Optimization of Return Trajectories for Orbital Transfer Vehicle between Earth and Moon

Yusuke Shibasaki*,
* University of Tokyo

Ryu Funase**, Yuichi Tsuda** and Jun'ichiro Kawaguchi**
** The Institute of Space and Astronautical Science,
Japan Aerospace Exploration Agency (ISAS/JAXA)

Abstract

In this paper, optimum trajectories in Earth Transfer Orbit (ETO) for a lunar transportation system are proposed. This paper aims at improving the payload ratio of the reusable orbital transfer vehicle (OTV), which transports the payload from Low Earth Orbit (LEO) to Lunar Low Orbit (LLO) and returns to LEO. In ETO, we discuss ballistic flight using chemical propulsion, multi-impulse flight using electrical propulsion, and aero-assisted flight using aero-brake. The feasibility of the OTV is considered.

1. Nomenclature

a	= semimajor axis [km]
a_{dr}	= radius direction component of thrust [km/s^2]
$a_{d\theta}$	= argument direction component of thrust [km/s^2]
a_{dh}	= normal direction of the orbit plane component of thrust [km/s^2]
a_T	= thrust acceleration magnitude [km/s^2]
e	= eccentricity [-]
f	= true anomaly [rad]
h	= altitude [km]
I_{sp}	= specific impulse [sec]
l	= mean longitude [rad]
M	= mean anomaly [rad]
n	= mean motion [rad/s]
p	= semi-latus rectum [km]
r	= orbital radius [km]
r_a	= altitude of apogee [km]
r_p	= altitude of perigee [km]
Q_{in}	= heating rate on stagnation point [W/m^2]
Q_{out}	= radiation energy [W/m^2]
R_e	= radius of the Earth [km]
R_N	= radius of curvature of nose [m]
$T_{\max.}$	= temperature of stagnation point [K]
t_f	= total flight time [sec]
t_p	= orbital period [sec]
w	= argument of periapsis [rad]
ε	= emissivity [-]
φ	= in-plane thrust steering angle [rad]
ρ	= density of atmosphere [kg/m^3]
σ	= coefficient of Stefan-Boltzmann [W/(m^{2}K^4)]

2. Introduction

The moon is recognized as an important destination for space science and exploration. To explore the moon efficiently, it is conceived that the special spacecraft is used in each segment such as LEO, Lunar/Earth Transfer Orbit (LTO/ETO), and LLO. For example, the OTV is the transportation spacecraft used in LTO/ETO. This paper aims at improving the payload ratio of the reusable OTV. The reusable OTV system is roughly composed of the payload module, the propulsion module, and the fuel cartridge. The payload module (included man) is separated from the OTV in LTO and is reached LLO by its single injection. The other modules (not included man) are returned to LEO through ETO.

In this research, we focus on the optimization problem of return trajectories with small fuel consumption, because the return flight in ETO is not constrained the flight time and the safety compared to the flight in LTO. In ETO, we discuss ballistic flight using chemical propulsion multi-impulse flight using electrical propulsion, and aero-assisted flight using aero-brake. The feasibility of the OTV is considered.

Fig.1 Image of Orbital Transfer Vehicle

3. Lunar Free Return Trajectory

A lunar free return trajectory is used a return reference orbit in ETO. The lunar free return trajectories have been studied and used since the Apollo era[1,2]. The trajectories have many beneficial characteristics for the OTV concept. For example, one characteristic is that the spacecraft is automatically returned to the Earth by only a single injection maneuver at the Earth departure and no further large delta-V is required except for small adjustment maneuver thereafter. Thus, the trajectories are very easy to operating the system such as the payload separation and very useful for saving the fuel consumption included the total operation.

The reference orbit in ETO is calculated using the patched-conic approximation.

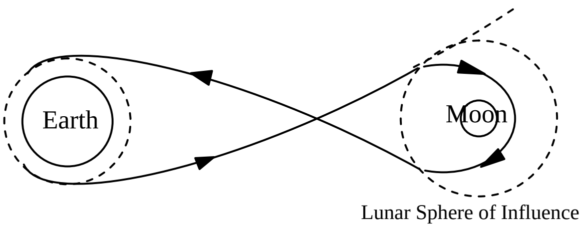

Fig.2 Lunar Free Return Trajectory

4. Optimum Trajectories using Low-Thrust in ETO

The optimum trajectories are designed from the periapsis in the elliptic orbit after the lunar flyby (Fig. 3). If the flight time is not constrained, the optimum control on the minimum fuel consumption is only the continuous injection near the periapsis. It is difficult to apply the optimum control the OTV system, because the flight time is very long. So, it is assumed that the OTV in ETO have the fuel to an extent and can always keep the thrust constant. It is also assumed that the OTV mass is constant, because the fuel consumption caused by low-thrust propulsion is very small compared to the initial mass. Then, the optimization problem on the minimum fuel consumption is equal to the optimization problem on the minimum flight time.

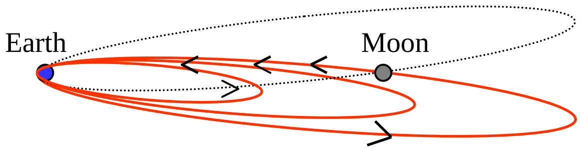

Fig.3 Section for Optimum Trajectory Design

4.1 Configuration

Now, the optimum control problem is to find the control history, u(t), which minimizes the following objective function J. Firstly, we consider the two-body problem in the two dimensions(in-plane) to simplify the problem.

$$J = \int_0^{t_f} dt \tag{1}$$

subject to the equation of motion in polar coordinate system (Gauss's form of the Lagrange's planetary equations)[2]

$$\begin{cases} \dfrac{da}{dt} = \dfrac{2a^2}{h}\left(esinfa_{dr} + \dfrac{p}{r} a_{d\theta} \right) \\ \dfrac{de}{dt} = \dfrac{1}{h}\left\{ psinfa_{dr} + \left[(p+r)cosf + re \right] a_{d\theta} \right\} \\ \dfrac{dl}{dt} = n - \dfrac{ae}{h(a+b)}\left[pcosfa_{dr} - (p+r)sinfa_{d\theta} \right] - \dfrac{2br}{ah} a_{dr} \end{cases} \tag{2}$$

$$\left(p = a\left(1 - e^2\right),\ r = \frac{p}{1 + ecosf},\ h = nab \right)$$

where is the mean longitude defined the following.

$$l = \Omega + w + M \tag{3}$$

In the case of the in-plane motion, $l = M$.

the control constraints

$$\begin{cases} a_{dr} = a_T sin\varphi \\ a_{d\theta} = a_T cos\varphi \end{cases} \quad \left(a_T = 10^{-7}\, km/s\right) \tag{4}$$

where the steering angle φ is defined in Fig.4.

the boundary conditions

$$\begin{cases} a(0) = a_0 = 273024 \\ e(0) = e_0 = 0.9759 \\ l(0) = l_0 = 0 \end{cases} \tag{5}$$

$$\begin{pmatrix} r_{a_0} \approx 85R_e \\ r_{p_0} = R_e + 200 \end{pmatrix}$$

$$\begin{cases} a(t_f) = a_{t_f} = 6778 \\ e(t_f) = e_{t_f} = 0 \\ l(t_f) = l_{t_f} = 2\pi \end{cases} \tag{6}$$

$$\left(r_{a_{t_f}} = r_{p_{t_f}} = R_e + 200\right)$$

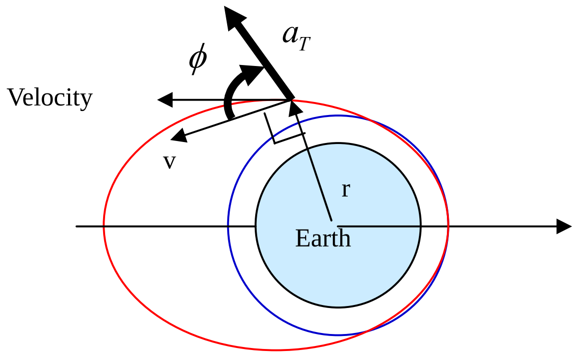

Fig.4 Definition of angle of thrust

The trajectory optimizes in the orbital period, and each of the optimum trajectories in the period is connected. Then, it is considered that the connected trajectory is the optimum trajectory in ETO.

4.2 Method of Optimization (DCNLP)

There are a lot of numerical methods for trajectory optimization [3]. In this paper, we approximate state variables by cubic polynomials, interpolate control variables linearly and use collocation to satisfy the differential equations. This method is called Direct Collocation with Nonlinear Programming (DCNLP).

This is method which converts the original optimum control problem into Nonlinear Programming (NLP) by discretizing the originally time continuous functions, and solves this NLP [4, 5]. Compared to the method based on calculus of variation, dealing with the variables directly, this method can be easily applied to various optimum control problems. Especially for the problems with state inequality constraints this is probably the most powerful numerical solver, since the method based on calculus of variation needs much effort to solve this kind of problem.

On the other hand, this method needs a lot of memory and long calculation time. But in this point the remarkable development of computers in these days has overcome these difficulties. The nonlinear problem is solved by Sequential Quadratic Programming(SQP) method.

4.3 Numerical Simulation in Quasi Circular Orbit

Firstly, we discuss the quasi circular orbit near the target circular orbit. The transfer time is on the order of hundreds of days, which results in thousands of orbital revolutions. So, it is difficult to optimize the trajectory in total flight time simultaneously. The trajectory should be optimized in the orbital period using DCNLP. The numerical condition is the following,

the objective function

$$J = \int_0^{t_p} dt \tag{7}$$

the equation of motion (2)
the control constraints (4)
the boundary conditions

$$\begin{cases} a(0) = a_0 = (R_e + 200) + \dfrac{r_{a_0} - r_{p_0}}{2} = 6678 \\ e(0) = e_0 = \dfrac{r_{a_0} - r_{p_0}}{2a_0} = 0.014975 \\ l(0) = l_0 = 0 \end{cases} \tag{8}$$

$$\left(r_{a_0} - r_{p_0} = 200\right)$$

$$\begin{cases} a(t_p) = a_{t_p} = free \\ e(t_p) = e_{t_p} = free \\ l(t_p) = l_{t_p} = 2\pi \end{cases} \tag{9}$$

These equations are normalized by using proper characteristic length L, characteristic time T, and so on. The results are shown in Fig.5 and Fig.6.

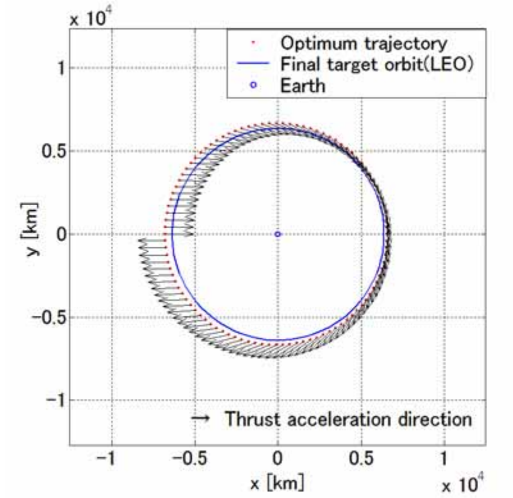

Fig.5 Optimum trajectory in quasi circular orbit (Inertial coordinate system)

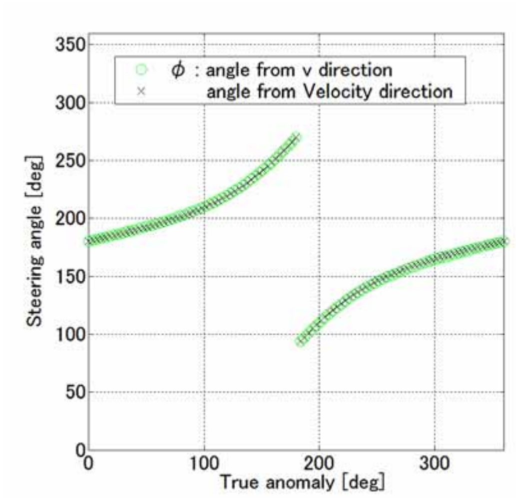

Fig.6 Optimum thrust steering angle profile

4.4 Numerical Simulation in Elliptic Orbit

Next, we discuss the elliptic orbit. The trajectory should be also optimized in the orbital period using DCNLP. The numerical condition is the following,

the objective function	(7)
the equation of motion	(2)
the control constraints	(4)
the boundary conditions	

$$\begin{cases} a(0) = a_0 = (R_e + 200) + \dfrac{r_{a_0} - r_{p_0}}{2} = 16578 \\ e(0) = e_0 = \dfrac{r_{a_0} - r_{p_0}}{2a_0} = 0.60321 \\ l(0) = l_0 = 0 \end{cases} \tag{10}$$

$$\left(r_{a_0} - r_{p_0} = 20000\right)$$

$$\begin{cases} a(t_p) = a_{t_p} = free \\ e(t_p) = e_{t_p} = free \\ l(t_p) = l_{t_p} = 2\pi \end{cases} \tag{9}$$

These equations are normalized by using proper characteristic length L, characteristic time T, and so on. The results are shown in Fig.7 and Fig.8.

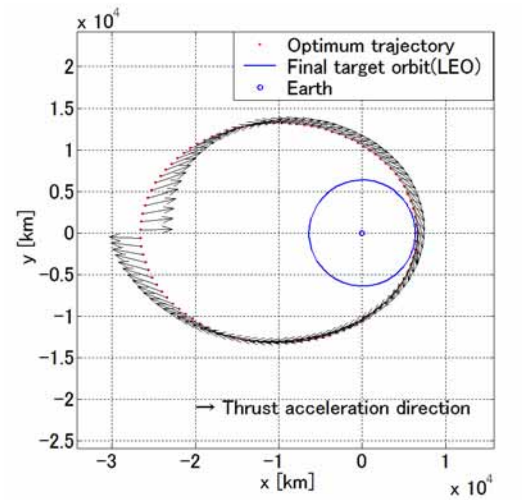

Fig.7 Optimum trajectory in elliptic circular orbit (Inertial coordinate system)

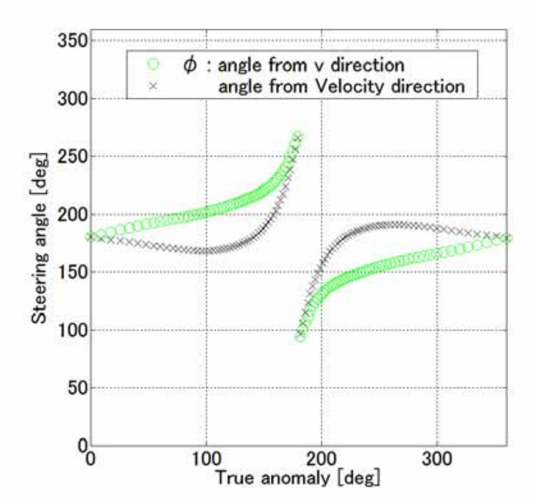

Fig.8 Optimum thrust steering angle profile

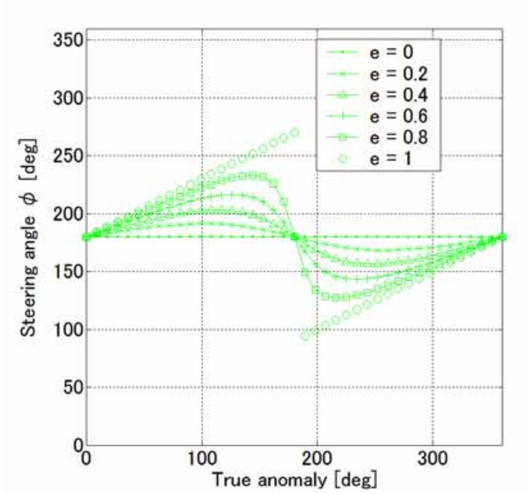

Fig.9 Optimum steering law for minimum $\dot{a}$

4.5 Analytical Optimum Steering Laws (In-plane motion)

In two dimensions, the semimajor axis and the eccentricity must be changed to reach the target LEO. To minimize the total flight time for the orbital transfer and meet the boundary conditions of the desired final orbit, the optimum thrust direction time history must be computed.

Firstly, the control law to minimize the time rate of change of the semimajor axis is considered, and is derived from the governing equation of motion for a.

$$\frac{da}{dt} = \frac{2a^2}{h}\left(esinfa_T sin\varphi + \frac{p}{r}a_T cos\varphi\right) \tag{11}$$

The optimum in-plane steering angle that maximizes the time rate of change of a is found by steering the partial derivative $\partial\dot{a}/\partial\varphi$ equal to zero.

$$\frac{\partial\dot{a}}{\partial\varphi} = \frac{2a^2}{h}\left(esinfa_T cos\varphi - \frac{p}{r}a_T sin\varphi\right) = 0 \tag{12}$$

Therefore, the steering law for minimum $\dot{a}$ is

$$\varphi_a = tan^{-1}\frac{esinf}{ecosf + 1} \tag{13}$$

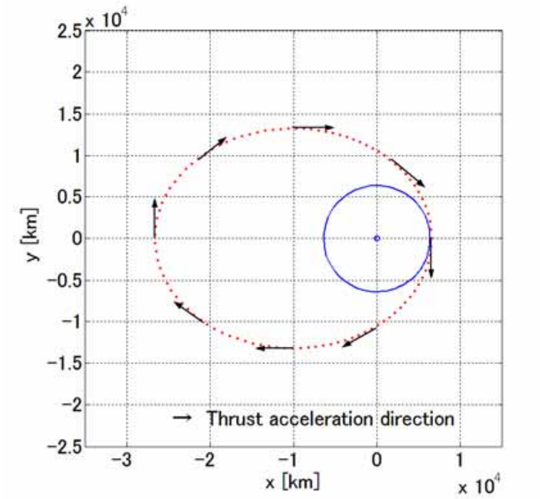

Fig.10 Optimum steering law for minimum $\dot{a}$ (Inertial coordinate system)

Next, the control law to minimize the time rate of change of the eccentricity is considered, and is derived from the governing equation of motion for e.

$$\frac{de}{dt} = \frac{1}{h}\left(psinfa_T sin\varphi + \left[(p+r)cosf + re\right]a_T cos\varphi\right) \tag{14}$$

Again, the partial derivative is set to zero. Therefore, the steering law for minimum $\dot{e}$ is

$$\varphi_e = tan^{-1}\frac{psinf}{(p+r)cosf + re} \tag{15}$$

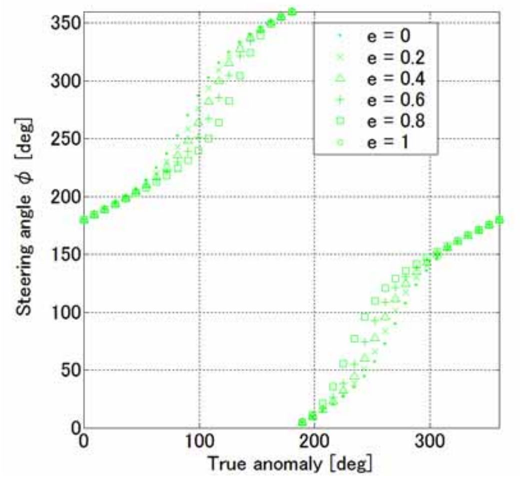

Fig.11 Optimum steering law for minimum $\dot{e}$
$\left(a = R_e + 200\right)$

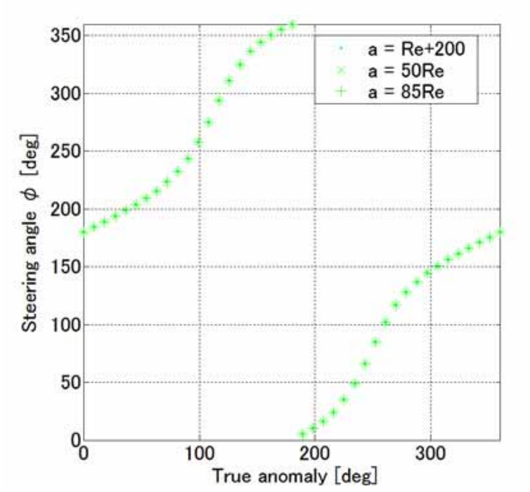

Fig.12 Optimum steering law for minimum $\dot{e}$
$\left(e = 0.5\right)$

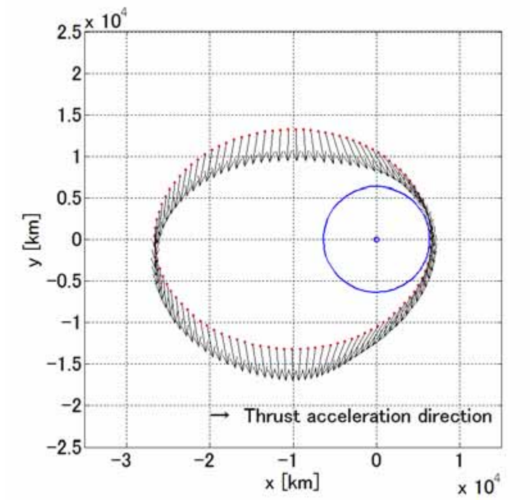

Fig.13 Optimum steering law for minimum $\dot{e}$
(Inertial coordinate system)
$\left(a = 16578, e = 0.60321\right)$

It is found that the steering law for minimum $\dot{e}$ depends on the eccentricity and doesn't depend on the semimajor axis from Fig.11 and Fig.12.

Finally, the control law to minimize the time rate of change of the height of apogee is considered, and is derived from the governing equation of motion for r_a.

$$\frac{dr_a}{dt} = (1+e)\frac{da}{dt} + a\frac{de}{dt} \tag{16}$$

Again, the partial derivative is set to zero. Therefore, the steering law for minimum $\dot{r}_a$ is

$$\varphi_{\dot{r}_a} = \tan^{-1}\frac{r\sin f\left\{2ae(1+e)+p\right\}}{r(p+r)\cos f + 2a(1+e)p + r^2 e} \tag{17}$$

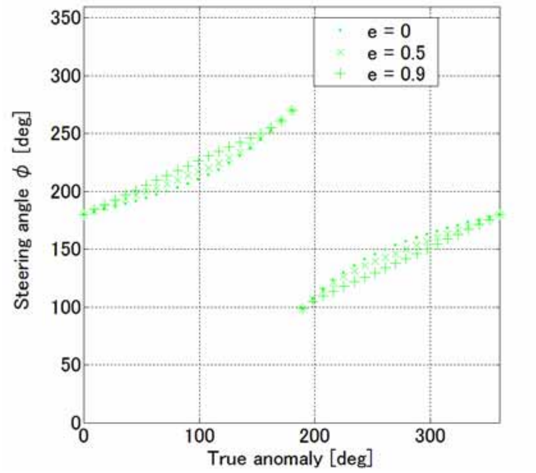

Fig.14 Optimum steering law for minimum $\dot{r}_a$

$\left(a = R_e + 200\right)$

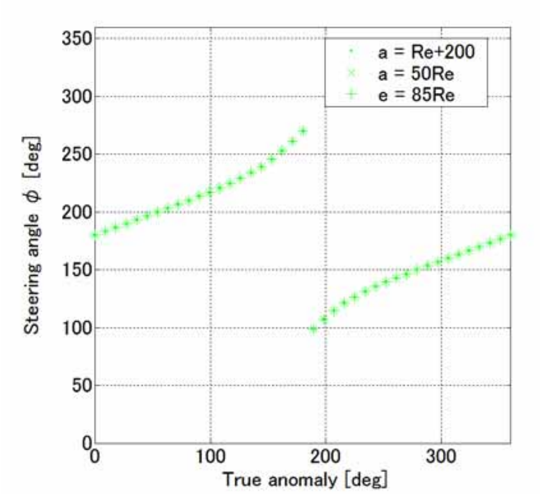

Fig.15 Optimum steering law for minimum $\dot{r}_a$

$\left(e = 0.5\right)$

It is found that the steering law for minimum $\dot{r}_a$ also depends on the eccentricity and doesn't depend on the semimajor axis from Fig.14 and Fig.15.

To determine if each equation (Eq.(13), (15), and (17)) minimizes or maximizes the value ($\dot{a}$, $\dot{e}$, or $\dot{r}_a$) is implicitly used the following boundary conditions.

$$\varphi_a\left(f=0\right) = \varphi_e\left(f=0\right) = \varphi_{r_a}\left(f=0\right) = \pi \qquad (18)$$

4.6 Analytical Optimum Steering Laws (Out--of-plane motion)

The control law to minimize the time of rate of change of the inclination is considered, and is derived from the governing equation of motion for i.

$$\frac{di}{dt} = \frac{r\cos(w+f)}{h} a_{dh} \qquad (18)$$

The partial derivative is set to zero. Therefore, the steering law for minimum i is

$$\varphi_i = \begin{cases} \pi/2 & \left(if\ \cos\left(w+f\right) > 0\right) \\ 3\pi/2 & \left(if\ \cos\left(w+f\right) < 0\right) \end{cases} \qquad (19)$$

When the angle $w+f$ is near the $\pm 90\,\text{deg}$, any out-of-plane steering is essentially wasted.

5. System Design for OTV

This paper aims at improving the payload ratio of the reusable OTV. The optimization problem is the minimization of the fuel consumption. In the case of the constant thrust, the optimization problem is equal to the minimization of the total flight time. To minimize the end time for orbital transfer, the optimum thrust direction time history must be considered. The optimum thrust direction for the minimization of the total flight time is investigated. In section 4.5, the three optimum steering laws for each parameter are obtained.

In our approach, each steering law is selected in the stage of the return trajectory for the OTV, because the operation of the OTV system should be simple.

Firstly, the initial value problem is computed for 60 days by using each optimum steering law, and the each result is compared. The state equations are numerically integrated by using a standard fixed step (1sec), fourth-order Runge-Kutta routine. The results are shown in Table.1 and Fig.16-19.

Table.1 Comparison among results of each steering law (after 60 days)

Steering law	$a[km]$	e	$r_a[km]$	$r_p[km]$
φ_a	35760	0.8227	65180	6340
φ_e	35860	0.8204	65280	6440
φ_{ra}	35770	0.8210	65140	6410

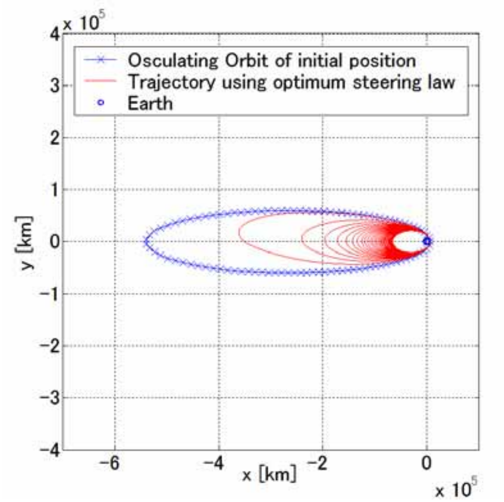

Fig.16 Trajectory using steering law for 60 days

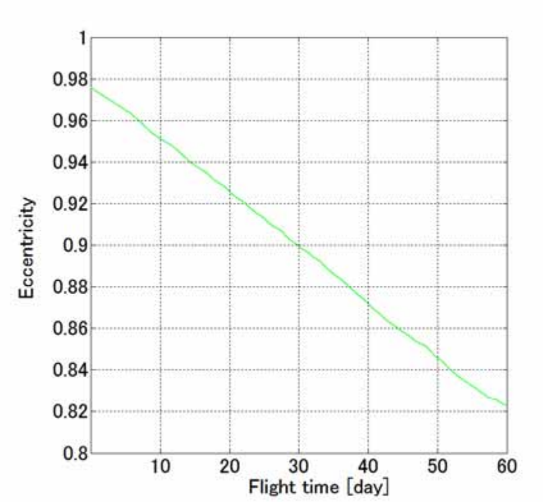

Fig.17 Change of eccentricity using steering law for 60 days

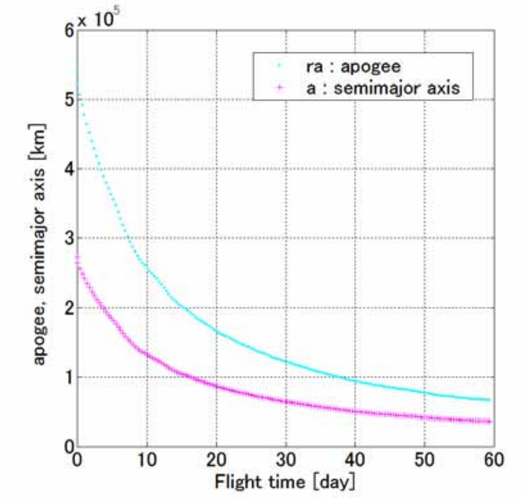

Fig.18 Change of apogee and semimajor axis using steering law for 60 days

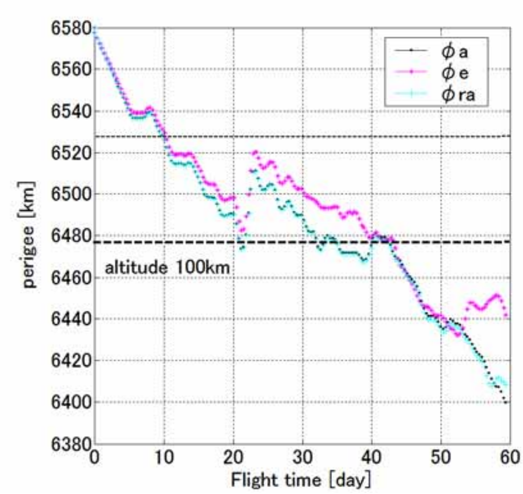

Fig.19 Change of perigee using steering law for 60 days

Then, in the case of the constant altitude of the perigee, the minimization of total flight time is equal to the minimization of the altitude of the apogee. Now, the altitude of the perigee is not constant, and it is difficult to decide that which steering law is the best. However, when the altitude of the perigee is nearly approximated constant, the optimum steering law is the law for minimum $\dot{a}$ or for minimum $\dot{r}_a$. There is not much difference between the minimum $\dot{a}$ law and the minimum $\dot{r}_a$ laws, but the optimum steering law in the first stage is selected the minimum $\dot{a}$ law, because it is easy to operate from the Fig. 10 in Section 4.5. However, the change of the perigee should be constrained, because the heating rate of the OTV caused by atmosphere is increased. It is assumed that the OTV system can't control the lift/drag ratio. Then, the operation in the next stage is to raise the altitude of the perigee. Now, when the altitude of the perigee is about 100~150km, the operation for the perigee up is done. For example, when the altitude of the perigee is smaller than 100 km, the thrust acceleration direction is controlled the velocity direction in the vicinity of the apogee.

Next, when the eccentricity or the semimajor axis is small, the optimum steering law is selected the minimum $\dot{r}_a$ law, because optimum steering angle profiles in the numerical simulation results of the case is similar to the minimum $\dot{r}_a$ law from the Fig.6, Fig.8, Fig.14, Fig.15.

Finally, when the semimajor axis is equal to the target orbit, the optimum steering law is selected the minimum $\dot{e}$ law.

We compare the low-thrust using the optimum thrust control strategy with the ballistic flight using the chemical propulsion.

Table.2 Comparison low-thrust with ballistic flight for OTV

Flight	$t_f[days]$	$\Delta m[kg](m_i = 5000kg)$
Ballistic	-	3250
Low-thrust (optimum)	410	1070

When the target inclination is different from the initial inclination, the operation to change the inclination must be done. It is considered that the operation is done after the arrival of the target orbit (LEO) or these steering laws are blended so that the orbital elements are controlled simultaneously, In our approach, the operation is done in the vicinity of the apogee in the first stage, because it is simple and easy to do the next operation.

6. Aero-assist (Atmospheric drag)

The effect of the atmospheric drag should be considered, because the altitude of the perigee is low. Firstly, the aero-brake is considered. When the lift/drag ratio can be changed, some optimum trajectories are considered [7,8,9]. However, it is assumed that the OTV system can't control the lift/drag ratio and the air drag acts only in a direction opposite to the velocity vector, because it is simple and easy to operate. It is assumed that the ballistic coefficient is constant 0.02m²/kg. The acceleration of the air drag is nearly equal to the electric propulsion in the altitude of 200km.

Next, the effect of the aerodynamic heating is considered. The top of the OTV is generally reached the maximum temperature. The heating rate on the stagnation point is calculated using the following equation[10].

$$Q_{in} = \frac{1.825 \times 10^{-4}}{\sqrt{R_N}} \sqrt{\rho(h)} V^3 \qquad (20)$$

The loss of the thermal energy is given by the following equation called the Stefan-Boltzmann law of radiation.

$$Q_{out} = \varepsilon \sigma T_{max.}{}^4$$
$$\left(\sigma = 5.67 \times 10^{-8}\right) \qquad (21)$$

The $T_{\max.}$ is calculated using $Q_{in} = Q_{out}$. The velocity of the OTV is the maximum at the perigee.

$$T_{max.} \le \left(\frac{1.825 \times 10^{-4} \Big/ \sqrt{R_N} \; \rho V_{max.}{}^3}{\varepsilon \sigma} \right)^{\frac{1}{4}} \qquad (22)$$

The model used for air density is a 1976 Standard Atmosphere. It is assumed that the value of emmisivity is 0.65 and the radius of curvature of the nose is 2m. The maximum velocity in the return trajectory is 10.9406 km/s at the initial perigee. Then, the maximum temperature of the OTV is 1361K at a height of 100km. It is possible to make a flight at a height of about 100km, because the allowable temperature limit of tempered glass is about 1800K.

7. Conclusion

The optimum return trajectory of the OTV is proposed. In this research, it is assumed that it is not included man in the reusable modules of the OTV. Then, we focus on the optimization problem with small fuel consumption. The control strategy is mainly divided into the three stages. The steering law is selected the minimum $\dot{a}$

law in the first stage, the minimum $\dot{r}_a$ law in the second stage, and the minimum $\dot{e}$ law in the final stage. When the perigee must be raised or the inclination must be changed, the steering law is selected the perigee-up law or the minimum $\dot{i}$ law. The problem, which is when it should be switched the minimum $\dot{a}$ law to the minimum $\dot{r}_a$ law, is left. It this investigation, it is switched the minimum $\dot{a}$ law to the minimum $\dot{r}_a$ law in the altitude of about 50,000 km.

The feasibility of the reusable OTV with solar electric propulsion is considered. The fuel consumption is greatly decreased using the low-thrust (Hall thruster). The weight of the OTV is increased by equipping solar electric propulsion with, however the increase of mass is a few hundreds kg. So, the improvement of the payload ratio is the remarkable. The flight time is also acceptable.

References

[1] V.A.Chobotov, "Orbital mechanics", AIAA Education Series, pp.317-325, 1991.

[2] Richard H. Battin, "An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition", AIAA Educational Series, pp 437-508, 1999

[3] John T. Betts, "Survey of Numerical Methods for Trajectories Optimization", *Journal of Spacecraft and Rockets*, Vol.21, No.2, pp. 193-207, 1998

[4] C.R. Hargarves and S.W. Paris, "Direct Trajectory Optimization Using Nonlinear Programming and Collocation", *Journal of Guidance, Control, and Dynamics*, Vol.10, No.4, pp. 338-342, 1987

[5] Paul J. Enright and Bruce A. Conway,"Optimal Finite-Thrust Spacecraft Trajectories Using Collocation and Nonlinear Programming", *Journal of Guidance, Control, and Dynamics*, Vol.14, No.5, pp. 981-985, 1991

[6] C.A.Kluever and S.R.Oleson, "Direct Approach for Computing Near-Optimal Low-Thrust Earth-Orbit Transfers", *Journal of Spacecraft and Rockets*, Vol.35, No.4, pp. 509-515, 1998

[7] K.D.Mease and N.X.Vinh, “Minimum-Fuel Aeroassited Coplanar Orbit Transfer Using Lift Modulation”, AIAA 83-2094, 1983

[8] J.A.Kechichian, M.I.Cruze, and E.A.Rinderle, “Optimization and Closed-Loop Guidance of Drag-Modulated Aeroassited Orbital Transfer”, AIAA 83-2093, 1983

[9] A.Miele, V.K.Basapur, and K.D.Mease, “Nearly-Grazing Optimal Trajectories for Aeroassited Orbital Transfer”, *Journal of Astronautical Science,* Vol.34, No.1, 1986

[10] R.C.Wingrove, “An Analysis Minimum Heat Trajectory for Entry at Hypersonic Speeds”, *Journal of Spacecraft and Rockets*, Vol.4, No.8, 1967