Solar sail propulsion systems enable a wide range of missions that require constant thrust or high delta-V over long mission times. One particularly challenging mission type is a comet rendezvous mission. This paper presents optimal low-thrust trajectory designs for a range of sailcraft performance metrics and mission transit times that enables a comet rendezvous mission. These optimal trajectory results provide a trade space which can be parameterized in terms of mission duration and sailcraft performance parameters such that a design space for a small satellite comet chaser mission is identified. These results show that a feasible space exists for a small satellite to perform a comet chaser mission in a reasonable mission time.

INTRODUCTION

With very few exceptions, spacecraft missions are designed based on impulsive thrust maneuvers produced conventional chemical propulsion systems. Scientists often devise mission objectives that are difficult to accomplish with current state-of-the-art propulsion technology. Missions such as asteroid surveys, high inclination solar orbits, and comet rendezvous place enormous demands on a typical reaction-mass propulsion system. Other missions demand an entirely new class of non-Keplerian orbits. Exotic missions such as station-keeping at artificial Lagrange points and orbits displaced from the ecliptic require a continual thrusting for the duration of the mission. These important missions cannot be achieved with conventional expendable propellants.

Moreover, the high cost of large-scale science missions - typically in the range of millions of dollars per kilogram of instrument payload - provides a significant incentive to investigate the potential for small (and mini-, micro-, or nano-) satellite missions. Considerable advances have been made in structures, avionics, power, and communication systems for smallsats, but to fully achieve the potential cost advantages for smallsats will require fundamental advances in smallsat propulsion systems. Solar sail propulsion systems offer much promise for enabling unconventional, non-Keplerian orbits, high delta-V orbits, and meeting the demands of smallsat applications. Solar sail propulsion utilizes the solar radiation pressure exerted by the momentum transfer of reflected photons. The integrated effect of a large number of photons is required to generate an appreciable momentum transfer which implies a large sail area. And since acceleration is inversely proportional to mass for a given thrust force, the mass of the sailcraft must be kept to a minimum.

Figure 1 illustrates how the solar radiation pressure is utilized for propulsion. Incident rays of sunlight reflect off the solar sail at an angle \( \theta \) with respect to the sail normal direction. Assume specular reflection from a perfectly flat sail membrane that produces two components of force, one in the direction of the incident sunlight and the second in a direction normal to the incident rays. When the force vectors are summed, the components tangent to the sail surface cancel and the components normal to the surface add to produce the thrust force in the direction normal to the sail surface. For a perfectly reflective 40 meter x 40 meter square sail at 1 AU from the sun, the solar radiation thrust force is approximately 0.03 Newtons.

* Engineer, Guidance, Navigation and Mission Analysis Branch, EV42, NASA Marshall Space Flight Center, Huntsville, AL 35812, (256) 544-1872, robert.w.stough@nasa.gov.

† Senior Engineer, Guidance, Navigation and Mission Analysis Branch, EV42, NASA Marshall Space Flight Center, Huntsville, AL 35812, (256) 544-3839, andrew.f.heaton@nasa.gov, AAS Member, AIAA Member.

‡ Branch Chief, Guidance, Navigation and Mission Analysis Branch, EV42, NASA Marshall Space Flight Center, Huntsville, AL 35812, (256) 544-1435, mark.s.whorton@nasa.gov, Associate Fellow AIAA.
The thrust derived from solar radiation pressure can be used to change the orbit of the sailcraft. If the sail is oriented such that the thrust force is opposite the direction of motion, as in Figure 1 for a heliocentric orbit, the orbit spirals inward. Conversely, if the thrust is in the direction of motion, the sailcraft orbit spirals outward. Orbit inclination changes result when a component of the thrust force is oriented perpendicular to the orbit plane.

COMET CHASER MISSION STUDY

Planetary science missions are costly endeavors. They are also unique, which makes cost comparisons between missions difficult. Nonetheless cost trends such as those illustrated in Figure 2 indicate that a typical science mission can be expected to cost on the order of one million dollars per kilogram of spacecraft mass, or alternatively around a quarter of that cost per kilogram of instrument payload. This leaves a potential "open door of opportunity" for low-cost science mission if the spacecraft mass can be considerably scaled down. To date, considerable progress has been made in avionics and structures for small satellites. The exception to this progress is small satellite propulsion systems, particularly for those missions that require high delta-V orbit changes such as a comet chaser. Solar sail propulsion systems perhaps afford a low-mass solution to this need for small satellites and enable missions in the small satellite open door of opportunity.

Solar sailcraft performance can be described in terms of a "characteristic acceleration", \( a_0 \), which is defined as

\[
a_0 = \frac{2 \eta PA}{m}
\]

where \( A \) is the sail area, \( P \) is the magnitude of the solar radiation pressure intensity at one astronomical unit, \( \eta \) is the sail efficiency, and \( m \) is total spacecraft mass. Figure 4 below illustrates the optimal solar-sail trajectory designs for a range of sailcraft characteristic acceleration levels. Each point on this curve represents a different optimal rendezvous trajectory and thus a different mission transit time for a range of typical characteristic acceleration values.

By writing the total system mass as the sum of the sail subsystem mass, \( m_s = K*A \), where \( K \) is the "sail areal density," and the "residual mass," \( m_r \), of the bus and payload, an upper limit on the residual mass can be expressed as a function of the sailcraft design parameters and the propulsive performance as

\[
m_r = \left( \frac{2 \eta P}{a_0} - K \right) A
\]

From this expression, a nonzero residual mass implies an inverse relationship between sail areal density and characteristic acceleration

\[
\frac{2 \eta P}{a_0} \geq K
\]

which defines the meaningful sailcraft system and mission design trade space illustrated by Figure 3. This figure illustrates the residual mass available for science instruments and bus subsystems given a particular sail areal density and mission transit time, from which various sailcraft concepts and configurations can be evaluated for the comet chaser mission. Comet chaser spacecraft with sail subsystems in the 5 kg range and bus+payload mass in the five kilogram range can potentially rendezvous with a comet in less than 10 years according to these analyses, with faster trip times achievable by mass savings in either the sail subsystem or the bus/payload subsystem. These sailcraft design concepts will be the subject of a future companion paper.
MODELS

The Solar Sail Spaceflight Simulation Software (S5) was used to quickly create and optimize solar sail trajectories. S5 was created by NASA's Jet Propulsion Laboratory (JPL) to support solar sail development by NASA's In-Space Propulsion Office. The intent of S5 was to raise the Technology Readiness Level of Solar Sail Propulsion systems by creating a test bed for both solar sail mission design and guidance, navigation and control (GNC) design.

S5 was built using JPL planetary ephemerides as well as including advanced Solar Sail optical models. S5 includes a recently added module named OPT. OPT optimizes the solar sail attitude relative to the sun to meet the user defined constraints/targets. OPT is a beta version and is currently being developed and tested. OPT utilizes SNOPT to perform all optimizations. Due to OPT's lack of validation, a separate simulation model was developed which utilizes the LaGrange Planetary Equations for solar sail propagation. All OPT trajectories were validated using the LPE propagator.

MISSION DESIGN

The scientific justification for chasing a comet is strong and the cost of solar sails is very low. The question remains, what comet should be targeted? The scientific community feels that any comet that could be followed in its orbit for an entire revolution would be greatly beneficial. While some comets might possess features that are more attractive than others, by definition we have to limit the trade space to comets that are in relatively low orbits compared to the majority.

Comet orbits are divided into three classes by the scientific community. Long-period comets are those with periods greater than 200 years. Short-period comets classically are those with periods of less than 200 years, but more recently they have been subdivided into two classes. “Jupiter short-period” comets are those with periods less than 20 years, while “Halley-comet short-period”, sometimes also called “intermediate period” comets, have periods between 20 and 200 years. The short period comets are thought to come from the Kuiper Belt, while long-period comets come from the Oort cloud.

Solar sails will not work well for the long-period or intermediate-period comets due to solar distance (and thus low radiation pressure). These classes of comets also have orbits measured in 10s or even 100s of years, and no one wants to be part of a mission that exceeds a human lifetime. Finally, the energy requirements to achieve the long- and intermediate-period classes of comets also make them undesirable. So the trade space of comets that could be considered is limited to the Jupiter-class comets.

At the time of this writing, the total number of comets that are considered periodic at all (meaning they have been observed through perihelion more than once) is 234. Of these, the great majority have undesirably high periods. Of the remaining comets, we also eliminated those with inclinations greater than 20 degrees, and restricted the period to less than 6 years and the maximum height of perihelion to 2 AU. With those restrictions, the comets considered to be attractive candidates declines to somewhere between 20 and 30, depending on how rigorously the limitations are enforced (for instance we might accept a very low-inclination comet that slightly violates the period restriction). A sampling of desirable candidates appears in Table 1.

Of the comets in Table 1, plus a few others, we conducted a study of simple ballistic delta-Vs in aid in narrowing the field. We computed the delta-V cost of a simple Hohmann transfer and also what the inclination change would cost and combined this into a single delta-V. We also assumed for this trade study, and for the overall study, that the starting point is from a 1 AU circular heliocentric orbit. Some results of this study appear in Table 2.
Table 1) Some Jupiter-class Comet Candidates

<table>
<thead>
<tr>
<th>Comet</th>
<th>SMA (AU)</th>
<th>Period (yr)</th>
<th>Perihelion (AU)</th>
<th>Inclination (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>125P Spacewatch</td>
<td>3.126018</td>
<td>5.53</td>
<td>1.523612</td>
<td>9.9858</td>
</tr>
<tr>
<td>73P/Schwassman-Wachmann</td>
<td>3.060407</td>
<td>5.35</td>
<td>0.939237</td>
<td>11.3959</td>
</tr>
<tr>
<td>46P/Wirtanen</td>
<td>3.092840</td>
<td>5.44</td>
<td>1.057654</td>
<td>11.77392</td>
</tr>
<tr>
<td>185P/Petriew</td>
<td>3.106031</td>
<td>5.47</td>
<td>0.937604</td>
<td>13.9745</td>
</tr>
<tr>
<td>182P/LONEOS</td>
<td>2.932476</td>
<td>5.02</td>
<td>0.979669</td>
<td>16.9051</td>
</tr>
<tr>
<td>9P/Tempel</td>
<td>3.121530</td>
<td>5.52</td>
<td>1.506167</td>
<td>10.5506</td>
</tr>
<tr>
<td>2P/Encke</td>
<td>2.218032</td>
<td>3.30</td>
<td>0.339269</td>
<td>11.7543</td>
</tr>
<tr>
<td>P/2006 U1 unnamed</td>
<td>2.776815</td>
<td>4.63</td>
<td>0.510629</td>
<td>8.4331</td>
</tr>
<tr>
<td>88P/Howell</td>
<td>3.112985</td>
<td>5.69</td>
<td>1.365252</td>
<td>4.3820</td>
</tr>
</tbody>
</table>

Table 2) Ballistic Delta-Vs for Comet Candidates

<table>
<thead>
<tr>
<th>Comet</th>
<th>Aphelion DV (km/s)</th>
<th>Ine DV (km/s)</th>
<th>Perihelion DV (km/s)</th>
<th>Total DV (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>125P Spacewatch</td>
<td>8.484</td>
<td>1.409</td>
<td>1.469</td>
<td>11.362</td>
</tr>
<tr>
<td>73P/Schwassman-Wachmann</td>
<td>8.780</td>
<td>1.479</td>
<td>0.194</td>
<td>10.451</td>
</tr>
<tr>
<td>46P/Wirtanen</td>
<td>8.747</td>
<td>1.541</td>
<td>0.178</td>
<td>10.466</td>
</tr>
<tr>
<td>185P/Petriew</td>
<td>8.833</td>
<td>1.781</td>
<td>0.197</td>
<td>10.813</td>
</tr>
<tr>
<td>182P/LONEOS</td>
<td>8.592</td>
<td>2.309</td>
<td>0.0668</td>
<td>10.968</td>
</tr>
<tr>
<td>9P/Tempel</td>
<td>8.490</td>
<td>1.482</td>
<td>1.426</td>
<td>11.399</td>
</tr>
<tr>
<td>2P/Encke</td>
<td>7.979</td>
<td>1.888</td>
<td>3.463</td>
<td>13.330</td>
</tr>
<tr>
<td>P/2006 U1 unnamed</td>
<td>8.694</td>
<td>1.122</td>
<td>1.943</td>
<td>11.759</td>
</tr>
<tr>
<td>88P/Howell</td>
<td>8.575</td>
<td>0.603</td>
<td>1.055</td>
<td>10.233</td>
</tr>
</tbody>
</table>

The results in Table 2 accomplish three things. First they provide a way of determining which comet will require the greatest total energy change to achieve rendezvous (though admittedly not precisely analogous to a low-thrust transfer). Second, the results allow an idea of how the energy transfer is distributed between inclination change and energy change, and which of those requires the greatest change. Third, it allows rough sizing of the solar sail capability needed to achieve those delta-Vs in each phase of the mission.

At this point, we can also apply some basic heuristic guidelines to further narrow the field and also to begin getting some insight on an initial guess at a fully optimized trajectory for one of the candidate comets. For instance, for a ballistic mission, an inclination change is typically accomplished at the highest point in the orbit, as far from the central gravity source as possible, but how does this compare with the lack of thrust available due to solar distance? Will the eventual optimized solution perhaps have clearly identifiable stages where changing the inclination or energy is the primary effect of the thrust from the sail? Table 2 did not answer these questions, but did help make better "initial guesses" at the optimized trajectory (always an important factor for convergence).

From past experience, an optimized trajectory usually bears some resemblance to its ballistic counterpart. For instance, a simple Earth-to-Mars transfer typically resembles a Hohmann transfer with
very long maneuver times. And to use a solar sail example, published trajectories for the Solar Polar Imager, a high-inclination mission to map the solar poles at 0.5 AU, break into clearly identifiable “pumping” (i.e., energy changing) and “cranking” (changing inclination) phases, although there is a small amount of overlap between the two. Thus, by looking at the ballistic Delta-V calculations in Table 2, we get some heuristic insight into initial guesses at the trajectory and guidance of a fully optimized trajectory.

As a follow-up to the ballistic Delta-V calculations, we also propagated portions of the solar sail trajectory in S5 for inclination-only (or cranking) and energy-change only (or pumping) segments. These studies accomplished three purposes. One, we got additional insight into how long it would take a solar sail to accomplish inclination change, aphelion-raising and perihelion-raising if that were all that needed to be accomplished. Two, it helped greatly in sail sizing and calculating the necessary characteristic acceleration of the solar sail. Three, we were able to test the S5 propagation software in a piece-wise fashion, and in doing so uncovered some pitfalls to avoid.

Using the results of the ballistic Delta-V studies and the suboptimal propagations in S5, we were able to achieve a somewhat better initial guess for the input into OPT, as well as gain insight into questions such as “What Time-of-Flight (TOF) is reasonable for a given sail performance”?. Finally, and most important, based on the results of these studies we also selected the Howell comet as the best candidate to test S5/OPT and demonstrate the feasibility of a solar sail comet mission with near-term sail technology.

**Solar Sail Optimization with OPT**

The low-thrust optimization portion of S5, referred to as OPT was used to converge fully optimized low-thrust trajectories. OPT is a direct method of optimization that uses the SNOPT package.
We achieved good results with OPT, after a great deal of iteration. Due to budget constraints, S5 was not fully beta-tested by JPL prior to delivery and so some additional work was required to streamline OPT. With some additional programming effort, OPT did converge to an Howell-like orbit for a range of characteristic accelerations of comets. Figure 4 is a plot of time of flight vs. characteristic acceleration. We note that the orbits in Figure 4 were matched for inclination and energy only, we did not attempt to time them precisely to make sure Howell was actually in the point on it's orbit that was intercepted. The goal of the particular study in Figure 4 was to demonstrate feasibility for a large number of different sails, not design a complete mission.

After an analysis of what might be feasible in the near future vs. reasonable TOF, we selected a 0.3 mm/s^2 characteristic as the best compromise. For this characteristic acceleration, we have a fully optimized trajectory.

LaGrange Planetary Equations

As mentioned above, we had to use S5 with some caution because technically it was still in Beta-test mode. As a result, we developed an analytical method of integrating the solar sail trajectory with the LaGrange Planetary equations. The method was to take a converged trajectory from OPT, and use the same characteristic acceleration and guidance in an independent Matlab integration to verify S5.

This method did validate the results from S5, although it also uncovered some issues with certain of the integrators included in S5. Thus, this was a good thing to do, as it not only aided the immediate study but also helped ensure proper use of OPT.

Fully Optimized Trajectory

Figure 5 illustrates a fully optimized trajectory that we discovered with OPT. The result in Fig. 5 is based on a 0.3 mm/s^2 characteristic acceleration and reasonable limits on the time of flight.

![Figure 5) Converged Rendezvous with Comet Howell](image)
A study of Figure 5 and other data shows how the solar sail is used to converge to the rendezvous point. A somewhat unanticipated result is that the sail initially reduces perihelion even though the perihelion of Howell lies outside the 1-AU starting point. This allows the sail to achieve a higher thrust for initial inclination and energy reduction. In contrast to the SPI result, the comet chaser does not reduce inclination and energy in essentially separate mission phases. Energy and inclination are reduced throughout the mission. However, there is more emphasis on inclination earlier in the mission, which matches the single-arc propagations above, which implied that inclination change was more effective at closer solar distances and that inclination change would be preferred earlier in the mission.

Figure 6) Guidance for Fully Optimized Comet Howell Rendezvous

Figure 6 presents the cone and clock angles for the sail. The pattern of cone and clock angles tends to vary as the orbit period, so the angles are maintained longer later in the mission as the period of the solar sail orbit gets larger. The early cone and clock angles also reflect the need to increase inclination and reduce energy early in the mission and are consistent with that goal.

CONCLUSIONS

A comet chaser mission is feasible in the near future using a solar sail. This mission will provide great benefits to space science at low cost if pursued.
Figure 1. Solar Radiation Thrust Force (NASA/JPL)

Figure 2. Mission Mass and Cost Trade
Figure 3. Sailcraft Mission Trade Space