

End-stage Transition Seminar, August 1993  
 Transition Models for Engineering Calculations

C. J. Fraser  
 Dundee Institute of Technology

**Abstract :-** While future theoretical and conceptual developments may promote a better understanding of the physical processes involved in the latter stages of boundary layer transition, the designers of rotodynamic machinery and other fluid dynamic devices need effective transition models now. This presentation will therefore centre round the development of some transition models which have been developed as design aids to improve the prediction codes used in the performance evaluation of gas turbine blading. All models are based on Narasimha's concentrated breakdown and spot growth hypothesis

The first model uses a correlation of the non-dimensional spot source rate density as a function of the local pressure gradient parameter and the freestream turbulence level at the onset of transition. Even although quite reasonable agreement is observed for a significant number of transitional flows, the rigidity of a correlation based on onset parameters leaves a lot to be desired. For gas turbine blade flows in particular, the pressure gradient may not only change rapidly in magnitude, but can also change in sign during transition. Since it was thought that this could have significant influence on the turbulent spot seeding rates, an alternative transition model was postulated in the differential form of a first order system. In the differential model the transition length parameter,  $\lambda$ , is left as a limited function of the streamwise distance,  $x$ .

ie. 
$$d\gamma/dx = \{2Ax_t / \lambda_{av}^2\} [1 - (x_t / \lambda_{av})(d\lambda/dx)] \exp(- Ax_t^2 / \lambda_{av}^2)$$

where  $\lambda_{av} = [1/x_t] \int_{x_\lambda=0}^{x_\lambda=0.25} \lambda dx$  ;  $A = 0.411$  and  $x_t = (x - x_{tran})$

Preliminary studies have indicated that  $d\lambda/dx$  may in some circumstances vary significantly at the start of transition, but usually tends to zero for  $\lambda > 0.2$ . Equally good agreement with the experimental data is therefore found by simply using the original Narasimha intermittency function with  $\lambda_{av}$  as defined above. These two latter empirical forms however, impart the required dynamical flexibility on the intermittency characteristics and uncouples the predicted distributions from the start of transition parameters.

Comparisons of the boundary layer integral parameters using intermittency weighted predictions and measured data are used as a basis for assessment and evaluation of the transition models developed.



## Zero Pressure Gradient Correlations

### Transition Models for Engineering Calculations

$$[R\theta]_{\text{tran}} = 0.85 [163 + e^{(6.91 - \theta)}]$$

C J Fraser

Department of Mechanical Engineering

Dundee Institute of Technology

Dundee

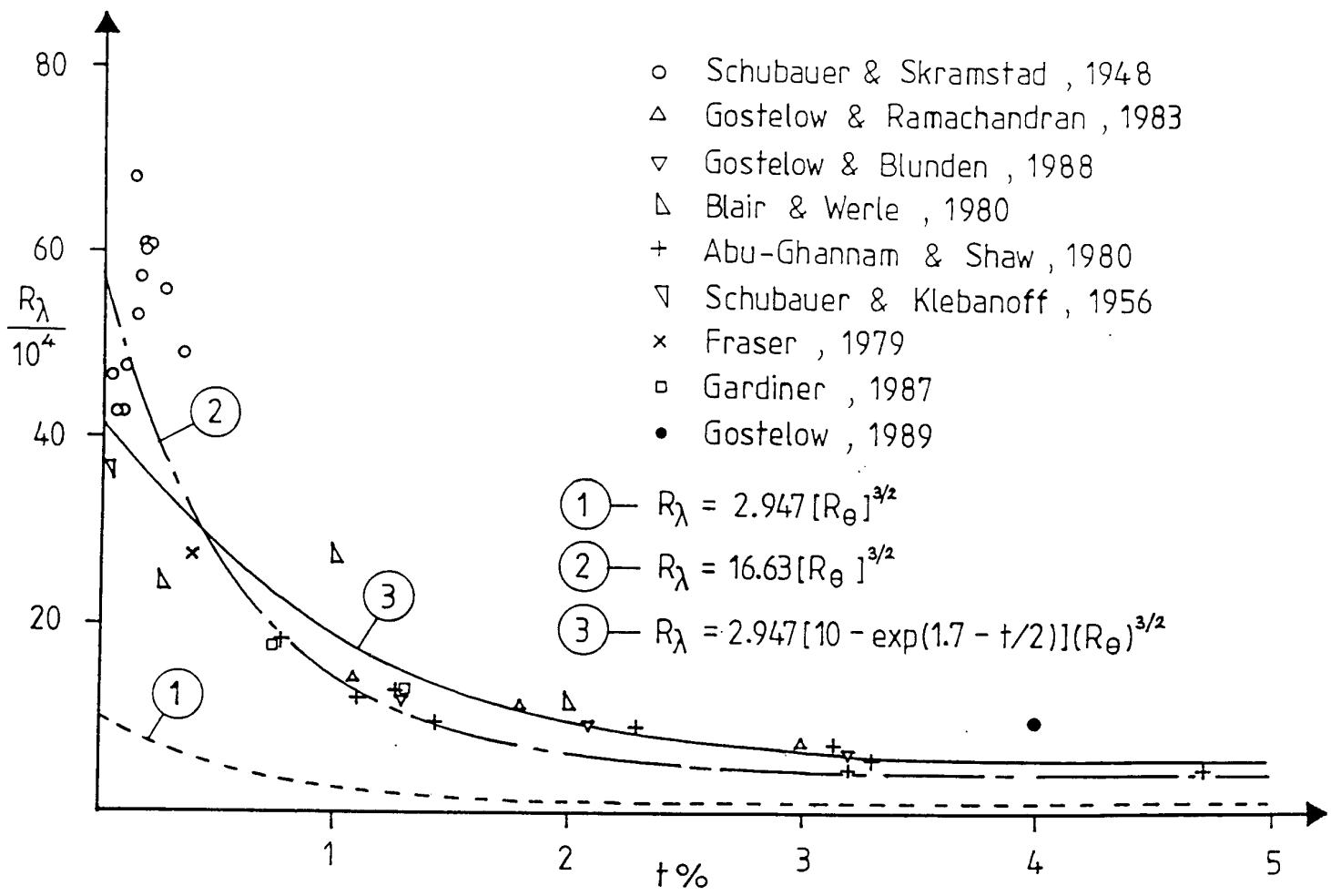
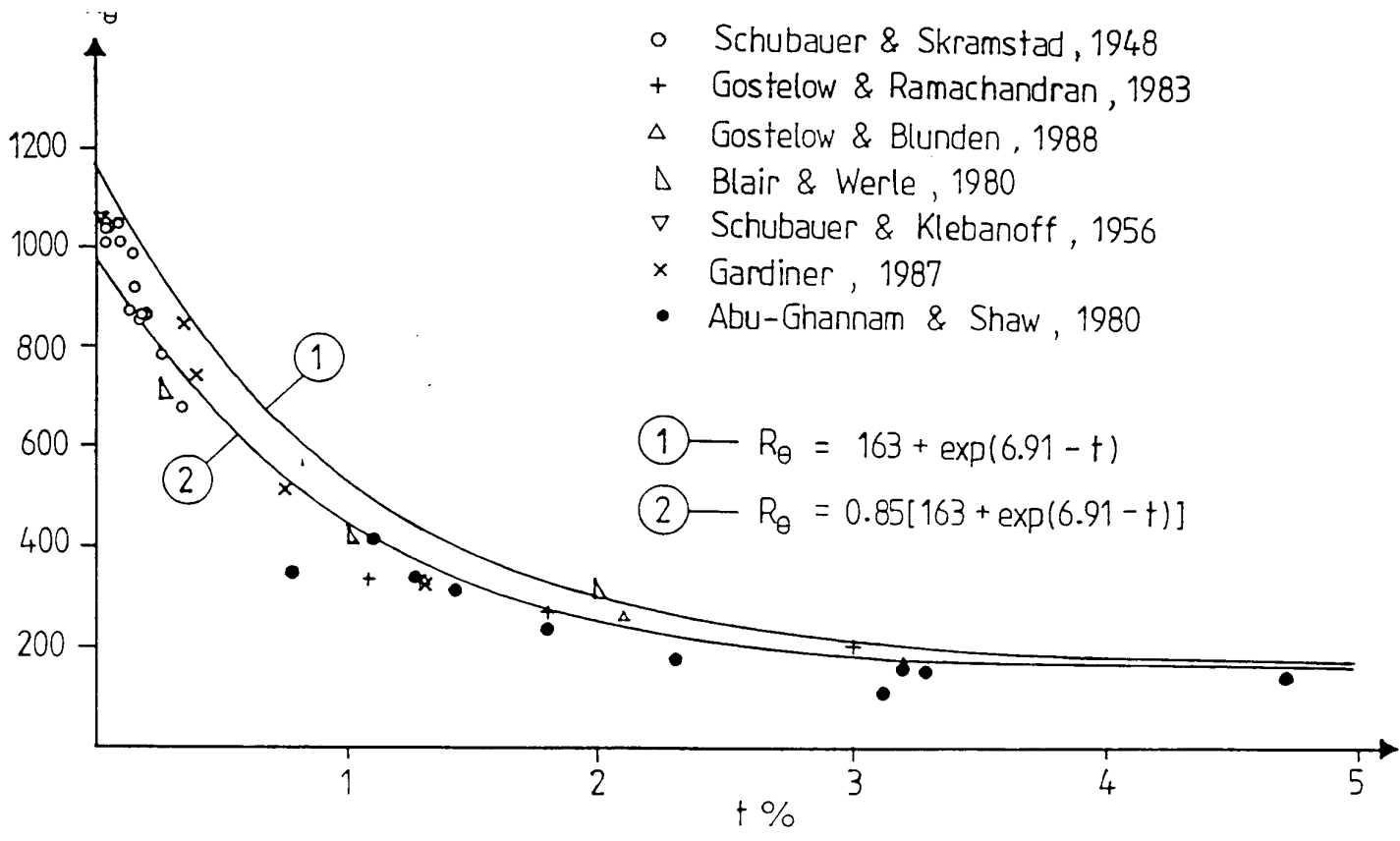
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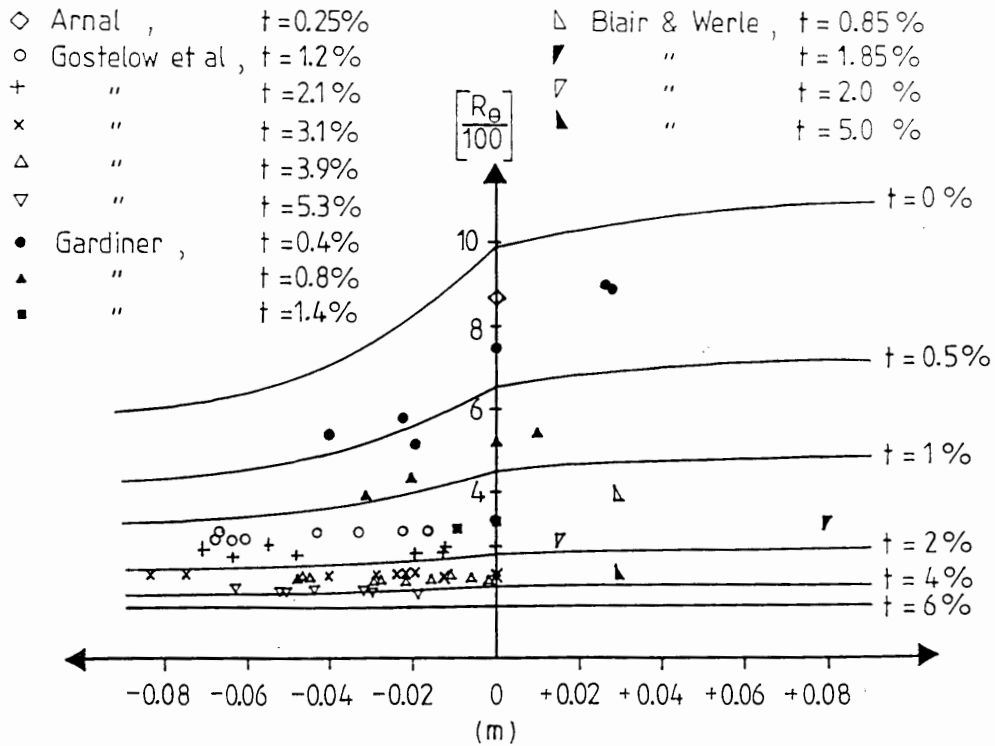
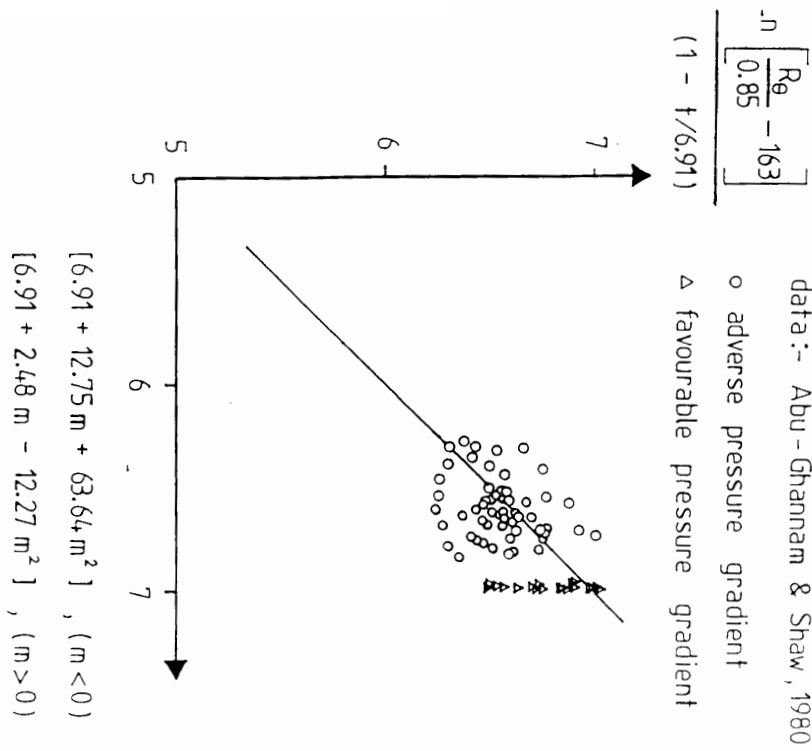
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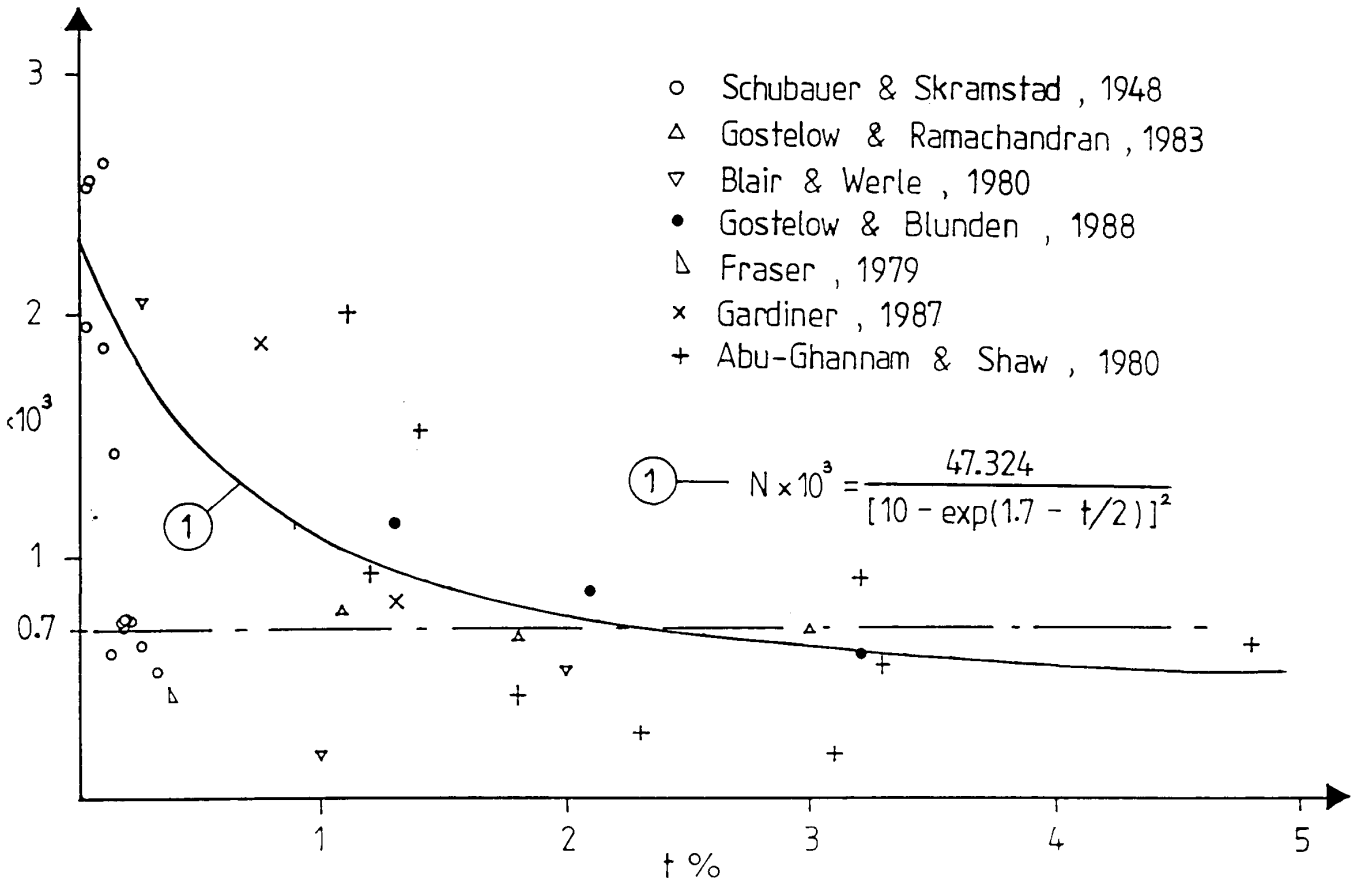
$$R_{\lambda} = 2.947 [10 - e^{(1.7 - \lambda/2)}] (R\theta)^{3/2}$$

$$\text{also } R_{\lambda} = \sqrt{[0.411 \cdot (R\theta)^3 / N_o]}$$

$$\therefore N_o \times 10^3 = 47.324 / [10 - e^{(1.7 - \lambda/2)}]^2$$







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**Non-zero Pressure Gradient Correlations**

$$[R\theta]_{\text{ran}} = 0.85 [163 + e^{g(m)}(1 - v_{6.91})]$$

$$g(m) = 6.91 + 12.75 m + 63.64 m^2 \text{ for } \{m < 0\}$$

$$g(m) = 6.91 + 2.48 m - 12.27 m^2 \text{ for } \{m > 0\}$$

$$\text{and } m = (\theta z / \nu) [dU/dx]$$

for  $\{m < 0\}$

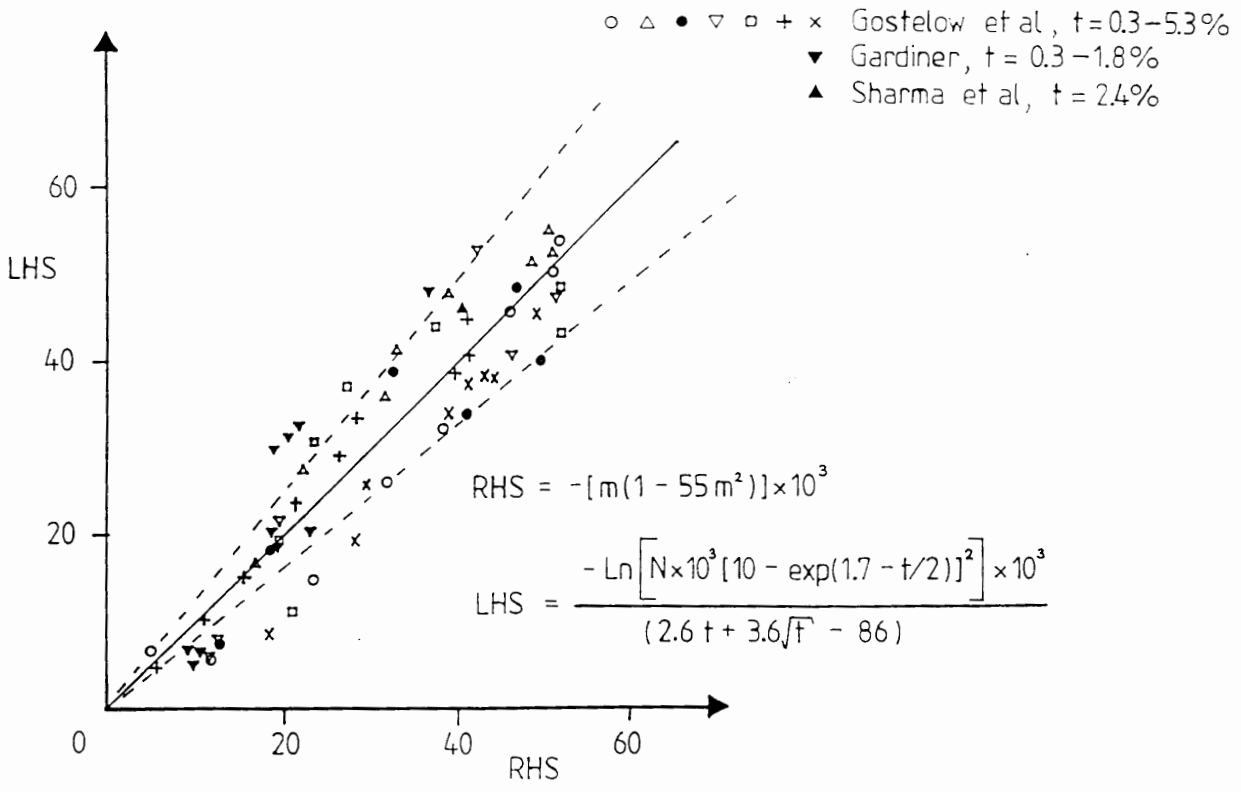
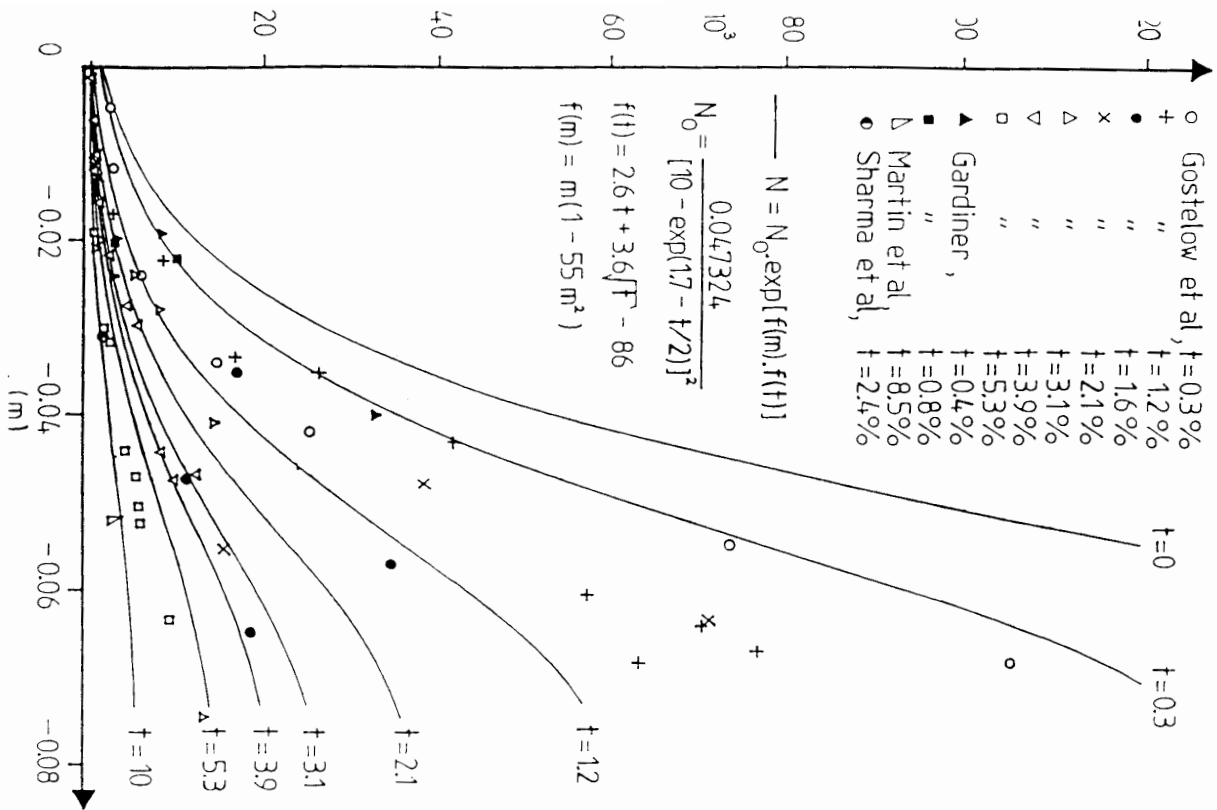
$$N = N_o \cdot e^{\{f(m) \cdot f(t)\}}$$

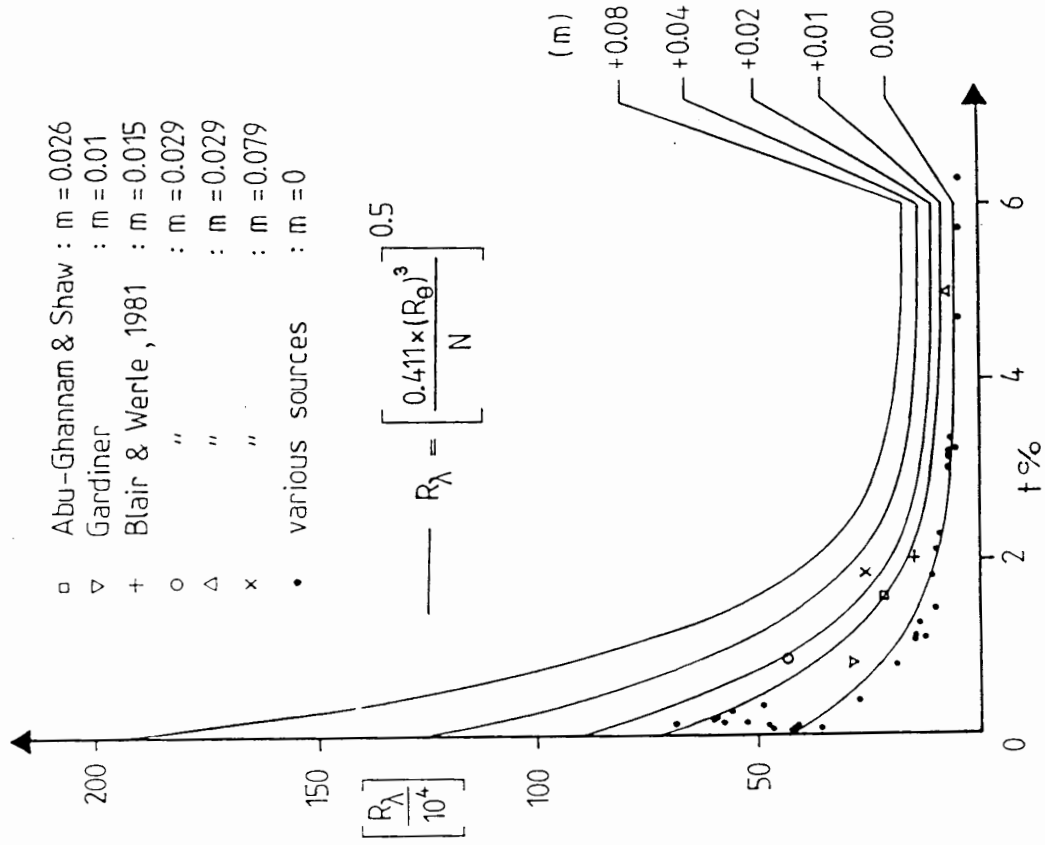
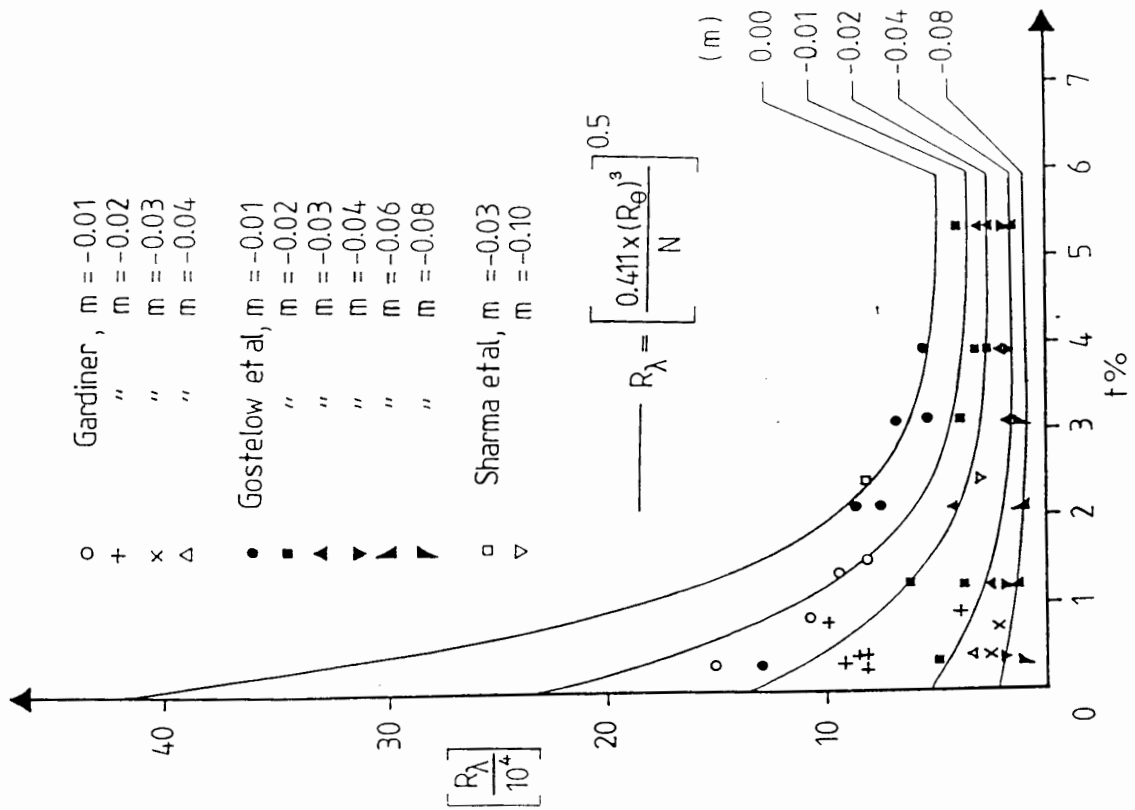
$$f(m) = m (1 - 55 m^2)$$

$$f(t) = 2.6 t + 3.6 \sqrt{t} - 86$$

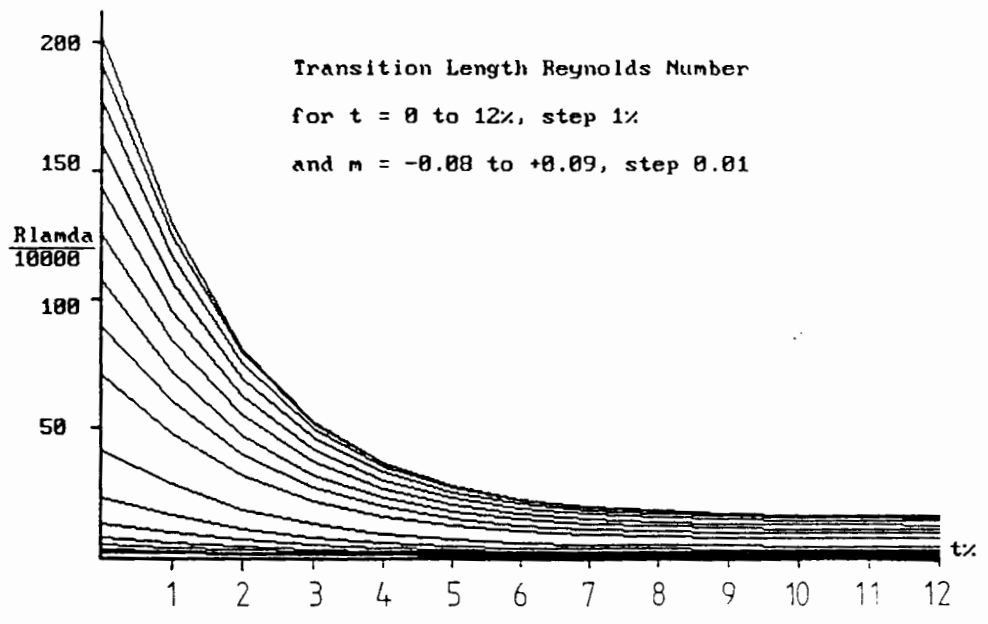
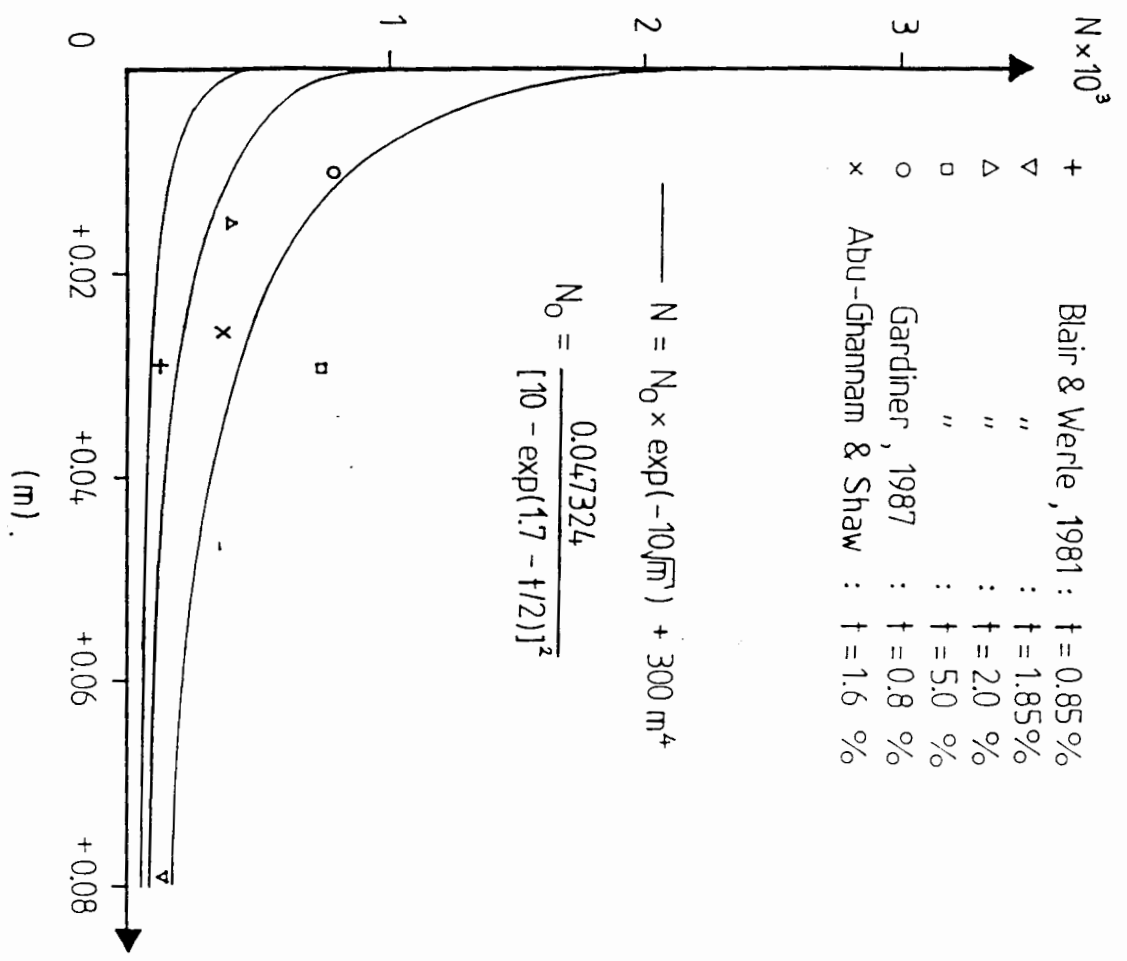
for  $\{m > 0\}$

$$N = N_o \cdot e^{t \cdot 10 \sqrt{m}} + 300 m^4$$

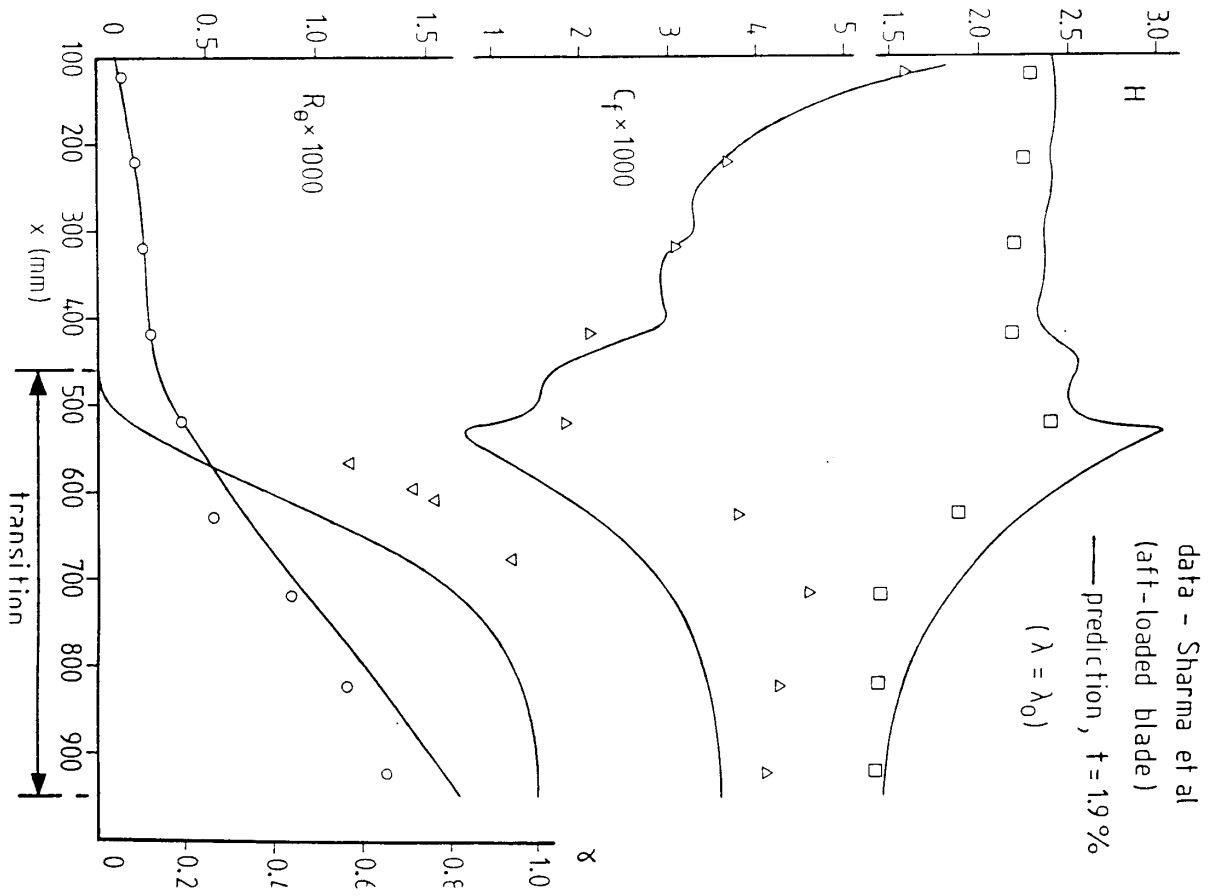
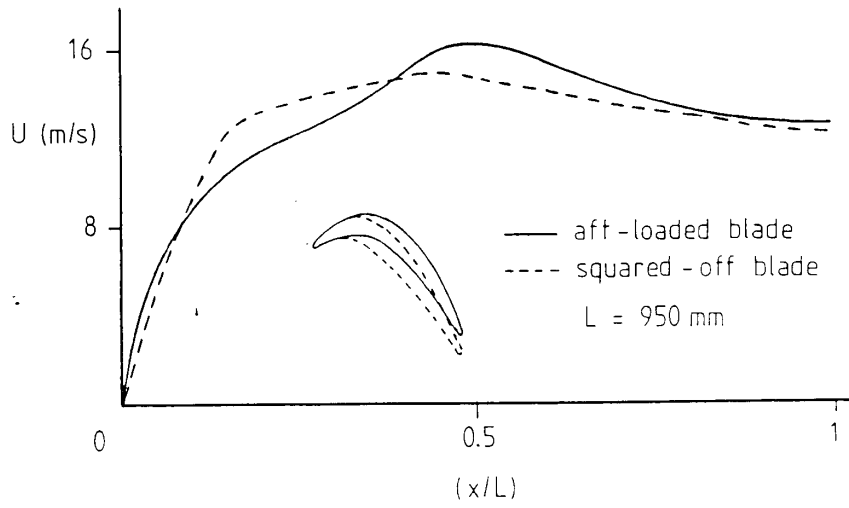








Freestream Velocity Distributions  
(from Sharma et al)



Intermittency Functions

$$\gamma = 1 - \exp(-Ax_1^2/\lambda^2)$$

where  $A = 0.411$  and  $x_1 = (x - x_{tran})$

Differential forms

$$dy/dx = \{2Ax_1/\lambda^2\} [1 - (x_1/\lambda)(d\lambda/dx)] \exp(-Ax_1^2/\lambda^2)$$

$$dy/dx = \{2Ax_1/\lambda_{av}^2\} [1 - (x_1/\lambda_{av})(d\lambda/dx)] \exp(-Ax_1^2/\lambda_{av}^2)$$

where  $\lambda_{av} = \int_{x_{y=0}}^{x_{y=1}} \lambda dx_1$

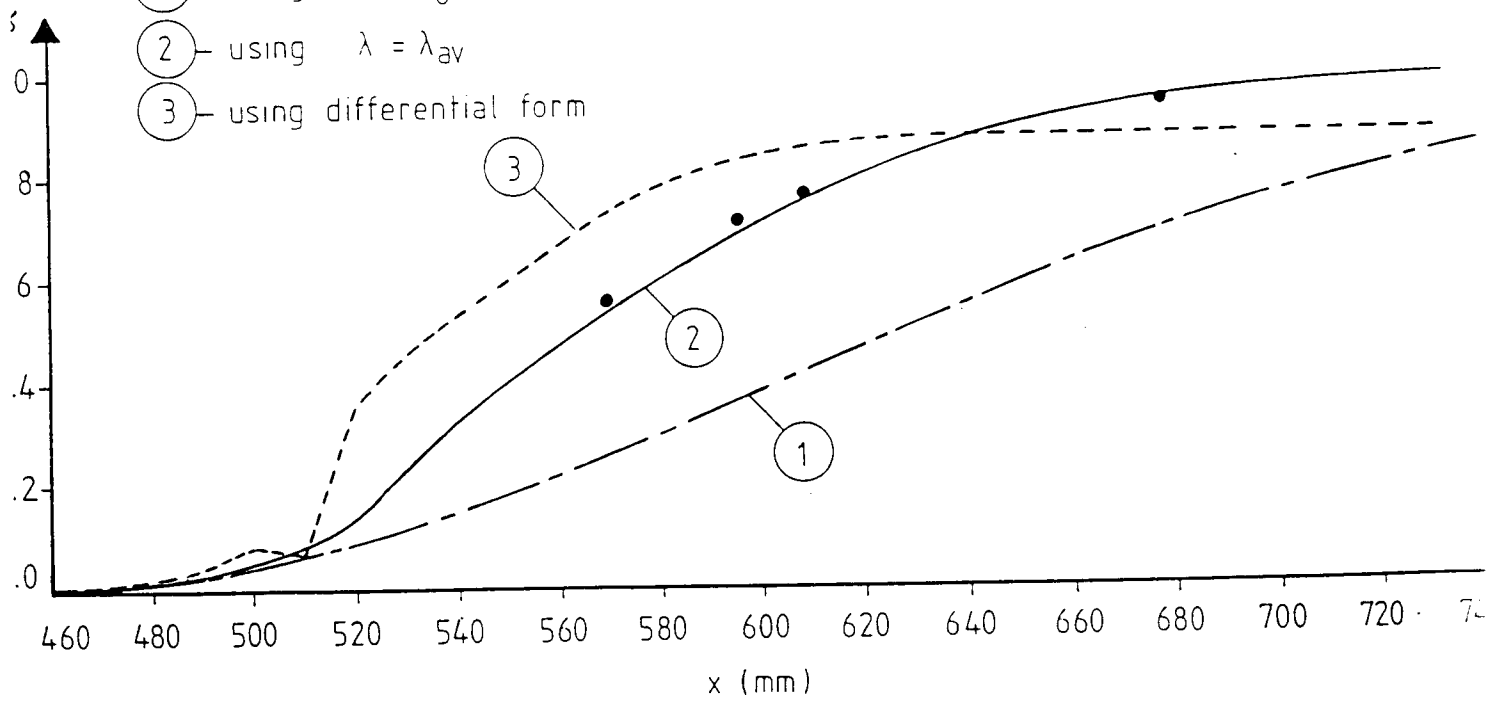
$$\gamma = 1 - \exp(-Ax_1^2/\lambda_{av}^2)$$

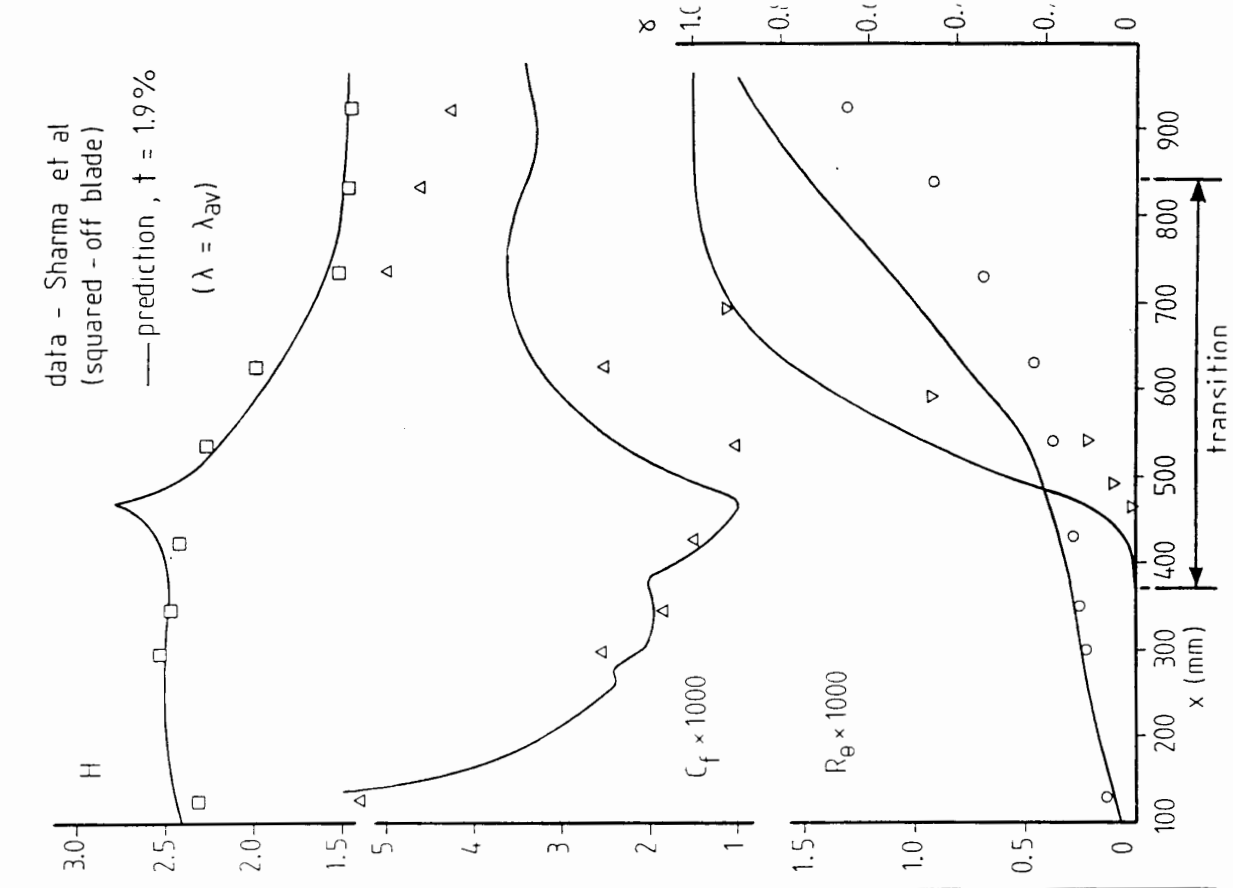
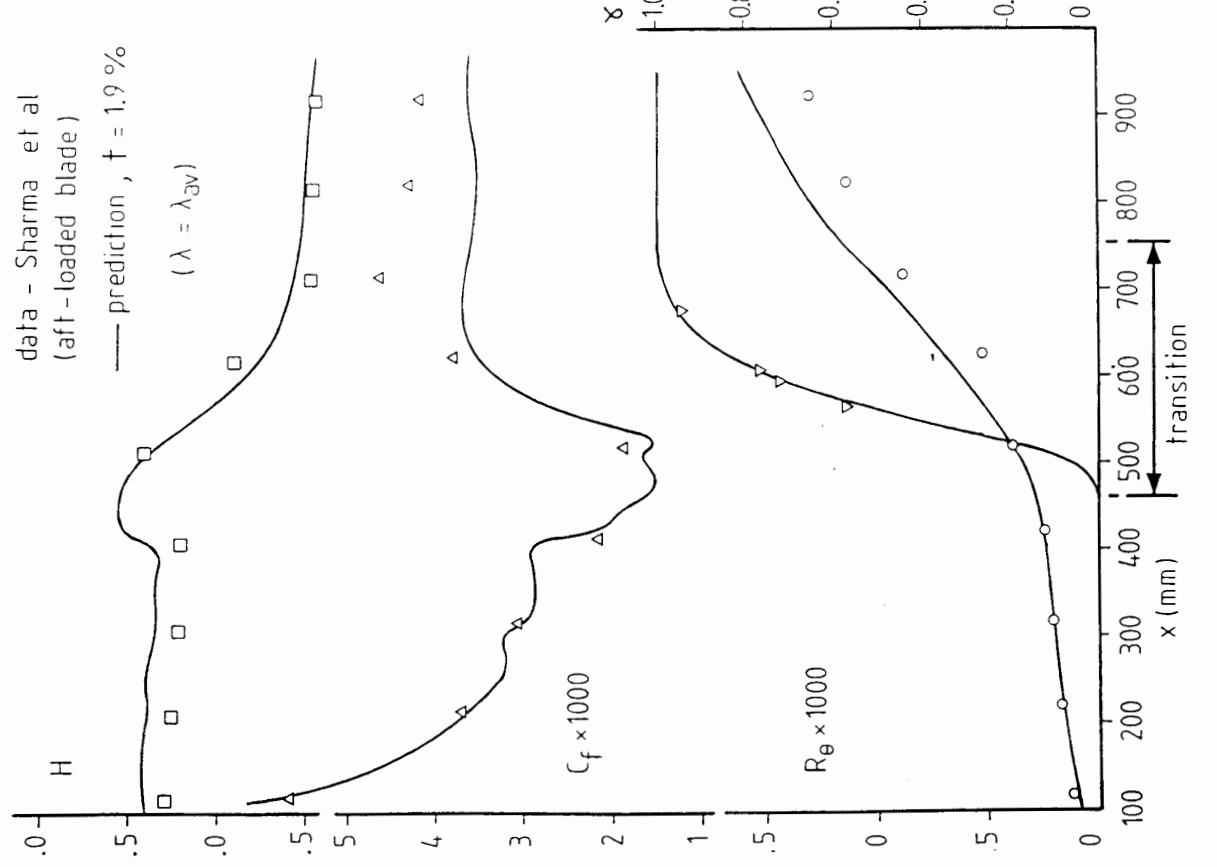
where  $\lambda_{av} = \int_{x_{y=0}}^{x_{y=0.25}} \lambda dx_1$

Intermittency Distributions

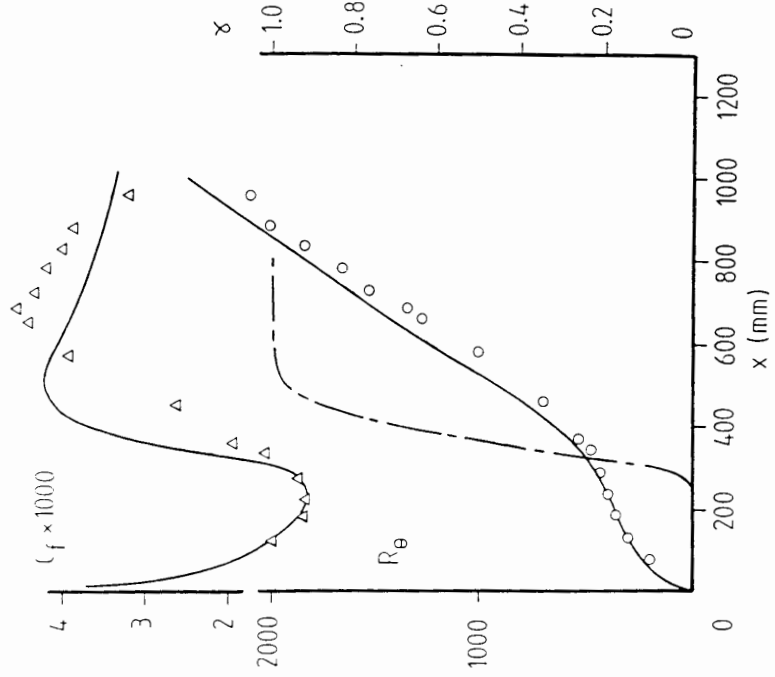
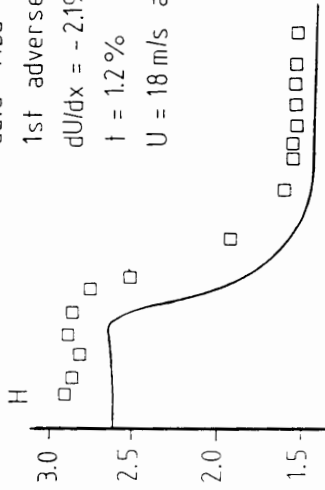
• data - Sharma et al, (aft-loaded blade)

- ① - using  $\lambda = \lambda_0$
- ② - using  $\lambda = \lambda_{av}$
- ③ - using differential form

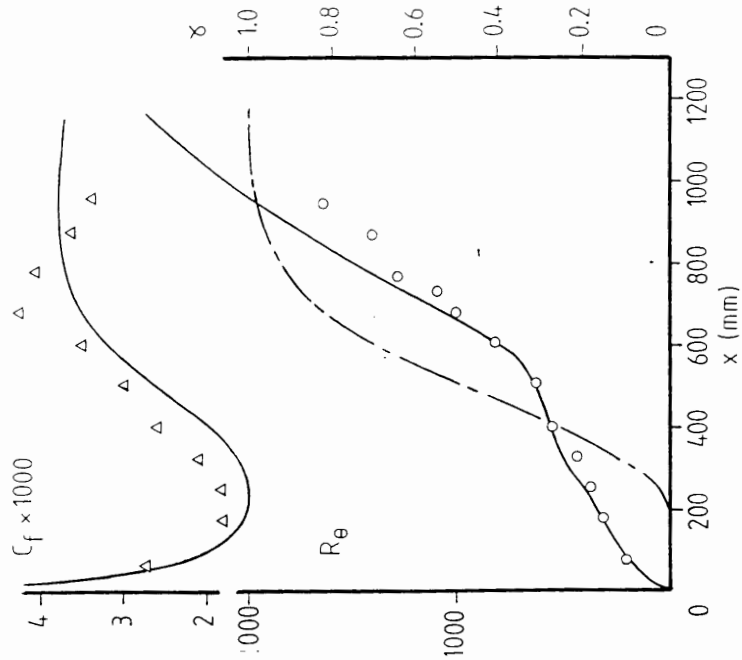
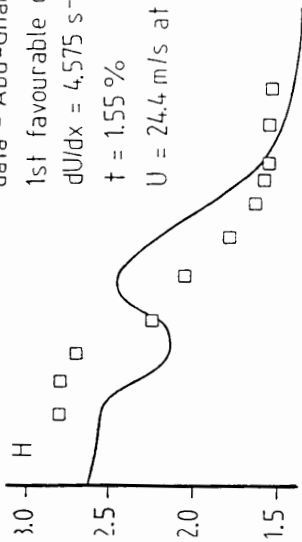




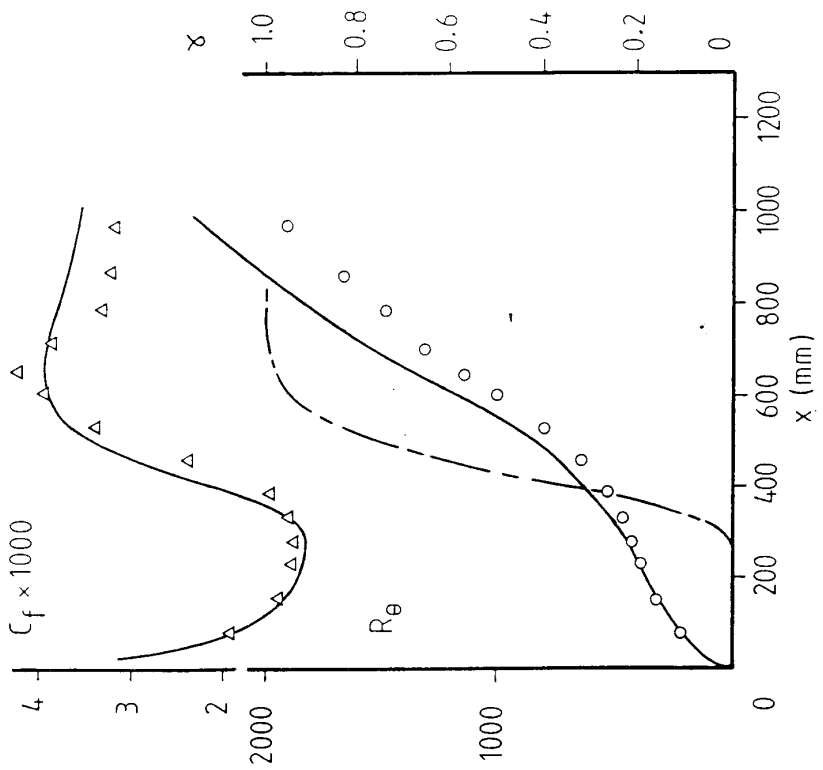
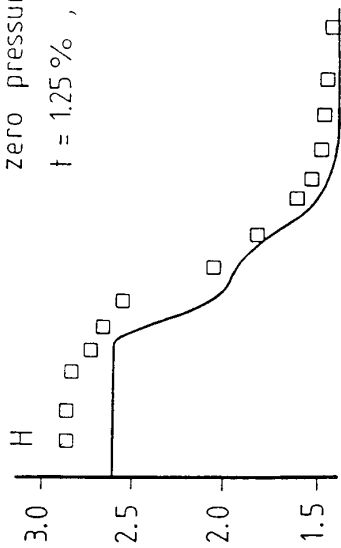
data - Abu-Ghannam & Shaw  
 1st adverse  $dp/dx$   
 $dU/dx = -2.19 \text{ s}^{-1}$ , (approx)  
 $t = 1.2\%$   
 $U = 18 \text{ m/s}$  at  $x = 1168 \text{ mm}$



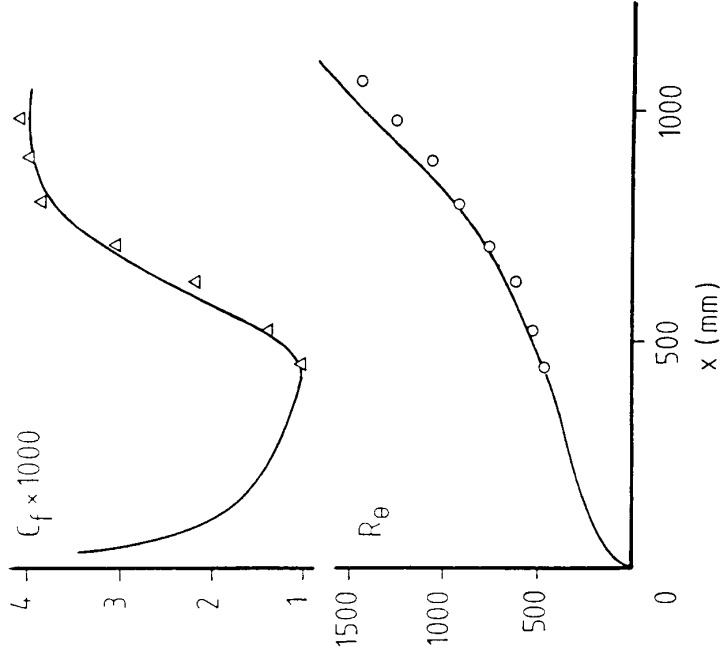
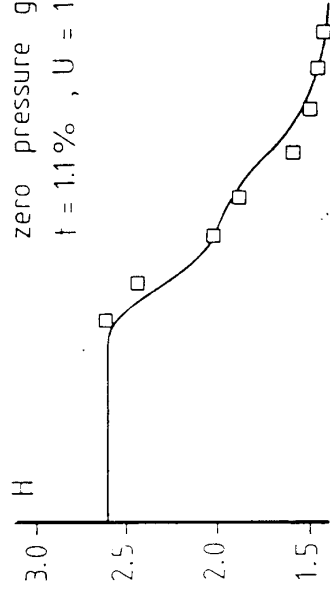
data - Abu-Ghannam & Shaw  
 1st favourable  $dp/dx$   
 $dU/dx = 4.575 \text{ s}^{-1}$ , (approx)  
 $t = 1.55\%$   
 $U = 24.4 \text{ m/s}$  at  $x = 1168 \text{ mm}$



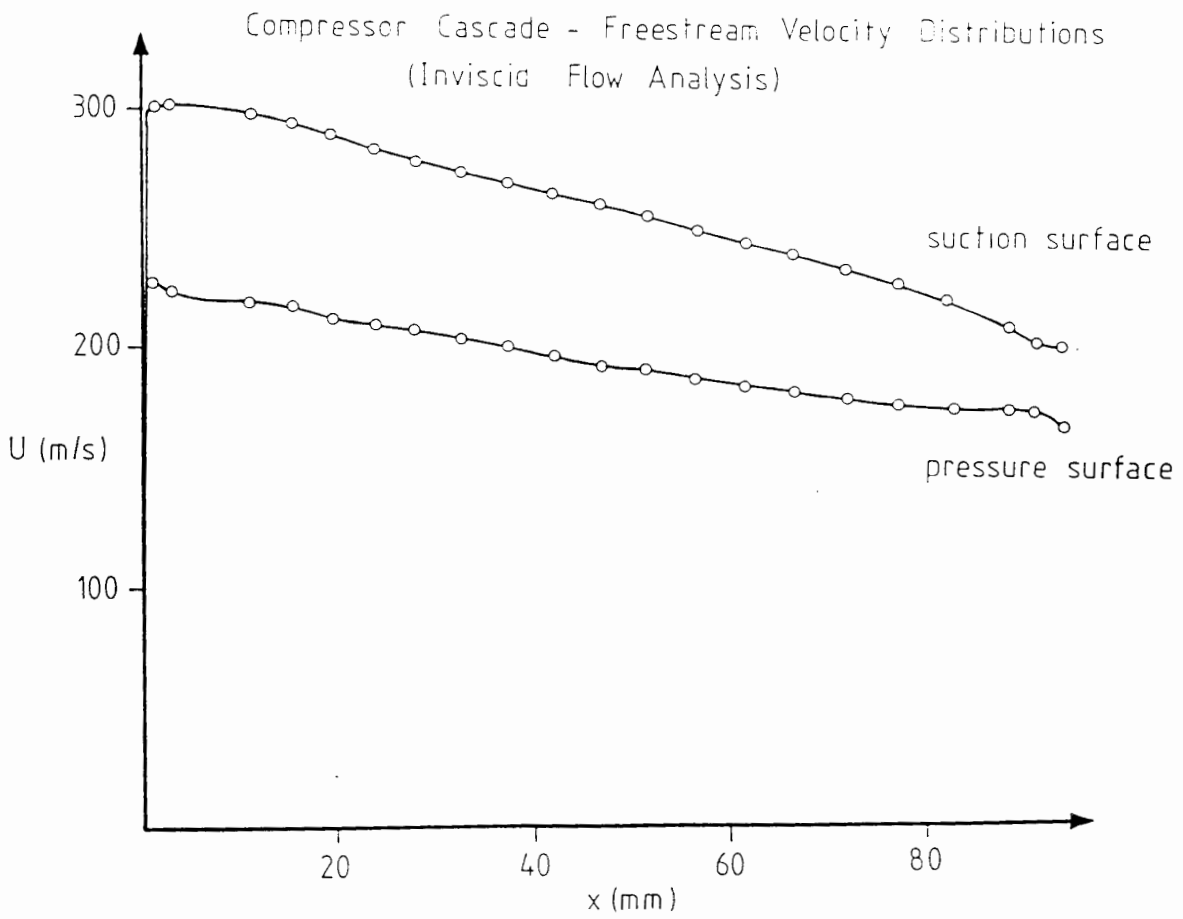
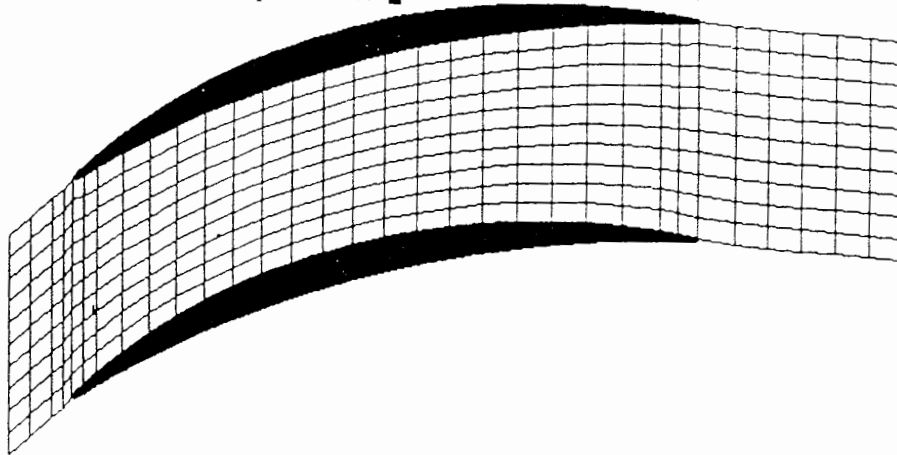
data - Abu-Ghannam & Shaw  
 zero pressure gradient  
 $t = 1.25\%$ ,  $U = 22$  m/s



data - Dhawan & Narasimha  
 zero pressure gradient  
 $t = 1.1\%$ ,  $U = 14.42$  m/s



Superimpose blades y/n ? y  
Press return to start computation? █



Compressor Cascade - Boundary Layer Analysis

