Nonlinear Theory and Breakdown

Frank Smith
University College-London
Department of Mathematics
Gower Street
England, London WC1E6BT

Abstract

The main points of recent theoretical and computational studies on boundary-layer transition and turbulence are to be highlighted. The work is based on high Reynolds numbers and attention is drawn to nonlinear interactions, breakdowns and scales. The research focusses in particular on truly nonlinear theories, i.e. those for which the mean-flow profile is completely altered from its original state. There appear to be three such theories, dealing with unsteady nonlinear pressure-displacement interactions (I), with vortex/wave interactions (II), and with Euler-scale flows (III). Specific recent findings noted for these three, and in quantitative agreement with experiments, are the following. Nonlinear finite-time break-ups occur in I, leading to sublayer eruption and vortex formation; here the theory agrees with experiments (Nishioka) regarding the first spike. II gives rise to finite-distance blowup of displacement thickness, then interaction and break-up as above; this theory agrees with experiments (Klebanoff, Nishioka) on the formation of three-dimensional streets. III leads to the prediction of turbulent boundary-layer micro-scale, displacement - and stress-sublayer-thicknesses.

R I Bowles and F T Smith

INTRODUCTION

This article summarizes nonlinear theory and computations on end-game transition, and in particular on the development of spikes. There are three main nonlinear regimes I-III as indicated on page 2.

Pages 3, 4 concern nonlinear theory on input wave amplitudes that are initially low. These can provoke vortex-wave interactions (I), which in turn lead to close comparisons with experiments in boundary-layer and channel-flow transition as shown.

Pages 5-7 address pressure-displacement interactions (II), produced at higher amplitudes, eg following I. Comparisons with channel-flow experiments (page 6) and with boundary-layer computations (page 7) are included, especially on the first spike, for which the theory yields an integral transition criterion (page 5).

Pages 7 (lower half)-8 describe the subsequent stages (nearer Euler scales III) effectively at still higher amplitudes, eg following II. Shorter scales and significant normal pressure gradients come into the reckoning within the spike. These lead to local vortex roll-ups whether in two or three spatial dimensions.

Relevant papers during 1990-93 are in the A I A A Jnl, Jnl of Fluid Mechs, Proc Roy Soc A, European Jnl of Mechs, Computat Phys Commns.

NONLINEAR THEORY / COMPUTATION / SPIKES (R.I.B.,F.T.S., UCI

Aims at physical understanding (scales), numerical aids, exper: links, parameter dependence —— at medium-to-large Re .

THREE MAIN NONLINEAR INTERACTIONS (⟶ END-GAME, via events in 3I

I. VORTEX-WAVE INTERACTIONS, at low input wave amplitudes

$$(u,v) = (u_0, \epsilon v_0)(x,\bar{y},\bar{z}) + \epsilon(u_1,v_1)(\bar{x},\bar{y},\bar{z}) + \ldots,$$

$$(x,\ y,\ z) = \epsilon(\bar{x},\ \bar{y},\ \bar{z}),\quad \epsilon \equiv Re^{-1/2}\ .$$

$\Rightarrow$ u_0-vortex eqs. are nonlinearly coupled with u_1-wave eq..

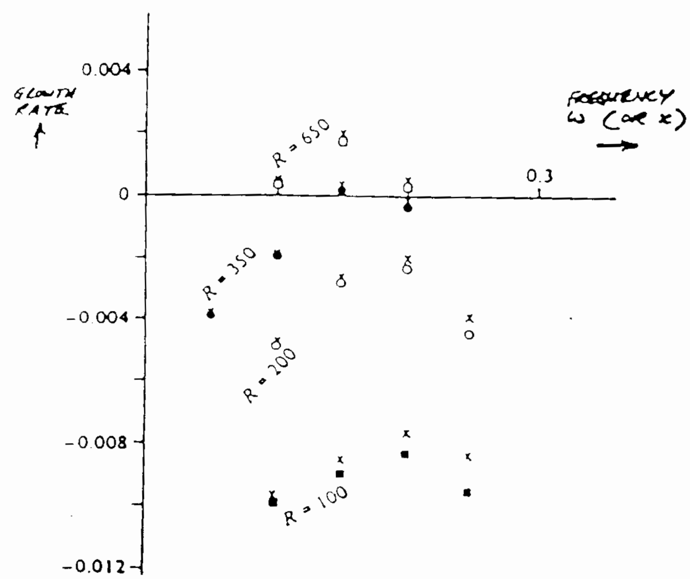
II. PRESSURE-DISPLACEMENT INTERACTIONS, at medium input

$\Rightarrow$ Interacting-boundary-layer eqs (e.g. nonlinear T.S. evolut

III. EULER-SCALE MOTION, at high input

$\Rightarrow$ Unsteady Euler eqs. (e.g. by-pass) .

Aside (on start-game) : stability at sub- & super-critical R_δ ($=\sqrt{2}$ |



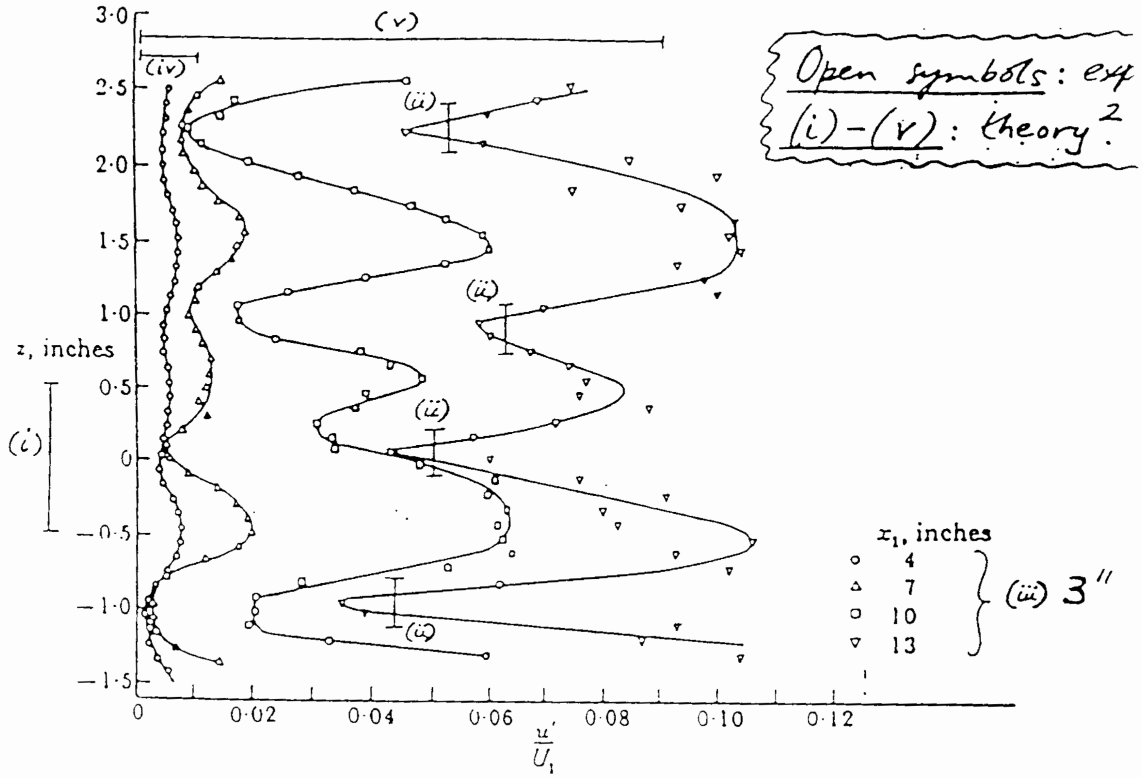
[J.F.M. 1984]

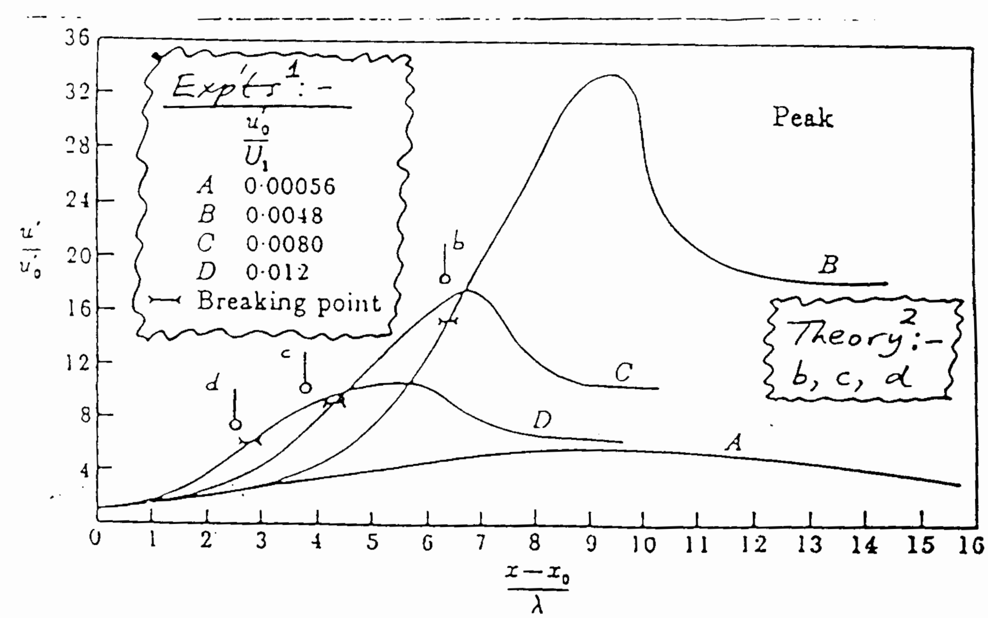
Comparisons between interactive and full solutions in the linear Blasius boundary layer.

Full solutions at R = 650, 350, 200, 100 are o , • , respectively, with x giving the corresponding interac results.

I. Stage i . 3D nonlinear evolution from near-2D input .

B. L.



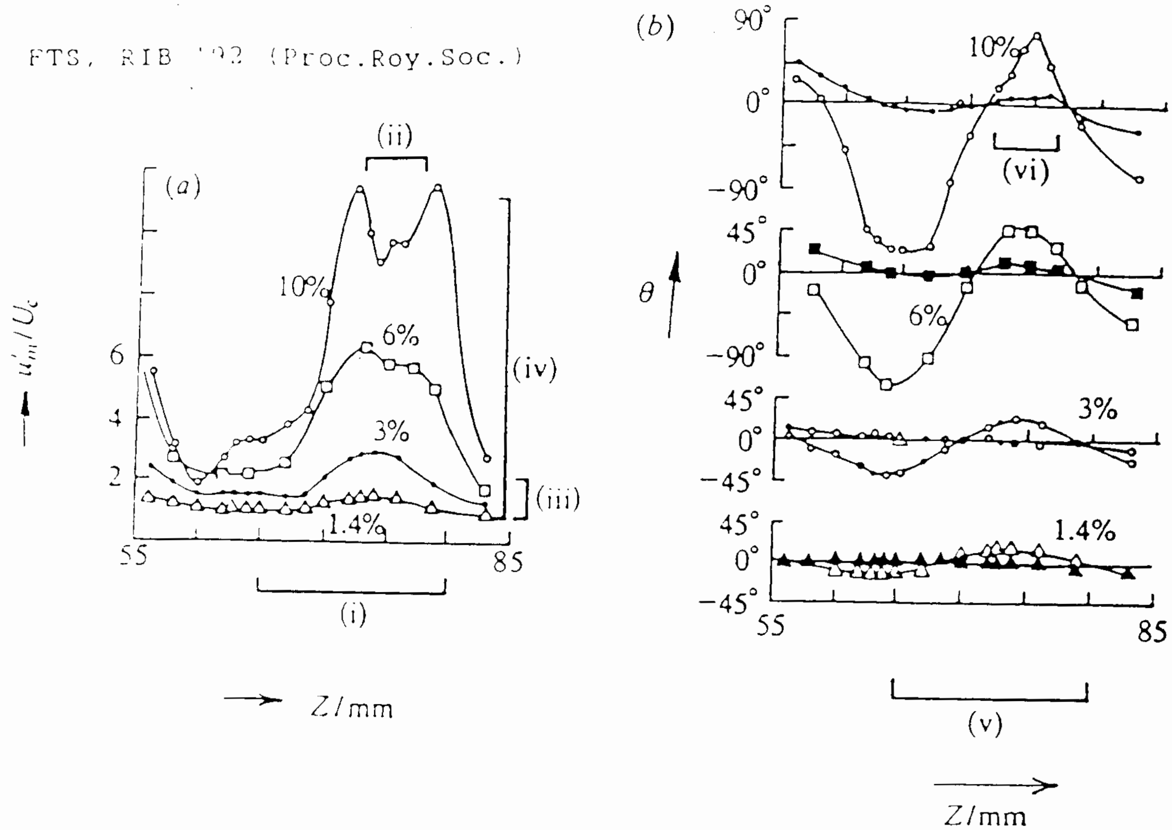


1. Klebanoff & Tidstrom '59 (e.g. ch. IX of "Lam. BL's")
2. Stewart & Smith '92 (J.F.M.)

$$\Downarrow \quad p_X - i p_{ZZ} = p - i p \varphi \quad \& \quad \varphi_{XX} = (|p|^2)_{ZZ} \,.$$

Channel flow

FTS, RIB '92 (Proc.Roy.Soc.)



Comparisons with the experiments of Nishioka *et al.* (1979), on stage

II. 2nd stage : 3D, strongly nonlinear

→ "finite-time break-up" (CRITERION ~~...~~)

As $t \to t_s-$, near $x = x_s$, in 2D,

$$\begin{cases} p \sim p_0 + (t_s - t)^{1/2}\, \tilde{p}(\tilde{x}) + \ldots, \\ u \sim u_0(y) + O(t_s - t)^{1/2}, \quad x - x_s = c(t - t_s) + (t_s - t)^{3/2} \end{cases}$$

F.T.S., MATHEMATIKA 1988

$$\Rightarrow \text{need } \int_0^\infty (u_0 - c)^{-2}\, dy = 0, \quad (*)$$

$$\& \text{ hence } \left(\frac{3\tilde{x}}{2} + \tilde{p}\right)\tilde{p}_{\tilde{x}} - \frac{1}{2}\tilde{p} = 0.$$

NISHIOKA ET AL EXPERIMENTS :-

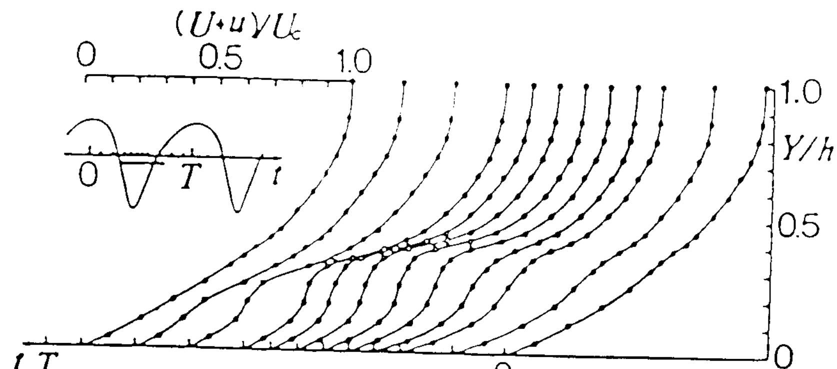


Fig. 5. Instantaneous velocity distributions at peak, at 9.4 % stage.

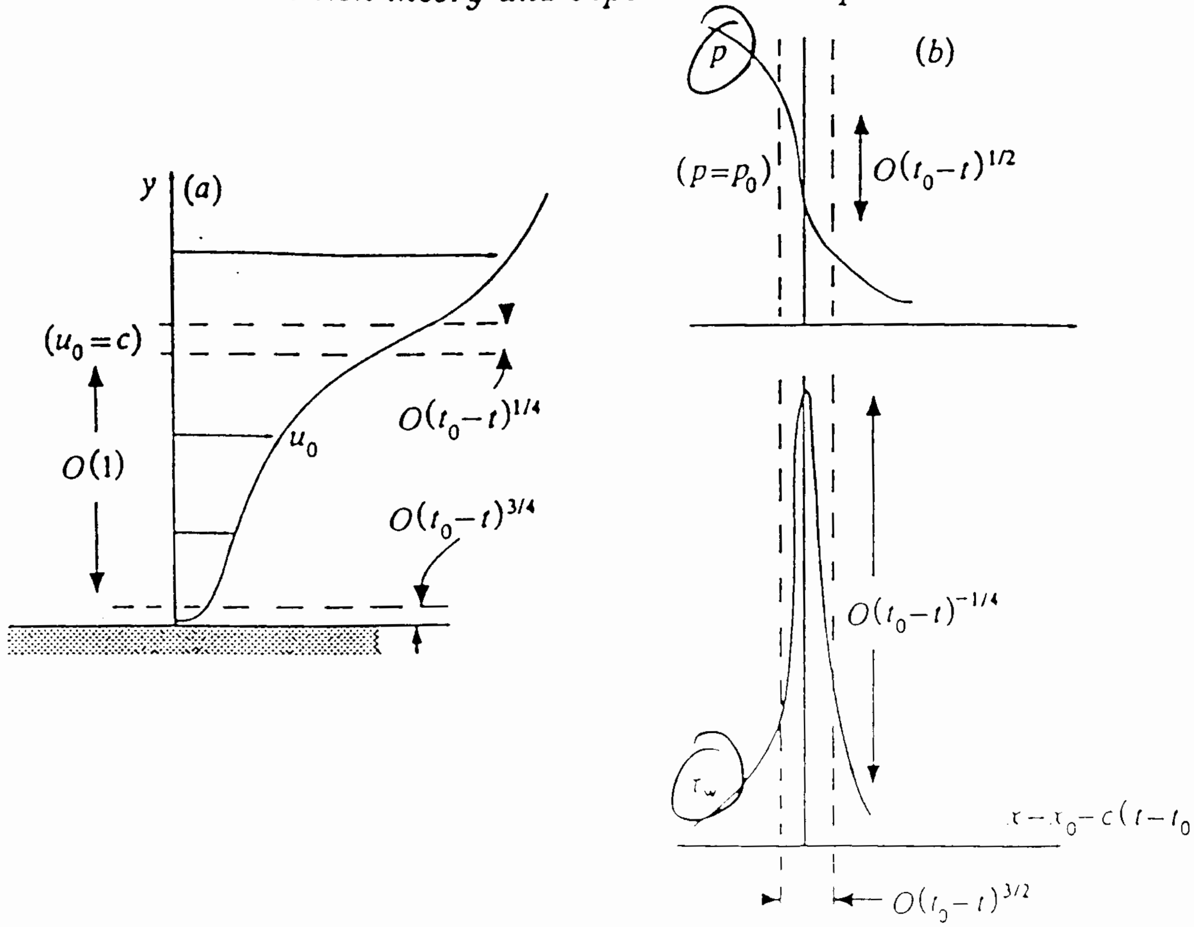


Figure . Schematic diagram for the transition stage II (strongly nonlinear).

Using ⊛

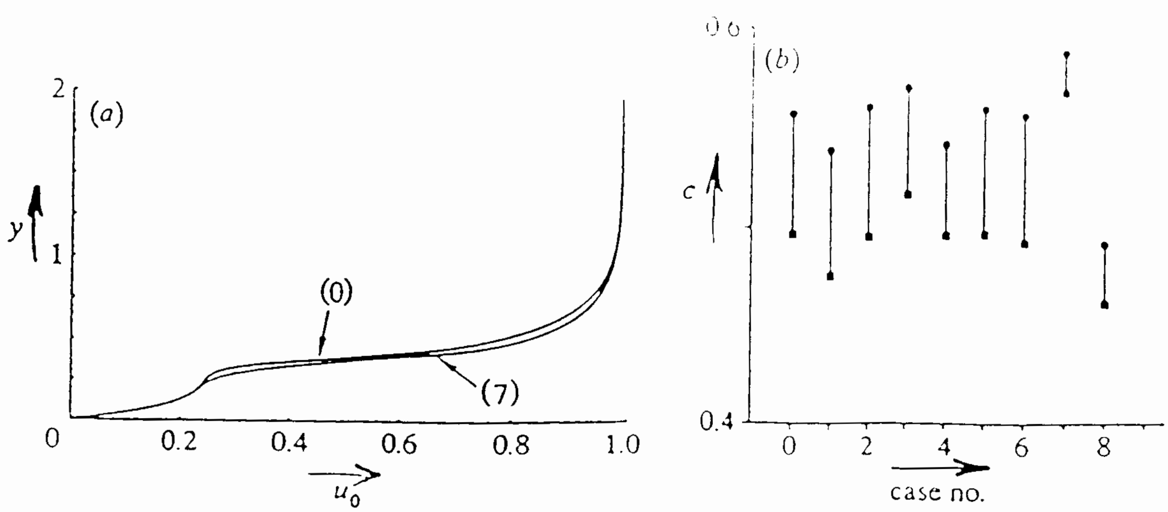


Figure . Comparisons with Nishioka *et al.*'s (1979) experiments, concerning the strongly nonlinear transition stage II.

7

THE SPIKE, IN B.L. CASE

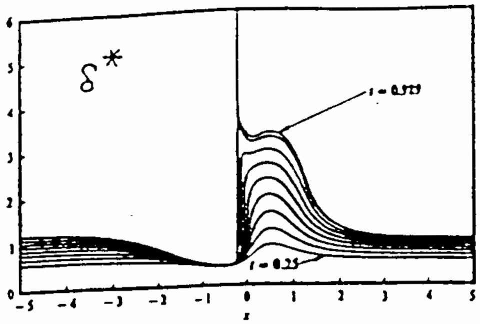
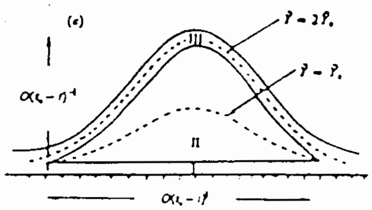
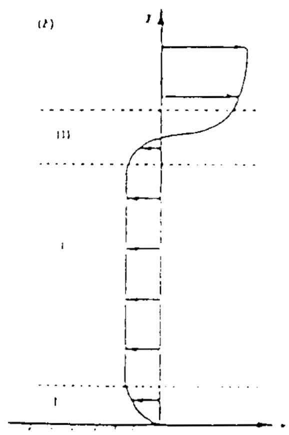


FIGURE 4. Temporal development of the displacement thickness; plotted curves are at $t = 0.25$ (0.10) 0.95 and $t_s = 0.989$.





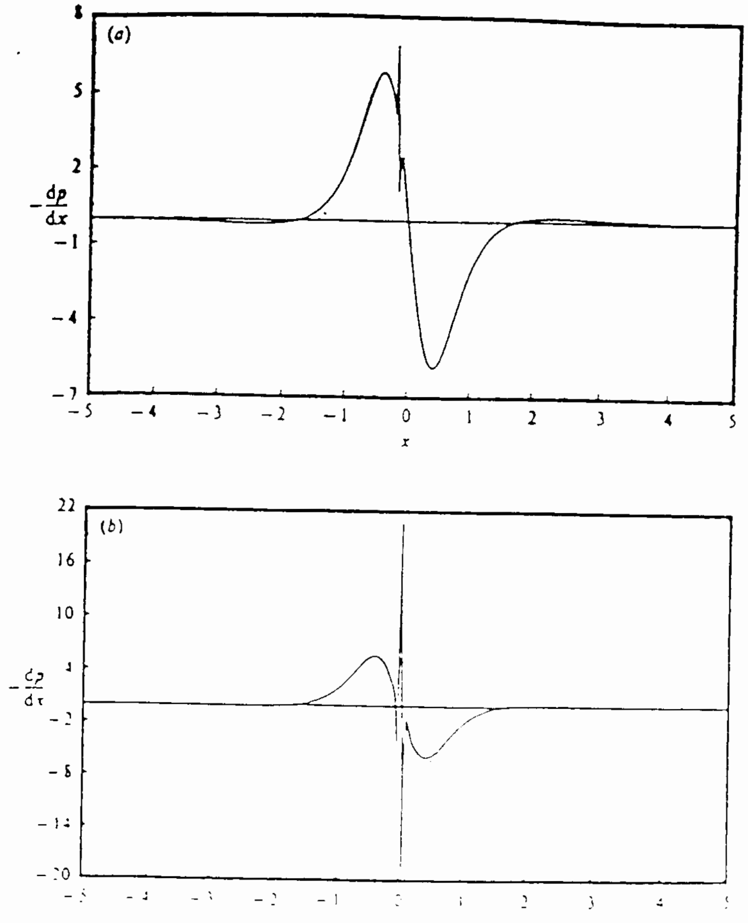
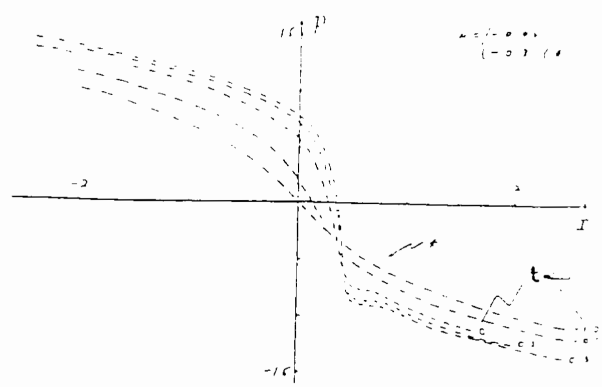


FIGURE 10. Mainstream pressure gradient at t_s (a) $Re = 10^{4}$; (b) $Re = 10^{6}$

(III) Subsequent stages in spike development



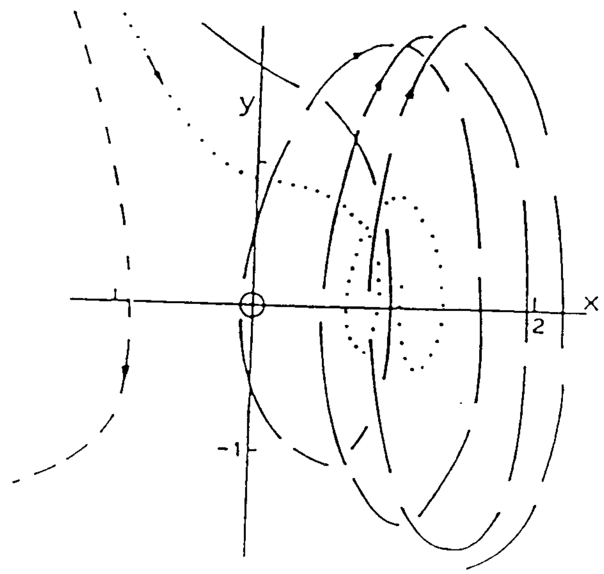
Inclusion of normal pressure gradient. (Hoyle <u>et</u> <u>al</u> 91-93)

$$p_t + pp_x = p_{xxx} + \mu p_x \int_{-\infty}^{\infty} \frac{p_t(s,t)}{p(x,t) - p(s,t)}\, ds$$

δ

Work in Progress

Local vortex formation and rollup (Hoyle et al 91-93) : –



Compressibility : –

$$\int_0^\infty \frac{\mathrm{d}y}{\rho_0(u_0 - c)^2} = 0 \quad .$$

3-D.
Pressure at breakdown
(Hoyle et al 91-93) : –