ANALYTICAL APPROACH VALIDATION FOR THE SPIN-STABILIZED SATELLITE ATTITUDE

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Abstract

An analytical approach for spin-stabilized spacecraft attitude prediction is presented for the influence of the residual magnetic torques and the satellite in an elliptical orbit. Assuming a quadripole model for the Earth's magnetic field, an analytical averaging method is applied to obtain the mean residual torque in every orbital period. The orbit mean anomaly is used to compute the average components of residual torque in the spacecraft body frame reference system. The theory is developed for time variations in the orbital elements, giving rise to many curvature integrals. It is observed that the residual magnetic torque does not have component along the spin axis. The inclusion of this torque on the rotational motion differential equations of a spin stabilized spacecraft yields conditions to derive an analytical solution. The solution shows that the residual torque does not affect the spin velocity magnitude, contributing only for the precession and the drift of the spin axis of the spacecraft. The theory developed has been applied to the Brazilian's spin stabilized satellites, which are quite appropriated for verification and comparison of the theory with the data generated and processed by the Satellite Control Center of Brazil National Research Institute. The results show the period that the analytical solution can be used to the attitude propagation, within the dispersion range of the attitude determination system performance of Satellite Control Center of Brazil National Research Institute.

1. Introduction

Emphasis within this paper is placed on an analytical approach for spin-stabilized spacecraft attitude prediction, considering the influence of the residual magnetic torques. Magnetic residual torques occurs due to the interaction between the Earth's magnetic field and the residual magnetic moment along the spin axis of the satellite. In spin stabilized satellites, equipped with nutation dumpers, such effect is usually the major perturbing torque.

It is assumed that the Earth´s magnetic field is given by the quadripole model and that the satellite is in an elliptical orbit. A spherical coordinates system fixed in the satellite is used to locate the spin axis of the satellite in relation to the terrestrial equatorial system. The direction of the spin axis is specified by its right ascension (α) and the declination (δ), which are represented in Fig. 1.

To compute the average components of the residual magnetic torque in the satellite body frame reference system (satellite system), an average over the mean anomaly is performed. The average torque includes the main effects associated with the residual magnetic torque. Developments are made in terms of the mean anomaly and first order terms in the eccentricity.

It is observed that the residual magnetic torque does not have component along the spin axis, however it has non-zero components in satellite body x-axis and y-axis . The inclusion of this torque on the rotational motion differential equations of a spin stabilized spacecraft yields conditions to derive an analytical solution.

In order to validate the analytical approach, the theory developed has been applied forthe spin stabilized Brazilian satellites (SCD1 and SCD2), which are quite appropriated for verification and comparison of the theory with the data generated and processed by the Satellite Control Center (SCC) of Brazil National Research Institute (INPE). In the numerical implementation of the analytical solution, the influences of the Earth oblateness in the orbital elements are taken in account.

The behavior of right ascension and declination of the spin axis with the time are shown. For the tests it is also observed the deviation between the actual SCC supplied spin axis and the analytically computed spin axis, for each satellite. Comparison are also developed with result for satellite in a circular orbit (Assis, 2004; Zanardi et al.,2005) and for the result using the dipole model for the geomagnetic field .

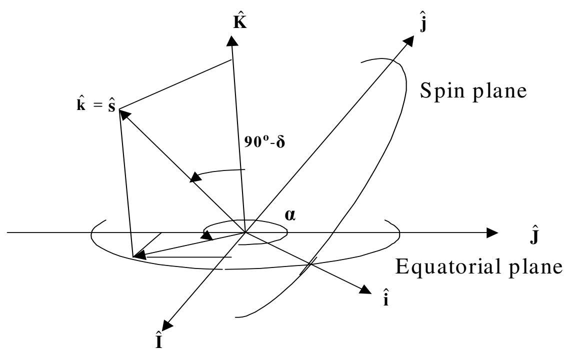

Figure 1 - Spin axis orientation ($\hat{s}$): Equatorial System ($\hat{I}, \hat{J}, \hat{K}$), Satellite System ($\hat{i}, \hat{j}, \hat{k}$), right ascension ($\alpha$) and declination ($\delta$) of the spin axis.

2. Residual Torque and Geomagnetic Field

Magnetic residual torque results from the interaction between the spacecraft's residual magnetic field and the Earth´s magnetic field. If $\vec{m}$ is the magnetic moment of the spacecraft and $\vec{B}$ is the geomagnetic field, the magnetic torques is given by (Wertz,1978):

$$\vec{N}_r = \vec{m} \times \vec{B} \,. \tag{1}$$

The quadripole model is assumed in this paper to describe the geomagnetic field. It is well known that the Earth's magnetic field can be obtained by the gradient of a scalar potential V (Wertz,1978):

$$V(r', \phi, \theta) = r_T \sum_{n=1}^{k} \left(\frac{r_T}{r}\right)^{n+1} \sum_{m=0}^{n} \left(g_n^m \cos m\theta + h_n^m \operatorname{sen} m\theta\right) P_n^m(\phi) \,, \tag{2}$$

where r_T is the Earth's equatorial radius, g_n^m, h_n^m are the Gaussian coefficients, $P_n^m(\phi)$ are the Legendre associated polinomian, r, ϕ,θ mean the geocentric distance, the local colatitude and local longitude.

In terms of spherical coordinates the geomagnetic field can be expressed by (Wertz,1978):

$$\vec{B} = B_r\,\hat{r} + B_\phi\,\hat{\phi} + B_\theta\,\hat{\theta}, \tag{3}$$

with:

$$B_r = -\frac{\partial V}{\partial r}, \qquad B_\phi = -\frac{1}{r}\frac{\partial V}{\partial \phi}, \qquad B_\theta = -\frac{1}{r\,\mathrm{sen}\,\phi}\frac{\partial V}{\partial \theta}. \tag{4}$$

For the quadripole model it is assumed *n* equal 1 and 2 and *m* equal 0,1 and 2 in Eq. (2). After straightforward computations, the geomagnetic field can be expressed by (Zanardi et al., 2005; Garcia,2007):

$$B_r = 2\left(\frac{r_T}{r}\right)^3 f_1(\theta,\phi) + 3\left(\frac{r_T}{r}\right)^4 f_2(\theta,\phi), \tag{5}$$

$$B_\phi = -\left(\frac{r_T}{r}\right)^3 f_3(\theta,\phi) - \left(\frac{r_T}{r}\right)^4 f_4(\theta,\phi), \tag{6}$$

$$B_\theta = -\frac{1}{\mathrm{sen}\,\phi}\{\left(\frac{r_T}{r}\right)^3 f_5(\theta,\phi) + \left(\frac{r_T}{r}\right)^4 f_6(\theta,\phi) + 2\left(\frac{r_T}{r}\right)^4 f_7(\phi,\theta)\}, \tag{7}$$

where the functions f_i, i =1,2 ,...7, are shown in Garcia (2007) and depend on the Gaussian coefficients $g_2^2, h_1^1, h_2^1, h_2^2$.

In the Equatorial system, the geomagnetic field is expressed by (Wertz,1978):

$$B_X = (B_r \cos\bar{\delta} + B_\phi\,\mathrm{sen}\,\bar{\delta})\cos\bar{\alpha} - B_\theta\,\mathrm{sen}\,\bar{\alpha}, \tag{8}$$

$$B_Y = (B_r \cos\bar{\delta} + B_\phi\,\mathrm{sen}\,\bar{\delta})\,\mathrm{sen}\,\bar{\alpha} - B_\theta \cos\bar{\alpha}, \tag{9}$$

$$B_Z = B_r\,\mathrm{sen}\,\bar{\delta} + B_\phi \cos\bar{\delta}, \tag{10}$$

where $\bar{\alpha}$ and $\bar{\delta}$ are the right ascension and declination of the satellite position vector, respectively, which can be obtained in terms of the orbital elements, B_r, B_ϕ and B_θ are given by (5), (6) and (7), respectively.

In a satellite system, in which one the axis z is along the spin axis, the geomagnetic field is given by (Kuga et al., 1987; Zanardi et al., 2005):

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}, \tag{11}$$

where

$$B_x = -B_X\,\mathrm{sen}\alpha + B_Y \cos\alpha, \tag{12}$$

$$B_y = -B_X\,\mathrm{sen}\delta\cos\alpha - B_Y\,\mathrm{sen}\,\delta\,\mathrm{sen}\,\alpha + B_Z \cos\delta, \tag{13}$$

$$B_z = -B_X \cos\delta\cos\alpha - B_Y \cos\delta\,\mathrm{sen}\,\alpha + B_Z\,\mathrm{sen}\,\delta, \tag{14}$$

and B_X, B_Y and B_Z are given by Eqs (8) – (10).

3. Mean Residual Torque for Spin Stabilized Satellite

For a spin stabilized satellite (which has the spin axis $\hat{s}$ along the geometry axis z), with appropriate nutation dampers, the magnetic moment is mostly aligned along the spin axis and the residual torque can be expressed by (Kuga et al, 1987):

$$\vec{N}_r = M_s \hat{k} \times \vec{B}, \tag{15}$$

where M_s is the satellite residual magnetic moment along the spin axis $\hat{s} = \hat{k}$.

By substituting the geomagnetic field (Eq. 11) in the Eq. (15), the instantaneous residual torque is expressed by:

$$\vec{N}_r = M_s \left(- B_y \hat{i} + B_x \hat{j} \right). \tag{16}$$

In order to obtain the mean residual torque it is necessary to integrate the instantaneous torque $\vec{N}_r$, given in Eq.(11), over one orbital period (T):

$$\vec{N}_{r_m} = \frac{1}{T} \int_{t_i}^{t_i + T} \vec{N}_r \, dt, \tag{17}$$

where t is time, t_i the initial time and T the orbital period.

In terms of the true anomaly (υ), the mean residual torque can be expressed by:

$$\vec{N}_{rm} = \frac{1}{T} \int_{\upsilon_i}^{\upsilon_i + 2\pi} \vec{N}_r \frac{r^2}{h} \, d\upsilon, \tag{18}$$

where υ_i is the true anomaly at instant t_i, r is the geocentric distance and h is the specific angular moment.

To evaluate the integral of Eq. (18) we can use spherical trigonometry properties, rotation matrix associated with the references systems and the elliptic expansions of the true anomaly in terms of the mean anomaly (Brouwer and Clemence, 1961), including terms up to first order in the eccentricity (e). Without loosing generality, for the sake of simplification of the integrals, we consider the initial time for integration equal to the instant that the satellite passes through perigee. After extensive but simple algebraic developments, the mean residual torque can be expressed by (Garcia,2007):

$$\vec{N}_{rm} = N_{xm} \hat{i} + N_{ym} \hat{j}, \tag{19}$$

with

$$N_{xm} = \frac{M_s}{2\pi} (A \,\text{sen}\, \delta \cos \alpha + B \,\text{sen}\, \delta \,\text{sen}\, \alpha - C \,\text{sen}\, \delta), \tag{20}$$

$$N_{ym} = \frac{M_s}{2\pi} (-D \,\text{sen}\, \alpha + E \cos \delta), \tag{21}$$

with A, B, C, D and E depending on ascending node orbit (Ω), orbital inclination (I), argument of perigee (ω), sideral time and right ascension (α) and declination (δ) of the spin axis (Garcia, 2007).

4. The Rotational Motion Equations and Analytical Solution

The variations of the angular velocity, the declination and the right ascension of the spin axis are given by Euler's equations (Kuga et al., 1987) :

$$\dot{W} = \frac{1}{I_z} N_z, \qquad \dot{\delta} = \frac{1}{I_z\ W} N_y, \qquad \dot{\alpha} = \frac{1}{I_z\ W\ \text{Cos}\,\delta} N_x, \tag{22}$$

where I_z is the moment of inertia along the spin axis, N_x, N_y and N_z components of the external torques in the satellite system.

By substituting the residual torque $\vec{N}_{rm}$, given by Eq. (11), in Eq.(22) it is possible to observe that the residual torque does not affect the satellite angular velocity (because its z-axis component is zero), while

$$\frac{d\delta}{dt} = \frac{N_{ym}}{I_z\ W} \qquad \text{and} \qquad \frac{d\alpha}{dt} = \frac{N_{xm}}{I_z\ W\ \cos\ \delta}\ . \tag{23}$$

The differential equations in Eq. (23) can be integrated assuming that the orbital elements (I, Ω,w) are held constant over one orbital period, and that all other terms on right-hand side of Eq.(23) (e. g. the attitude angles (α, δ) and angular velocity (W)) are equal to initial values (α_o, δ_o, W_o). Then for one orbit period the analytical solution of Eq. (23) can simply be expressed as:

$$\delta = k_1 t + \delta_0 \qquad \text{and} \qquad \alpha = k_2 t + \alpha_0\,, \tag{24}$$

where

$$k_1 = \frac{N_{ym}}{I_z\ W_o} \qquad \text{and} \qquad k_2 = \frac{N_{xm}}{I_z\ W_o\ \cos\ \delta_o}\ . \tag{25}$$

Therefore the residual torque causes a drift in the satellite spin axis. The solution given by Eq.(24) is assumed to be valid for one orbital period. Thus, every orbital period, the attitude angles (α, δ) must be update with this theory results and the orbital data must be updated, taking into account at least the main influence of the Earth oblateness. With this approach the analytical theory will be close to the real attitude behavior of the satellite.

5. Applications

The theory developed has been applied to the spin stabilized Brazilian satellites (SCD1 and SCD2) for verification and comparison of the theory against data generated by the Satellite Control Center (SCC) of INPE. Operationally, SCC attitude determination comprises: sensors data pre-processing, preliminary attitude determination and fine attitude determination (Orlando et al., 1998; Kuga et al., 1999). The pre-processing is applied to each set of data of the attitude sensors collected every satellite pass over the ground station. Afterwards, from the whole preprocessed data, the preliminary attitude determination produces estimates the angular velocity vector every satellite pass over a given ground station. The fine attitude determination takes (one week) a set angular velocity vector and estimates dynamical parameters (angular velocity vector, residual magnetic moment and Foucault parameter). Those parameters are further used in the attitude propagation to predict the need of attitude corrections. Over the test period there isn't attitude corrections. The numerical comparison is shown for the quadripole and the dipole model for the geomagnetic field and for the results to the circular orbit (Assis,2004). It is important to observe that by analytical theory the spin velocity is considered constant during 24 hours. Also, the orbital elements and spin velocity supplied by the SCC were updated daily in the attitude propagation program.

Results for SCD1 satellite

The initial conditions of attitude had been taken on 22 of August of 1993 to the 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC). The table on the Appendix A shows the results with the data from SCC and computed values by the present analytical theory and by the analytical theory for circular orbit (Assis,2004), using the dipole and quadripole model.

The behavior of the error deviation for analytical solution and CCS data for right ascension (α) and declination (δ) along time are shown in Fig. 2 and 3, respectively.

The mean error deviation for right ascension and declination are shown in Table 1 for different period of time. It is possible to note that mean error increases with the time simulation.

Over the 5 days test period and with the quadripole model and elliptical orbit the difference between theory and CCS data has mean error deviation in right ascension of 0.1616^o and -0.4677^o in the declination, which are within the dispersion range of the attitude determination system performance of INPE's control center.

Table 1 – Mean deviation for different time simulation and SCD1.: INPE's Satellite Control Center Data (index CCS), computed results with quadripole model and elliptical orbit (index QE), computed results with dipole model and elliptical orbit (index DE), computed results with quadripole model and circular orbit (index QC), computed results with dipole model and circular orbit (index QE),

Time Simulation (days)	**11**	**5**	**2**
$\alpha_{CCS} - \alpha_{QE}$ (^{o})	-1.5882	0.1616	-0.0151
$\alpha_{CCS} - \alpha_{DE}$ (^{o})	-1.5864	0.1620	-0.0150
$\alpha_{CCS} - \alpha_{QC}$ (^{o})	-1.0779	0.1824	-0.0258
$\alpha_{CCS} - \alpha_{DC}$ (^{o})	-1.5863	0.1620	-0.0150
$\delta_{CCS} - \delta_{QE}$ (^{o})	-1.0896	-0.4677	-0.1449
$\delta_{CCS} - \delta_{DE}$ (^{o})	-1.0900	-0.4680	-0.1450
$\delta_{CCS} - \delta_{QC}$ (^{o})	-1.1707	-0.4542	-0.1376
$\delta_{CCS} - \delta_{DC}$ (^{o})	-1.0900	-0.4680	-0.1450

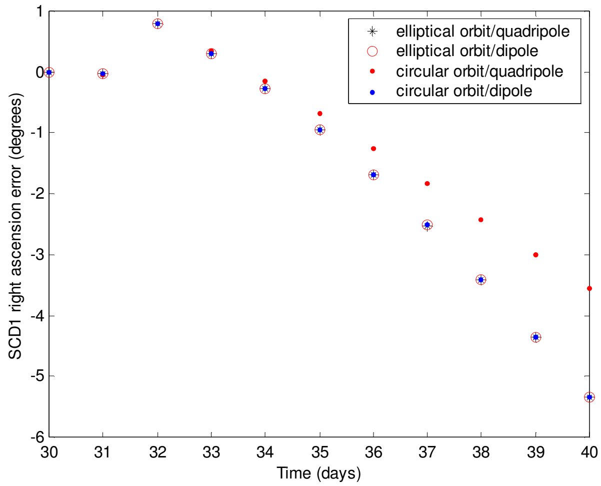

Figure 2 – SCD1 right ascension error evolution.

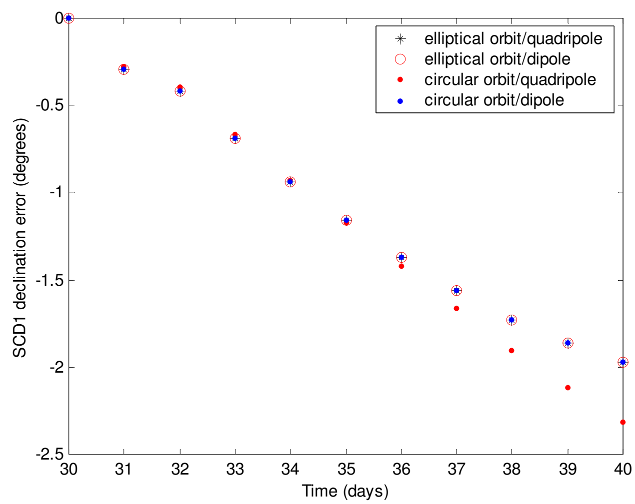

Figure 3 – SCD1 declination error evolution.

Results for SCD2 satellite

The initial conditions of attitude had been taken on 12 December 2002 at 00:00:00 GMT, supplied by the SCC. In the same way for the SCD1, the Table in the Appendix B presents the results with the data from SCC and computed values by elliptical orbit and circular orbit. The behavior of the error deviation for analytical solution and CCS data for attitude angles (α and δ) along time are shown in Fig. 4 and 5, respectively.

The mean error deviations are shown in Table 2 for different period of time. For this satellite there is no significant difference between the dipole and quadripole model.

Over the test period of the 12 days and with the quadripole model and elliptical orbit the difference between theory and CCS data has mean error deviation in right ascension of -0.1267^{o} and -0.1358^{o} in the declination, which are within the dispersion range of the attitude determination system performance of INPE's control center.

Table 2 – Mean error for different time simulation and SCD2 (similar notation of Table 1)

Time Simulation (days)	12	8	5	2
α_{CCS} - α_{QE} (^{o})	- 0.1267	- 0.0200	0.0018	0.0010
α_{CCS} - α_{DE} (^{o})	- 0.1267	- 0.0200	0.0180	0.0100
α_{CCS} - α_{QC} (^{o})	- 0.1334	- 0.2468	0.0153	0.0094
α_{CCS} - α_{DC} (^{o})	- 0.1267	- 0.0200	0.0180	0.0100
δ_{CCS} - δ_{QE} (^{o})	- 0.1358	- 0.0925	- 0.0520	- 0.0010
δ_{CCS} - δ_{DE} (^{o})	- 0.1358	- 0.0925	- 0.0520	- 0.0100
δ_{CCS} - δ_{QC} (^{o})	- 0.1312	- 0.0894	- 0.0502	- 0.0096
δ_{CCS} - δ_{DC} (^{o})	- 0.1358	- 0.0925	- 0.0520	- 0.0100

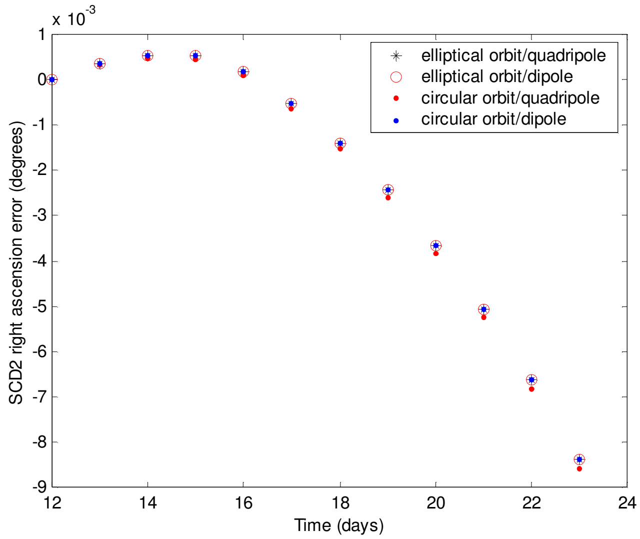

Figure 4 – SCD2 right ascension error evolution.

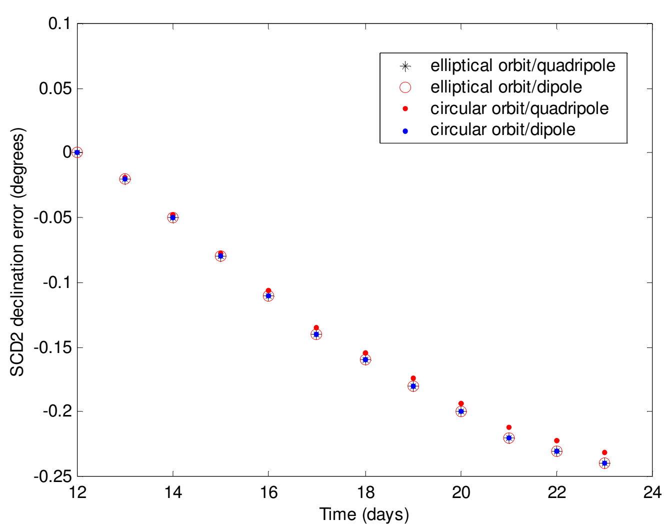

Figure 5 – SCD2 declination error evolution.

Pointing deviation

For the validation of the analytical theory it is important to observe the deviation between the actual SCC supplied spin axis and the analytically computed spin axis, for each satellite. It can be computed by:

$$\theta = \mathrm{Cos}^{-1}\left(\hat{i}\cdot\hat{i}_c + \hat{j}\cdot\hat{j}_c + \hat{k}\cdot\hat{k}_c\right) \tag{26}$$

where $(\hat{i}, \hat{j}, \hat{k})$ indicates the unity vectors computed by SCC and $(\hat{i}_c, \hat{j}_c, \hat{k}_c)$ indicates the unity vector computed by present theory.

The Fig. 6 and 7 present the pointing deviation for the period of test for SCD1 and SCD2, with the dipole and the quadripole model.

For the SCD1, using the quadripole model and the time simulation of 11 days the mean pointing deviation was around 1.2^o, which isn't within the dispersion range of the attitude determination system performance of INPE's control center (0.5^o). Then for SCD1 the period of the application of the theory is restrict for 5 days, with the mean pointing deviation around 0.4735^o.

On the other hand, for the SCD2, using the quadripole model and the time simulation of 12 days the mean pointing deviation was around 0.154^o, which is within the dispersion range of the attitude determination system performance of INPE's control center. Therefore for SCD2 these analytical approach can be used for more than 12 days.

Anyway the period of applications of these theory depends on the precision mission of the satellite.

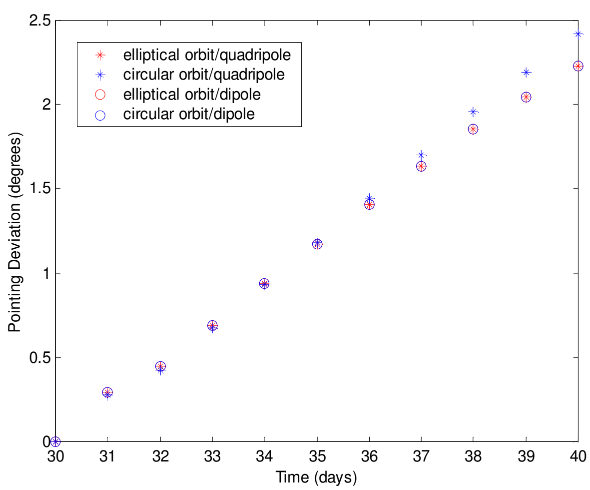

Figure 6 – SCD1 Pointing deviation evolution.

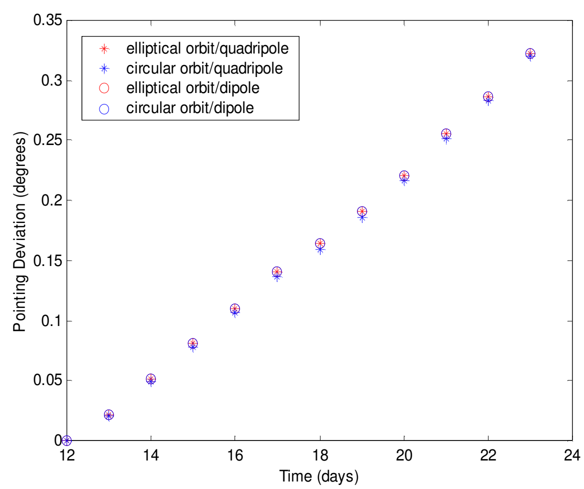

Figure 7 – SCD2 Pointing deviation evolution.

6. Summary

In this paper an analytical approach was presented to the spin-stabilized satellite attitude propagation taking in account the residual magnetic torque. The mean components of this torque in the satellite body reference system have been obtained and the theory shows that there is no residual torque component along the spin axis (z-axis). Therefore this torque does not affect the spin velocity magnitude, but it can cause a drift in the satellite spin axis. The results agree with those presented in Thomas and Capellari (1964), which were obtained through another approach using the instantaneous torque, and with Zanardi et al. (2005), which used the inclined dipole model for geomagnetic field.

The theory was applied to the spin stabilized Brazilian's satellites SCD1 and SCD2 in order to validate the analytical approach, using dipole and quadripole model for geomagnetic field.

The result of the 5 days simulations SCD1 shows a good agreement between the analytical solution and the actual satellite behavior. The difference between theory and CCS data has mean error deviation in right ascension of 0.1616^o and -0.4677^o in the declination, and the mean pointing deviation was around 0.4735^o, which are within the dispersion range of the attitude determination system performance of INPE's control center.

For the satellite SCD2, over the 12 days test period, the difference between theory and CCS data has mean error deviation in right ascension of -0.1334^o and -0.1312^o in the declination, and the mean pointing deviation was around 0.154^o, which are within the dispersion range of the attitude determination system performance of INPE's Control Center. Therefore for SCD2 this analytical approach can be used for more than 12 days.

Thus the procedures are useful for modeling the dynamics of spin stabilized satellite attitude perturbed by residual magnetic torques but the time simulation depend on the precision required for satellite mission.

ACKOWLEDGMENTS

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APPENDIX A: Simulation Results for 11 days period for SCD1 satellite (in degree): INPE's Satellite Control Center Data (index CCS), computed results with quadripole model and elliptical orbit (index QE), computed results with dipole model and elliptical orbit (index DE), computed results with quadripole model and circular orbit (index QC), computed results with dipole model and circular orbit (index QE),

Day	α_{CCS}	α_{QE}	$\alpha_{CCS} - \alpha_{QE}$	α_{DE}	$\alpha_{CCS} - \alpha_{DE}$	α_{QC}	$\alpha_{CCS} - \alpha_{QC}$	α_{DC}	$\alpha_{CCS} - \alpha_{DC}$
22 /08 /93	282.70	282.700000	0.0000	2.82700000	0.0000	282.700000	0	282.700000	0
23 /08 /93	282.67	282.700159	-0.0302	2.82700000	-0.0300	282.721641	-0.051641	282.700000	-0.030000
24 /08 /93	283.50	282.699947	0.8001	2.82700001	0.8000	282.715057	0.784943	282.700001	0.799999
25 /08 /93	283.01	282.700441	0.3096	2.82700001	0.3100	282.673686	0.336314	282.700001	0.309999
26 /08 /93	282.43	282.701475	-0.2715	2.82700002	-0.2700	282.587710	-0.157710	282.700002	-0.270002
27 /08 /93	281.76	282.701926	-0.9419	2.82700002	-0.9400	282.452550	-0.692550	282.700002	-0.940002
28 /08 /93	281.01	282.701866	-1.6919	2.82700003	-1.6900	282.263087	-1.253087	282.700003	-1.690003
29 /08 /93	280.18	282.702400	-2.5224	2.82700003	-2.5200	282.015751	-1.835751	282.700003	-2.520003
30 /08 /93	279.29	282.703577	-3.4136	2.82700004	-3.4100	281.709074	-2.419074	282.700004	-3.410004
31 /08 /93	278.34	282.704318	-4.3643	2.82700004	-4.3600	281.343930	-3.003930	282.700004	-4.360004
01 /09/ 93	277.36	282.704411	-5.3444	2.82700005	-5.3400	280,924018	-3,564018	282.700005	-5.340005
Day	δ_{CCS}	δ_{QE}	$\delta_{CCS} - \delta_{QE}$	δ_{DE}	$\delta_{CCS} - \delta_{DE}$	δ_{QC}	$\delta_{CCS} - \delta_{QC}$	δ_{DC}	$\delta_{CCS} - \delta_{DC}$
22 /08 /93	79.64	79.640000	0.0000	79.64000000	0.0000	79.640000	0	79.64000000	0
23 /08 /93	79.35	79.639886	-0.2899	79.64000003	-0.2900	79.625125	-0.275125	79.64000003	-0.290000
24 /08 /93	79.22	79.639416	-0.4194	79.64000005	-0.4200	79.617200	-0.397200	79.64000005	-0.420000
25 /08 /93	78.95	79.639471	-0.6895	79.64000007	-0.6900	79.618154	-0.668154	79.64000007	-0.690000
26 /08 /93	78.70	79.639907	-0.9399	79.64000010	-0.9400	79.630291	-0.930291	79.64000010	-0.940000
27 /08 /93	78.48	79.639830	-1.1598	79.64000012	-1.1600	79.655185	-1.175185	79.64000012	-1.160000
28 /08 /93	78.27	79.639304	-1.3693	79.64000014	-1.3700	79.694000	-1.424000	79.64000014	-1.370000
29 /08 /93	78.08	79.639225	-1.5592	79.64000016	-1.5600	79.747315	-1.667315	79.64000016	-1.560000
30 /08 /93	77.91	79.639637	-1.7296	79.64000019	-1.7300	79.814993	-1.904993	79.64000019	-1.730000
31 /08 /93	77.78	79.639662	-1.8597	79.64000021	-1.8600	79.896164	-2.116164	79.64000021	-1.860000
01 /09/ 93	77.67	79.639122	-1.9691	79.64000023	-1.9700	79,989178	-2,319178	79.64000023	-1.970000

APPENDIX B: Simulation Results for 12 days period for SCD2 satellite (in degree, similar notation of the APPENDIX A)

Day	α_{CCS}	α_{QE}	α_{CCS} - α_{QE}	α_{DE}	α_{CCS} - α_{DE}	α_{QC}	α_{CCS} - α_{QC}	α_{DC}	α_{CCS} - α_{DC}
12 /02 /02	278.71	278.710000	0.0000	278.71000000	-0.0000	278.710000	0	278.710000000	0
13 /02 /02	278.73	278.709999	0.0200	278.71000000	0.0200	278.711301	0.018699	278.710000001	0.02000000
14 /02 /02	278.74	278.710000	0.0300	278.71000000	0.0300	278.712675	0.027325	278.710000003	0.03000000
15 /02 /02	278.74	278.710000	0.0300	278.71000000	0.0300	278.714079	0.025921	278.710000004	0.03000000
16 /02 /02	278.72	278.709999	0.0100	278.71000001	0.0100	278.715470	0.004530	278.710000006	0.01000000
17 /02 /02	278.68	278.709999	-0.0300	278.71000001	-0.0300	278.716800	-0.036800	278.710000007	0.00300000
18 /02 /02	278.63	278.710000	-0.0800	278.71000001	-0.0800	278.718026	-0.088026	278.710000009	0.00800000
19 /02 /02	278.57	278.710001	-0.1400	278.71000001	-0.1400	278.719110	-0.149110	278.710000008	-0.14000001
20 /02 /02	278.50	278.710000	-0.2100	278.71000001	-0.2100	278.720022	-0.220022	278.710000011	-0.21000001
21 /02 /02	278.42	278.709999	-0.2900	278.71000001	-0.2900	278.720740	-0.300740	278.710000013	-0.29000001
22 /02 /02	278.33	278.710000	-0.3800	278.71000001	-0.3800	278.721252	-0.391252	278.710000014	-0.38000001
23 /02 /02	278.23	278.710002	-0.4800	278.71000002	-0.4800	278.721562	-0.491562	278.710000015	-0.48000002
Day	δ_{CCS}	δ_{QE}	δ_{CCS} - δ_{QE}	δ_{DE}	δ_{CCS} - δ_{DE}	δ_{QC}	δ_{CCS} - δ_{QC}	δ_{DC}	δ_{CCS} - δ_{DC}
12 /02 /02	63.47	63.470000	0.0000	63.470000000	0.0000	63.470000	0	6.3470000000	0
13 /02 /02	63.45	63.469998	-0.0200	63.470000000	-0.0200	63.469153	-0.019153	6.3470000000	0.02000000
14 /02 /02	63.42	63.470002	-0.0500	63.470000001	-0.0500	63.468246	-0.048246	6.3470000001	0.05000000
15 /02 /02	63.39	63.470005	-0.0800	63.470000001	-0.0800	63.467304	-0.077304	6.3470000001	0.08000000
16 /02 /02	63.36	63.470002	-0.1100	63.470000001	-0.1100	63.466358	-0.106358	6.3470000001	-0.11000000
17 /02 /02	63.33	63.470000	-0.1400	63.470000002	-0.1400	63.465436	-0.135436	6.3470000002	-0.14000000
18 /02 /02	63.31	63.470003	-0.1600	63.470000002	-0.1600	63.464568	-0.154568	6.3470000002	-0.16000000
19 /02 /02	63.29	63.470006	-0.1800	63.470000002	-0.1800	63.463780	-0.173780	6.3470000002	-0.18000000
20 /02 /02	63.27	63.470004	-0.2000	63.470000003	-0.2000	63.463093	-0.193093	6.3470000003	-0.20000000
21 /02 /02	63.25	63.470000	-0.2200	63.470000003	-0.2200	63.462524	-0.212524	6.3470000003	-0.22000000
22 /02 /02	63.24	63.470002	-0.2300	63.470000003	-0.2300	63.462083	-0.222083	6.3470000003	-0.23000000
23 /02 /02	63.23	63.470006	-0.2400	63.470000003	-0.2400	63.461771	-0.231771	6.3470000003	-0.24000000