ANALYSIS OF CAPTURE TRAJECTORIES TO THE VICINITY OF LIBRATION POINTS

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Spacecraft capture trajectories to the periodic orbits of the L1 and L2 points in the restricted Hill three-body problem are studied. The specific focus is on transfer to these vicinities from interplanetary trajectories. This application is motivated by future proposals to place "Deep Space ports" at the Earth and Mars L1 or L2 points. These spaceports are considered as candidate gateways for interplanetary transfers in the future. We utilize stable manifolds for capture trajectories to periodic orbits around the libration points. As a result, the cost of capture into a periodic orbit is also reduced relative to direct capture into a parabolic orbit. The way of linking between interplanetary transfer trajectories and the stable manifold is also discussed.

INTRODUCTION

There has been great interest in the libration points of the circular restricted 3-body problem (CR3BP), which are located where the gravity of the first and second massive bodies and centrifugal force are balanced. In particular, the position of L1 and L2, which lie on the line connecting the two masses, can be considered equivalent to the boundary of the gravity field of the smaller mass of the two. Since an object around these points can maintain the same orientation with respect to the two masses, transfers between the primary body and the libration point have been investigated extensively in the past[1-6]. In fact, several astronomical satellites have already utilized such Halo/Lissajous orbits around the L1 and L2 points of the Sun-Earth system[7]. Moreover, a transfer to the interior and exterior region of the Earth is relatively simple by addition of energy to a spacecraft in the vicinity of the L1 and L2 points. Therefore, these points are also considered as candidate gateways for interplanetary transfers in the future[8-13]. Recently, the analysis and design of transfer orbits using invariant manifolds associated to periodic orbits around the libration points have been a topic of study[14-20].

Furthermore, if a spaceport is also built around the L1 or L2 points of a target celestial body, the fuel required for landing or for entry orbit at the target celestial body can be supplied there to the spacecraft arriving from interplanetary space, or we can transfer cargo there to another spacecraft which is used exclusively for landing or for an entry orbit. In this way, constructing a spaceport in the vicinity of L1 or L2 of the Sun-Earth and Sun-Target celestial body system can separate the transportation system into three regions: transfer inside the gravity field of the Earth, transfer inside the gravity field of a target celestial body and the interplanetary transfer phase (see Fig. 1). Moreover, this system facilitates round-trip exploration using spaceports as relay points and leads to a reusable transportation system[8, 11]. In the past, the escape trajectories from the libration points of Sun-Earth system also have been examined[21-23]. However,

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capture trajectories from an interplanetary trajectory to the vicinity of the L1 and L2 points of a target celestial body are not fully understood. Thus, spacecraft capture trajectories to the vicinity of the L1 and L2 points in the planar restricted Hill three-body problem are analyzed in this paper.

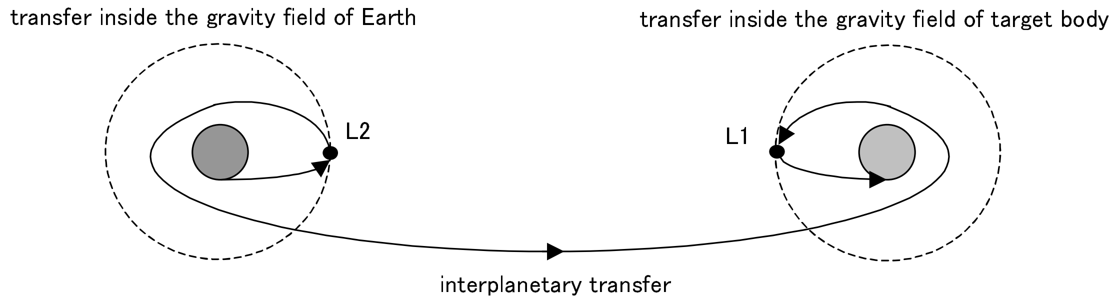


Figure 1: Vision of an Interplanetary Transfer in the Future

HILL THREE-BODY MODEL

The physical system considered is the restricted Hill three-body model. This model is a limiting case of the circular restricted three-body problem (CR3BP) and describes the dynamics of a massless particle attracted by two point masses revolving around each other in a circular obit (see Fig. 2). In fact, the Hill model can be obtained from the CR3BP by setting the center of the coordinate system to be at the secondary body and then assuming that the distance of the satellite from the center is small compared to the distance between the target body and the sun. The resulting equations of motion provide a good description for the motion of a spacecraft in the vicinity of the L1 and L2 libration points of the smaller body[24].

Equations of Motion

The equations of motion for spacecraft in this planar Hill model are given by[25, 26,]

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = -\frac{\mu}{r^3} x \tag{1}$$

$$\ddot{y} + 2\omega\dot{x} = -\frac{\mu}{r^3} y \tag{2}$$

where $r = \sqrt{x^2 + y^2}$ is the distance from the center of second smaller body to a spacecraft, ω is the angular velocity of the secondary body (m2) about the primary body (m1), and μ is the gravitational parameter of the secondary body.

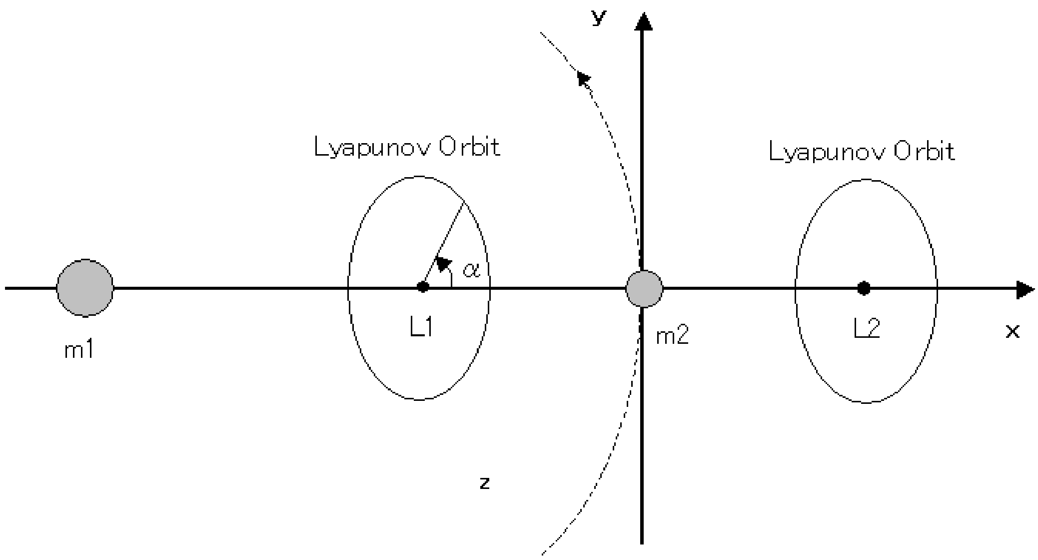


Figure 2: Geometry of the planar Restricted Hill Three-Body Model

Libration Points

In the circular restricted Three-body problem model there are five points where the gravity of the first and second massive bodies and centrifugal force acting on S/C are balanced, which are called libration points. In the restricted Hill Three-body problem only the L1 and L2 libration points exist, but they are symmetric about the origin with coordinates $x = \pm(\mu/3\omega^2)^{1/3}$, $y = 0$.

Jacobi Integral

Equations (1-2) have an integral of motion similar to the CR3BP. The following equation denotes the *Jacobi integral*, which is a conservative quantity determined from the initial conditions,

$$J = \frac{1}{2}v^2 - \frac{\mu}{r} + \frac{3x^2}{2}\omega \tag{3}$$

where $v = \sqrt{\dot{x}^2 + \dot{y}^2}$ is the velocity of the particle in the rotating frame. This constant has a deep influence on the dynamics of motion. The condition $v^2 \geq 0$ in Eq. 3 impose a restriction on the allowable position for the motion at any given value of J. Setting $v = 0$ defines the zero-velocity surface, which sets a physical boundary of the allowable motion (Fig. 2). In particular, the critical value of J at L1 and L2 defines the energy at which the zero-velocity surfaces open at L1 and L2, and is expressed as:

$$J_{L1,2} = -\frac{1}{2}(9\mu\omega)^{2/3} \tag{4}$$

Normalization

Next, we normalize the above equations setting the unit length and the unit time as follows:

$$l = \left(\frac{\mu}{\omega}\right)^{1/3} \text{ and } \tau = \frac{1}{\omega} \tag{5}$$

The normalized equations of motion are then

$$\ddot{x} - 2\dot{y} - 3x = -\frac{x}{r^3} \tag{6}$$

$$\ddot{y} + 2\dot{x} = -\frac{y}{r^3} \tag{7}$$

This normalization allows us to eliminate all free parameters from the equations, thus, computations performed for them can be scaled to any physical system by multiplying by the unit length and time, which only depend on the properties of the primary and secondary bodies. Moreover, this normalization is equivalent to $\omega = 1$ and $\mu = 1$, thus, the normalized x coordinate of libration points and the normalized value of J at L1 and L2 are equal to

$$x = \pm(1/3)^{1/3} = \pm 0.693... \tag{8}$$

$$J_{L1,2} = \frac{1}{2} 9^{2/3} = 2.16337... \tag{9}$$

Table 1 gives the normalized radius for most planets of the solar system.

Table 1

NORMALIZED RADIUS OF THE SOLAR SYSTEM PLANET

Planet	Mass (× 10^23 kg)	Gravitational parameter (× 10^5 km^3/s^2)	Mean motion (rad/s)	Normalized radius
Mercury	0.3302	0.220329	8.27 *10^-7	0.007663
Venus	4.869	3.248889	3.24 *10^-7	0.00415
Earth	5.9742	3.986345	1.99 *10^-7	0.002955
Mars	0.64191	0.428321	1.06 *10^-7	0.002173
Jupiter	1899	1267.127	1.68 *10^-8	0.000933
Saturn	568.8	379.5375	6.76 *10^-9	0.000641
Uranus	86.86	57.9582	2.67 *10^-9	0.000253
Neptune	102.4	68.32742	1.21 *10^-9	0.000148

Lyapunov Orbit

There exist periodic orbits near the libration points in two-dimensional space[27-31] called Lyapunov orbits, respectively, whose sizes depend on the value of the Jacobi constant (see Fig. 3). In this paper, we first analyze capture trajectories to a planar Lyapunov orbit to outline our procedure and then will consider the three-dimensional periodic orbit transfer case for future work.

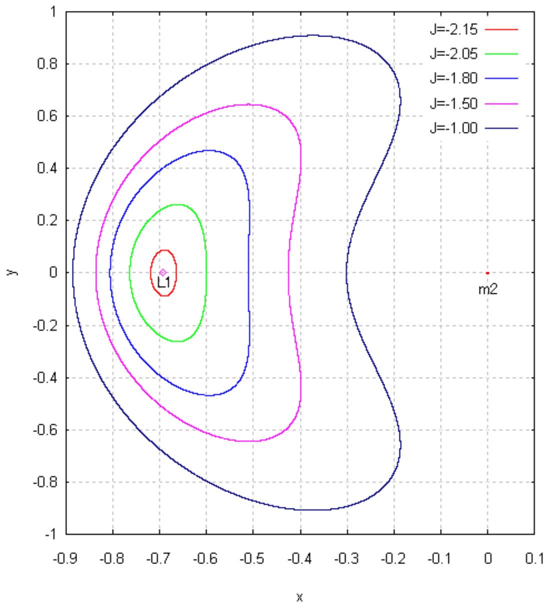


Figure 3: Lyapunov Orbits around L1

Invariant Manifolds

Above-mentioned periodic orbits, Lyapunov orbits, are not stable completely. There exist invariant structures associated with Lyapunov orbits, called unstable and stable manifolds (see Fig. 4). These are very sensitive and are affected by initial conditions. We exploit this stable manifold for capture trajectories to periodic orbits around the libration points.

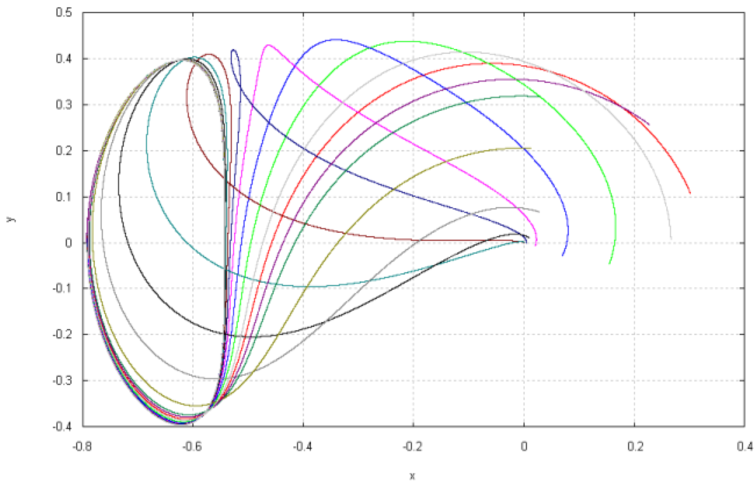


Figure 4: Stable Manifold of Lyapunov orbit

TRANSFERS BETWEEN SECONDARY BODIES AND LIBRATION POINT ORBITS

Assumption of Capture Trajectories

In this paper, we assume that capture trajectories are trajectories that enter the sphere of influence of a target body from interplanetary space and have a close flyby to the target body. At closest approach an impulsive maneuver is performed to put into the spacecraft on the stable manifold that leads to capture to a periodic orbit in the vicinity of L1 or L2 of the target body. The reason why an impulsive maneuver is performed near the surface of the target body (periapsis) is because it is the energetically most efficient place to reduce the approach energy, and may also be reduced by using an aero assist with the planetary atmosphere. After orbiting around the target body a few times, S/C is placed into the Lyapunov orbit. At this time, an infinitesimal impulsive maneuver is necessary to keep the Lyapunov orbits, but it is negligible. In this way, the stable manifolds are used for capture trajectories to Lyapunov orbits.

Definition of Periapsis Points

Periapsis points are the closest points of stable manifolds to the target body. In this study, we investigate the first four periapsis passage points of stable manifolds propageted backwards in time. Fig. 5 shows the first four periapsis passage points of one example trajectory of the stable manifold (J = -2.15).

Based on this result, Fig. 6 plots the first four periapsis point's locations of the stable manifold for J = -2.15. We can see that each periapsis points region spread out and these periapsis location depends on the size of Jacobi constant..

Fig. 7 shows the relation between the minimum periapsis distance and the value of J. The minimum periapsis distance means the distance from the origin to the periapsis point, which is closest from the origin in each periapsis points. It is found that the minimum periapsis distance decreases as the value of J increases. Moreover, each minimum periapsis distance becomes smaller than 0.007663 (which is the largest normalized planetary radius, Mercury) when the value of J is large enough. Thus the stable manifold of first four periapsis passage point can intersect the surface of any of the planets in the solar system.

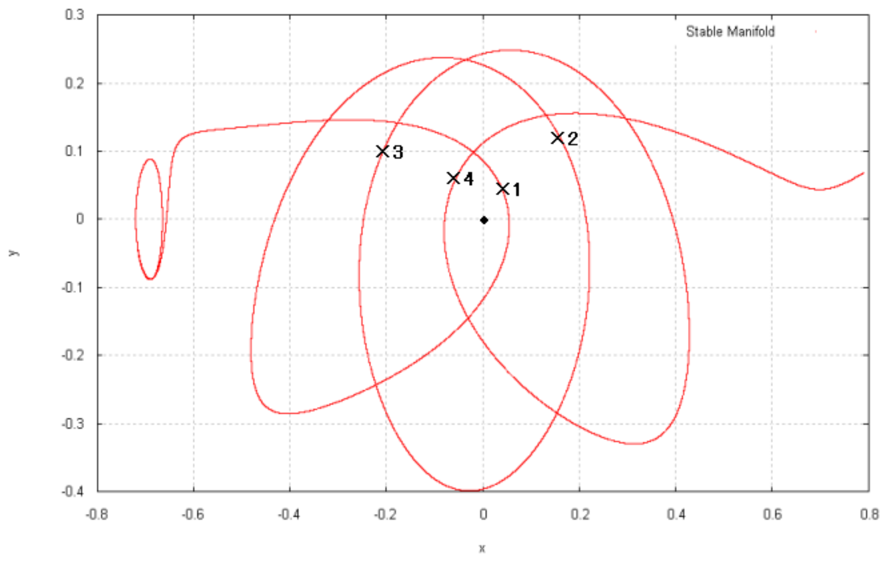
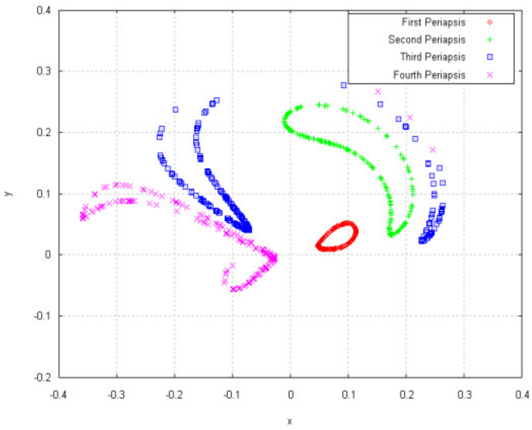
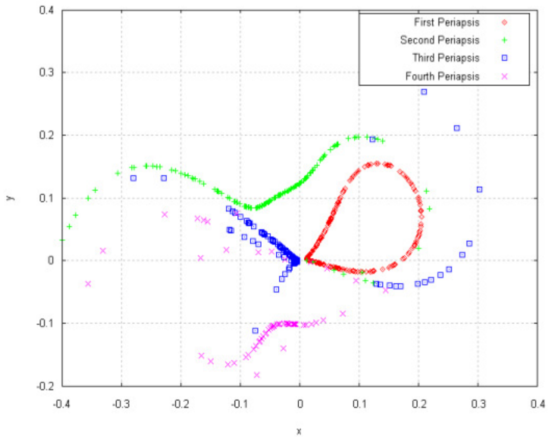


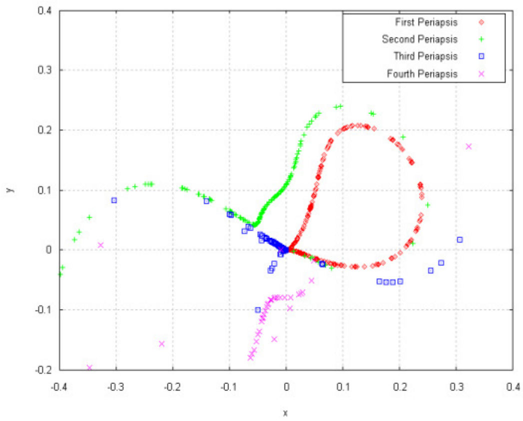
Figure 5: First Four Periapsis Passage Points of one example trajectory of the stable manifold propagated backward for J = -2.15.



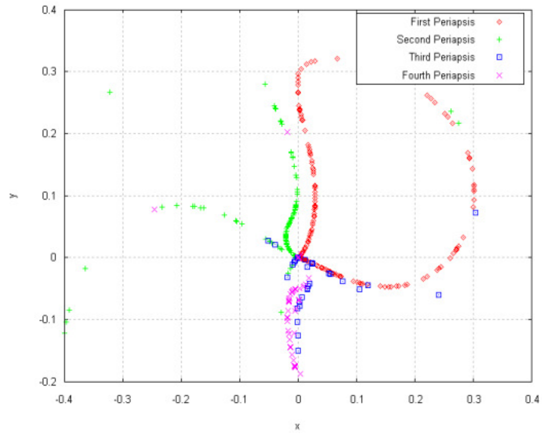
a) J = -2.15



b) J = -2.05



c) J = -1.95



d) J = -1.85

Figure 6: First Four Periapsis Location.

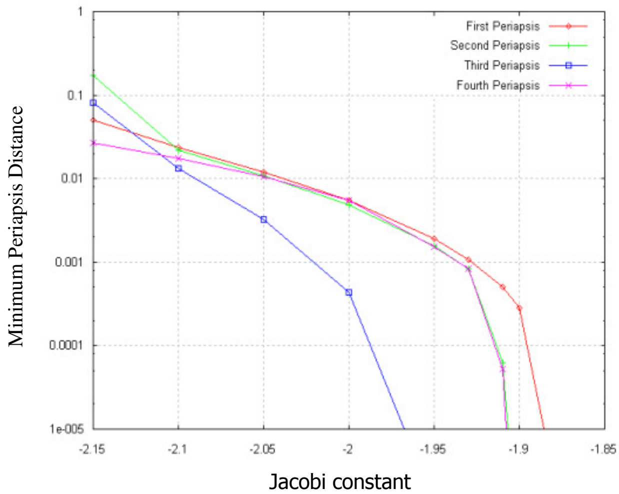


Figure 7: Minimum Distance as a Function of Size of Lyapunov Orbit (Value of Jacobi Constant)

APPLICATION TO EARTH-MARS TRANSFER

In this section, a patched conic approximation for the interplanetary transfer is applied to our study of capture trajectories to Lyapunov orbits planar dimensional cases. We focus our attention on a transfer from the Earth to Mars. However, these results can be applied to other planets of the solar system as well. We compare the cost of transfer into the Lyapunov orbit with the cost of parabolic capture to Mars, assuming the 2-body problem. At this time, a trajectory correction maneuver is assumed at the periapsis near the surface of the Mars ($h \cong 200$ km) after an interplanetary transfer.

From Fig. 7 and Table 1, the stable manifold intersects the surface of Mars for first periapsis when Jacobi constant is equal to –1.961. Fig. 8 shows that the minimum first periapsis point of stable manifold intersects the surface of Mars (J = -1.961). However, the location of periapsis after interplanetary space is determined from interplanetary transfer trajectory (see Fig. 9). In case of 200 km periapsis altitude, angle β is equal to 51.1 deg. Thus, it is impossible to link the interplanetary trajectory and the stable manifold at the first periapsis point. On the other hand, the third periapsis point of stable manifold are more likely to connect to the periapsis point after interplanetary space. Fig. 10 shows the relation between angle β of the third periapsis of stable manifold and Jacobi constant, and Fig. 11 plot the relation between them. We can see that angle β of the third periapsis becomes 51.1 deg when J is –1.898. Fig. 12 shows the stable manifold starting from the third periapsis (in backward integration) in case of J = -1.898.

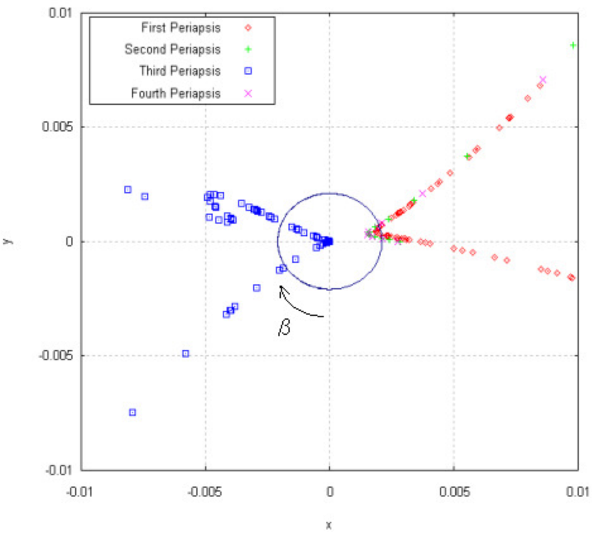


Figure 8: Periapsis Location for J = -1.961: ◯ is the Surface of Mars.

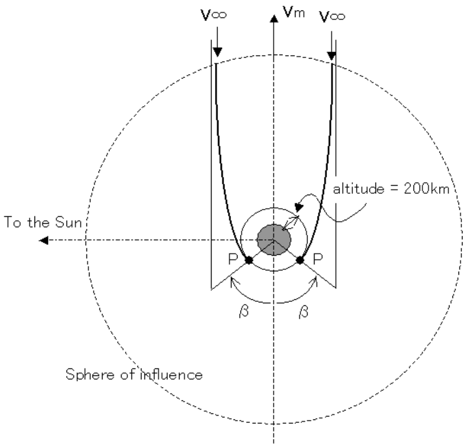
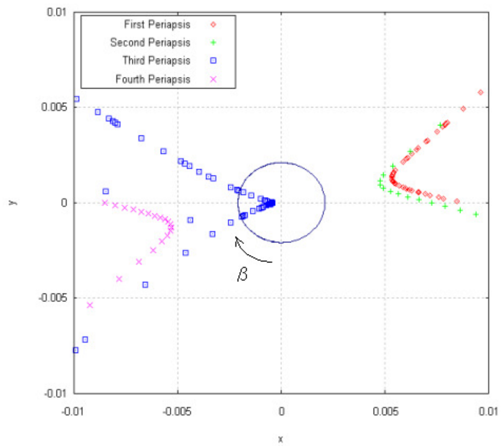
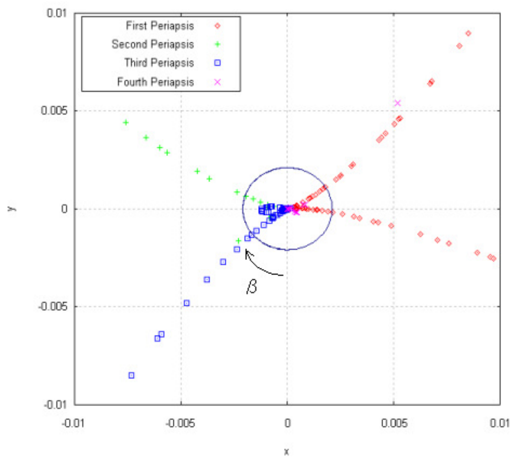


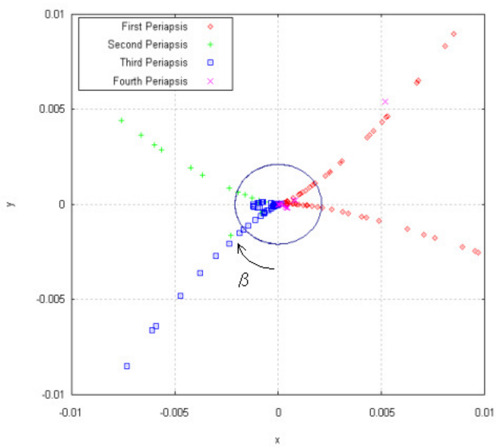
Figure 9: Periapsis Location (angle β) after Interplanetary Space for Hohmann Transfer.



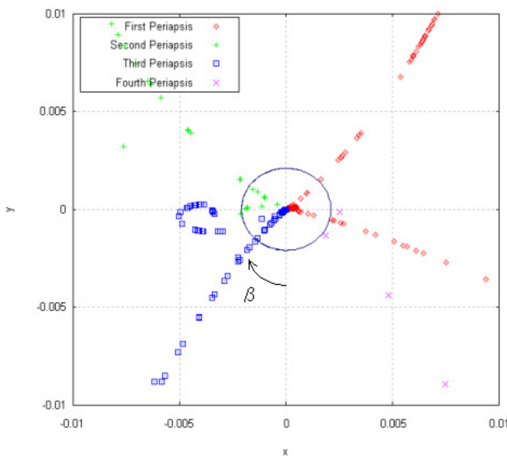
a) J = -2.00



b) J = -1.95



c) J = -1.90



d) J = -1.85

Figure 10: Relation between Angle β of Third Periapsis of Stable Manifold and Jacobi Constant.

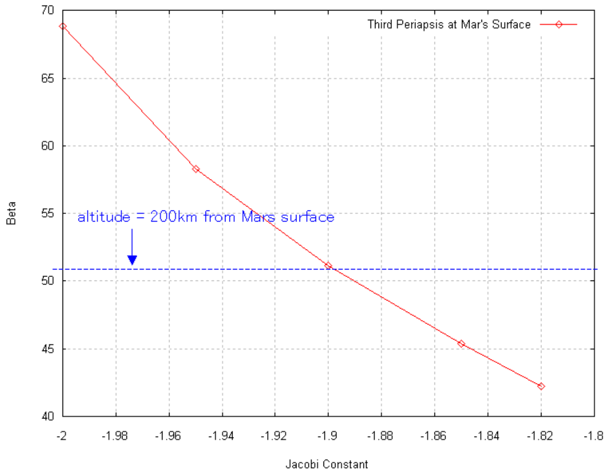


Figure 11: Angle β of Third Periapsis of Stable Manifold plotted against Jacobi Constant.

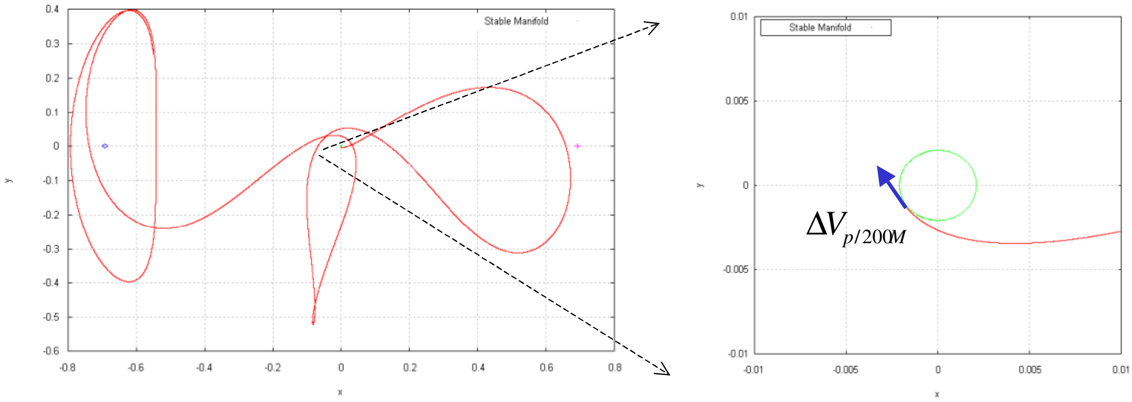


Figure 12: Stable Manifold until Third Periapsis in case of J = -1.898.

The hyperbolic arrival velocity of a spacecraft with respect to Mars is $v_{\infty/M}$ = 2.648 km/s if we assume a simple Hohmann transfer from Earth to Mars, and use the patched conic approximation for arrival at Mars. Subsequently, the velocity near the surface of Mars, $v_{p/M}$, is expressed as:

$$v_{p/M} = \sqrt{v_{\infty/M} + (2\mu_M/(r_M + 200))} \tag{12}$$

where μ_M is the gravitational parameter of Mars and r_M is the radius of Mars, leading to a periapsis velocity $v_{p/M}$ = 5.551 km/s. On the other hand, the velocity at the third periapsis passage point of stable manifold from Lyapunov orbit (J = -1.898) is 5.009 km/s. Therefore, the required $\Delta V_{p/M}$ for entry to the Lyapunov orbit with J= -1.898is 5.551 - 5.009 = 0.542 km/s. It is instructive to compare the ΔV for capture to the periodic orbit with simple 2-body capture criterion at Mars: $\Delta V_{p/M} = v_{p/M} - 2^{1/2}\, v_{c/M}$ = 0.673 km/s. Thus capture to the Lyapunov orbit allows a significant improvement even with respect to capture into a parabolic orbit, or in other words after our capture maneuver is performed the spacecraft is initially on a hyperbolic orbit, but one that transitions to a bound periodic orbit. Therefore, the required ΔV for capture to Lyapunov orbit is reduced by over 19% from a simple capture maneuver. If a spaceport is located at Sun-Mars Lyapunov orbit, the interplanetary cargo ship does not need to carry the fuel after departing from spaceport, that is, fuel for injection into Mars circular orbit ($\Delta V_{C/M}$ = 1.56 km/s).

CONCLUSIONS

This paper investigates capture trajectories to Lyapunov/orbits in the planar Hill three-body problem. We concentrate on transfer into these vicinities from interplanetary trajectories. The characteristics of the relation between the minimum periapsis distance and the size of Lyapunov orbits are obtained using the manifold trajectories from Lyapunov orbits by backward integration. Numerical results show that the manifold trajectories of the first four periapsis passage point from Lyapunov orbits can intersect the surface of any of the planets in the solar system if the value of Jacobi integral is large enough. Thus this technique can be used to reduce the cost of capture at any of the planets in the solar system.

As an example we consider transfers from an interplanetary transfer into a Lyapunov orbit near the Mars surface. It was found that the required velocity increment for could be reduced by approximately 19 % compared with the capture into a parabolic orbit although connection point between the periapsis point after interplanetary space and stable manifold trajectories.

Extension to three-dimension (capture to Halo orbit) and analysis of aero assist using planetary atmosphere are future work.

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