CNES APPROACHING GUIDANCE EXPERIMENT WITHIN FFIORD

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Abstract

This article outlines the relative orbit control (guidance algorithm and its preliminary performances tests evaluation) that will be tested by the CNES Team on FFIORD (Formation Flying In Orbit Ranging Demonstration) onboard PRISMA mission.
After a brief summary of the PRISMA mission context, the paper provides a full description of the rendezvous function involved in the approaching guidance experiment. This FFIORD onboard function is detailed in terms of on-board algorithmic method (basic algorithm and enhanced alternative), sensibility analysis used to construct manoeuvre plans and preliminary tests results.

1 PRISMA MISSION AND FFIORD EXPERIMENT

PRISMA is a "technology in-orbit testbed mission" [Ref. 1] for demonstrating formation flying and rendezvous technologies, as well as flight testing of sensor and actuator technologies. The mission is funded by the Swedish National Space Board (SNSB), and is undertaken as a multilateral project with additional contributions from CNES(Centre National d'Etudes Spatiales), DLR (Deutsche Zentrum für Luft- und Raumfahrt) and DTU(Danmarks Tekniske Universitet). The prime contractor is the Swedish Space Corporation (SSC), responsible for design, integration and operation of the space and ground segments, as well as implementation of in-orbit experiments involving Autonomous Formation Flying, Homing, Rendezvous, and Three Dimensional Proximity Operations including Final Approach and Recede.

Launch is expected in the first half of 2009 as secondary payload into a sun-synchronous LEO orbit of about 700 km, and the expected mission lifetime will be approximately 8 months. All flight operations will be controlled by SSC via the Kiruna ground station. This mission will consist of two spacecraft. One Main spacecraft of 140 kg with full 3-axis reaction wheel based attitude control and 3-axis delta-V capability and one simplified Target of 40 kg with coarse 3-axis attitude control based on magnetometers, sun sensors and magnetic torques.

The FFIORD experiment [Ref. 3] led by CNES includes the RF sensor-based relative navigation and guidance functions, including proximity operations (close station keeping and translations), stand-by on relative "football" orbit, rendezvous and collision avoidance (This last function is developed by EADS Astrium within a partnership with CNES – [Ref. 5]). The onboard software is implemented as a Matlab/Simulink® library delivered to SSC and then autocoded into C code with the Real Time Workshop Embedded coder®.

One of the objectives of the FFIORD experiment is to validate in flight the guidance functions. These functions are used mainly for closed loop manoeuvres, where they provide:

- the desired Delta-V and the time of its execution for sparse manoeuvres,
- the calculated reference trajectory in the case of continuous thrusting.

In particular this experiment will allow evaluating the in-flight performances and autonomy level of these algorithms in preparation of future formation flying operational applications such as SIMBOL-X (a French/Italian/German X-ray telescope mission).

2 RENDEZVOUS EXPERIMENT

The purpose of the approaching experiment is to demonstrate the capability to perform a safe rendezvous of the MAIN spacecraft from a distance of about 10 km from the TARGET spacecraft relying on the RF navigation system and exercising a large degree of autonomy. The rendezvous will be achieved several times with different initial states and scenarios of manoeuvres. For each test the performance will be evaluated based on 2 criteria: offset between the desired goal and the position reached, and deviation of the delta-V cost with respect to the expected value.

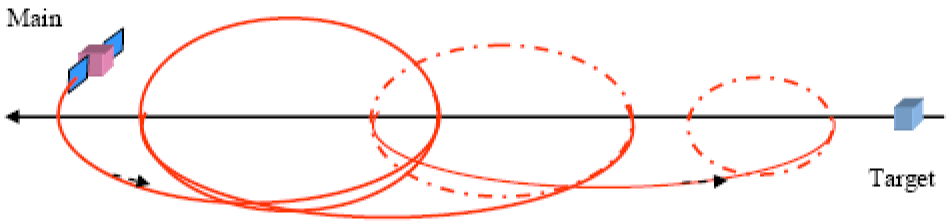


Figure 1: Rendezvous operations

The rendezvous must span over several orbits in order to minimize the delta-v cost and thus to allow a few rehearsals throughout FFIORD experiment. The duration of a rendezvous will be in the range of 12 hours for an average cost of 20 cm/s.

In this experiment we do not consider essential to implement a fully autonomous system that does not need any planning from the ground since future formation flying like SIMBOL-X will not use such a feature. As soon as the rendezvous is initiated, our objective is to demonstrate the capability to run the whole rendezvous without ground intervention and adjust the satellite trajectory to circumstances even though the on-board computing of manoeuvres follows some uploaded plan.

3 RENDEZVOUS BASIC ALGORITHM

Consequently, the rendezvous algorithm has been designed to work with the onboard estimated relative state and a pre-defined manoeuvre plan $\{t_k, \Delta V_k\}$ (with k ≤10). The general principle is the following: an initial manoeuvres plan is uploaded by TC and the on-board algorithm update autonomously this plan with respect to the current estimated relative state $X(t_c)$ with the objective to get to the target $X(t)$ on time t and without changing the dates of manoeuvres. The model of the relative motion used by the rendezvous function relies on Clohessy-Wiltshire equations (also called Hill equations) :

$$\boxed{X(t) = A(t-t_c)\,X(t_c) + B(t-t_c)\,\gamma + \sum_{k=1}^{n} A(t-t_k)\,\Delta V_k} \qquad (1)$$

with :

- A and B : Clohessy-Wiltshire matrices in the LVLH (Local Vertical Local Horizontal) frame of the target spacecraft. B represents the drag effect, it is optionnal.
- γ: differential drag acceleration (regarded as constant during all the rendezvous phase),
- $\Delta V_k = {}^t(0,0,0,\Delta V_{k,x},\Delta V_{k,y},\Delta V_{k,z})$: manoeuvre n°k in the LVLH frame of the target spacecraft,
- t_c : current date
- t_k : date of each manoeuvres of the plan ($n \leq 10$)
- t : date associated to rendezvous point

If we split the matrix $A(t)$ in block matrices like: $A(t) = \begin{pmatrix} A_1(t) & A_2(t) \\ A_3(t) & A_4(t) \end{pmatrix}$ and we rename ΔV_k in the following way $\Delta V_k = {}^t(\Delta V_{k,x},\Delta V_{k,y},\Delta V_{k,z})$, equation (1) can be rewritten more simply as:

$$\sum_{k=1}^{n} M_k.\Delta V_k = C(t,t_c) \qquad (2)$$

with: $C(t,t_c) = X(t) - A(t-t_c).X(t_c) - B(t-t_c).\gamma$ and $M_k = \begin{pmatrix} A_2(t-t_k) \\ A_4(t-t_k) \end{pmatrix}$.

Naturally, the role of the rendezvous function is to compute at each time step the serial of ΔV_k that enable to get $X(t)$ at t with the minimum consumption of propellant. Thanks to the expression (2) of the Clohessy-Wiltshire model, and a quadratic criteria for the minimization of the consumption, the problem solved on-board by the rendezvous algorithm can be reduced into a standard least square problem as follow:

$$\begin{cases} \sum_{k=1}^{n} M_k.\Delta V_k = C(t,t_c) \\ \min(\sum_{k=1}^{n} \|\Delta V_k\|^2) \end{cases} \Leftrightarrow \begin{cases} M.DV = C(t,t_c) \\ \min(\|DV\|^2) \end{cases}$$

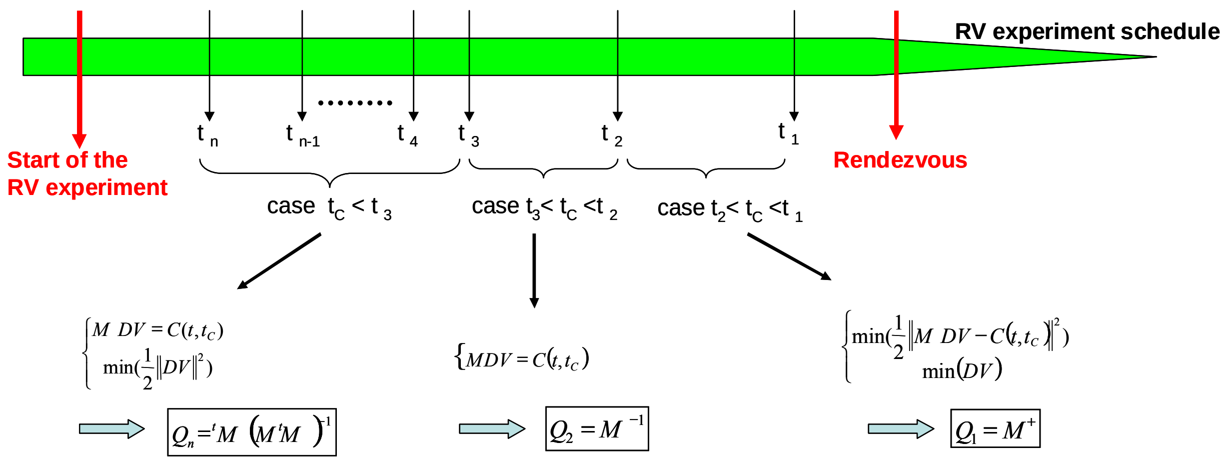
with : $M = (M_1, M_2, \ldots, M_{n-1}, M_n) \in \mathrm{M}_{6,3n}(\Re)$ and $DV = {}^t(\Delta V_1, \Delta V_2, \ldots, \Delta V_{n-1}, \Delta V_n) \in M_{3n,1}(\Re)$.

The solution of this standard least square problem is linear as following :

$$\Delta V = Q_n . C(t,t_c) \text{ with } Q_n = \begin{cases} {}^tM.(M.{}^tM)^{-1} & if\ n > 2 \\ M^{-1} & if\ n = 2 \\ M^{+} & if\ n = 1 \end{cases} \qquad (3)$$

(Note that another standard criterion like $\min\left(\sum_{k=1}^{n} \|\Delta V_k\|\right)$ instead of the quadratic criterion would have lead to a standard linear programming problem whose simplex solution is linear as well. The simplex alternative has the advantage to compute the manoeuvre dates which make it more autonomous but it is more difficult to implement on-board because of its sensibility to the manoeuvres dates. It is anyway another option to be studied).

Finally, equation (3) sum up the on-board rendezvous algorithm mechanism: at each time step the algorithm compute $C(t,t_c)$ and multiply it with Q_n to deduce the serial of oncoming ΔV_k. The Q_n can be pre-calculated on ground at the beginning of rendezvous phase (the problem of its invertibility for the computation is considered on §5). Q_n is different for each step of the plan depending on the number of remaining manoeuvres. The diagram below sum up the different phases of the rendezvous function involving the different Q_n during the experiment:



This onboard rendezvous guidance function needs an initial ground manoeuvre plan (mainly, manoeuvres dates t_k and associated pseudo inverse matrices Q_n, , Q_{n-1} , … , Q_2 , Q_1 . the ground computed ΔV_k are only used to monitor the onboard computed ΔV_k). Today, tool to be used to build this ground manoeuvre plan and its associated matrices will be certainly tool , so called DRAGON (**D**étermination des **R**endez-vous par **A**lgorithmes **G**énéraux d'**O**ptimisation **N**umérique), used at CNES to optimize rendezvous between near-circular and near-coplanar orbits. This tool, resulting from a tight collaboration with KIAM (Keldysh Institute for Applied Mathematics), has been involved for several years in phasing manoeuvres calculation in the frame of ATV mission (Automated Transfer Vehicle) and ATV – ISS rendezvous.

4 ENHANCED RENDEZVOUS ALGORITHM

As indicated above, in FFIORD experiment we use a linear model, so called Clohessy-Wiltshire model, as a closed-form solution to the differential solution of relative motion on near-circular orbit. To obtain an enhanced rendezvous algorithm while keeping a robust and simple mechanism such as a linear mechanism, we introduce and justify in this paragraph a polynomial change of variable on the Clohessy-Wiltshire model that improve the accuracy of the model while keeping its linear structure.

4.1 UNSTABLE VARIABLE AND LIMITS OF THE LINEAR MODEL

To do so, let's come back to the origin of the Clohessy-Wiltshire model. The relative dynamics of the main spacecraft is studied in the LVLH frame of target spacecraft orbiting the Earth on a circular orbit (pulsation ω, radius R) without any other perturbations. Assuming these hypotheses, we represent the relative motion by the following dynamical system:

$$\dot{X} = F(X) \tag{4}$$

In order to study the motion of the main spacecraft around the target we develop the dynamic model (4) in the vicinity of the origin. Applying the Taylor development of F around 0 at the order 2 (lawful since F is C^∞ in 0 thanks to the hypotheses) it gives :

$$(4) \Leftrightarrow \dot{X} = AX + F_2(X) + o\left(|X|^3\right) \tag{5}$$

$$\text{where } A = \left.\frac{DF}{DX}\right|_{X=0} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2\omega \\ 0 & -\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3\omega^2 & -2\omega & 0 & 0 \end{pmatrix} \text{ and } F_2(X) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3\omega^2 \frac{zx}{R} \\ -3\omega^2 \frac{zy}{R} \\ 3\omega^2 \frac{z^2}{R} - \omega^2 \frac{3(x^2+y^2)}{2R} \end{pmatrix}.$$

Note that in this development, the second term $F_2(X)$ is made of homogeneous polynomials of degree 2 in X coordinates. In particular $F_2(X) = o\left(|X|^2\right)$.

To obtain a clearer representation of the dynamics we write (5) it the basis that give A in its Jordan canonical form. With the linear change of variable: $Y = M^{-1}X$ we obtain equation (5) in the Jordan basis of A :

$$(5) \Leftrightarrow \dot{Y} = BY + \widetilde{F}_2(Y) + o\left(|Y|^3\right) \tag{6}$$

$$\text{where } M = \begin{pmatrix} -3 & 0 & \frac{2i}{\omega} & -\frac{2i}{\omega} & 0 & 0 \\ 0 & 0 & \frac{i}{2\omega} & -\frac{i}{2\omega} & \frac{i}{2\omega} & -\frac{i}{2\omega} \\ 0 & -\frac{2}{\omega} & \frac{1}{\omega} & \frac{1}{\omega} & 0 & 0 \\ 0 & -3 & 2 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -i & i & 0 & 0 \end{pmatrix}, \quad B = M^{-1}AM = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & i\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -i\omega & 0 \\ 0 & 0 & 0 & 0 & 0 & i\omega \end{pmatrix}$$

and $\widetilde{F}_2(Y) = M^{-1}F_2(MY)$ .Note that $\widetilde{F}_2(Y)$ is also made of homogeneous polynomials of degree 2 in Y coordinates (direct consequence of $\widetilde{F}_2$ definition) .In particular $\widetilde{F}_2(Y) = o\left(|Y|^2\right)$.

The on-board algorithm presented in §3 solves only the linear part of the dynamics $\dot{Y} = BY$. The result is a linear model that represents the first order of the dynamics in the vicinity of the origin. In the Jordan basis it gives:

$$M^{-1}.X(t) = Y(t) = \begin{pmatrix} 1 & t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-i\omega t} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{i\omega t} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-i\omega t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{i\omega t} \end{pmatrix} \cdot \begin{pmatrix} y_{1,0} \\ y_{2,0} \\ y_{3,0} \\ y_{4,0} \\ y_{5,0} \\ y_{6,0} \end{pmatrix} \tag{7}$$

As expected this solution is simple, robust and easily computed on-board the spacecraft. One can recognize here the well known Clohessy-Wiltshire model or Hill model (summed up by $\dot{Y} = BY$). It represents an approximation of the dynamical system (4) at the order 1. Indeed the difference R between the approximated system (7) and the original system (4) can be written at the first order as a vector of homogeneous polynomials of degree 2 which is an order 2 in $|Y|$:

$$R(Y) = R(y_1, y_2, y_3, y_4, y_5, y_6) = \widetilde{F}_2(Y) + o\left(|Y|^3\right) = o\left(|Y|^2\right)$$

Consequently the system (7) and its associated order 1 solution should be relevant as long as the approximation of order 1 is true. Simulations results show that it turns out to be relevant only when $\frac{|Y|}{R_T} \le 0.15\%$ (i.e. in the case of FFIORD it is relevant in a ball of 10 km). This unexpected feature has its origin in the linear solution. Indeed the linear solution of (7) has an unstable variable $y_1(t) = y_{1,0} + y_{2,0} \cdot t$ that increases rapidly with the time. The consequence is that the variable $y_1(t)$ makes the rest $R(y_1, y_2, y_3, y_4, y_5, y_6)$ increase drastically with time. So it makes the linear approximation be quickly unrepresentative of the real dynamics (4).

4.2 NO LINEAR SOLUTION TO SOLVE THE DYNAMICS UNTIL THE SECOND ORDER

In order to increase the validity domain of the on-board solution and meet PRISMA requirements the most simple idea is to look for onboard resolutions of system (4) that solve it until the second order, i.e. that solves the following system:

$$\dot{Y} = BY + \widetilde{F}_2(Y) \tag{8}$$

The most simple (and classical) way to do so is to find a well suited change of variable that reduces system (8) to an integrable system. Since the second order $\widetilde{F}_2(Y)$ is made of polynomials, we look for a polynomial change of variable. Since $\widetilde{F}_2(Y)$ is homogeneous of degree 2 the following type of change of variable is sufficient:

$$\begin{array}{ccc} \mathfrak{R}^6 & \xrightarrow{C_1} & \mathfrak{R}^6 \\ Y & \longrightarrow & Y + P(Y) \end{array}$$ (9) where P(Y) is made of homogeneous polynomials of degree 2.

Unfortunately, as we show here after, such a polynomial change of variable can not reduce system (8) to an integrable system.

Indeed, let's apply this change of variable to the system (6). We obtain:

$(4) \Leftrightarrow (I + DP(Y))\dot{Y} = B \cdot (Y + P(Y)) + \widetilde{F}_2(Y + P(Y)) + o\left(|Y + P(Y)|^3\right)$ where $DP(Y)$ stand for the Jacobian matrix of $P(Y)$.

Besides, $\widetilde{F}_2(Y + P(Y)) = \widetilde{F}_2(Y) + o\left(|Y|^3\right)$ and $o\left(|Y + P(Y)|^3\right) = o\left(|Y|^3\right)$ since $\widetilde{F}_2(Y)$ and $P(Y)$ are homogeneous polynomials of degree 2.

So the system (6) becomes $(I+DP(Y))\dot{Y}=B\cdot Y+B\cdot P(Y)+\widetilde{F}_2(Y)+o\left(|Y|^3\right)$. By multiplying each side by $(I-DP(Y))$ we obtain :

$$\dot{Y}-DP^2(Y)\cdot\dot{Y}=B\cdot Y+B\cdot P(Y)+\widetilde{F}_2(Y)-DP(Y)\cdot B\cdot Y-DP(Y)\cdot B\cdot P(Y)-DP(Y)\cdot\widetilde{F}_2(Y)+o\left(|Y|^3\right)$$

Besides $DP(Y)=o(|Y|)$ since $P(Y)$ is composed of homogeneous polynomial of degree 2.

So, with the change of variable, system (4) take the final form:

$$\dot{Y}=B\cdot Y+B\cdot P(Y)-DP(Y)\cdot B\cdot Y+\widetilde{F}_2(Y)+o\left(|Y|^3\right)$$

This system is integrable (in the sense "linearly integrable") until the second order if $B\cdot P(Y)-DP(Y)\cdot B\cdot Y+\widetilde{F}_2(Y)=0$ i.e. if we can find an homogeneous polynomial of degree 2 in H_2 (vectorial space generated by the homogeneous polynomial of degree 2) such that $B\cdot P(Y)-DP(Y)\cdot B\cdot Y=-\widetilde{F}_2(Y)$ (10).

Let's introduce the operator L_B such that $L_B(Y)=B\cdot Y$. L_B induces an endomorphism $ad\,L_B$ on H_2 defined by $ad\,L_B(Q)=[Q,L_B]=DL_B.Q-DQ.L_B$ for any Q in H_2 .(In this definition one can recognize the Lie bracket in the notation [,]).

With these definition condition (10) can be rewritten as follow: $(10)\Leftrightarrow ad\ L_B(P(Y))=-\widetilde{F}_2(Y)$. It means that condition (10) is verified as far as we can find an antecedent to $-\widetilde{F}_2(Y)$ in H_2 by the endomorphism $ad\,L_B$. One can show (It is too long to be done here and it is not the main point of this explanation) that $ad\,L_B$ is not inversible on H_2 (the reason comes from the eigenvalue 0 of B). So no antecedent to $-\widetilde{F}_2(Y)$ can be found. The consequence is that it does not exist a change of variable of the type (9) to integrate the system (4) until the second order (Note that this is not incompatible with the possibility to obtain high-order analytical solutions in particular cases. As an example, high-order analytical solutions can be developed in the case of periodic relative trajectories [Ref. 8 and 9]).

Consequently, to sum up this paragraph, the dynamic can not be solved until the second order with a polynomial change of variables. So, a simple polynomial resolution of the second order dynamics can not be implemented on-board to enlarge the domain of validity of the PRISMA control algorithm.

4.3 ENHANCED LINEAR MODEL OF THE RELATIVE DYNAMICS

Nevertheless the previous development enables to find ways to enhance the first order on board resolution of the dynamics. If condition (10) doesn't have a solution, it exists however some homogeneous polynomial Q of degree 2 in H_2 such that: $ad\ L_B(Q(Y))-\widetilde{F}_2(Y)=\widetilde{R}(y_2,y_3,y_4,y_5,y_6)$ (11) where $\widetilde{R}(y_2,y_3,y_4,y_5,y_6)$ is independent with respect to the unstable variable y_1.

Indeed we show with Maple (the Maple sheets are available on request to the authors) that the homogeneous polynomial of degree 2: $Q(Y)=M^{-1}\cdot T(M\cdot Y)$ verify condition (11) if T is chosen as the classical curvilinear change of variable (at the second order) defined as follow:

$$T:\quad \begin{array}{ccc}\Re^6 & \xrightarrow{T} & \Re^6\\ (t_1,t_2,t_3,t_4,t_5,t_6) & \longrightarrow & \left(\frac{t_1t_3}{R},\frac{t_2t_3}{R},-\frac{t_1^2+t_2^2}{2R},\frac{t_4t_3+t_1t_6}{R},\frac{t_5t_3+t_2t_6}{R},-\frac{t_1t_4+t_2t_5}{R}\right)\end{array}$$

Consequently by applying on-board the polynomial change of variable C_2 defined as following: $\begin{array}{ccc}\Re^6 & \xrightarrow{C_2} & \Re^6\\ Y & \longrightarrow & Y+Q(Y)\end{array}$ on the dynamics and then solving the linear system (7), the system that is solved is totally equivalent to the following system in Y:

$$(I + DQ(Y))\dot{Y} = B \cdot Y + B \cdot Q(Y) \quad (12)$$

This system differs from system (4) only at the second order. Indeed by multiplying each side by $(I - DP(Y))$ we show that system (12) is equivalent to the following system $\dot{Y} = B \cdot Y + B \cdot Q(Y) - DQ(Y) \cdot B \cdot Y + o(\|Y\|^3)$. Thanks to the endomorphism $ad\, L_B$ it can be summed up by the equation: $\dot{Y} = B \cdot Y + ad\ L_B(Q(Y)) + o(\|Y\|^3)$. Since Q verifies condition (11), system (12) is finally rewritten: $\dot{Y} = B \cdot Y + \widetilde{F}_2(Y) + \widetilde{R}(y_2, y_3, y_4, y_5, y_6) + o(\|Y\|^3)$ (13).

This time, the difference between the on-board computed system (system (13)) and system (4) is $\widetilde{R}(z_2, z_3, z_4, z_5, z_6)$. It is still a rest of the second order since $\widetilde{R} \in H_2$ but it is not dependent on the unstable variable $y_1(t)$. So this time it ensures the expected domain of validity over the whole ball $\frac{|Y|}{R} \leq 1\%$ that occurs usually when only stable variables are involved. Consequently, by applying on-board the change of variable, the accuracy of the model has been enhanced without changing the rendezvous method. It improve the rendezvous function in accuracy and it enlarges its validity domain.

5 SENSIBILITY OF RENDEZVOUS FUNCTION

In this paragraph, we develop the analytical sensibility matrices of the rendezvous algorithm. As in previous paragraph, we develop here the FFIORD rendezvous algorithm in the Jordan base to have a clearer representation of the dynamics, to put on relief the impact of the unstable variable of the dynamics on the sensibility of the algorithm and to facilitate the calculation.

5.1 ON-BOARD ΔV COMPUTATION IN THE JORDAN BASIS

With the linear change of variable: $Y = M^{-1}X$ we obtain indeed the Clohessy-Wiltshire model (1) in the Jordan basis as following:

$$Y(t) = M^{-1}.X(t) = M^{-1}.A(t - t_0).M.M^{-1}X(t_0) + \sum_{k=1}^{n} M^{-1}.A(t - t_k).M.M^{-1}\Delta V_k$$

$$\text{i.e.}\quad Y(t) = J(t - t_0).Y(t_0) + \sum_{k=1}^{n} J(t - t_k).M^{-1}\Delta V_k \quad (14)$$

To simplify the calculation, we transform equation (14) in a symmetric equation with respect to t and t_0, by multiplying (14) on the left by the coefficient $J(t_n - t)$, where t_n is the date of last manoeuvre. It gives the equivalent equation for the motion:

$$\boxed{J(t_n - t).Y(t) = J(t_n - t_0).Y(t_0) + \sum_{k=1}^{n} J(t_n - t_k).M^{-1}\Delta V_k} \quad (15)$$

We follow the same process as in the third paragraph to deduce from model (15), the on-board computation of the Clohessy-Wiltshire model. We develop it in the Jordan basis:

- First we isolate the variables $(\Delta V_{k,i})_{i=x,y,z}$ and we rewrite $J(t_n - t_k).M^{-1}$ in block matrices $\in \mathrm{M}_{3,3}(\mathbb{C})$:

$$\left(\Delta V_k = \begin{pmatrix} \Delta V_{k,x} \\ \Delta V_{k,y} \\ \Delta V_{k,z} \end{pmatrix}\right)_{k=1\ldots n} \quad \text{and} \quad J(t_n - t_k).M^{-1} = \begin{pmatrix} J_1(t_n - t_k) & J_2(t_n - t_k) \\ J_3(t_n - t_k) & J_4(t_n - t_k) \end{pmatrix}$$

- Then we gather the factor of the variables $(\Delta V_{k,i})_{i=x,y,z}$ in matrices $N_{n,k}$: $N_{n,k} = \begin{pmatrix} J_2(t_n - t_k) \\ J_4(t_n - t_k) \end{pmatrix}$

- Then equation (13) can be written as following:

$$(15) \Leftrightarrow J(t_n - t).Y(t) = J(t_n - t_0).Y(t_0) + \sum_{k=1}^{n} N_{n,k}.\Delta V_k$$

- And finally by gathering the ΔV_k in a global variable $DV_{k,n}$ and the matrices $N_{n,k}$ in a global matrix $M_{n,k}$ we obtain the compact following expression for the dynamics:

$$\boxed{J(t_n - t).Y(t) = J(t_n - t_0)Y(t_0) + M_{n,1}.DV_{1,n}} \quad (16)$$

where: $DV_{k,n} = \begin{pmatrix} \Delta V_k \\ \vdots \\ \Delta V_n \end{pmatrix} \in \mathrm{M}_{3(n-k+1),1}(\Re)$ and $M_{p,l} = (N_{p,l} \cdots N_{p,p}) \in \mathrm{M}_{6,3p}(\Re)$.

Note here that in equation (16), in opposition to the variable Y which is written in the Jordan basis to separate clearly the unstable variable from the others, the variable DV is kept written in the LVLH frame to facilitate the interpretations on the sensibility matrices elements.

Then, we follow again the same process as in paragraph 3 to deduce from model (16), the onboard computation of the ΔV in the Jordan basis at each time step of the algorithm. At each date t_c the rendezvous algorithm receive from the navigation the main spacecraft state, $Y(t_c)$ (t_c is the date associated to current time). Considering this state as the initial state, the algorithm compute the serial of ΔV in the variable DV that ensure the rendezvous at t in $Y(t)$ (t is the date associated to rendezvous target point). Naturally, as in paragraph 3, the solution is divided into 3 cases that depend on the number of remaining manoeuvers: for $t_{p-1} \le t_c < t_p$

$$\boxed{DV_{p,n} = Q_p.[J(t_n - t).Y(t) - J(t_n - t_c).Y(t_c)]} \text{ with } Q_p = \begin{cases} {}^tM_{n,p}.(M_{n,p}.{}^tM_{n,p})^{-1} & if\ p \le n-2 \\ M_{n,p}^{-1} & if\ p = n-1 \\ M_{n,p}^{+} & if\ p = n \end{cases} \quad (17)$$

Note in equation (17) that at each time step t_c such that $t_{p-1} \le t_c < t_p$ the rendezvous algorithm compute in the variable $DV_{p,n}$ the whole serial of $(\Delta V_k)_{p \le k \le n}$ that ensure the rendezvous. Actually in this serial of ΔV only the first one ΔV_p will be applied by the spacecraft propulsion system while the other $(\Delta V_k)_{p<k\le n}$ will be recomputed after t_p before being applied. Consequently the equation of the ΔV_p applied is actually:

$$\boxed{\Delta V_p = S(Q_p).[J(t_n - t).Y(t) - J(t_n - t_c).Y(t_c)]} \quad (18)$$

with S:
$$\begin{array}{ccc} M_{g,h}(R) & \xrightarrow{S} & M_{3,h}(R) \\ Q = \begin{pmatrix} q_{1,1} & \cdots & q_{1,h} \\ \vdots & \ddots & \vdots \\ q_{g,1} & \cdots & q_{g,h} \end{pmatrix} & \longrightarrow & \begin{pmatrix} q_{1,1} & \cdots & q_{1,h} \\ q_{2,1} & \cdots & q_{3,h} \\ q_{3,1} & \cdots & q_{3,h} \end{pmatrix} \end{array}$$

5.2 SENSIBILITY OF THE ΔV COMPUTATION

We develop here the sensibility of the ΔV computation with respect to the navigation errors and the propulsion errors.

For practical purposes, the $DV_{p,n}$ is computed on-board every seconds between t_{p-1} and t_p. In order to lighten the notations, we will consider that $t_c = t_p$ (considering $t_c = t_p - \delta t$ seconds would just add to the equation a negligible term like $J(t_c - t_p)$).

Consequently equation (18) that gives the expression of the applied ΔV_p is changed in the following equation:

$$\Delta V_p = S(Q_p).[J(t_n - t).Y(t) - J(t_n - t_p).Y(t_p)]$$

If we add in this equation the error of propulsion $\delta\Delta V_p$ at t_p it gives:

$$\Delta V_p = \delta\Delta V_p + S(Q_p).[J(t_n - t).Y(t) - J(t_n - t_p).Y(t_p)] \tag{19}$$

In this equation $Y(t_p)$ is the main spacecraft state at t_p. According to equation (16) its expression is theoretically:

$$Y(t_p) = J(t_p - t_0)Y(t_0) + M_{p-1,1}.DV_{1,p-1} = J(t_p - t_0)Y(t_0) + \sum_{k=1}^{p-1} N_{p-1,k}\Delta V_k$$

If we add the navigation error $\delta Y(t_k)$ at t_k, the propulsion error $\delta\Delta V_k$ and we replace ΔV_k by its expression (19) it gives:

$$Y(t_p) = \delta Y(t_p) + J(t_p - t_0).Y(t_0) + \sum_{k=1}^{p-1} N_{p-1,k}.S(Q_k).[J(t_n - t).Y(t) - J(t_n - t_k).(Y(t_k) + \delta Y(t_k)) + \delta\Delta V_k] \tag{20}$$

Consequently the expression of ΔV_p with respect to the errors of navigation and propulsion is obtained by inserting expression (20) in expression (19). It gives:

$$\boxed{\begin{aligned}\Delta V_p = {} & \delta\Delta V_p + S(Q_p).[J(t_n - t).Y(t) - J(t_n - t_0).Y(t_0) - J(t_n - t_p).\delta Y(t_p)] \\ & - S(Q_p)\left[J(t_n - t_p)\sum_{k=1}^{p-1} N_{p-1,k}.S(Q_k).[J(t_n - t).Y(t) - J(t_n - t_k).(Y(t_k) + \delta Y(t_k)) + \delta\Delta V_k]\right]\end{aligned}} \tag{21}$$

Note that equation (21) is consistent with the strategy: ΔV_p depends only on the first $p-1$ manoeuvres. It is then sensible only on the first $p-1$ errors of propulsion ($(\delta\Delta V_k)_{k<p}$) and on the first p errors of navigation ($(\delta Y(t_k))_{k\leq p}$).

The onboard navigation provide the rendezvous algorithm with the variables $(Y(t_k) + \delta Y(t_k))_{k\leq p}$, which is the state vector of the main spacecraft in the local LVLH frame attached to the target, at the time of computation t_k. The sensibility matrix of the $DV_{1,n}$ with respect to the navigation errors is thus given by the Jacobian matrix of the variable $DV_{1,n}$ with respect to $\delta Y(t_k)$. It gives the following block matrix of $\mathrm{M}_{3n,6n}(\Re)$:

$$S_{DV/\delta Y} = \begin{pmatrix} \dfrac{\partial \Delta V_1}{\partial \delta Y(t_1)} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \dfrac{\partial \Delta V_n}{\partial \delta Y(t_1)} & \cdots & \cdots & \dfrac{\partial \Delta V_n}{\partial \delta Y(t_n)} \end{pmatrix} \text{where } \frac{\partial \Delta V_p}{\partial \delta Y(t_k)} = \begin{cases} -S(Q_p).J(t_n - t_p) & \text{if } p = k \\ S(Q_p).J(t_n - t_p).N_{p-1,k}.S(Q_k).J(t_n - t_k) & \text{if } p < k \\ 0 & \text{if } p > k \end{cases}$$

The propulsion is represented in the rendezvous algorithm by the variables $(\Delta V_k + \delta\Delta V_k)_{k<p}$, which is the ΔV executed by the main spacecraft in the local framework attached to the target, at the time t_k. The sensibility matrix of the $DV_{1,n}$ with respect to the propulsion errors is thus given by the Jacobian matrix of the variable $DV_{1,n}$ with respect to $\delta\Delta V_k$. It gives the following block matrix of $\mathrm{M}_{3n,3n}(\Re)$:

$$S_{DV/\delta\Delta V} = \begin{pmatrix} \frac{\partial \Delta V_1}{\partial \delta\Delta V_1} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \frac{\partial \Delta V_n}{\partial \delta\Delta V_1} & \cdots & \cdots & \frac{\partial \Delta V_n}{\partial \delta\Delta V_n} \end{pmatrix} \text{ where } \frac{\partial \Delta V_p}{\partial \delta\Delta V_k} = \begin{cases} 1 & \text{if } p = k \\ -S(Q_p).J(t_n - t_p).N_{p-1,k}.S(Q_k) & \text{if } p < k \\ 0 & \text{if } p > k \end{cases}$$

5.3 SENSIBILITY OF THE TARGETED STATE

We develop here the sensibility of the targeted state $Y(t)$ with respect to the navigation errors and the propulsion errors. The purpose of this paragraph is to put on relief the impact of the different errors on the algorithm performances with respect to the targeted state. By developing equation (20) until $Y(t)$,t date associated to targeted rendezvous, we obtain the expression of $Y(t)$ with respect to the errors of navigation and propulsion as follow:

$$\boxed{Y(t) = J(t-t_0).Y(t_0) + J(t-t_n).\sum_{k=1}^{n} N_{n,k}.S(Q_k).[J(t_n - t).Y(t) - J(t_n - t_k).(Y(t_k) + \delta Y(t_k)) + \delta\Delta V_k]} \quad (22)$$

By the same argument as the one used for the calculation of the ΔV sensibility matrix , the sensibility matrix of the targeted state $Y(t)$ with respect to the navigation errors is also given by the Jacobian matrix of the variable $Y(t)$ with respect to $\delta Y(t_k)$. It gives the following block matrix of $\mathrm{M}_{6n,6}(\Re)$:

$$S_{Y(t)/\delta Y} = \begin{pmatrix} \frac{\partial Y(t)}{\partial \delta Y(t_1)} \\ \vdots \\ \frac{\partial Y(t)}{\partial \delta Y(t_n)} \end{pmatrix} \text{ where } \frac{\partial Y(t)}{\partial \delta Y(t_k)} = -J(t-t_n).N_{n,k}.S(Q_k).J(t_n - t_k)$$

Once again the same argument as the one used for the calculation of the ΔV sensibility matrix. the sensibility matrix of the targeted state $Y(t)$ with respect to the propulsion errors is also given by the Jacobian matrix of the variable $Y(t)$ with respect to $\delta\Delta V_k$. It gives the following block matrix of $\mathrm{M}_{6n,3}(\Re)$:

$$S_{Y(t)/\delta\Delta V} = \begin{pmatrix} \frac{\partial Y(t)}{\partial \delta\Delta V_1} \\ \vdots \\ \frac{\partial Y(t)}{\partial \delta\Delta V_n} \end{pmatrix} \text{ where } \frac{\partial \Delta Y(t)}{\partial \delta\Delta V_k} = J(t-t_n).N_{n,k}.S(Q_k)$$

5.4 CONCLUSION ON THE SENSIBILITY ANALYSIS OF THE RENDEZVOUS FUNCTION

The numerical computation of the sensibility matrices for each case of rendezvous will obviously give the exact sensibility factor of each ΔV_k and of $Y(t)$ with respect to each error of propulsion or navigation. It will give in each case the interpretation of the behaviour of the rendezvous scenario with respect to the errors.

From an analytical point of view, we can already highlight two conditions on the manoeuvres dates t_k that ensure homogeneous terms in the different sensibility matrices. Consequently these conditions will be sufficient to ensure a robust behaviour of the rendezvous algorithm.

Indeed, all the coefficients of the sensibility matrices are products of terms like $S(Q_k)$, $J(t_l - t_m)$ and $N_{p,q}$.

On one side the terms of type $J(t_l - t_m)$ and $N_{p,q}$ are of the same type since $N_{p,q} = \begin{pmatrix} J_2(t_p - t_q) \\ J_4(t_p - t_q) \end{pmatrix}$. They contain exclusively terms of type constant or of type $\omega(t_l - t_m)$ or of type $e^{i\omega(t_l - t_m)}$. These terms are homogeneous as long as the only divergent type which is $\omega(t_l - t_m)$ stay close to 1. Basically, this means that the strategy must ensure that there is not too many time (more than several orbits of the target spacecraft) between two manoeuvres. This give us the first condition for a robust behaviour of the rendezvous algorithm:

CONDITION 1: the dates of manoeuvres t_k must not be separated by more than a few periods of the target spacecraft orbits .
The physical interpretation of this condition is that when you let too many time between two manoeuvres the error due to the navigation or the propulsion is integrated in the dynamics which contains a divergent variable. The risk is that the errors diverge with this variable and get very high. As the sensibility matrices suggest it, by avoiding too much time between the manoeuvres you ensure a regular control of the errors and a small dependence of your algorithm with respect to divergent error.

On the other side the terms of type $S(Q_k)$ are of another type. If they derive as well from the type $J(t_l - t_m)$, they contain above all an inverse procedure that need some restrictions to stay homogeneous. The restriction is that the inverted matrices involved in the calculation of $S(Q_k)$ must be indeed invertible. Two cases appear: if $k \le n-2$ $Q_k = {}^tM_{n,k} . (M_{n,k} . {}^tM_{n,k})^{-1}$ and the matrix that should be invertible is $M_{n,k} . {}^tM_{n,k}$, while if $k = n-2$ $Q_k = M_{n,k}^{-1}$ and the matrix that should be invertible is $M_{n,k}$. In both cases a sufficient condition for the invertibility of the matrices is that the columns of $M_{n,k}$ form a free family of vectors. Since $M_{n,k} = (N_{n,k} \cdots N_{n,n})$ a sufficient condition is that the columns of the $(N_{n,k})_{1 \le k \le n}$ form a free family of vectors. Considering the expression of $N_{n,k}$ an obvious sufficient condition is that $(t_n - t_m) \ne (t_n - t_q) \left[\frac{\pi}{\omega} \right]$ for $m, q \in [1, n]$ (it ensures that the exponentials of different vectors are free with respect to each other). This gives us the second sufficient condition of robustness for the rendezvous algorithm:

CONDITION 2: the dates of manoeuvres t_k must not be separated by an integer number of half periods of the target spacecraft orbits.

The physical interpretation of this condition is that locating two of the manoeuvres at an integer number of half periods make your algorithm loose some controllability parameters on the dynamics while its degrees of freedom (number of $\Delta V_{k,x,y\ or\ z}$) is kept equal. Between the two manoeuvres, the control that is done on some of the dynamics variables is redundant. The rendezvous algorithm loose efficiency (loss of controllability parameters) gets instable (the dynamical system is no more invertible and the sensibility matrix raise drastically) and sometime even loose the total control of the dynamics (in the case of the last two manoeuvres at an integer number of half periods the system has no solution and at least one of the dynamics variable is out of control).

An important point is that the two conditions listed before are sufficient conditions for a robust behaviour of the rendezvous algorithm. But they are as well necessary conditions in the case of a two manoeuvre transfer. Consequently, for any transfer, the last two manoeuvres should absolutely verify these 2 conditions. If not, a variable in the spacecraft dynamics may remain out of control at the end of the transfer.

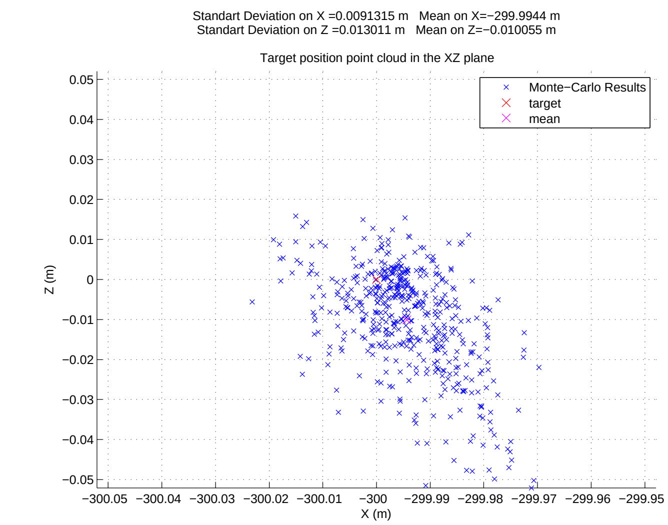
6 PRELIMINARY PERFORMANCES RESULTS

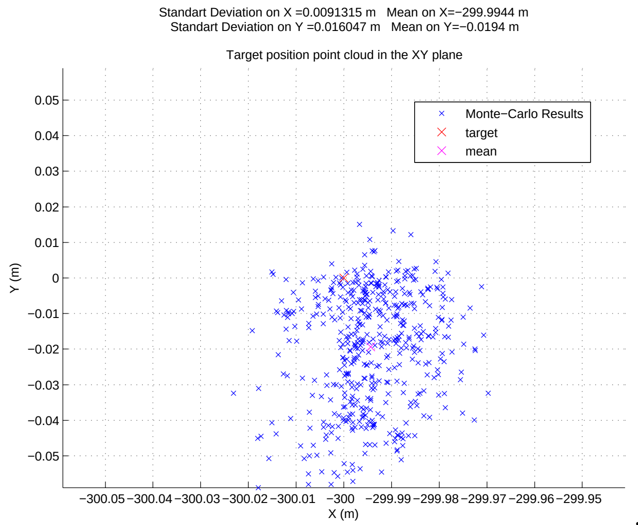
Monte Carlo simulations are currently performed to assess accuracy and robustness of the onboard rendezvous function. In order to lead these tests, an accurate simulator was developed by CNES under Matlab/Simulink®. This testbed environment makes possible to analyse the impact of the following perturbations :

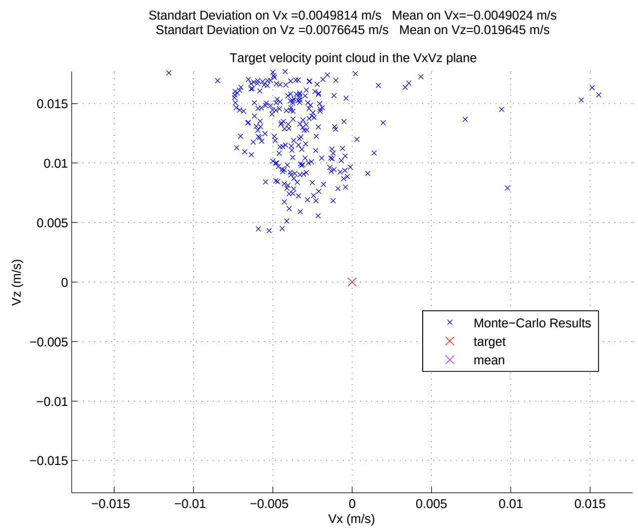
- perturbations such as geopotential's higher orders, atmospheric drag, luni-solar attraction and Sun radiation pressure,
- eccentricity of the TARGET: an eccentricity up to 0.004 is expected instead of a null eccentricity, as assumed in the Clohessy-Wiltshire equations (In simulation tests, eccentricity interval variation is equal to [0, 0.01]) ,
- navigation : the FFIORD navigation function with a statistical model of navigation errors is used inside this testbed. The FFRF subsystem is specified to provide the following information every seconds : inter-satellite distance (accuracy = 1 cm) and azimuth and elevation of line-of-sight between two satellites (specified accuracy < 1°). Considering these inputs, the navigation function delivers an estimated distance (X_{LVLH}) with a residual noise lower than 2 mm for short and long range and an error of position on Y_{LVLH}, Z_{LVLH} components of about 5 cm for short range (0 to 1000 m) and 60 cm for long range (1000 m to 10 000 m) [Ref. 3] ,
- Manoeuvre realization errors: MIB (Minimum Impulse Bit), splitting of ΔV on pulses, and so on.

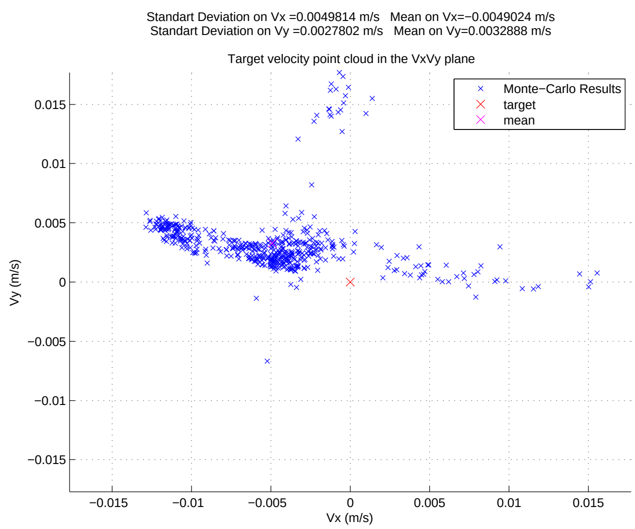
In this paragraph, we present preliminary results obtained on some rendezvous. :

- **Four manoeuvres Rendezvous [-200 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$ → [-300 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$:**
    - o Position and velocity accuracy wrt theoretical TARGET :









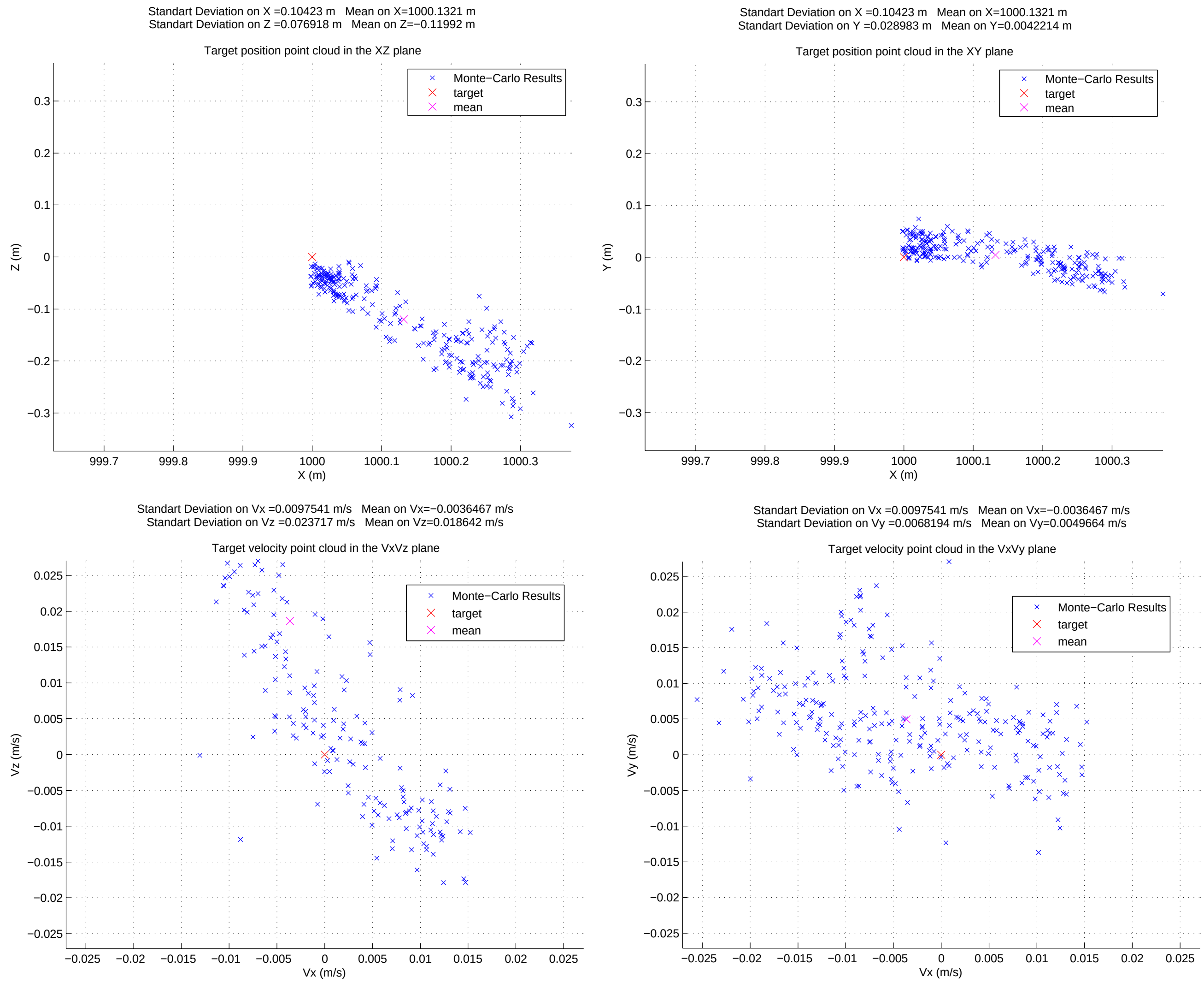
- o Maneuvers updated by on-board function:

Maneuvers dates: t_0 + 30 s, t_0 + 6676 s, t_0 + 7436 s, t_0 + 8176 s and date associated to TARGET point : t_0 + 8206 s (Orbital period of the TARGET : 5920 s)

ΔV components (LVLH)	ΔV_1 (m/s)			ΔV_2 (m/s)			ΔV_3 (m/s)			ΔV_4 (m/s)		
	Mean	St. dev	Initial ground value	Mean	St. dev	Initial ground value	Mean	St. dev	Initial ground value	Mean	St. dev	Initial ground value
X	0.006	0.0006	0.0056	-0.002	0.0035	-.0058	0.001	0.0006	$4.5\ 10^{-5}$	-0.01	0.0077	0.00016
Y	$-5.0\ 10^{-5}$	$2.6\ 10^{-6}$	0.0	$-8.1\ 10^{-5}$	$3.9\ 10^{-5}$	0.0	0.0002	0.0003	0.0	0.003	0.0029	0.0
Z	0.003	$4.1\ 10^{-5}$	0.0029	0.009	0.006	0.0041	0.008	0.003	0.0018	0.023	0.013	-0.0028

- **Six manoeuvres Rendezvous [5000 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$ → [1000 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$** :

  - o Position and velocity accuracy wrt theoretical TARGET :

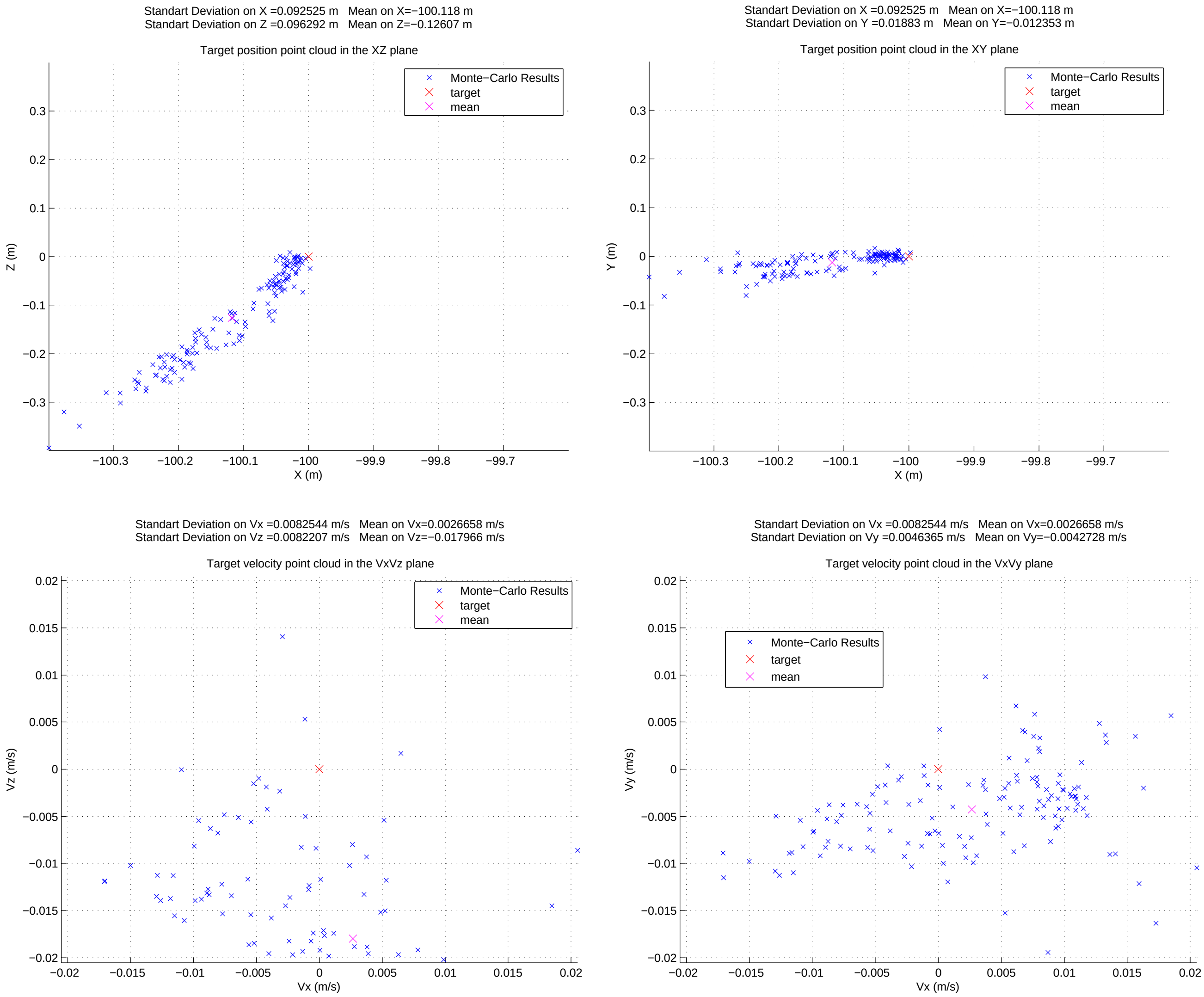


  - o Maneuvers updated by on-board function:

Maneuvers dates: t_0 + 30 s, t_0 + 18795 s, t_0 + 26201 s, t_0 + 33118 s, t_0 + 34104 s, t_0 + 34597 s and date associated to TARGET point : t_0 + 34627 s (Orbital period of the TARGET : 5920 s).

Maneuvers cumulated mean values of $\Delta V_{1..4}$: (0.15 m/s, 0.008 m/s, 0.25 m/s).

- **Nine manoeuvres Rendezvous [-10 000 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$ → [- 100 m, 0 m, 0 m, 0 m/s, 0 m/s, 0 m/s]$_{LVLH}$:**
  - Position and velocity accuracy wrt theoretical TARGET :



  - Maneuvers updated by on-board function:

Maneuvers dates: t_0 + 30 s, t_0 + 18138 s, t_0 + 33941 s, t_0 + 46452 s, t_0 + 54354 s, t_0 + 61926 s, t_0 + 62915 s, t_0 + 63902 s, t_0 + 64550 s and date associated to TARGET point : t_0 + 64590 s (Orbital period of the TARGET : 5920 s).

Maneuvers cumulated mean values of $\Delta V_{1..9}$: (0.26 m/s, 0.009 m/s, 0.25 m/s).

7 CONCLUSION

This article has presented the rendezvous function involved in the FFIORD guidance experiment. The initial performances and robustness tests show a correct behaviour of the algorithm. They tend to show that the resulting accuracy on the targeted state reached by the algorithm is sufficient to comply FFIORD requirements .

Nevertheless, this approach seems limited to near-circular orbits. And it is as well dependent on the ground segment for the computation of the maneuvers dates which limits the function autonomy. So, to prepare mission such as SIMBOL-X flying formation (high elliptic orbit, $e \approx 0.8$) other methods are studied, especially methods based on Model Predictive Control [Ref. 6 and 4] method and state transition matrices (Yamanaka–Ankersen closed-form solution - [Ref. 7]), to cope with the eccentricity effects and to investigate the onboard optimization of maneuvers dates .

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