

# ORBIT DETERMINATION SYSTEM FOR LOW EARTH ORBIT SATELLITES

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## Abstract

The IAI/MBT Precise Orbit Determination system for Low Earth Orbit satellites is presented. The system is based on GPS pseudorange and carrier phase measurements and implements the Reduced Dynamics method. The GPS measurements model, the dynamic model, and the least squares orbit determination are discussed. Results are shown for data from the CHAMP satellite and for simulated data from the ROKAR GPS receiver. In both cases the one sigma 3D position and velocity accuracy is about 0.2 m and 0.5 mm/sec respectively.

## Introduction

Satellite tracking and orbit determination are essential elements of most satellite missions. Knowledge of the spacecraft position at any time is a requirement for communication, mission planning and for geolocation purposes.

Several Orbit Determination systems for Low Earth Orbit satellites have been implemented in IAI/MBT. Both, on-board and ground station systems are used. The on-board system is based on the ROKAR GPS receiver. The ground segment orbit determination is based mostly on GPS samples, with a backup mode, which uses the tracking antenna measurements in case GPS signals are not available.

The GPS code or GRAPHIC (code + phase) measurements form the basis for several types of integrated solutions for satellite orbit determination, using either dynamic or kinematic models (or a combination of the two). The existence of independent undetermined parameters requires an overall solution using batch post-processing or real-time processing techniques. The batch solutions use least-squares methods, while the real-time solutions apply Kalman filters.

The dynamic approach is to use force and satellite models in order to compute the satellite's acceleration. The satellite's position as a function of time is then computed by numerical integration. This result is compared with the orbit predicted by the GPS measurements. In the batch least-squares solution, the independent force parameters are chosen so as to minimize the differences between the predicted trajectory and the actual measurements. In Yunck's "Kinematic Orbit Determination" [1], a Kalman filter is used to apply geometric corrections to the dynamic trajectory as a result of the GPS measurements. In Yunck's "Reduced Dynamic Orbit Determination" [1], these corrections are both geometric and dynamic in nature.

Montenbruck has proposed a “Kinematic” solution [2], which uses no dynamic model at all (as opposed to Yunck’s kinematic solution). It merely computes a least squares solution for all the locations and the biases relative to the GPS predictions. High accuracy solutions are obtained, however, kinematic methods are vulnerable and sensitive to bad measurements or bad geometry.

Montenbruck et al. have proposed a “Reduced Dynamic” solution [3], which involves estimation of empirical accelerations on top of a precise deterministic force model. It is composed of the dynamic models and the purely kinematic solution and combines the best of both worlds. Not only the accuracy of GPS measurements may be fully exploited but it also has a high robustness offered by dynamical orbit determination techniques.

In this paper we describe the Precise Orbit Determination system for Low Earth Orbit Satellites that has been implemented in IAI/MBT for future missions. The concept is based on the Reduced Dynamics approach. In the following, we discuss the GPS measurements model, the dynamic model, and the least squares orbit determination. We show results for data from the CHAMP satellite and for simulated data from the ROKAR GPS receiver.

## Precise Orbit Determination

### *ROKAR GPS receiver*

The real-time navigation position and velocity accuracy provided by the ROKAR GPS receiver is about 5-10 meters and 2-3 cm/sec respectively. Since this accuracy may be inadequate for geolocation implementations and for on-board orbit prediction, a real time orbital filter is implemented in the receiver. The orbital filter implements an Extended Kalman Filter (EKF) algorithm, which generates the refined estimates on the basis of the 3D fixed PVT solution supplied by the GPS receiver and the orbit dynamic equations. The orbital filter reduces the velocity error to 1 cm/sec and in the absence of sufficient visibility conditions (e.g. when the antenna is obscured) the GPS receiver uses the orbital filter to generate the extrapolation estimate of the orbit for aided navigation (i.e. faster reacquisition).

The ROKAR GPS receiver provides L1 C/A code and carrier phase tracking on 12 channels with accuracies of 0.8 meters and 1 mm respectively. The code measurement is composed of the true distance between a GPS satellite and the receiver antenna, clock offsets of both, the receiver and the GPS satellites, the ionospheric path delay, and the receiver noise of 0.8 meters. The carrier-phase measurements have much lower noise of 1 mm, but contain an unknown bias, which must be estimated as part of the orbit determination process. This bias is different for each observed GPS Satellite but constant between epochs during uninterrupted carrier-phase tracking. These measurements are downlinked and processed offline by Least Squares Fit (LSF) for precise orbit determination.

## ***GPS data and GRAPHIC method***

The main error sources of space navigation based on a single frequency GPS receiver are: ionospheric range delay, inaccurate GPS ephemeris and clock data, and receiver noise.

The ionospheric range delay is the most significant error source even when a ionospheric model is used in the post processing of GPS data. Therefore an alternative approach must be implemented to reduce this error source. As a result of the ionospheric layer characteristic, its effect on the carrier-phase measurements and on the code measurements is equal in magnitude but opposite in sign. By using the arithmetic mean of code and carrier measurements, the ionospheric path delays can be fully eliminated. This measurement called GRAPHIC (Group And Phase Ionospheric Calibration) [1], exhibits a noise level of about half the pseudorange code noise, i.e., 0.4 m for the ROKAR GPS receiver.

Inaccuracy in the GPS satellites orbit is the second error source that has to be handled. For post processing GPS navigation implementations, the GPS satellites ephemerides are known with high precision and are used for precise orbit determination. The Center for Orbit Determination in Europe (CODE) provides special GPS ephemeris products [4]. The GPS orbit data is available in the standard SP3 format on a 15 min grid with typical position error of 5 cm. By using 9<sup>th</sup> order Lagrange interpolation, intermediate positions of similar accuracy can be calculated.

The Center for Orbit Determination in Europe (CODE) also provides high rate (30 sec) GPS satellites clock drifts in a standard CLK file [4]. By using such a high rate clock product the range modeling error is less than 1 cm.

Updated differential code biases for the GPS satellites (DCB file) and the relative position of the GPS satellites antenna from the center of mass (Antex file) are also used.

## ***Dynamical model***

The additional use of orbit knowledge from the equations of motion may substantially improve the orbit determination accuracy. By using the fundamentals of Newtonian mechanics, given an initial position and velocity vector, the satellite's orbit can be computed at arbitrary times by performing a double integration of the satellite accelerations over time. This computation is called orbit propagation and is composed of two main procedures that work consecutively.

The first one, the acceleration model, calculates the satellite instantaneous acceleration as a function of time, position, and velocity. The acceleration model has to describe the forces that act on the satellite faithfully because it affects significantly the prediction accuracy. The force model has to include the main forces that act on LEO satellites: earth gravity, atmospheric drag, and luni-solar gravity. Smaller forces like dynamic solid tide, solar radiation pressure, and albedo can be considered, however, their contribution is negligible and in many cases the inaccuracy in the main forces models is bigger than all these small forces together. The acceleration model in this work includes EGM96 of degree and order 70 as the Earth gravity model [5], CIRA72 as the atmospheric density model [6], Moon and Sun gravity with low precision Solar and

Lunar coordinates [7]. The dependence of the cross section area on satellite attitude is taken into account in the drag model.

The other procedure is the numerical integration of the instantaneous acceleration for the solution of the equation of motion. The differential equation can be handled by common integration methods but second sum methods are the most suitable for orbit calculation. Therefore, Gauss-Jackson [8, 9] of order 5 with step size of 30 seconds is used in this work. There is no need to use a more accurate method because the inaccuracies, which result from the acceleration model, are much bigger than those from the numerical integrator.

Orbit propagation of 10 hours for a LEO satellite at altitude of 400 Km above earth produces an error of 0.5 – 1 Km mainly due to inaccuracies in the force model. In order to reduce this error and to get a more reliable orbit, some parameters that characterize the forces at work might be determined as part of the estimation process. For example, a drag coefficient,  $C_D$ , and a solar radiation pressure coefficient,  $C_R$ , act as adjustable scaling factors in many orbit determination systems [1, 7]. However, even when these two empirical constants are estimated properly and the measurements have a very small error, there are errors of 2 - 5 meters in the estimated orbit.

In order to account for these deficiencies in the dynamic model, more degrees of freedom have to be enabled. Piecewise constant synthetic accelerations in the radial, tangential, and normal directions are estimated as part of the orbit determination process to compensate for faults in the acceleration model [3].

The interval size for the synthetic accelerations has to be chosen wisely: on one hand short intervals increases the number of degrees of freedom and overweigh the GPS measurements, on the other hand, long intervals may not compensate sufficiently for the lacks in the acceleration model. Intervals of 5 – 15 minutes duration have been found to be suitable, and were implemented in the MBT ground orbit determination system.

### ***Least squares orbit determination***

The notion of least squares estimation in the context of orbit determination is to find a set of model parameters for which the square of the difference between the modeled observations and the actual measurements becomes minimal. Using these parameters one can derive the position and velocity of the spacecraft at any instant within the time interval of the measurements and for some time into the future.

The unknown variables that are estimated during the orbit determination process are:

- Initial satellite position and velocity
- The amplitude of the empirical accelerations
- Receiver clock offsets at each measurement epoch
- Carrier-phase bias for each arc of continuous tracking of a single GPS satellite

The practical solution of the least squares orbit determination problem is complicated by the fact that the observation model is a highly non-linear function of the unknown variables, which makes it difficult or impossible to locate the minimum of the loss function without additional information. Therefore, calculating a priori values for the unknown variables and estimating only small corrections to these initial values, simplifies the least squares problem considerably. As this is a nonlinear problem, we

reformulate it as one of computing a linear correction to the initial guess. Strictly speaking, this is still not a linear problem, but if the nominal trajectory is sufficiently close to the true trajectory, it will be in the “linear regime”, where a linear correction is adequate. Yet, some iterations are required to cope with the non-linearity of the orbit determination problem.

Estimation of the drag coefficient,  $C_D$ , in addition to the empirical accelerations is problematic due to the coupling between them. The estimated value of  $C_D$  is highly dependent on its weight relative to the weight of the empirical accelerations in the estimator. Without strict calibration, the obtained  $C_D$  value is unreliable and meaningless. In order to avoid this problem, first, the orbit is estimated using synthetic accelerations with a constant  $C_D$  coefficient. Later on, the drag coefficient is estimated from the accurate orbit without estimating synthetic accelerations. When this approach is used, an accurate trajectory of 0.2 meters and 0.5 mm/sec, and a precise drag coefficient are obtained because the empirical accelerations and the atmospheric drag are now decoupled.

## Results

The validation of the ground orbit determination was performed in two stages. The first stage in the process was based on data from the CHAMP satellite [10]. The orbit of the CHAMP satellite is known to a very high precision due to a high quality dual frequency GPS Receiver located on the satellite and advanced orbit determination techniques. However, only single frequency data was used in the IAI/MBT orbit determination. In the second stage of tests data was generated in a simulation setup which included a Spirent hardware in the loop GPS simulator that transmitted RF signals to the ROKAR GPS receiver.

In the CHAMP based validation tests 48 orbital data arcs of GPS measurements in 2004 have been processed and analyzed. Each orbital data arc contains 10 hours of data. Inputs of the Precise Orbit Determination system were: RINEX file (L1 frequency GPS measurements every 30 seconds), precise GPS ephemerides, and high rate (30 sec) GPS satellite clock data. Input data was obtained from CODE, The Center for Orbit Determination in Europe.

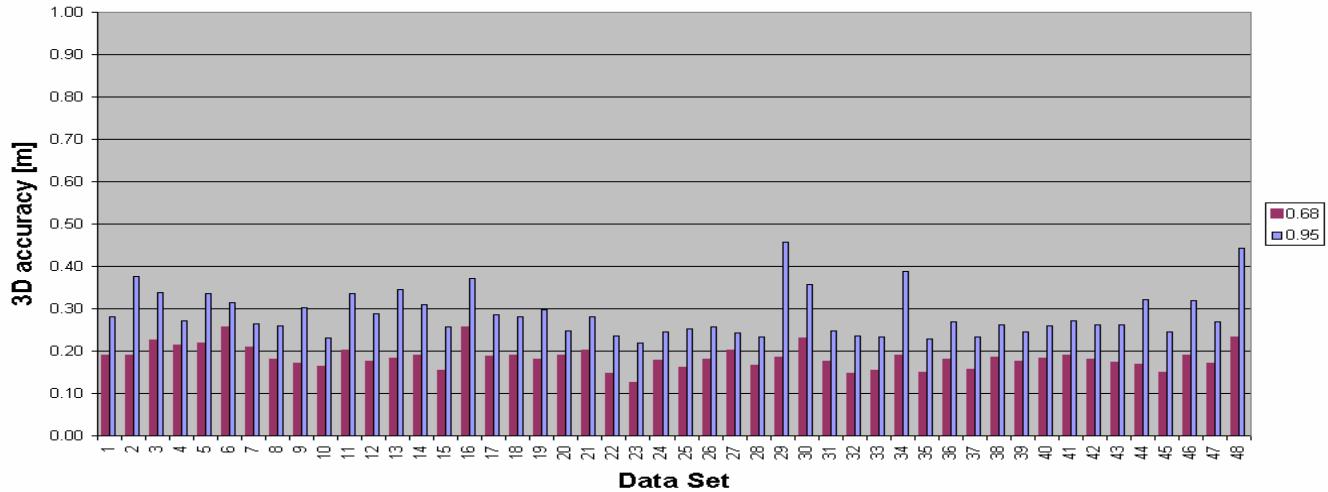
The Champ orbit was estimated from the GPS data in three cases:

- Optimal - using all the available measurements
- Realistic - GPS antenna is considered to be obscured due to the satellite cruise law and only 75% of the available GPS measurements are used
- Near real-time - doesn't incorporate clocks data (corresponds to 3 hours delay instead of 17 hours in the optimal case)

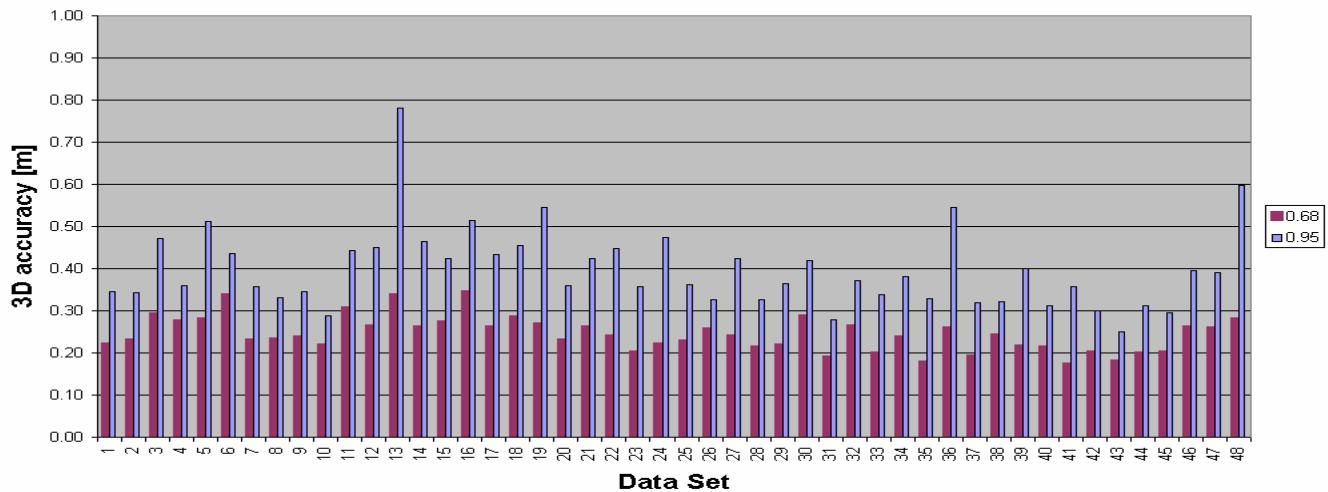
In the optimal case, a typical position accuracy of less than 20 cm 1-sigma has been achieved. Due to the robustness of the algorithm, the results of the realistic case were only slightly worse - 25 cm 1-sigma. In the near real time case a better than 40 cm, 1-sigma accuracy was achieved. In all cases the velocity accuracy was less than 0.5 mm/sec.

The results are summarized in the following graphs:

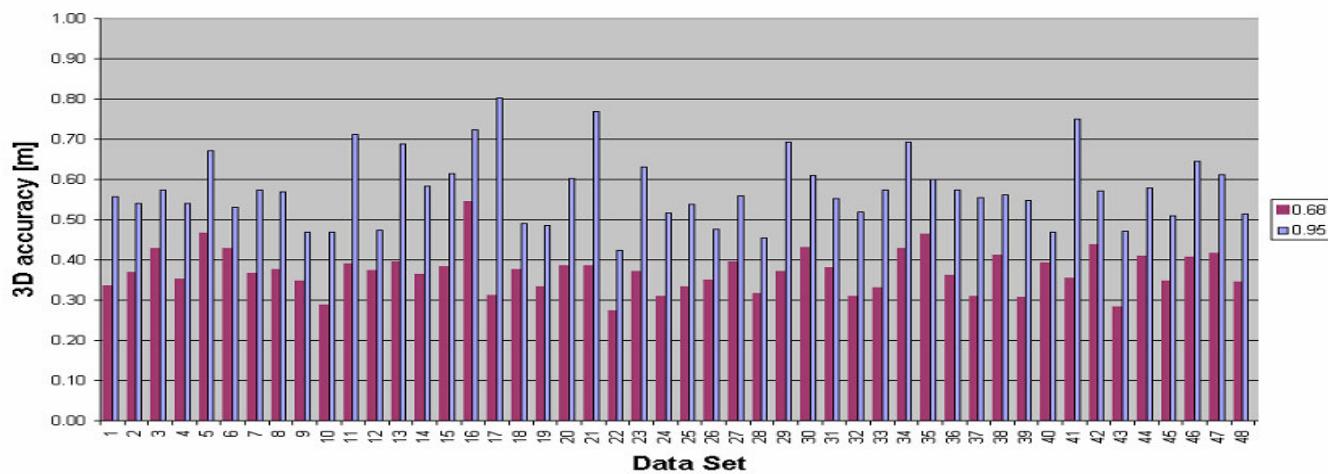
**3D Accuracy - full GPS constellation**



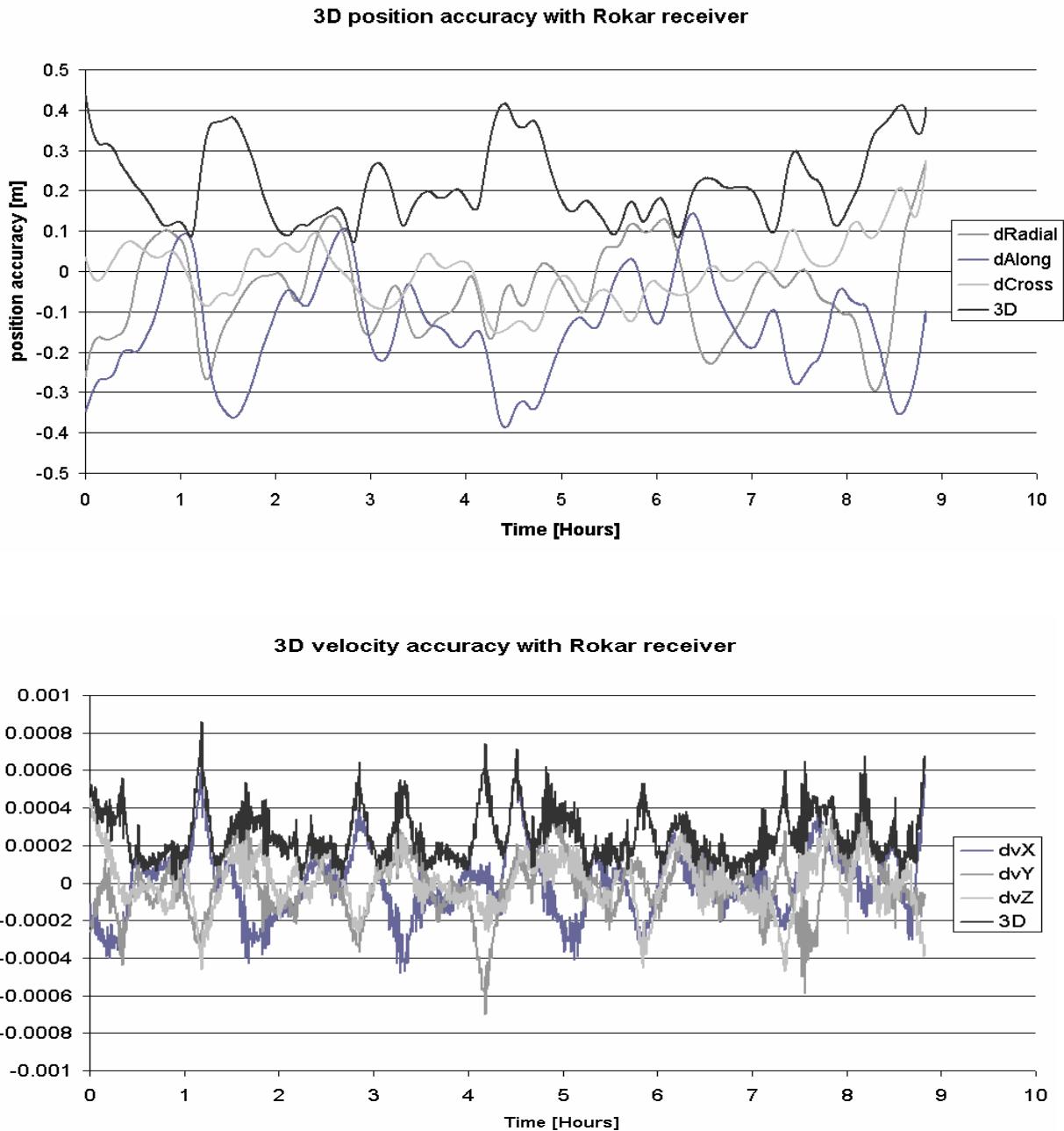
**3D Accuracy - Inferior GPS constellation**



**3D Accuracy - Near Real Time**



In the second validation phase, simulated scenarios of satellite orbit and attitude, and extreme ionospheric conditions were generated in the GPS simulator. The simulated orbit was circular with inclination of 40 degrees and with altitude of 470 Km above earth. The satellite cruise law was implemented so that the GPS antenna received only part of the GPS signals according to its attitude. The simulator generated the GPS RF signals according to the scenario and transmitted them to the antenna of the receiver. The receiver measured and processed these signals and the collected data was used later on as input to the orbit determination system. The results are summarized in the following graphs:



## **Summary**

The IAI/MBT Precise Orbit Determination system for Low Earth Orbit satellites has been presented. The performance was demonstrated on single frequency data of the CHAMP satellite and on simulated data from the ROKAR GPS receiver. The combination of GRAPHIC measurements with the dynamic model and GPS ephemeris and clock data has been shown to improve the precision of the position and velocity knowledge up to a 3D accuracy of 0.2 meters and 0.5 mm/sec respectively, which are appropriate for most satellite applications.

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