Motion parameters determination of the SC and Phobos in the project Phobos-Grunt

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Introduction

The SC “Phobos-Grunt” flight is planned to 2009 in Russia with the purpose to deliver to the Earth the soil samples of the Mars satellite Phobos. The mission will pass under the following scheme [1-4]: the SC flight from the Earth to the Mars, the SC transit on the Mars satellite orbit, the motion round the Mars on the observation orbit and on the quasi-synchronous one [5], landing on Phobos, taking of a ground and start in the direction to the Earth. The implementation of complicated dynamical operations in the Phobos vicinity is foreseen by the project. The SC will be in a disturbance sphere of gravitational fields from the Sun, the Mars and the Phobos. The SC orbit determination is carried out on a totality of trajectory measurements executed from ground tracking stations and measurements of autonomous systems onboard space vehicle relatively the Phobos. As ground measurements the radio engineering measurements of range and range rate are used. There are possible as onboard optical observations of the Phobos by a television system and ranges from the SC up to the Phobos surface by laser locator. As soon as the Phobos orbit accuracy is insufficient for a solution of a problem of landing its orbit determination will be carried out together with determination of the SC orbit. Therefore the algorithms for joint improving of initial conditions of the SC and the Phobos are necessary to determine parameters of the SC relative the Phobos motion within a single dynamical motion model. After putting on the martial satellite orbit, on the Phobos observation orbit, on the quasi-synchronous orbit in the Phobos vicinity the equipment guidance and the following process of the SC orbit determination relatively Phobos requires a priori knowledge of the Phobos orbit parameters with sufficiently high precision. These parameters should be obtained beforehand using both all modern observations and historical ones.

Motion model and equations.

There are two approaches for the motion model construction: kinematical and dynamical. These nominations were introduced by Sincler [6]. The feature of the kinematical approach is the availability of a redundant amount of parameters, updated on measurements, which will be not always agreed with the dynamics laws. The kinematical approach assumes creation of the analytical theory any way taking into account the various disturbance factors. For example, in early Sincler’s works [7,8] the Sun perturbations on a long time span was calculated using undisturbed Kepler motion of Mars. In the Jacobson work [9] the Mars orbit elements were approximated by polynomials of the second order. The kinematical approach is realized also in works [10,11]. In this sense the Chapront-Touze theory [12,13] is most advanced: representing Phobos coordinates as Fourier serieses with arguments as linear combinations of constants, it takes into account practically all disturbance factors, including influence of a Phobos gravitational field on its progressive movement round Mars.

The number of the solve-for parameters can be significant (in work [9] makes 14). A step to the dynamical approach was made in [14].

In the present work the dynamical approach is used in which a computational model is based on the material body motion equations. The necessity of such approach is stipulated by the following circumstances: the joint improvement of the SC motion parameters and the Phobos ones should be processed within the single frame of a high-precision motion model. As soon as for the SC motion description is required a numerical integration of the motion equations, the similar model should be used for Phobos.

The model takes into account: perturbations from the Mars gravity field represented as expansion of a potential on surface harmonics up to the 8x8 degrees, influences of the Sun, planets and Deimos, the Phobos secular acceleration, uncentrality of the Phobos gravity field (harmonics C20, C21, S21)

The planet ephemeredes are calculated with using of the JPL DE405 planet motion theory results [15]. The Deimos coordinates are calculated from the Chapront-Touze ESADE theory [13] which are accessible from the site [16] of L'INSTITUT DE MÉCANIQUE CÉLESTE ET DE CALCUL DES ÉPHÉMÉRIDES. The meanings of coefficients of the Mars potential expansion are accepted in the correspondence with work [17]. The orientations of Mars and Phobos are determined under the International Astronomical Union (IAU) recommendations [18]. The spherical harmonic coefficients of the Phobos gravity field expansion are taken from work [19], where they are determined under the Phobos mass homogeneity assumption.

The Phobos improving parameters are the orbit elements $\bar{E}_{Ph} = \{a, \varphi_1, \varphi_2, i, \Omega, \tau_\Omega\}$, where a semimajor axis, $\varphi_1 = e\sin\omega$, $\varphi_1 = e\cos\omega$ (ω - the longitude of pericentre), i - the orbit inclination to equator of the inertial system J2000, Ω -the longitude of an ascending node, τ_Ω - moment of the ascending node passage, the gravitational constant of Phobos μ, and its secular acceleration ν, referred to the initial epoch. The motion is described by the Lagrange equations for the Phobos orbit elements and ones of the SC.

$$\frac{d\bar{E}_{Ph}}{dt} = F_{Ph}(t, \bar{E}_{Ph}, \mu)$$

There are used measurements of Phobos made by the SC onboard facilities and for theirs calculation it is necessary to know the SC trajectory. The SC motion calculation is carried out by the similar way for elements $\bar{E}_{SC}$

$$\frac{d\bar{E}_{SC}}{dt} = F_{SC}(t, \bar{E}_{SC}, \mu)$$

The equations are integrated numerically to get the Phobos coordinates and the SC ones on a measurement moment. The method of a numerical integration of the eighth order of accuracy [20] is used.

Especially it is necessary to tell the Phobos secular acceleration ν. Opened in 1945 [21] it testifies to the Pbobos power loss, owing to the tide bulge emergence inside Mars. Because of computing difficulties the influence of this tide forces is not openly written in the motion equations. Their calculation is carried out within the framework of the traditional approach. On the current time moment a correction to the Phobos coordinates is added for displacement along a longitude. The Phobos orbit position is corrected because of the secular acceleration by adding $\nu(t - t_0)^2/2$ to the i-th moment of the ascending node passage τ^i_Ω, where t the current time moment and t_0 the initial epoch. Such approach ensures eligibility, possibility of a direct comparison with results of other theories and computing efficiency.

To resolve the parameter determination task it is necessary to know also values of derivatives from elements $\bar{E}_{Ph}$ on a measurement moment with respect to elements $\bar{E}^0_{Ph}$ on the initial epoch moment. For this purpose the equations in variations numerically are integrated

$$\frac{d}{dt}\left(\frac{\partial \bar{E}_{Ph}}{\partial \bar{E}^0_{Ph}}\right) = G_{Ph}(t, \bar{E}_{Ph}, \mu) \qquad \frac{d}{dt}\left(\frac{\partial \bar{E}_{SC}}{\partial \bar{E}^0_{SC}}\right) = G_{SC}(t, \bar{E}_{SC}, \mu)$$

$$\frac{d}{dt}\left(\frac{\partial \bar{E}_{Ph}}{\partial \mu}\right) = G^{\mu}_{Ph}(t, \bar{E}_{Ph}, \mu) \qquad \frac{d}{dt}\left(\frac{\partial \bar{E}_{SC}}{\partial \mu}\right) = G^{\mu}_{SC}(t, \bar{E}_{SC}, \mu)$$

Measurements

Under the theory construction there are used the optical measurements on the time span 1877-1989 years with data taken from sources in the work [11], the Phobos onboard television measurements from SC Mariner-9 (1971-1972 yrs.) [22], Viking (1976-1978 yrs.) [23], Phobos-2 (1989 yr.), Mars Express (2005 yr.) [24], the measurements of Mars Orbiter Laser Altimeter (MOLA) from Mars Global Surveyor (MGS) (1998 yr.) [25], the measurements of an angular distance between Phobos center and center of the solar disk during Phobos's passages as they were seen from American rovers Spirit and Opportunity sitting on the Mars surface (2004 yr.) [26]. The trajectory measurements of the SC being in the Phobos vicinity contain the important information about its gravitational constant. Such measurements were used for space vehicle Phobos-2 and MGS, as they approached with Phobos and changed one's own trajectories under its gravity. The types of measurements are shown in a Fig. 1.

To the Phobos optical astrometry measurements made from the Earth, the angular measurements in a rectangular frame with respect to the Mars center in the telescope picture plane are relevant. The directions of this system axeses can both to coincide with a North Pole direction and a direction «east - west» and to be turned around the sight line on some angle. One can be given as a proper angular distance i.e. module of a Mars- Phobos vector s, then this vector coordinates x, y, a, d, and also a position angle between a direction on the Mars North Pole and direction on Phobos P. To obtain the calculated significance ψ^c these values are expressed through equatorial coordinates of Mars and Phobos bounded to the measurement epoch, or to J1900.0, J1950.0, and J2000.0. To obtain ψ^c it is necessary to transition from a frame, in which rectangular coordinates of the object received by integration and ones of Earth and Mars (J2000) are available, to the specific epoch of measurements. For the Earth measuring point the transition from known geographical coordinates in the measurement epoch is carried out. For transition in the fixed epoch a precessions is only necessary, for transition in the current epoch of date a nutation is added. As the measurements are differential, calculations of aberrations at the expense the observer motion caused both the Earth rotation (daily aberration) and the Earth movement on orbit (stellar aberration) are not necessary. In calculations of differential measurements the Earth and Mars vectors respectively the Sun participate, which undertake from the theory of the Sun system planet motions JPL DE405 [15].

The SC television measurements are made in a picture plane orthogonal to the TV camera optical axes of view with knowledge of coordinates of basic stars (that increases the measurement accuracy) or without them (knowing the SC attitude). The Phobos center angular coordinates relatively the SC are determined. The SC coordinates, excluding Phobos-2 and MGS, were determined by high-precision radio engineering measurements from the Earth during their flights, therefore their coordinates relatively Mars are taken from the indicated sources on a measurement moment. For Phobos-2 and MGS by radio engineering measurements (ones are designated on Fig.1 as gravitational) the adjustment of the Phobos and SC coordinates were made, and by SC Phobos-2 the improving of the Phobos gravitational constant μ as well. Besides the SC MGS fulfilled distance measurements up to the Phobos surface. The coordinates of the under locator point are known, therefore it is possible to take into account a hypsometry.

There are measurements on Fig.1, which will be made in forthcoming mission Phobos-Grunt. They are similar to the ones above described and shown by dotted line. It is necessary to notice, that after probe landing the radio measurements will be passed with it from Earth, which are essential for the error decreasing of a starting from Phobos returned SC and for improving the Phobos gravity field parameters.

For all optical and radio engineering measurements made from Earth, the measurement weights were calculated by accuracy, which is informed by the source. For optical observations from space vehicles Mariner-9, Viking, Mars Express the error magnitude was corrected by the formula offered in work [23], which introduces dependence of error magnitude from a distance between space vehicle and Phobos. The Soviet SC Phobos-2 measurements were fulfilled from a close distance, on TV pictures there were no basic stars, therefore their exactitude has appeared low: the measurement error o makes 0.5-1 degrees. After processing magnitudes of average weighted errors for each type of measurements do not exceed 1.

Technique of determination of parameters

Under theory construction alongside with Phobos elements the SC motion parameters were updated, namely Phobos-2 and MGS, which close approached to Phobos .The set of improved parameters $\bar{Q}$ include: the Phobos orbit elements $\bar{E}^0_{Ph}$, the Phobos gravitational constant μ, the Phobos secular acceleration ν, Phobos-2 motion parameters $\bar{E}^0_{SC1}$ and parameters of maneuvers, MGS motion parameters $\bar{E}^0_{SC2}$.

The determination of theory parameters $\bar{Q}$ by the method of a maximum probability [27] demands minimization of a functional $F\left(\bar{Q}\right)=\sum_{i=1}^{M}\bar{\xi}_i^T\left(\bar{Q}\right)P_i\bar{\xi}_i\left(\bar{Q}\right)$, where $\bar{\xi}_i\left(\bar{Q}\right)=\bar{\psi}_i^o-\bar{\psi}_i^c\left(\bar{Q}\right)$, $i=1,2,...,M$, and $\bar{\psi}_i^o$ is $i-$th a measurement -vector with covariance matrix of a priori errors K_i and $P_i=K_i^{-1}$, $\bar{\psi}_i^c$ is the calculated significance of a measurement-vector. For minimization F the iteration method is used. On each step the linear system $\left(\sum_{i=1}^{M}\left(\frac{\partial\bar{\xi}_i}{\partial\bar{Q}}\right)^T P_i\left(\frac{\partial\bar{\xi}_i}{\partial\bar{Q}}\right)\right)\Delta\bar{Q}=\left(\sum_{i=1}^{M}\left(\frac{\partial\bar{\xi}_i}{\partial\bar{Q}}\right)^T P_i\bar{\xi}_i\right)$ is solved and parameters $\bar{Q}_m=\bar{Q}_{m-1}+\Delta\bar{Q}$ are updated.

Anyone of types of measurements used in the theory construction depends on the own assemble of updated parameters $\bar{q}$, which is a subset, but does not coincide a full set $\bar{Q}$. Therefore the common system of the linear equations $A\bar{Q}=B$ may be divided on some subsystems to obtain all population:

1. Subsystem Phobos, ν ;
2. Subsystem Phobos-2 (with adjusting of maneuvers), Phobos, μ,ν ;
3. Subsystem MGS, Phobos.

Each i-th subsystem represents a problem of processing of the certain set of measurements on each iteration. It begins by preparation of necessary data - measurements, initial conditions $\bar{q}_i^0$ and etc. and is finished by shaping of a matrix A_i and right sides B_i of the normal equations for this subsystem $A_i\bar{q}_i=B_i$, which will enter to a common system $A\bar{Q}=B$ as the constituents. The first subsystem ensures processing all ground optical measurements of Phobos and of Phobos passing over the Sun. It makes improving of the Phobos motion parameters on an interval more then 100 years and updates the initial conditions for two other subsystems before the beginning of each following iteration. The second subsystem conducts processing trajectory measurements of the SC Phobos-2 range rates and onboard television measurements of Phobos. The scope of this subsystem activity is distributed to the time span from the beginning of a transfer orbit, on which space vehicle has left after realization of a braking maneuver and transition into Mars satellite orbit, and to the quasi-synchronous orbit (an end of the SC activity existence). The parameters of these orbits are indicated in Table1. The SC motion description on all orbit sites is made within the framework of a single trajectory, on which there are sections of passive movement and maneuvering. This trajectory is described by the SC orbit elements in the initial time moment on a transfer orbit and by parameters of maneuvers, and also by meanings of Phobos gravity field constant and of its secular acceleration. Each maneuver is described by six parameters ensuring calculation of the state vector increments at the maneuver completion moment. The priori significances of maneuver parameters are included in processing, the errors of their fulfillment being counted. This a priori information is represented in the form of a covariance matrix. The third subsystem is intended for processing the MGS measurements. The long of an interval of measurements taken in processing makes some days, the motion model being described within the framework of passive movement.

Schematically shaping of a common system is represented on Fig.2. Each element of a common system is received by summation of appropriate elements of subsystems, which is shown by identical gray shade. Besides on a Fig. 2 there are the place of the future subsystem SC3 Phobos-Grunt and parameters for determination from this subsystem.

Results

After processing measurements on the time span 1877-2005 years the Phobos orbit updated meanings of elements, secular acceleration and Phobos’s gravitational constant have been obtained. With use of these parameters the numerical integration of the motion equations have been fulfilled i.e. calculation of coordinates and velocities in epoch J2000 on an interval 1989-2015 years. The obtained data are written on the magnetic carrier as spreadsheets with a step 0.5 hours and can be used in the form of spreadsheets during preparation to the Phobos-Grunt mission.

Table 2 contains updated meanings of Phobos's secular acceleration and one's gravitational constant. For a comparison the outcomes of other authors are indicated.

The coordination of the theory with observations is characterized by residuals of the measured values from calculated. On Figures 3-8 the residuals for optical measurements made from the Earth are shown. In main, the residuals do not exceed an actual accuracy of measurements. On Figures 9-10 the residuals for onboard television measurements are represented. In an initial kind these measurements are given as right ascension and declination with respect axeses of the inertial system (B1950.0, J2000). To avoid distortions in right ascensions originating near to poles, they were transferred in angular coordinates x, y in a plane that is orthogonal to the line-of-sight. In the whole the coordination of these measurements with the theory corresponds to an eccuracy. The measurements of the SC Phobos-2 have the greatest deviations. It is explained by absence of basic stars on TB pictures. On the other hand these pictures were made from a close distance and therefore they represent the greatest value.

There are residuals of trajectory radio measurements of the SC Phobos-2 and MGS with calculated meanings on updated orbit on Fig 11-12. These are measurements of Doppler frequency displacements of signals transformed into significances of a range rates. In average, the deviations for Phobos-2 make approximately 1sm/sec. As a minimal distance of the SC Phobos-2 Phobos makes less than 200 kms, such errors ensure a possibility of Phobos's gravitational constant improving and give some presentation about its position on orbit relatively Mars. Deviations of the SC MGS laser measurements from calculated significances with using ephemeris JPL mar033-7.bsp and orbit of space vehicle MGS obtained in JPL have made approximately 4 kms [25]. We have assumed, that these deviations can be explained both inaccuracy of Phobos's ephemeris and errors in the MGS orbit determination. In this connection the attempt was undertaken to update simultaneously parameters of Phobos and MGS motions by a population of trajectory measurements MGS and Phobos's laser measurements from MGS within the framework of described above technique. As a result of a solution of a problem the MGS initial vector practically was not changed (coordinates were changed less, than on 500 m). The coordination improving of the measured and calculated meanings of laser measurements was reached at the expense of a modification of the Phobos orbit. These residuals are represented on Fig.13. As visual case there are shown also residuals obtained without taking account the hypsometry of a Phobos surface.

In the Table 3 there are results of a comparison of the measured and calculated meanings of parameters of Phobos's passages on the Sun disk which were observed from the Mars surface by American rovers Spirit and Opportunity in a 2004. In total 4 passages were fixed and for each one the measurement was generated. As such measurement the angular distance between center of the Sun and the Phobos center is accepted. The time moment of the measurement binding corresponds to a middle of a time span from a beginning of the first contact of Phobos with an edge of the Sun disk up to the ending of the second one. In the table the results of counts executed on basis of present theory, on theories of Jacobson (2006г) and of Chapront-Touze [16] are indicated. In recalculation to a distance the largest residual does not exceed 7 km - top of the table. Other residuals are significantly less.

Comparison of results

The direct comparison of the Phobos coordinates in epoch J2000 on an interval 1989-2016 years with data of the Jacobson theory and of the Chapront-Touze theory is executed. The results are represented in a fig. 14-19. As it is visible, the divergences between Jacobson's and KIAM's results do not exceed 15 km.

Accuracy estimation

The accuracy estimation of the Phobos coordinates in an orbital frame has been executed. The axeses of this system are directed along a Phobos's radius-vector, along the Phobos velocity, orthogonal to the orbit plane. The errors (3σ) on the time span 1989-2016 years are shown on Figures 20-22.

Conclusions

1. Intended to support the Phobos –Grunt mission the dynamical numerical theory of the Phobos motion is constructed.
2. The motion model relies on the most authentic constants of a field of Mars, on coordinates of a pole of Mars and velocity of their modification, on other astronomical constants. It ensures compatibility of this Phobos motion model with the SC motion model in the vicinities of Mars and Phobos. The interval of measurements envelops period 128 years. The optical measurements from the Earth, the SC TV optical and laser measurements, optical measurements from a surface of Mars, radio engineering observations from the Earth of the SC motion are included in composition of measurements
3. The comparison of the theory data with observations and with ones of other similar theories is executed and it is shown sufficiently good agreement.
4. The accuracy estimation shows acceptable Phobos ephemerid precision

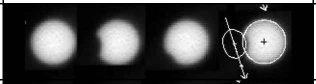
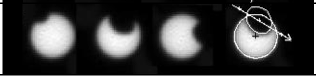
Table 1. The SC Phobos-2 orbits.

Section number	1	2	3	4	5	6
Span 1989	29.01-12.02	12.02-15.02	18.02-07.03	07.03-15.03	15.03-21.03	21.03-27.03
Period	3d 7h.13m	3d 14h 29m	0d 08h 01m	0d 08h 02m	0d 08h 01m	0d 07h 39m
Eccentricity	0.904	0.792	0.012	0.003	0.003	0.03

Table 2. Phobos's secular acceleration ν and gravitational constant μ

Author	Year	ν $grad/(day)^2 \cdot 10^{-8}$	μ $grad/(day)^2 \cdot 10^{-8}$
KIAM	2007	0.9359 ± 0.0123	7.158 ± 0.001
Konopliv et al.	2006		7.16 ± 0.005
Lainy et al.	2005	0.9422	
Emelianov et al..	1993	0.9666	
Ivanov et al..	1990	0.9144	
Chapront-Touze	1989	0.9303	
Jacobson et al.	1989	0.9306	
Morley	1988	0.9527	
Sinclair	1989	0.9272	
Sinclair	1972	0.7195	
Shor et al..	1988	0.984	
Shor	1971	1.01	

Table 3 Results of residuals in passing Phobos on the Sun disk for three theories

Date of 2004 year	The angular distance measurement on a middle time moment	The measurement *(grad)*	Residuals *(grad)*		
			Jacobson	Chapront-Touze	KIAM
03/07		0.23	0.03	0.02	0.07
03/10		0.04	-0.09	-0.10	-0.04
03/12		0.02	-0.05	-0.06	-0.01
04/18		0.12	-0.02	-0.01	0.01

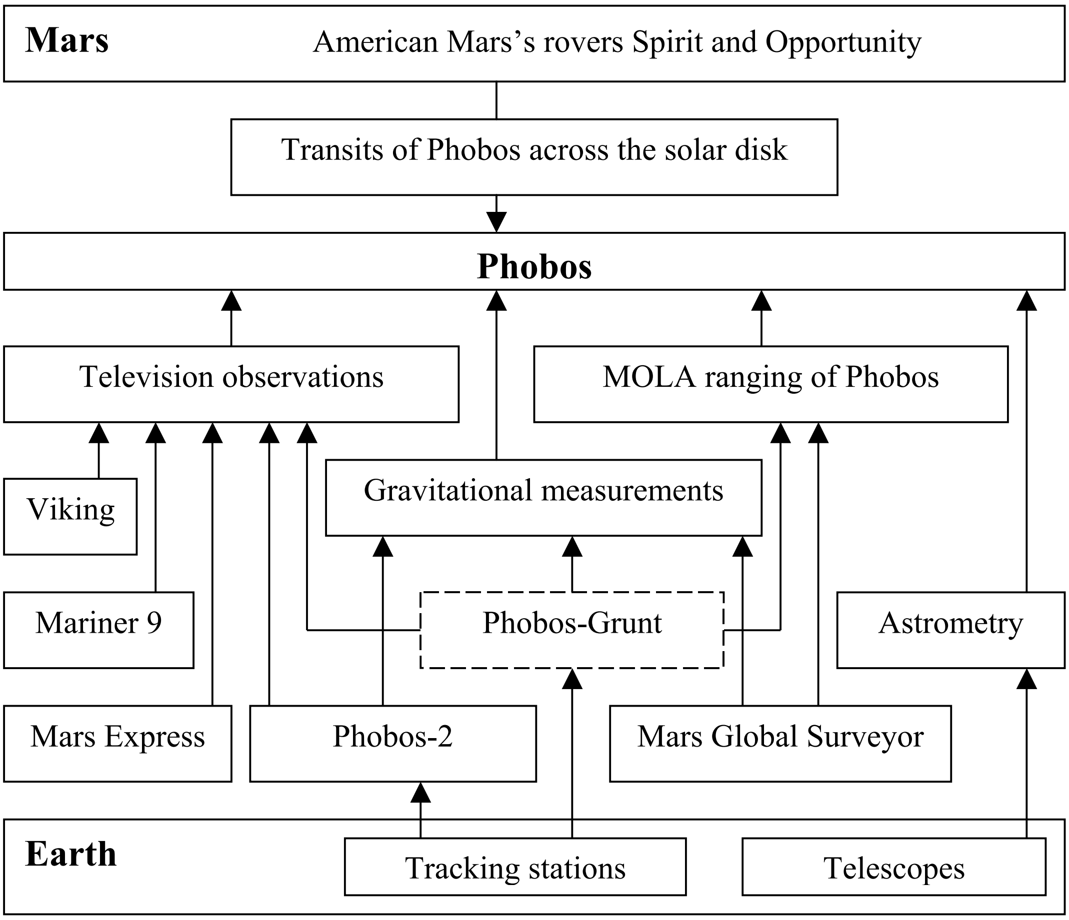


Fig. 1 Types of measurements used for the theory construction.

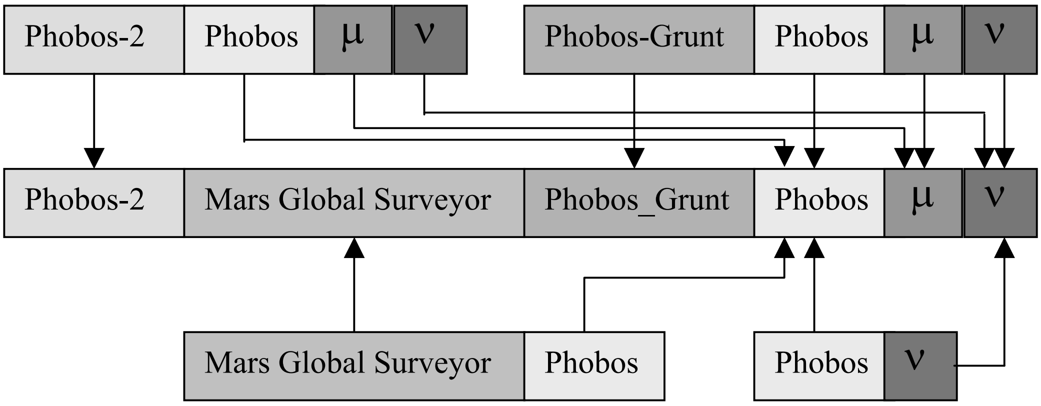


Fig. 2 The scheme of shaping of the common normal system of equations $A\bar{Q} = B$ to derive parameters $\bar{Q}$ on the current iteration.

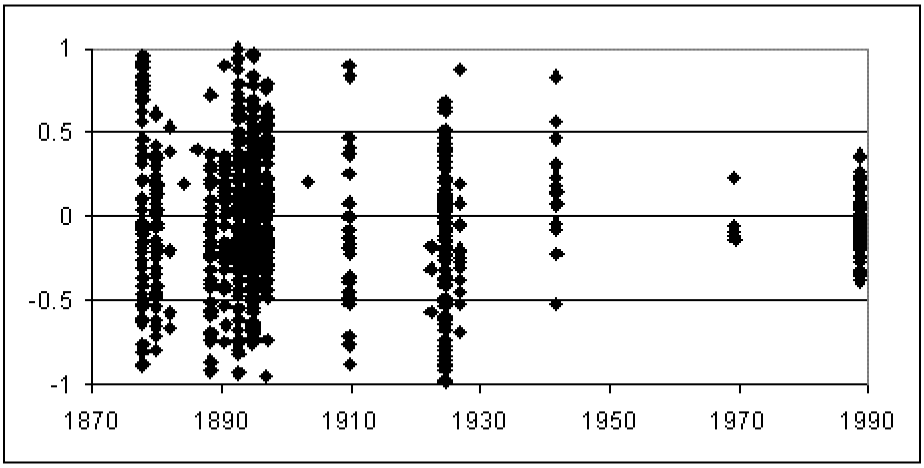


Fig. 3. Residuals of the Phobos angular distances s from Mars center. Years are on the abscissa axis, angular seconds are on the ordinate axis.

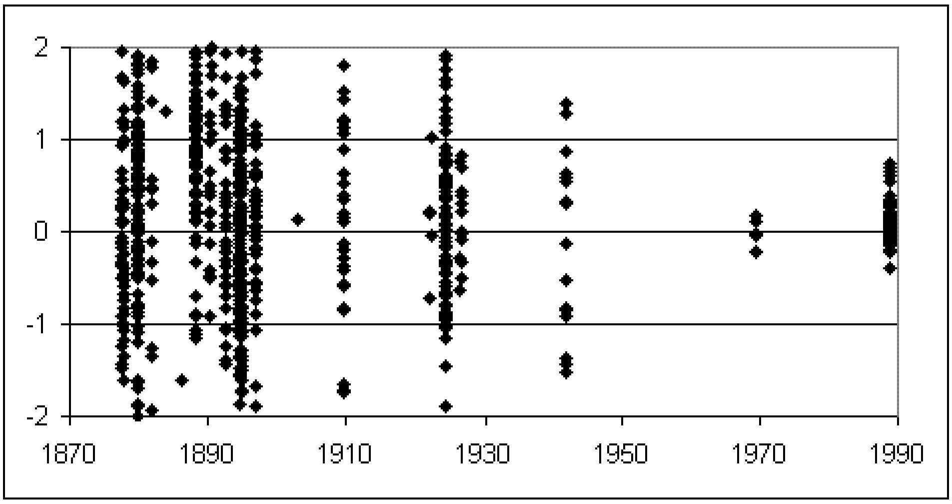


Fig. 4.Residuals of position angles P. Years are on the abscissa axis, grades are on the ordinate axis.

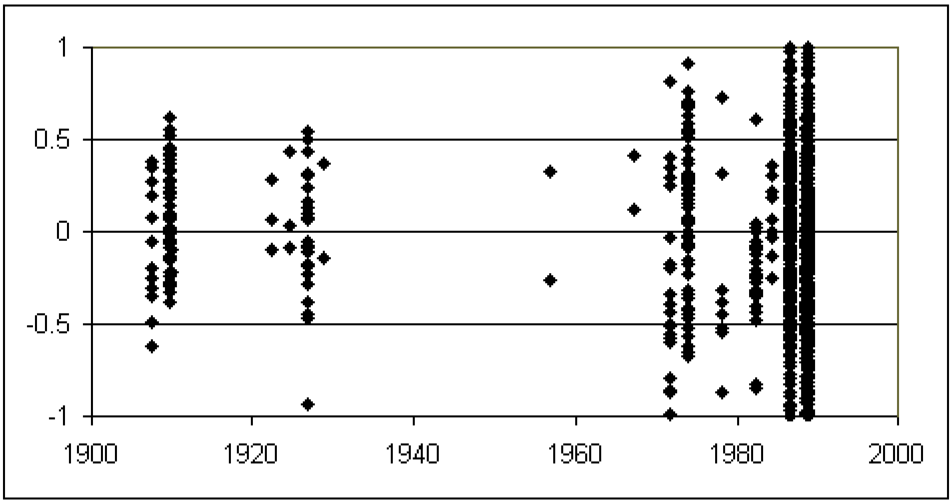


Fig. 5. Residuals for coordinates x. Years are on the abscissa axis, angular seconds are on the ordinate axis.

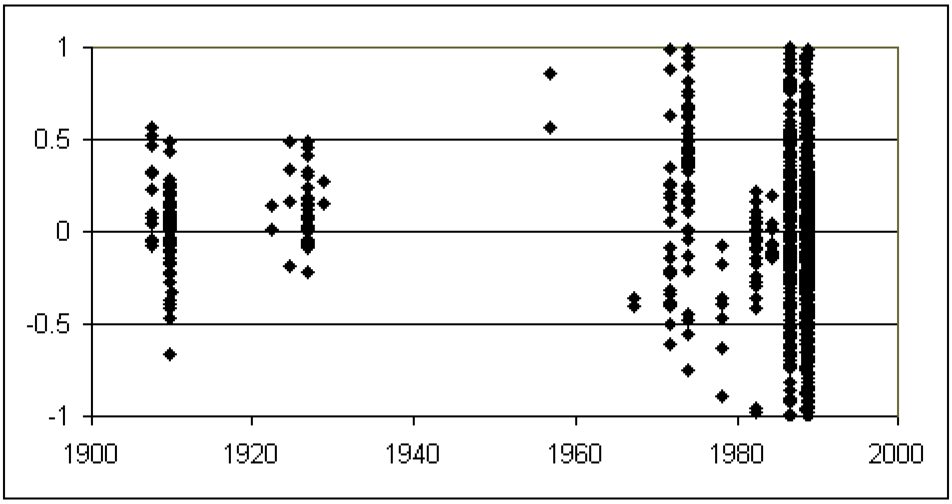


Fig. 6. Residuals for coordinates y. Years are on the abscissa axis, angular seconds are on the ordinate axis.

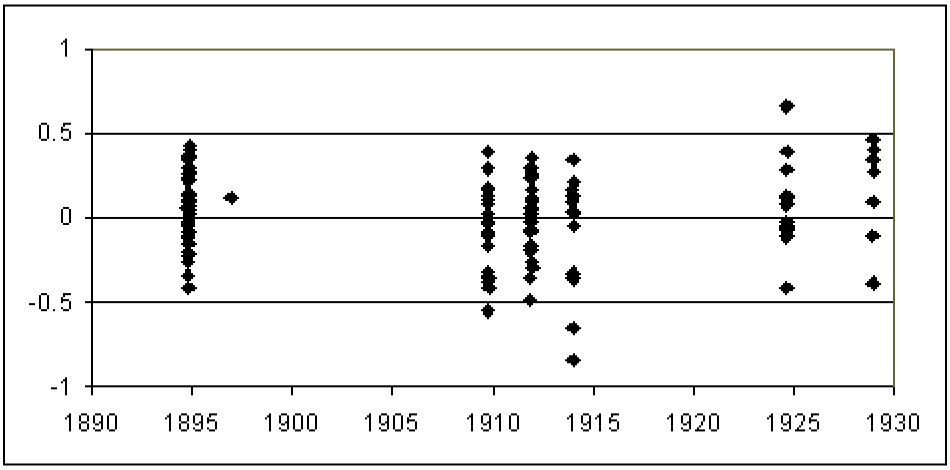


Fig. 7.Residuals for coordinates a. Years are on the abscissa axis, angular seconds are on the ordinate axis.

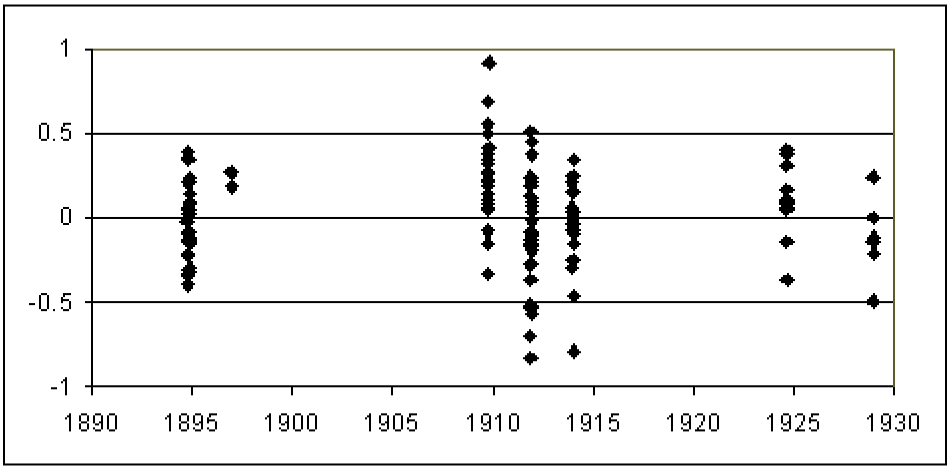


Fig. 8. Residuals for coordinates d. Years are on the abscissa axis, angular seconds are on the ordinate axis.

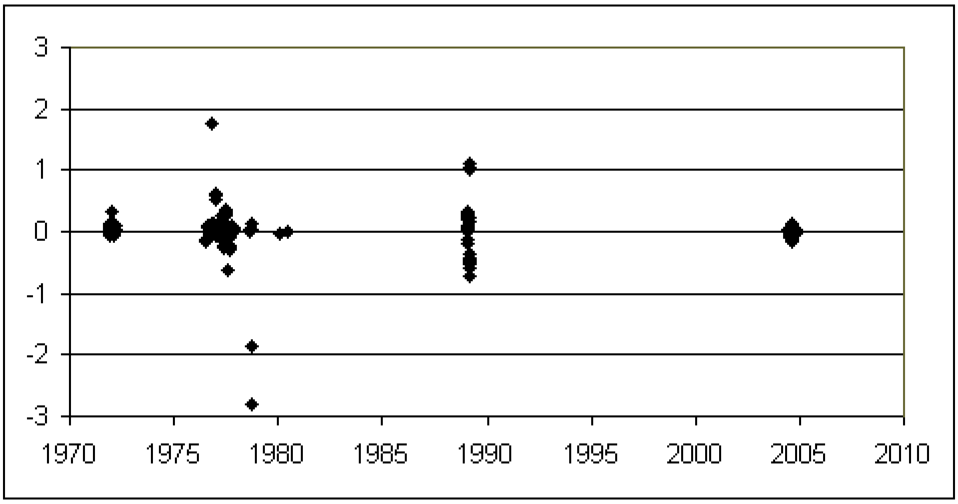


Fig. 9. Residuals for coordinates on the X axis in the picture plane of the SC TV system.. Years are on the abscissa axis, grades are on the ordinate axis.

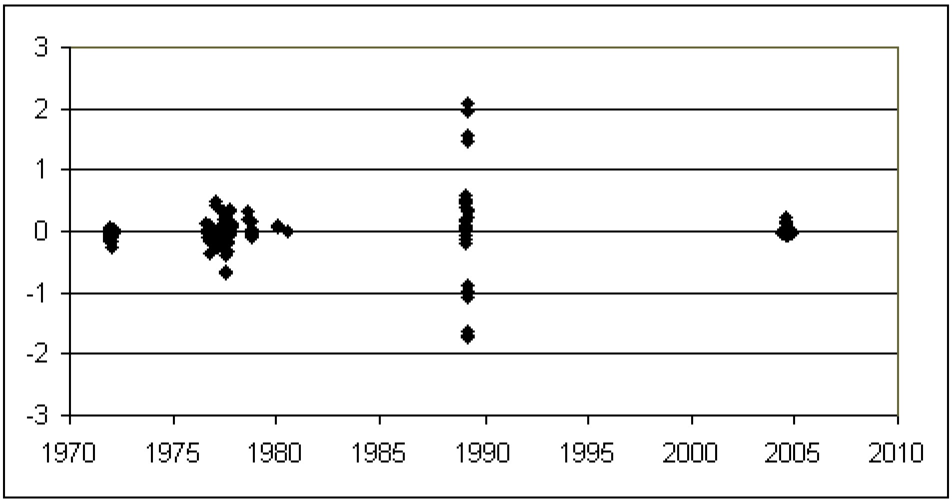


Fig. 10. Residuals for coordinates on the Y axis in the picture plane of the SC TV system.. Years are on the abscissa axis, grades are on the ordinate axis.

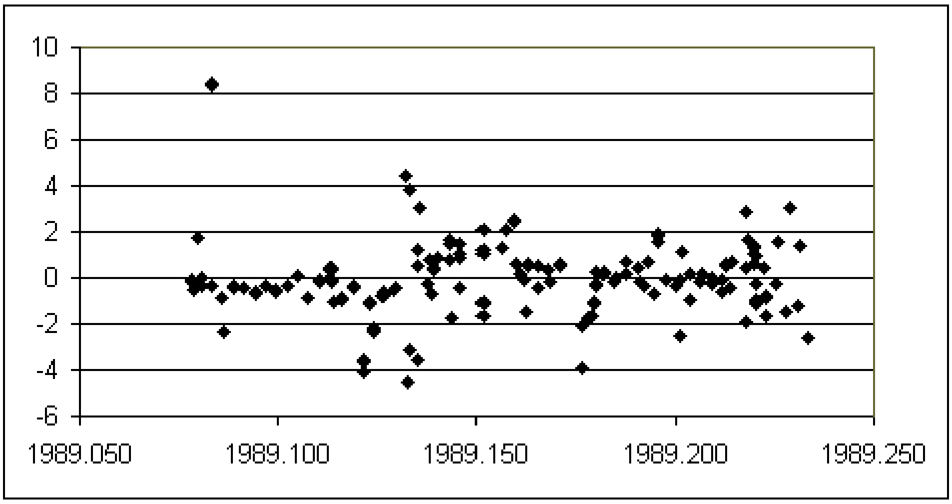


Fig. 11. Residuals for the SC Phobos-2 range rates. Time is in year fractions are on the abscissa axis, the ordinate axis in sm/sec.

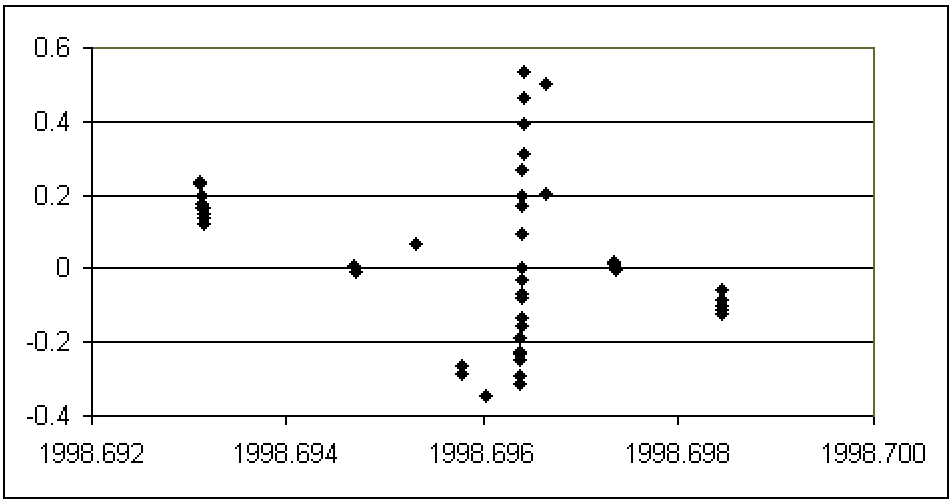


Fig. 12. Residuals for the SC MGS range rates. Time is in year fractions are on the abscissa axis, the ordinate axis in sm/sec.

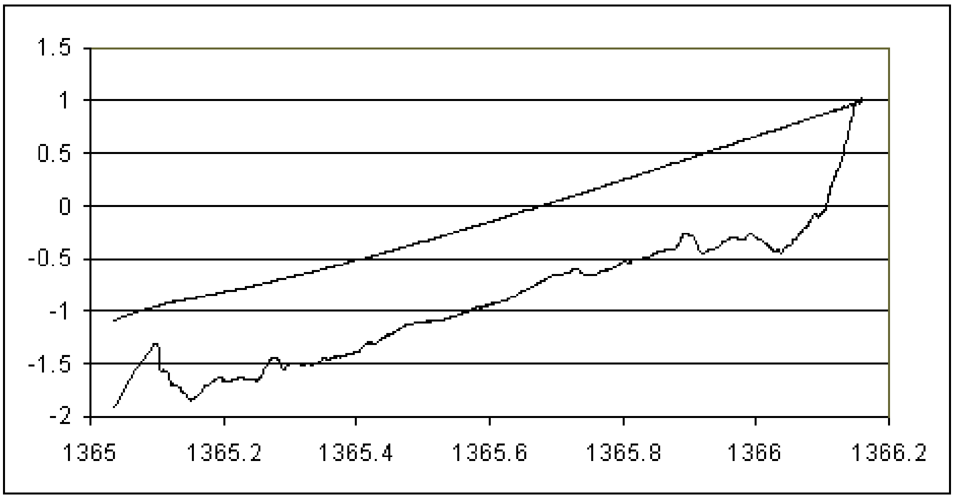


Fig.13. Residuals for SC MGS – Phobos's surface ranges, the relief is counted (smoothed line) and without relief. Time is in thousand sec from the beginning of the date 1998/09/13, the ordinate axis is in km.

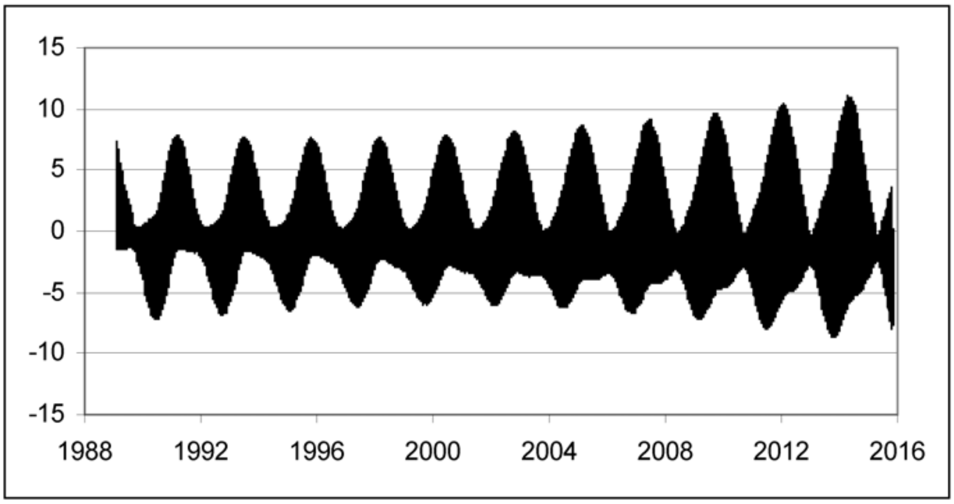


Fig. 14. Years are on the abscissa axis, residuals on the X coordinate between Jacobson and KIAM theories in km.

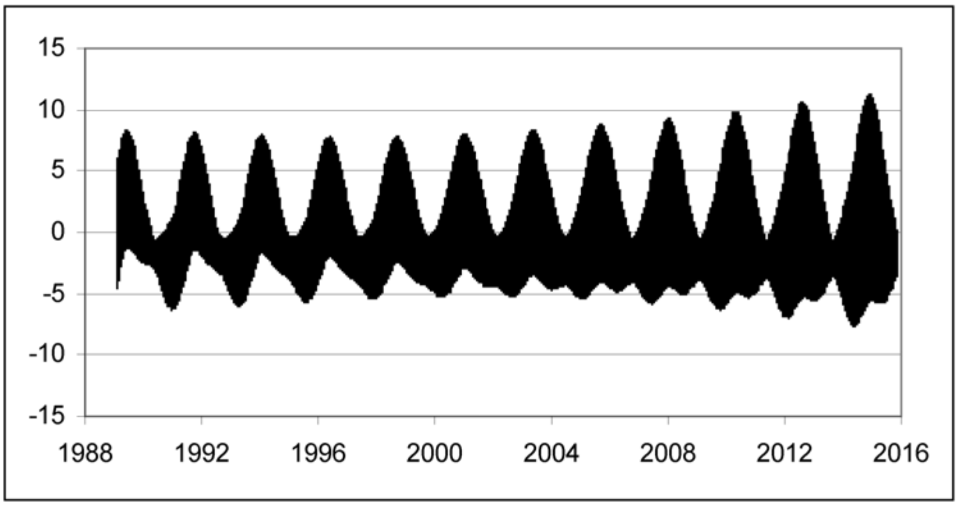


Fig. 15. Years are on the abscissa axis, residuals on the Y coordinate between Jacobson and KIAM theories in km.

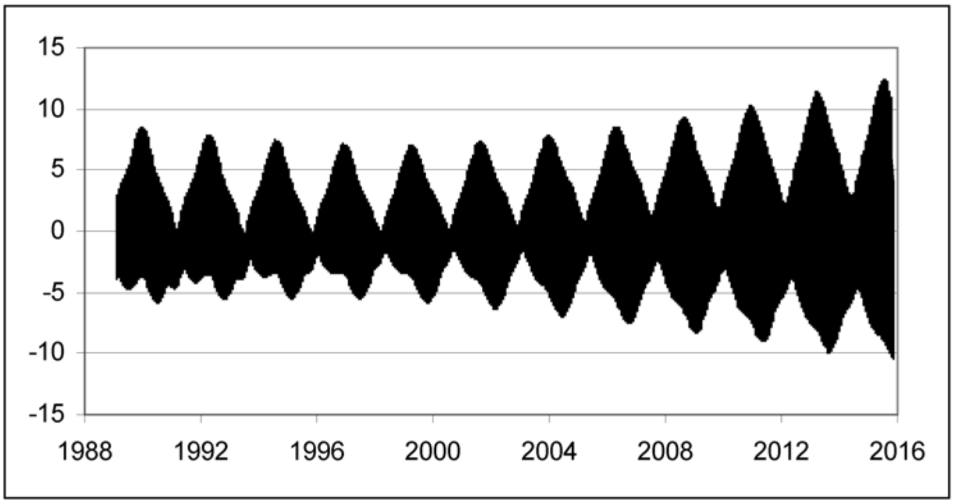


Fig. 16. Years are on the abscissa axis, residuals on the Z coordinate between Jacobson and KIAM theories in km.

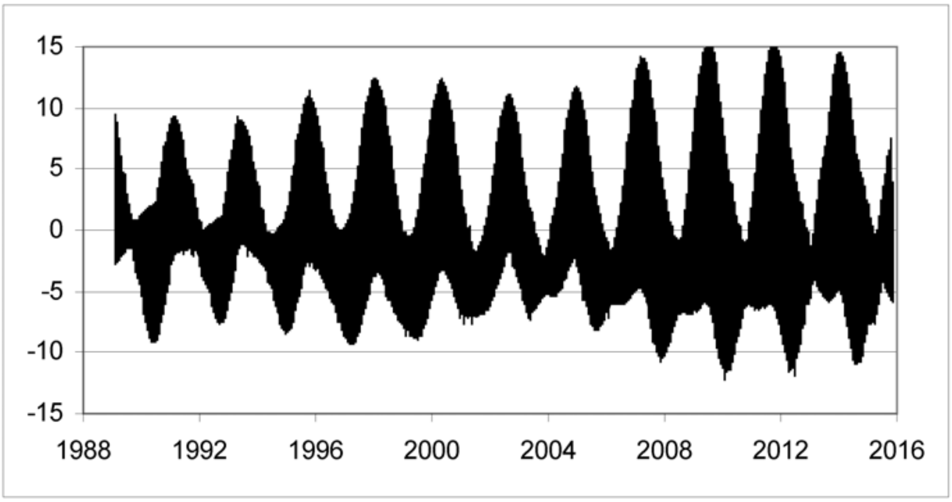


Fig. 17. Years are on the abscissa axis, residuals on the X coordinate between Chapront-Touze and KIAM theories in km.

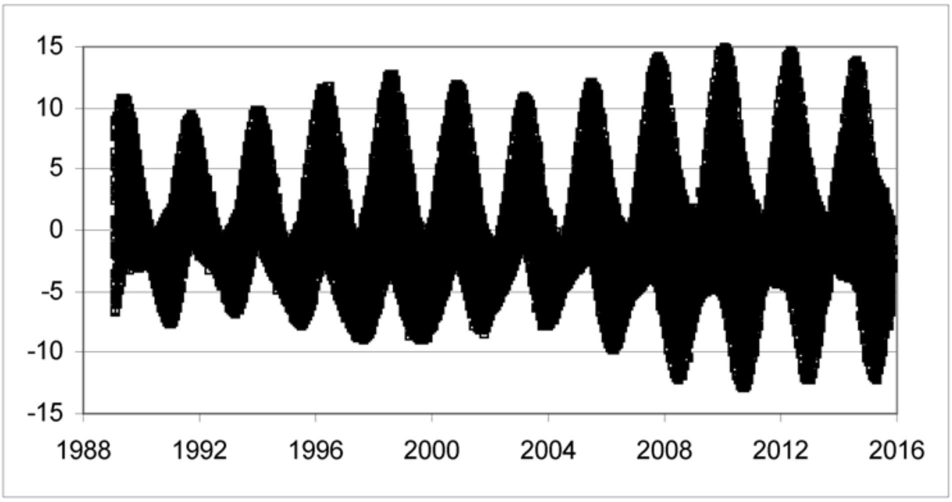


Fig. 18. Years are on the abscissa axis, residuals on the Y coordinate between Chapront-Touze and KIAM theories in km.

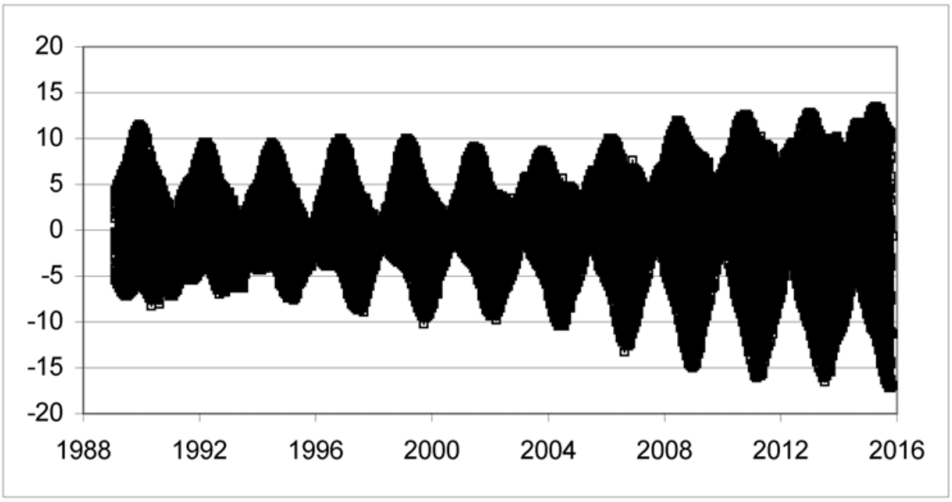


Fig. 19. Years are on the abscissa axis, residuals on the Z coordinate between Chapront-Touze and KIAM theories in km.

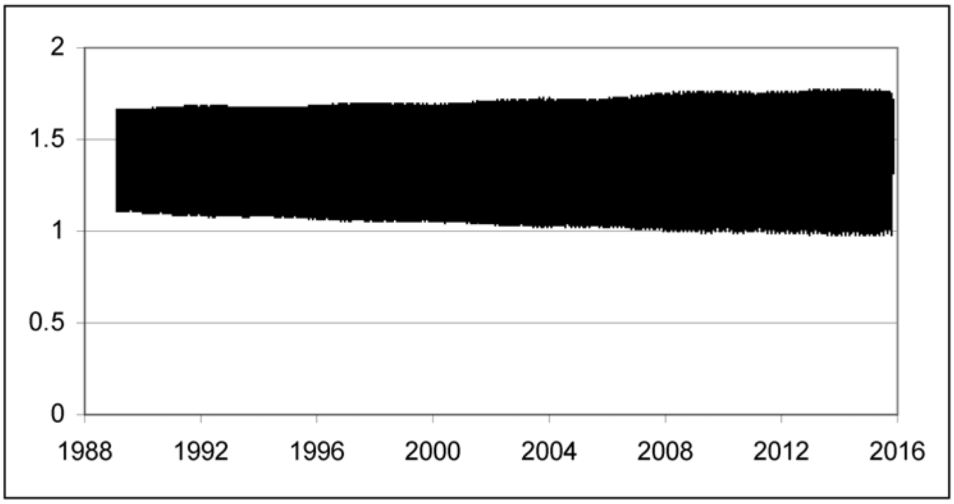


Fig. 20. The errors along the Pbobos's radius-vector in km.Years are on the abscissa axis.

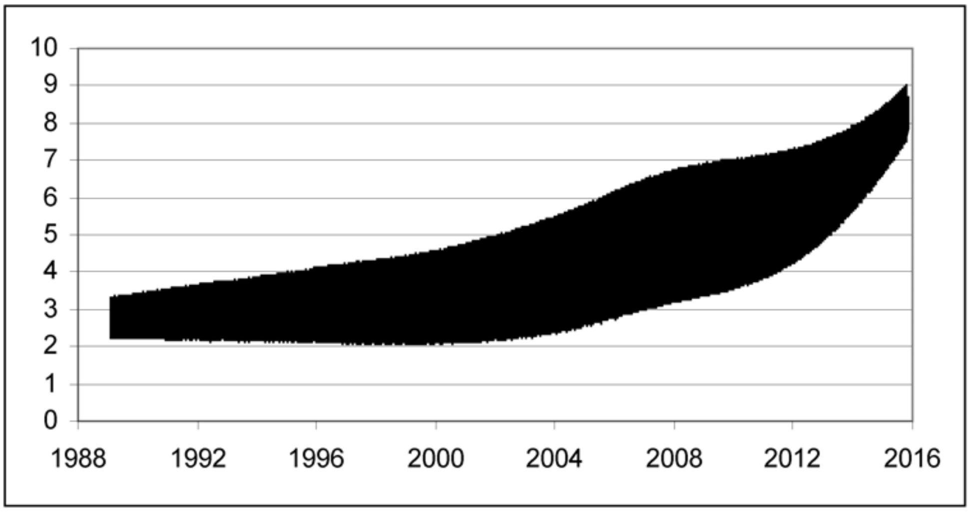


Fig. 21. The errors along the Phobos velocity in km. Years are on the abscissa axis.

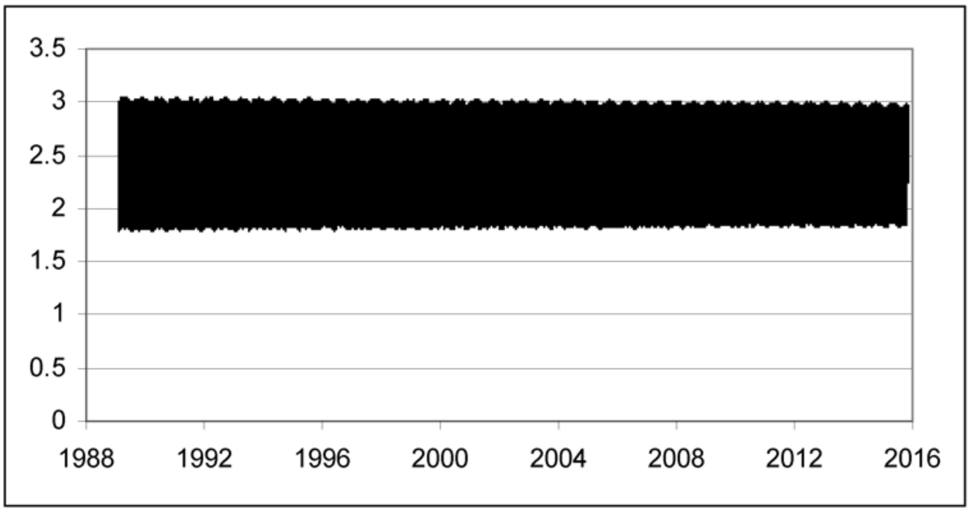


Fig. 22. The errors orthogonal to the orbit plane in km. Years are on the abscissa axis.

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