LONG TERM MEAN LOCAL TIME OF THE ASCENDING NODE PREDICTION[†]

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Abstract

Significant error has been observed in the long term prediction of the Mean Local Time of the Ascending Node on the Aqua spacecraft. This error of approximately 90 seconds over a two year prediction is a complication in planning and timing of maneuvers for all members of the Earth Observing System Afternoon Constellation, which use Aqua's MLTAN as the reference for their inclination maneuvers. It was determined that the source of the prediction error was the lack of a solid Earth tide model in the operational force models. The Love Model of the solid Earth tide potential was used to derive analytic corrections to the inclination and right ascension of the ascending node of Aqua's Sun-synchronous orbit. Additionally, it was determined that the resonance between the Sun and orbit plane of the Sun-synchronous orbit is the primary driver of this error. The analytic corrections have been added to the operational force models for the Aqua spacecraft reducing the two-year 90-second error to less than 7 seconds.

Nomenclature

D: the disturbing body; Sun or Moon
U_D: *Potential function for disturbing body D*
μ_d : *Gravitational parameter for D*
$\vec{R}_D$: *Position vector of D*
θ_D: *Argument of latitude of D*
Ω_D: *Right Ascension of Ascending Node of D*
i_D: *inclination of D*
R_e: *radius of Earth*
$\vec{r}$: *Position vector of spacecraft expressed in classical Keplerian orbital elements a, e, i,* Ω, ω, ν
k_2: *Love Number; measures elasticity of the Earth*

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I. Introduction

Aqua is the lead spacecraft in the Earth Observing System (EOS) Afternoon Constellation. This constellation is a series of five spacecraft that fly in a loose formation to provide coordinated observation of the Earth. One of the parameters used to describe the orbits of members of the constellation is the Mean Local Time of the Ascending Node (MLTAN). Each spacecraft in the constellation is required to maintain a specific MLTAN range to avoid potential conjunctions with other constellation constituents. Due to orbital perturbations, the MLTAN will drift requiring periodic correction. Since the MLTAN drift rate is caused primarily by the J2 perturbation to the Earth's gravity and this perturbation is a function of orbital inclination, the MLTAN rate is controlled through inclination maneuvers. In order to maintain the relative geometry between the constellation members, inclination maneuvers are performed on all spacecraft in the constellation at approximately the same time.

As the lead spacecraft in the constellation, long-term predictions of Aqua's MLTAN are used to determine the date when the MLTAN requirements will be violated. Inclination maneuvers must be performed before this date, close to one of the equinoxes. These long-term predictions have typically been on the order of two years. Errors of ~90 seconds of MLTAN after 2 years have been observed in these predictions when compared to the definitive orbit. This can result in as much as 100 days of error in predicting the requirements violation date. Aqua is only capable of performing inclination maneuvers in the Spring and Fall near an equinox due to orbital night length constraints and the desire to maximize burn efficiency. Therefore, a 100-day prediction error in requirements violation may require an inclination series to be performed six months earlier than originally planned. Inclination campaigns for the Afternoon Constellation typically take six months to a year to plan and coordinate all five missions' inclination maneuvers and their science operations. The uncertainty in MLTAN prediction adds complexity to the coordination and planning effort across the Afternoon Constellation. Understanding the source of the MLTAN error growth and accounting for it in EOS Flight Dynamics System (FDS) MLTAN prediction methods and software improved the long-term prediction of Aqua's MLTAN drift and consequently improved the coordination of the Afternoon Constellation inclination maneuvers.

This paper will demonstrate the current EOS FDS MLTAN prediction capabilities using the standard operational force models and integrators. Comparison of the operational force models with other propagators suggested that the lack of a solid Earth-tide model was the cause of the MLTAN prediction error. A simple model of the solid Earth-tide potential is used to investigate the effects on Sun-synchronous orbits. Finally, equations are developed that correct the MLTAN prediction.

II. Current MLTAN Prediction Capabilities

The Mean Local Time of the Ascending Node (MLTAN) of an orbit is defined as the angle between the orbit's ascending node and the mean Sun as shown in Figure 1. A

Sun-synchronous orbit, such as Aqua's, is designed to maintain a constant MLTAN by matching the J2 nodal rate of the satellite with the nodal rate of the mean Sun. The MLTAN is often presented in units of time with 12:00 PM – or noon – describing a Sun-synchronous orbit that places the Sun directly at zenith when the spacecraft is at the ascending node. Orbital perturbation caused by the Sun and the Moon will cause the actual MLTAN of a spacecraft to deviate from a fixed value. The Aqua spacecraft is required to maintain a MLTAN between 13:30 – 13:45 to provide a nearly constant geometry despite these deviations.

To predict when the MTLAN will violate the acceptable limits, an ephemeris is generated using Aqua's operational force models in the Earth Observing System Flight Dynamics System (EOS FDS) software. The core of EOS FDS is the commercial-off-the-shelf (COTS) software package FreeFlyer®. These force models include a 30 zonal x 30 tesseral Joint Gravity Model (JGM) gravity field model, Jachhia-Roberts or Harris-Priester atmospheric density model, solar radiation pressure (SRP), and lunar and solar 3rd body gravitational effects.

Figure 2 shows the predicted MLTAN resulting from an ephemeris computed with these force models, excluding drag, along with the definitive (truth) MLTAN computed from the best estimate trajectory. After 650 days of propagation, 90 seconds of error in MLTAN is observed. This 90-second prediction error results in the spacecraft reaching a given MLTAN nearly 100 days earlier than was predicted by the operational force models.

The MLTAN prediction error was first noted in predictions following the Fall 2004 inclination campaign. Predictions made immediately after these maneuvers showed that Aqua would not violate its MLTAN limits until the Spring of 2007. However, refined predictions during 2005 began to show the MLTAN growing more quickly. Eventually, it was determined that the next inclination campaign would be required in the Fall of 2006 as opposed to the spring of 2007.

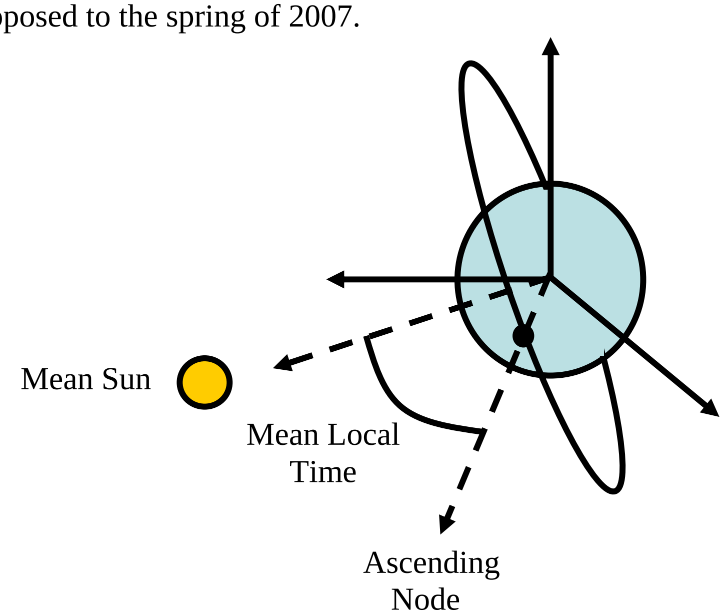

Figure 1. Definition of Mean Local Time of the Ascending Node

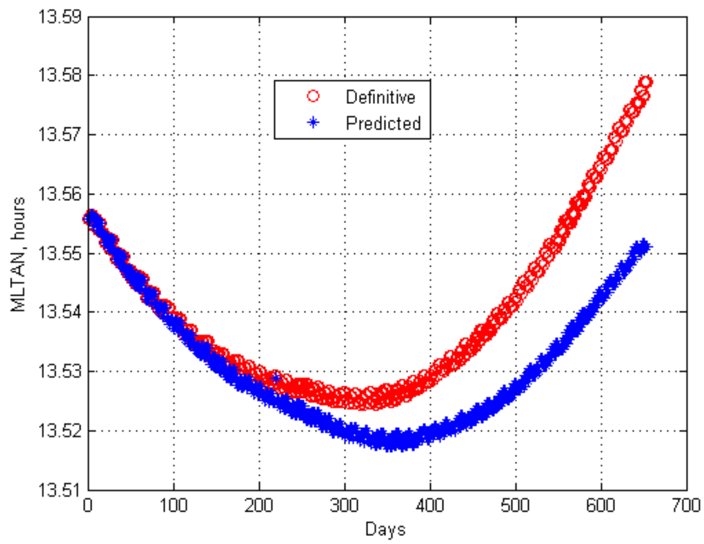

Figure 2. Comparison of Predicted and Definitive (Truth) MLTAN Over a 650-Day Propagation

To determine if there was any sensitivity of the MLTAN prediction to initial conditions, several one year propagations were made and compared to the definitive. MLTAN and inclination comparisons of the definitive orbit ephemeris against four 1-year propagated arcs anchored at various dates during 2004 and 2005 are shown in Figure 3 and Figure 4. The 1-year prediction error in MLTAN is observed to be between 24 and 32 seconds in Figure 3. It is observed in Figure 4 that the inclination prediction error rate appears to decrease around the summer solstice. Therefore, predictions that start around this time period have less initial inclination error that is carried throughout the propagation, resulting in less MLTAN prediction error. This behavior is observed in the arc anchored in May. The decrease in inclination error happens shortly after the start of the propagation meaning less initial error is carried through the propagation. Figure 3 clearly shows that the MLTAN prediction from the May – anchored arc has the least error.

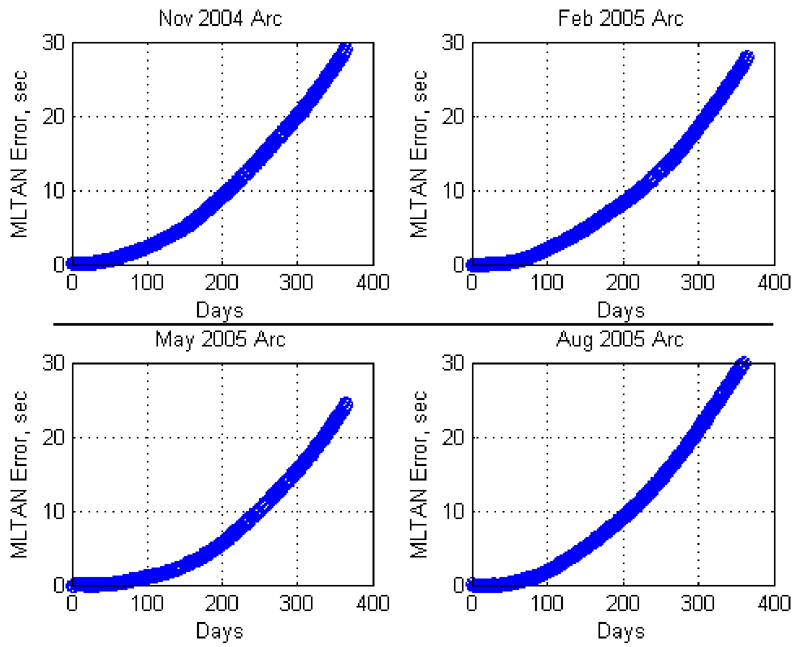

Figure 3. MLTAN Prediction Error for Several Starting Dates Demonstrating Seasonal Dependence of Prediction Error

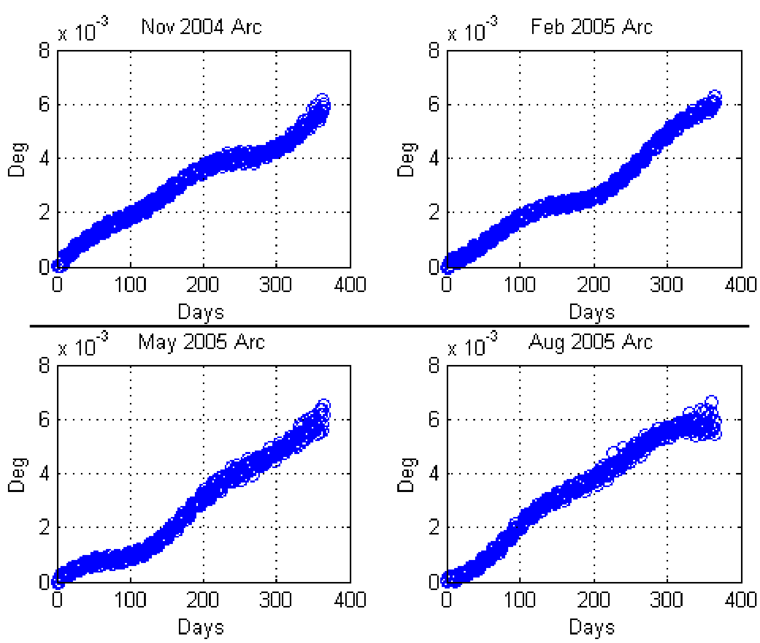

Figure 4. Inclination Prediction Error for Several Starting Dates

An initial investigation into the source of the MLTAN prediction error focused on the operational force model used in EOS FDS. Adjusting force model parameters such as the SRP coefficient to compensate for the error did not yield the required improvement in MLTAN prediction accuracy. Next, the Goddard Trajectory Determination System (GTDS) was used to compare against the operational propagators and force models in EOS FDS. GTDS has the option to include a solid Earth tide model which models the redistribution of the Earth's mass due to the gravitational attraction of the Sun and the Moon. It was determined that including the solid Earth tide model in the numerical propagation could reduce the MTLAN error from approximately 30 seconds over one year to less than 5 seconds.

III. Evaluating the Solid Earth Tide Effects within the Aqua Force Model

The GTDS comparison strongly suggested that the lack of a solid Earth tide model in the Aqua EOS FDS force model as the source of the MLTAN prediction error. Given this, a solid Earth tide model was analytically derived based on the Love model for the specific case of the EOS Aqua mission. This solid Earth tide model was used to derive an MLTAN error rate prediction model that favorably compared with the error between the predicted and observed MLTAN evolution as shown in Figure 2. The solid Earth tide model was incorporated within the Aqua force model and yielded numeric results with significantly reduced MLTAN prediction errors when compared to the definitive orbit.

To investigate the effects of a solid Earth tide model within the operational force models, a simple solid Earth tide potential given by the Love Model (1) is used:

$$\sum_D U_D = \sum_D \frac{k_2 \mu_D R_e^5}{2R_D^3 r^3}\left[3(\hat{R}_D \cdot \hat{r})^2 - 1\right] = \sum_D C_D \left[3(\hat{R}_D \cdot \hat{r})^2 - 1\right] \quad \textbf{(1)}$$

where C_D is constant under the assumption of circular orbits.

If the assumption is made that the Moon's orbit about the Earth and the Earth's orbit about the Sun are circular, the unit position vector of the disturbing body can be written in terms of inclination, right ascension of the ascending node, and argument of latitude of the disturbing body:

$$\hat{R}_D = \begin{Bmatrix} \cos\theta_D \cos\Omega_D + \sin\theta_D \sin\Omega_D \cos i_D \\ \cos\theta_D \sin\Omega_D + \sin\theta_D \cos\Omega_D \cos i_D \\ \sin\theta_D \sin i_D \end{Bmatrix} = \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix} \quad \textbf{(2)}$$

Likewise, the position vector for the spacecraft orbit about the Earth can be written in terms of its orbital elements.

$$\hat{r} = \begin{Bmatrix} \cos\nu(\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i) + \sin\nu(-\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i) \\ \cos\nu(\sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i) + \sin\nu(-\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i) \\ \cos\nu\sin\omega + \cos\nu\cos\omega\sin i) \end{Bmatrix} \quad \textbf{(3)}$$

When considering the effects of the Sun and Moon on the solid Earth tide, there are four fundamental frequencies:

1. The orbital period of the spacecraft about the Earth; Approximately 98 minutes for EOS missions
2. The orbital period of the Moon about the Earth; Approximately 27 days.
3. The orbital period of the Sun about the Earth; Approximately 365 days.
4. The motion of the Moon's orbit in the inertial (J2000) reference frame; Approximately 18 years;

Given the four frequencies listed above, it is reasonable to assume that during one orbit of the spacecraft the Sun and Moon are nearly constant. Therefore, the potential given by Equation 1 can be averaged over one spacecraft orbit while holding the components of the disturbing potential as constants:

$$\begin{aligned}\overline{U}_D = C_D\{3[\frac{{R_x}^2}{2}\left(\cos^2\Omega + \sin^2\Omega\cos^2 i\right) + \frac{{R_y}^2}{2}\left(\sin^2\Omega + \cos^2\Omega\cos^2 i\right) + \frac{{R_z}^2}{2}\sin^2 i \\ + R_x R_y \cos\Omega\sin\Omega\sin^2 i - R_x R_z \sin\Omega\cos i\sin i + R_y R_z \cos\Omega\cos i\sin i] - 1\}\end{aligned} \quad \textbf{(4)}$$

It was hypothesized that the error in MLTAN is driven by the right ascension of the ascending node (RAAN) rate due to the J2 perturbation. Since the J2 driven RAAN rate is primarily a function of inclination, the average potential developed in Equation 4 was used in the Lagrange planetary equation for the inclination rate

$$\frac{di}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\left\{\cos i\frac{\partial\overline{U}_D}{\partial\omega} - \frac{\partial\overline{U}_D}{\partial\Omega}\right\} \quad \textbf{(5)}$$

The partial derivative of the averaged potential with respect to the orbit argument of perigee is zero and the partial derivative with respect to the RAAN is given by:

$$\begin{aligned}\frac{\partial\overline{U}_D}{\partial\Omega} &= 3C_d[\cos\Omega\sin\Omega\sin^2 i\left(R_y^2 - R_x^2\right) \\ &+ R_x R_y \sin^2 i\left(\cos^2\Omega - \sin^2\Omega\right) \\ &- \cos i\sin i\left(R_x R_z\cos\Omega + R_y R_z\sin\Omega\right)]\end{aligned} \quad \textbf{(6)}$$

The orbital inclination is written in terms of the inclination rate given by Equation 5 by noting that the inclination is dominantly linear. The cosine of the inclination can then be expanded in a Taylor series:

$$i(t) = i_0 + \frac{di}{dt}t$$

Therefore

$$\cos(i(t)) = \cos(i_0 + \frac{di}{dt}t) \approx \cos(i_o) - \sin(i_o)\frac{di}{dt}t \quad \textbf{(7)}$$

The RAAN rate caused by the J2 perturbation is given by the classic result:

$$\frac{d\Omega}{dt} = \frac{-3nR_e^2 J_2}{2p^2}\cos(i) \quad \textbf{(8)}$$

The Taylor Series expansion of the cosine function given in Equation 7 is then substituted into Equation 8. Note that we now have a function for the RAAN rate including the linear inclination rate due to the solid Earth tide effects given in Equation 5.

$$\frac{d\Omega}{dt} = \frac{-3nR_e^2 J_2}{2p^2}[\cos(i_0) - \sin(i_0)\frac{di}{dt}t] \quad \textbf{(9)}$$

The error in MLTAN due to the solid Earth tide is the difference between the nominal J2 RAAN drift rate and that computed including the inclination drift rate. This difference results in an equation for the MLTAN error rate due to the solid Earth tide, assuming zero eccentricity to:

$$\frac{d\delta MLTAN}{dt} = \frac{3nR_e^2 J_2}{2a^2}\sin(i_0)\frac{di}{dt}t \quad \textbf{(10)}$$

Integrating Equation 10 once yields Equation 11 for the MLTAN error. Note that this equation is quadratic in time, which matches the observed error growth in Figure 3.

$$\delta MLTAN(t) = \frac{3nR_e^2 J_2}{4a^2}\sin(i_0)\frac{di}{dt}t^2 \quad \textbf{(11)}$$

To correct for the lack of an Earth tide model in the Aqua operational force models, Equations 5, 6, and 11 can be used to develop the corrections shown in Equations 12 and 13. At each time step during the numerical integration of the standard force models, the inclination correction δi, and RAAN correction $\delta\Omega$, are computed and added to the current state. The integration step size is given by Δt.

$$\delta i = \frac{di}{dt}\Delta t \quad \textbf{(12)}$$

$$\delta\Omega = \frac{3nR_e^2 J_2}{4a^2}\sin(i_0)\frac{di}{dt}\Delta t^2 \quad \textbf{(13)}$$

The results of including the corrections in Equations 12 and 13 in the propagation are shown in Figure 5. Comparison of Figure 5 with Figure 3 shows that these corrections reduce the MLTAN prediction error from 25 – 30 seconds over 1 year to less than 6 seconds over 1 year. It is seen that this model overcorrects for the MLTAN, however, the magnitude of the error is reduced.

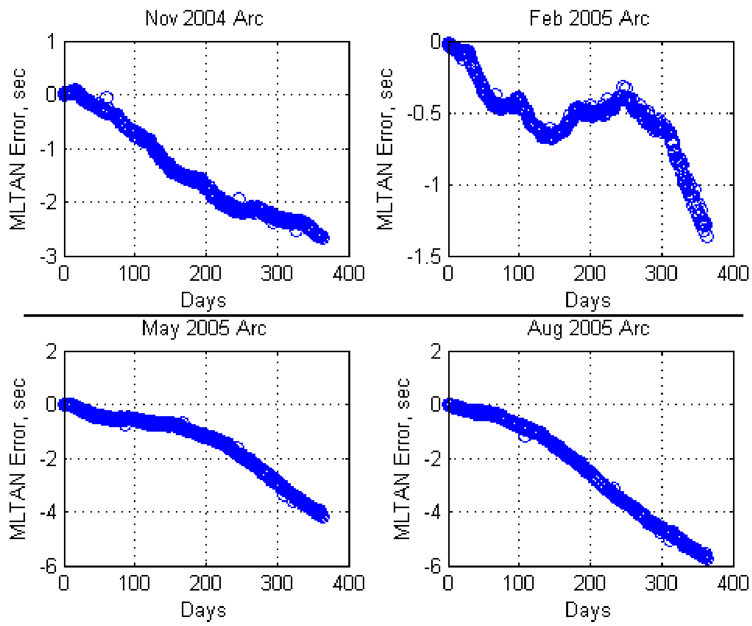

Figure 5. Improved MLTAN Predictions

IV. Linear Correction Equations

The corrections in Equations 12 and 13 can be further simplified under certain assumptions. First it is assumed that the RAAN of the disturbing body is zero. This is true for the Sun, and will be shown to be inconsequential for the Moon. Second it is assumed that the RAAN of the spacecraft and the Argument of Latitude of the disturbing body are linear functions of time:

$$\Omega(t) = \Omega_0 + \dot{\Omega}t \tag{14}$$

$$\theta_D(t) = \theta_{D0} + \dot{\theta}_D t \tag{15}$$

Using these assumptions, Equation 6 can be written as an explicit function of time:

$$\begin{aligned} \partial \overline{U}_D / \partial\Omega = 3C_D\{&0.25\sin^2 i[-\sin(2\Omega_0 + 2\dot{\Omega}t)\sin^2 i_D \\ &+ \underline{\cos(2\theta_{D0}+2\dot{\theta}_D t)\sin(2\Omega_0 + 2\dot{\Omega}t)}(-1-\cos^2 i_D) \\ &+ 2\underline{\sin(2\theta_{D0}+2\dot{\theta}_D t)\cos(2\Omega_0 + 2\dot{\Omega}t)}\cos i_D] \\ &+ 0.5\cos i\sin i\sin i_D[\sin(2\theta_{D0}+2\dot{\theta}_D t)\cos(\Omega_0 + \dot{\Omega}t) \\ &+ \cos i_D(\sin(\Omega_0 + \dot{\Omega}t) - \cos(2\theta_{D0}+2\dot{\theta}_D t)\sin(\Omega_0 + \dot{\Omega}t))]\} \end{aligned} \tag{16}$$

Using Equation 16 along with Equation 5 allows us to write the inclination rate as an explicit function of time, which could then be integrated to find the inclination as a function of time. Integration of the majority of terms in Equation 16 will result in periodic terms. However, when the RAAN rate, $\dot{\Omega}$, and the Argument of Latitude rate, $\dot{\theta}_D$, are in resonance, integration of the underlined terms in Equation 16 will result in linear terms. The Sun-synchronous orbit flown by Aqua is designed to create such a resonance. Therefore, it is reasonable to assume that the inclination drift is driven primarily by the resonance between these two terms. Keeping only the resonant terms and substituting in the Obliquity of the Ecliptic for i_D the correction equation can be simplified to

$$\delta i(t) = \frac{3\sin i_0 C_{sun}}{8na^2\sqrt{1-e^2}}(\sin(2\theta_{D0} - 2\Omega_0)(\cos 23.5 + 1)^2)t \quad \textbf{(17)}$$

$$\delta RAAN = -\frac{9\sin^2 i_0 C_{sun} J_2 R_E^2}{32a^4}\sin(2\theta_{D0} - 2\Omega_0)(\cos 23.5 + 1)^2 t^2 \quad \textbf{(18)}$$

Equations 17 and 18 reflect corrections to the inclination and RAAN of the spacecraft orbit due solid Earth tide created by the Sun. The Moon is not considered because no resonant frequencies exist between the motion of the Moon and the Sun-synchronous orbit. The MLTAN propagation error when using the corrections given by Equation 17 and 18 are shown in Figure 6. While the error in MLTAN prediction error can still be on the order of seven seconds over a one year prediction as in the May 2007 arc, this is still a significant improvement over the uncorrected 25-30 second error observed in Figure 3.

The significant improvement provided by these correction equations to long-term MLTAN prediction is observed in Figure 7. The definitive, predicted, and corrected MLTAN predictions show that the corrected equations provide a significantly more accurate prediction. The nearly 100 day error in predicting MLTAN violations has been eliminated allowing accurate planning of future inclination maneuvers.

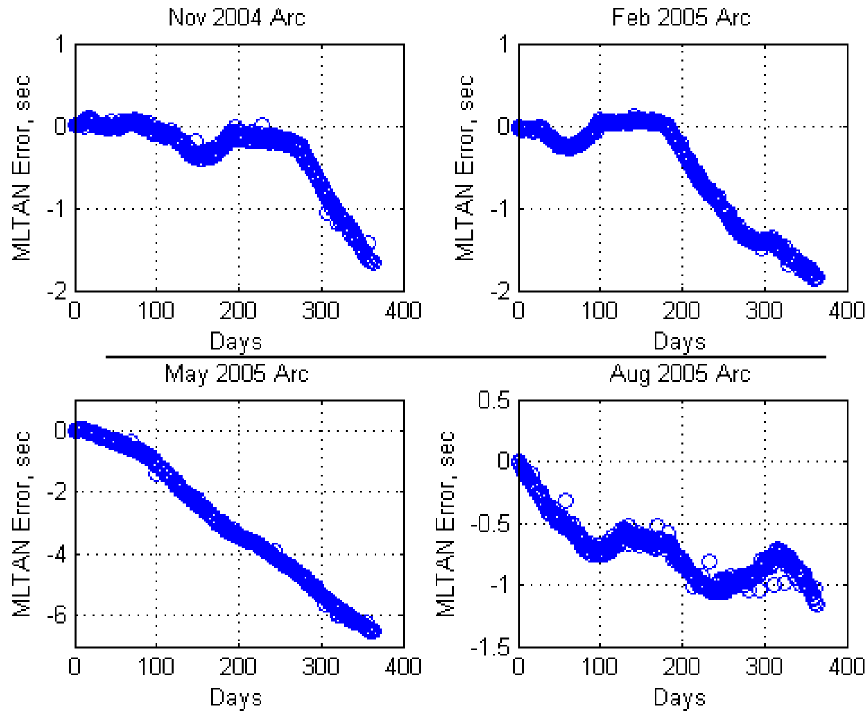

Figure 6. Improved MLTAN Prediction Using Linear Correction Equations

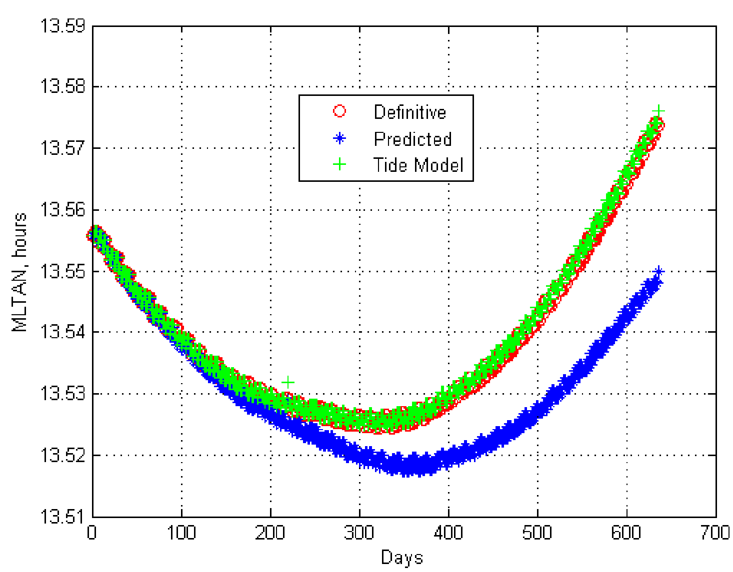

Figure 7. Definitive, Predicted, and Corrected Predicted MLTAN Predictions

Additionally, the solid-Earth tide model may explain the dependence of the prediction error on the date of the beginning of the arc. A simplified explanation of how a disturbing body creates a solid Earth tidal force on the spacecraft orbit is shown in Figure 8. The disturbing bodies' gravitational force pulls on the solid Earth creating a lumped redistribution of the planet's mass. This redistribution creates a new component of the gravitational force F. It is obvious from the graphic that the amount of the perturbing force that contributes to the spacecraft orbital inclination change is dependent on the angle alpha, which is the minimum angle between the spacecraft orbit plane and the vector from the center of the Earth to the disturbing body.

The Sun-synchronous orbit flown by the Aqua spacecraft has an inclination of 98.2 degrees. This orbit will have yearly minimum and maximum alpha angles due to the north-south transition of the Sun over the year. The minimum angle, and therefore minimum amount of disturbing force contributing to inclination change occurs at the summer solstice when the Sun is in the northern hemisphere due to the orbit's 98.2 degree inclination. This explains the reduction in inclination error rate observed in the inclination error plots of Figure 4. Additional analysis is needed to fully understand the seasonal dependence of the prediction errors.

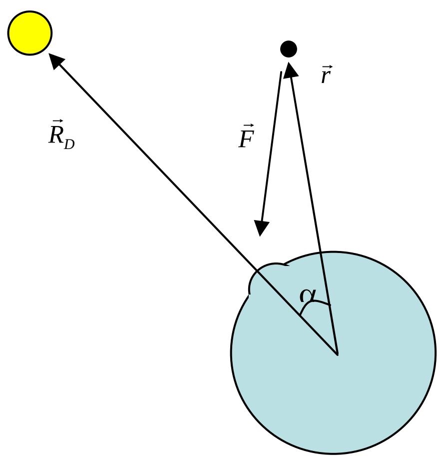

Figure 8. Simplified Explanation of Solid Earth Tidal Forces on a Spacecraft

V. Conclusion

The long-term prediction errors in Mean Local Time of the Ascending Node for the Aqua spacecraft were investigated. The MLTAN prediction error using the normal EOS FDS operational force models was on the order of 90 seconds over a two year propagation period. These errors led to increased complexity when attempting to coordinate future inclination maneuvers with the EOS Afternoon Constellation. Analysis has determined that the lack of a solid Earth tide model in the EOS FDS is the primary source of error in the current force models. The Love Model for the solid Earth tide potential was investigated and it was shown that the resonance between the Sun and spacecraft orbital plane created by a Sun-synchronous orbit is the primary driver of the observed inclination error. The inclination error then drives the MLTAN error through the J2 perturbation on the RAAN. Analytical correction equations for the inclination and RAAN were derived, simplifying assumptions were made, and were demonstrated to significantly reduce the error in the MLTAN error for Aqua. These corrections can now be used to provide increased accuracy in long range MLTAN predictions which will simplify the planning and timing of future coordinated inclination maneuver campaigns for the EOS Afternoon Constellation.

References

1 “Error Models for Solid Earth and Ocean Tidal Effects in Satellite Systems Analysis,” Wolf Research and Development Corporation, Contract No. NAS 5-11735-Mod 57, July 1972