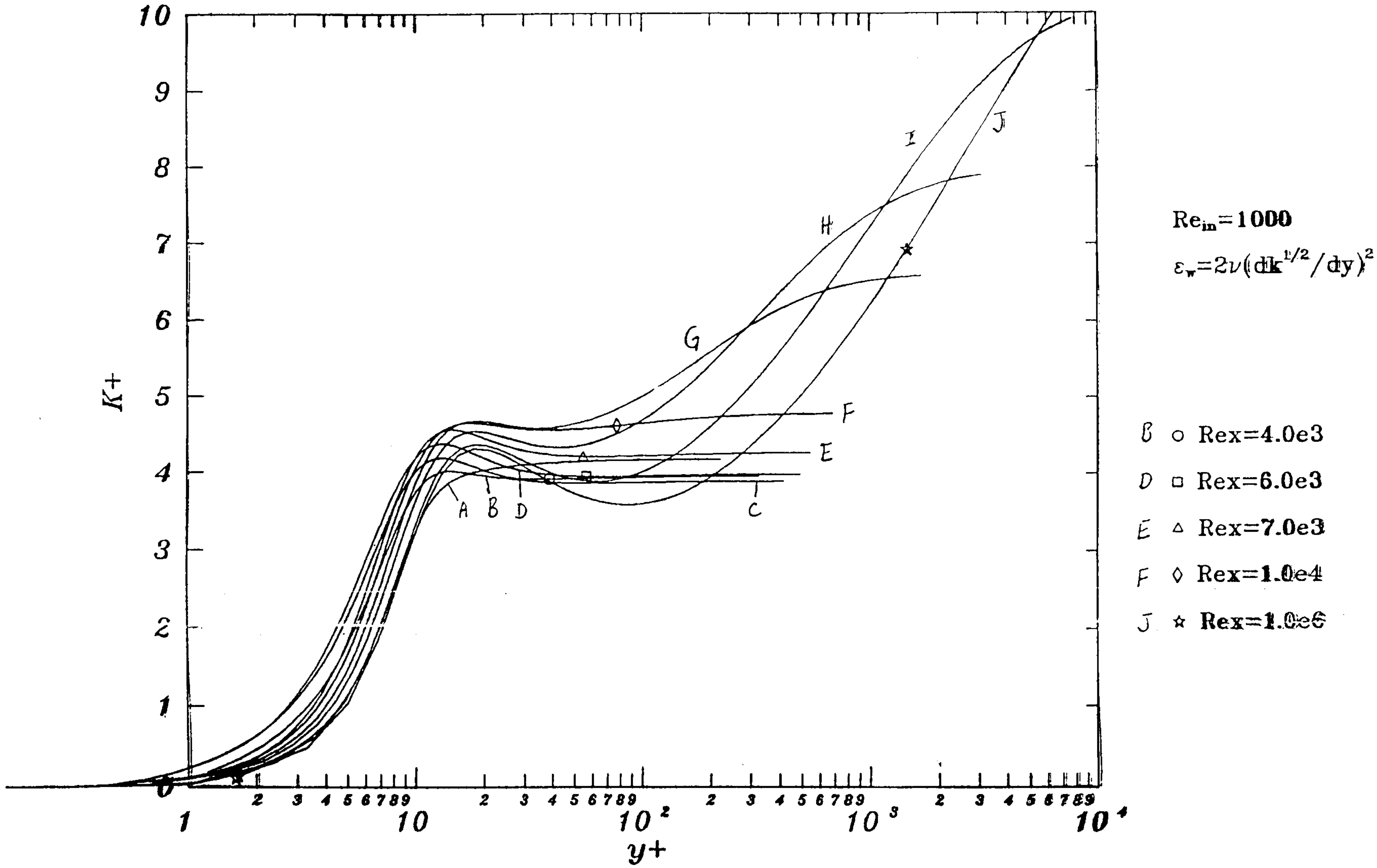


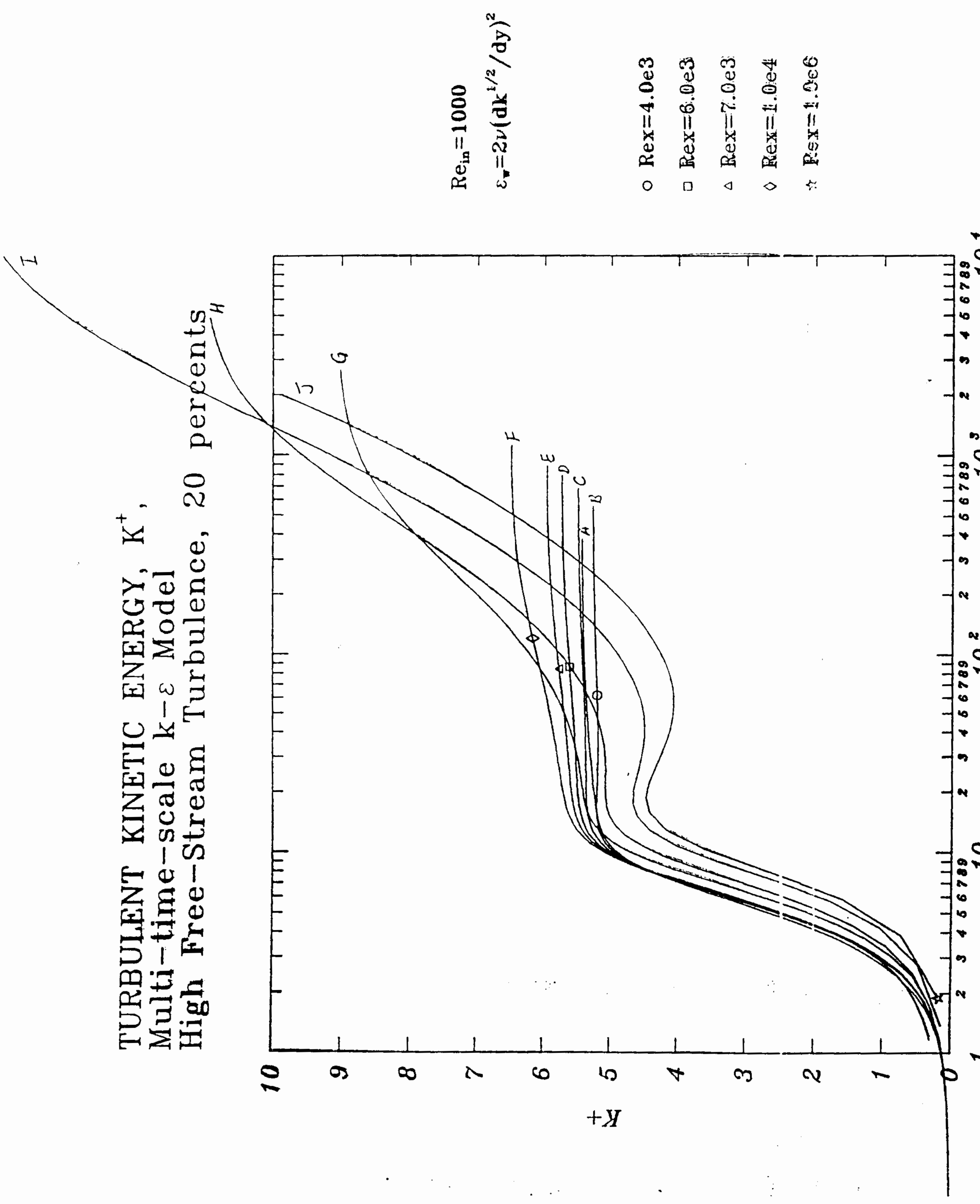
DISSIPATION, E_T^+,
Multi-time-scale k−ε Model
ERCOFTAC Test Case T3A
$K_p=0.7K_{edge}$
$Re_{in}=100$
○ Rex=5.0e4
□ Rex=1.0e5
△ Rex=1.5e5
◇ Rex=1.7e5
☆ Rex=2.0e5
+ Rex=3.0e5
× Rex=4.0e5
* Rex=5.0e5
ET+
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
y+
1
10
10^2
10^3

TURBULENT KINETIC ENERGY, K^+,
Multi-time-scale k-ε Model
High Free-Stream Turbulence, 12 percents



TURBULENT KINETIC ENERGY, K^{+},
Multi-time-scale k-ε Model
High Free-Stream Turbulence, 15 percents




TURBULENT KINETIC ENERGY, K$^+$,
Multi-time-scale k-ε Model
High Free-Stream Turbulence, 20 percents
Re_{in}=1000
$\varepsilon_w=2\nu(dk^{1/2}/dy)^2$
○ Rex=4.0e3
□ Rex=6.0e3
△ Rex=7.0e3
◇ Rex=1.0e4
☆ Rex=1.0e6
K+
y+
A
B
C
D
E
F
G
H
I
J

DIRECTIONS

- CONTINUE EXPERIMENTAL STUDIES TO PROVIDE PHENOMENOLOGICAL DATA FOR MODELING VALIDATION

- REFINE MODELS (MTS, etc) TO BE USED FOR TURBINE BLADE DESIGN CALCULATIONS

- LOOK HARD AT "RECEPTIVITY" ISSUE TOWARD IDENTIFYING PHYSICS OF BYPASS INITIATION

EXPERIMENTS – BREUER, KENDALL

ANALYSIS COMPUTATION – LANDAHL, HENNINGSON SCHMID HERBERT (PSE)