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Dispersive Elements for Enhanced Laser Gyroscopy and Cavity Stabilization

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ABSTRACT

We analyze the effect of a highly dispersive element placed inside a modulated optical cavity on the frequency and amplitude of the modulation to determine the conditions for cavity self-stabilization and enhanced gyroscopic sensitivity. We find an enhancement in the sensitivity of a laser gyroscope to rotation for normal dispersion, while anomalous dispersion can be used to self-stabilize an optical cavity. Our results indicate that atomic media, even coherent superpositions in multilevel atoms, are of limited use for these applications, because the amplitude and phase filters work against one another, i.e., decreasing the modulation frequency increases its amplitude and vice-versa. On the other hand, for optical resonators the dispersion reversal associated with critical coupling enables the amplitude and phase filters to work together. We find that for over-coupled resonators, the absorption and normal dispersion on-resonance increase the contrast and frequency of the beat-note, respectively, resulting in a substantial enhancement of the gyroscopic response. Under-coupled resonators can be used to stabilize the frequency of a laser cavity, but result in a concomitant increase in amplitude fluctuations. As a more ideal solution we propose the use of a variety of coupled-resonator-induced transparency that is accompanied by anomalous dispersion.

Keywords: whispering gallery modes, optical resonators, optical gyroscopes, laser stabilization, dispersion.

I. INTRODUCTION

The Sagnac phase shift is transformed into a frequency shift between two counter-propagating modes inside an optical cavity. These modes may be made to interfere at a location outside the cavity, producing a beat-note. Ideally, for the measurement of motion, only the Sagnac effect contributes to the measured beat frequency. However, cavity instabilities, noise, and/or mode coupling due to back-scattering or diffential loss (or gain) can substantially reduce gyro sensitivities, or even result in a dead-band. In recent years there has been an interest in the development of highly dispersive materials whose resonant features can speed up, slow down, stop, store, or reverse the propagation of pulses of light[1-8] as a result of their substantially modified group velocities. We discuss the use of such dispersive elements placed within a laser cavity for the purpose of: (*i*) compensating undesirable phase shifts such as those due to cavity instabilities, and (*ii*) enhancing desirable phase shifts such as that due to rotation.

The use of modified group velocities for the enhancement of an interferometric optical gyroscope has been discussed by several authors in recent years[9-12]. Shahriar has pointed out that the enhancement predicted for an interferometric gyro containing a slow-light medium, applies only to relative rotation measurements, not to measurements of absolute rotation where the source and gyroscope are co-rotating and the Sagnac effect is independent of refractive index[13]. The splitting of modes in a ring cavity, on the other hand, does depend on refractive index, and so it is possible to achieve a dispersion-related enhancement for absolute rotation only in this case. Yet the case of an intracavity dispersive element in a laser gyro is not considered in these works, and is still not thoroughly understood. A major shortcoming has been that the absorptive response of the medium has not been considered.

We derive the equations for the modulation spectroscopy of optical resonators incorporated into a laser cavity, as shown in Figure 1, and demonstrate that such a resonator acts as an amplitude and phase filter, affecting (demodulating and shifting) the frequency components of the cavity instability or Doppler shift. Thus, we consider the modulation to be either deleterious (such as a cavity instability) or desirable (such as a rotation, index of refraction, or other thing to be sensed), and we derive the conditions under which the modulation may be enhanced or diminished.

II. MODULATION SPECTROSCOPY

Consider an arbitrary modulation of carrier frequency $\omega_0 = ck_0$ and modulation frequency $\omega_m = ck_m$. The modulated electric field

$$E(z,t) = \tilde{E}(z,t)\exp[ik_0 z - \omega_0 t + \varphi_0(z,t)] + c.c. \tag{1}$$

can be written in terms of it's carrier and modulated components as

$$\tilde{E}(z,t) = E_0(z,t) + \tilde{E}_m(z,t)\,, \tag{2}$$

where the modulated field can be expressed as the interference of two side-bands

$$\tilde{E}_m(z,t) = E_m(z,t)e^{i\varphi_m(z,t)}[\cos(\psi)e^{i(k_m z-\omega_b t)} + \sin(\psi)e^{-i(k_m z-\omega_b t)}] \tag{3}$$

where

$$\omega_b^0(z,t) = \omega_m t - \theta(z,t)/2 \tag{4}$$

is the beat-frequency, $\theta(z,t) = \varphi_+(z,t) - \varphi_-(z,t)$ is the phase difference, and $\tan(\psi) = E_-(z,t)/E_+(z,t)$ is the amplitude ratio of the interfering side-bands respectively, and subscripts are indicative of frequency components. Equation (3) is that of an ellipse, whose rate of rotation is proportional to the beat frequency, and whose shape is related to the relative modulation,

$$M(z,t) = E_m(z,t)/E_0(z,t) \tag{5}$$

The modulation ellipse evolves with propagation through a dispersive medium. The effective beat-frequency and relative modulation amplitude, respectively evolve according to

$$\Omega_b^p = \omega_b^0 - \frac{1}{2\tau_c p}\sum_{j=1}^{p}[\Phi_+^j - \Phi_-^j], \tag{6}$$

and

$$M^p(z) = \frac{E_m(z)}{E_0(z)} = \left\{\left[\frac{E_+}{E_0}\right]^2 \prod_{j=1}^{p}\frac{\tau_-^j}{\tau_0^j} + \left[\frac{E_-}{E_0}\right]^2 \prod_{j=1}^{p}\frac{\tau_-^j}{\tau_0^j}\right\}^{\frac{1}{2}}. \tag{7}$$

where superscripts indicate the round-trip number. The second term in Equation (6) represents the contribution due to the dispersive medium. The effect of the dispersive medium is considered by the use of a complex transfer function $\tilde{\tau}(\omega) = \tau(\omega)\exp[i\Phi(\omega)]$ where $\Phi(\omega)$ is the effective phase shift introduced by the medium, and $\tau(\omega)$ is the transmission spectrum. Hence the medium acts as both an amplitude and phase filter. The phase filter affects the beat-frequency, whereas the amplitude filter affects the relative modulation. The AM/FM signal ratio is determined roughly by the real and imaginary parts of Equation (3), and also evolves with propagation through the medium.

III. SELF-STABILIZATION OF AN OPTICAL CAVITY

It is clear from Equation (6) that dispersion alters the beat-frequency, anomalous dispersion decreases the beat-frequency, while normal dispersion increases it. Ignoring all forms of dispersion in the cavity except for the dispersive element, the effective group-index for the j^{th} pass is

$$N_g^j = c\frac{dK}{d\omega} = N + \omega\frac{dN}{d\omega} = n_c\left[1 + \frac{1}{\tau_c}\frac{d\Phi^j}{d\omega}\right], \qquad (8)$$

then the group-index averaged over p passes is

$$N_g^p = \frac{1}{p}\sum_{j=1}^{p} N_g^j = n_c\left[1 + \frac{1}{\tau_c p}\sum_{j=1}^{p}\frac{d\Phi^j}{d\omega}\right] \qquad (9)$$

Hence, the sensitivity enhancement of the gyro is just determined by the group-index of one of the side-bands averaged over p passes, i.e.,

$$\frac{N_g^p}{n_c} = \frac{d\Omega_b^p}{d\omega_b^0} \qquad (10)$$

A single under-coupled resonator displays anomalous dispersion on-resonance, similar to that of a two-level atom. It is possible to use this anomalous dispersion to decrease the beat-frequency, and the sensitivity to perturbations in the cavity, and thereby stabilize a laser against the occurrence of unwanted sidebands. This is shown in Figure 1, where the beat frequency and relative modulation are plotted for several different values of the dispersion. All of the calculations in this section utilize a carrier frequency of $\omega_0 = 10^{14} Hz$, a cavity of $1/\tau_c = 100 MHz$ repetition rate, modulation frequency of $\omega_m = 10 KHz$, and a resonator quality factor of $Q = 5\times10^7$, corresponding to a resonance linewidth of 2 *MHz*. This corresponds to 10,000 effective phase-shifts per modulation cycle due to the presence of the resonator in the cavity. Note that the beat frequency undergoes a number of oscillations proportional to the dispersion, and reaches a steady-state value, of $\Omega_b^\infty/\omega_b^0 = 1/\left(2 - N_g^1\right)$. If more initial terms are included prior to the perturbation, the oscillations will be averaged out, and more passes will be required to reach a steady-state.

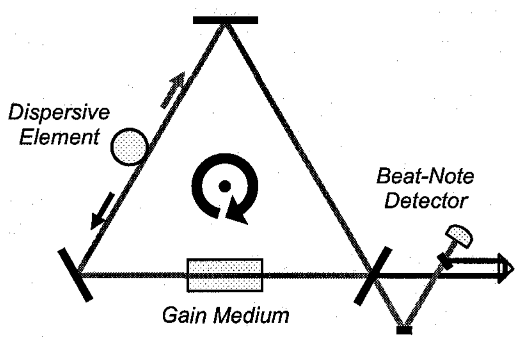


Figure 1. The rotation-induced Doppler-shift of two counter-propagating modes in a ring laser gyro is enhanced by a dispersive element.

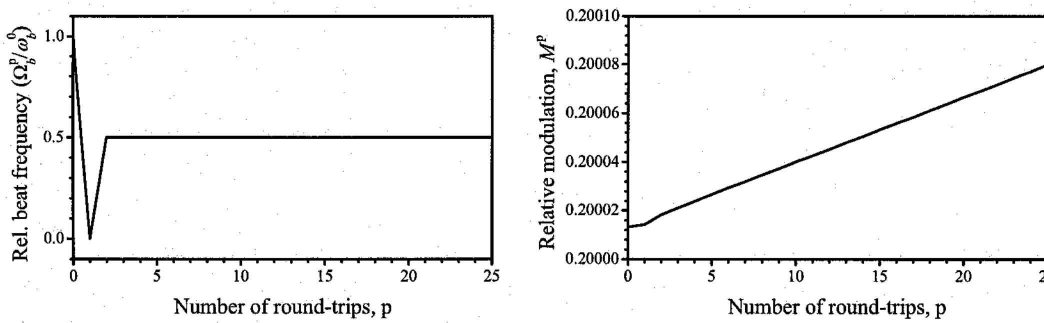


Figure 2a. Evolution of the beat frequency with cavity round trip for an under-coupled resonator with an initial dispersion of $[\Phi_+^1 - \Phi_-^1]/2\tau_c = \omega_b^0$, corresponding to an initial effective group index, $N_g^1 = 0$. The steady-state value value is $\Omega_b^\infty = \omega_b^0/2$.

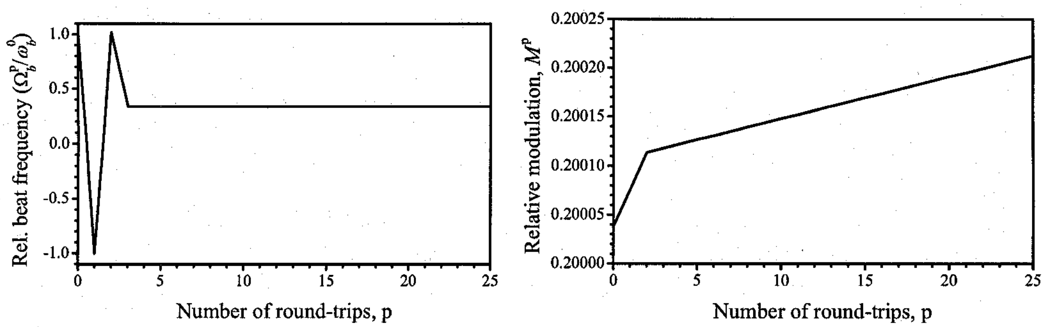


Figure 2b. Evolution of the beat frequency for an under-coupled resonator with an initial dispersion of $[\Phi_+^1 - \Phi_-^1]/2\tau_c = 2\omega_b^0$, corresponding to an initial group index, $N_g^1 = -1$. The steady-state value is $\Omega_b^\infty = \omega_b^0/3$.

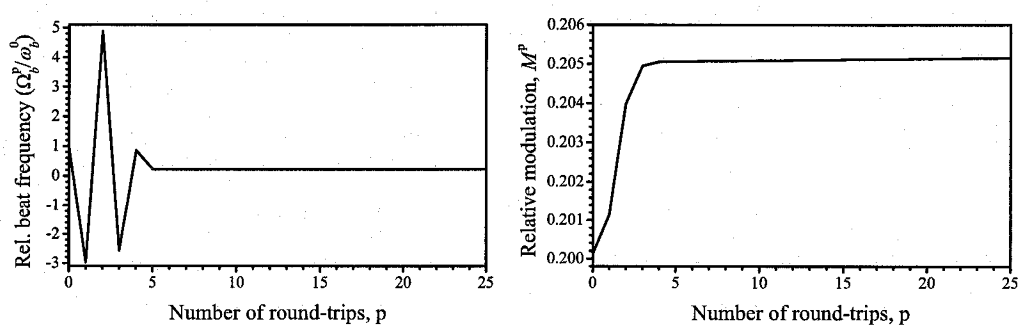
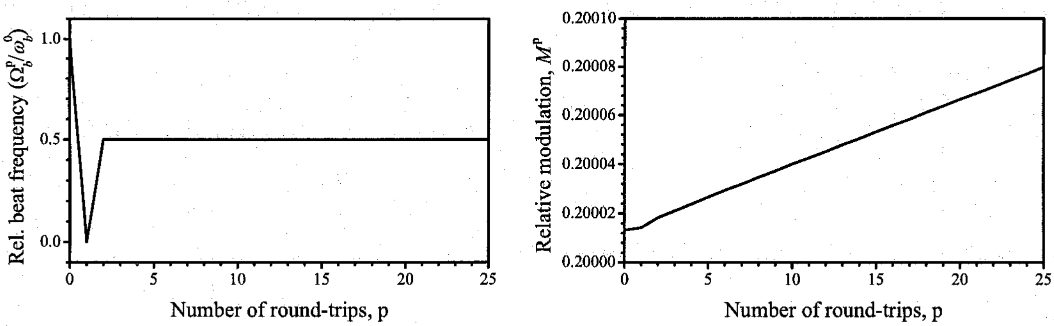


Figure 2c. Evolution of the beat frequency for an under-coupled resonator with an initial dispersion of $[\Phi_+^1 - \Phi_-^1]/2\tau_c = 4\omega_b^0$, corresponding to a group index, $N_g^1 = -3$. The steady-state value is $\Omega_b^\infty = \omega_b^0/5$.

One problem with this approach is that although the frequency of the side-bands is reduced, their relative amplitude increases, as evident by the increase in the relative modulation. These results are also shown in Figure 2, for the case of $N_g^1 = 0$. Note that the carrier frequency is attenuated to a greater degree by the medium than the side bands. A more ideal solution would be to use a medium with both anomalous dispersion, and greater transparency at the carrier frequency than at the side-bands, however this type of medium is uncommon. Coupled-resonator-induced transparency



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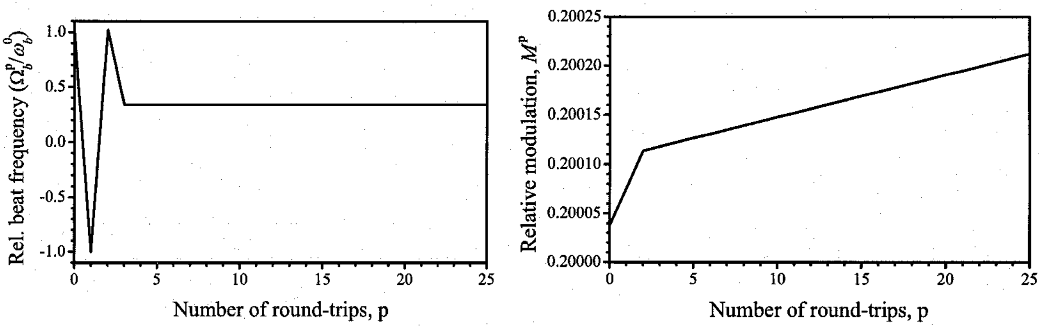

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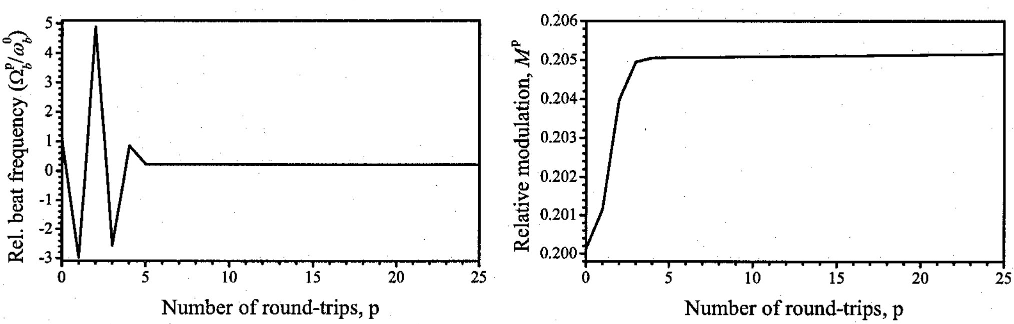

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(CRIT) is an effect that occurs in optical resonators and is analogous to EIT. But in contrast to EIT, which has normal dispersion at line center, CRIT can occur with either normal or anomalous dispersion, depending on the coupling between the resonators. It is only in the case of CRIT with anomalous dispersion that the amplitude and phase filters will decrease both the frequency and amplitude of the modulation, leading to complete cavity self-stabilization.

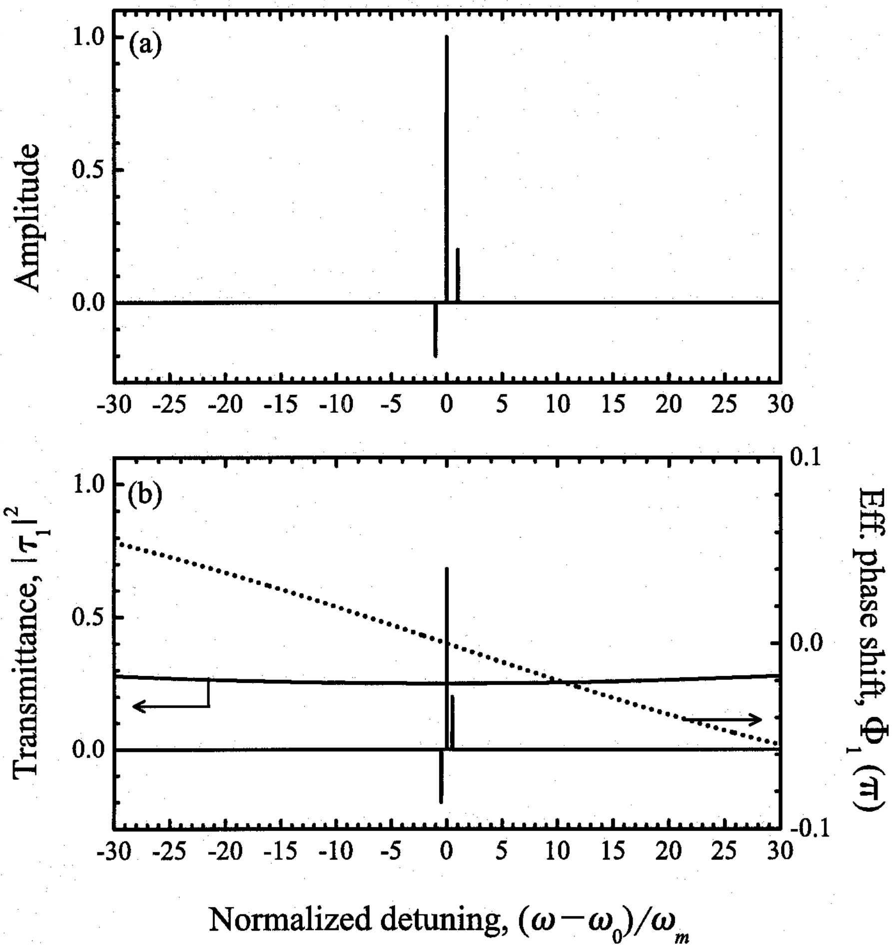

Figure 3. Self-stabilization of FM side-bands for the case of $N_g^1 = 0$. The beat-frequency decreases, but the modulation amplitude increases. (TOP) Initial modulation frequency, and (BOTTOM) after steady-state is obtained. The effective phase shift and transmittance are also shown.

IV. ENHANCED GYROSCOPIC RESPONSE

Note that the phase filter effects any frequency shift that may occur in the cavity. This frequency shift may be due to instability, or it may be a Doppler shift due to rotation of the cavity. The Doppler shift due to rotation can be considered to be simply a strong $(M >> 1)$ AM modulation where the two sidebands represent the two standing-wave eigenmodes, and the carrier frequency results from dissipative effects such as backscattering and/or differential loss. Ideally, for the measurement of rotation, the sidebands will have the same frequencies as the counter-propagating traveling-wave modes, and the residual carrier frequency will be unimportant in comparison with the strong modulation. However, any form of coupling, whether it be conservative (specular reflections) or dissipative (diffuse reflections or differential gain or loss) will cause the sideband frequencies to deviate from the frequencies of the counter-propagating modes, and therefore lead to a reduction in gyro sensitivity at low rates of rotation. Moreover, any dissipative coupling (in the form of back-scattering and/or differential gain or loss) will feed frequencies between the two side modes. If the back-scattering is sufficiently strong, these central frequencies can injection lock the side bands, eliminating the Doppler shift and leading to a dead-band. The amplitude filter effects the relative modulation, and hence the relative size of the dead-band. Therefore, to determine whether a dispersive element improves the performance of a laser gyro, in addition to the modification of the modulation frequency by the dispersion, the modification of the relative modulation by the absorption must also be taken into account.

The amplitude and phase filters can work in concert, or against one another. For atomic media, the amplitude and phase filters work against one another, i.e., decreasing the modulation amplitude increases its frequency and vice-versa. In the case of two-level atoms, this is obvious. For non-inverted two-level atoms, the absorption is higher on-resonance, which attenuates the carrier frequency more than the sidebands, increasing the relative modulation, while at the same time the anomalous dispersion decreases the magnitude of the beat frequency. If the two-level atom is inverted, the amplification is higher on-resonance, decreasing the relative modulation, but now the dispersion is normal, which increases the modulation frequency.

The same is true of coherently prepared three-level atoms. Consider the three prototypical atomic coherence effects of electromagnetically-induced absorption (EIA)[14], electromagnetically-induced transparency (EIT)[15], and gain-assisted superluminality (GAS)[16,17] from a double-gain line. The case of EIA is similar to an absorbing two-level atom, i.e., absorption accompanied by anomalous dispersion on-resonance increase the relative modulation, but decrease the magnitude of the modulation frequency. For the case of EIT, on the other hand, the transparency and normal dispersion on-resonance result in an increased modulation frequency, but decreased relative modulation, i.e., the beat-frequency is enhanced, but so is the relative importance of the dead-band. Finally, in the case of GAS, the higher gain for the sidebands increases the relative modulation, but the anomalous dispersion reduces the modulation frequency.

Therefore, atomic media are of limited usefulness for laser gyro enhancement. The same is not true for optical resonators. The subtle yet important distinction between these materials is that for atomic media the dispersion reverses only when the medium possesses gain, whereas for resonators the dispersion reverses at the critical coupling (even when it is non-amplifying). Indeed, single under-coupled optical resonators display anomalous dispersion (analogous to a two level atom), whereas single over-coupled resonators display normal dispersion on-resonance, i.e., a reversal from anomalous to normal dispersion occurs at critical coupling. Hence, resonators can be non-amplifying ***and*** possess normal dispersion (provided they are over-coupled), whereas atoms must be inverted to possess normal dispersion at resonance. Hence for non-amplifying over-coupled resonators, the amplitude and phase filters can work in concert. In this case the on-resonance absorption increases the relative modulation, while the normal (rather than anomalous) dispersion also increases its frequency, which is beneficial for gyro sensitivity enhancement as we have already discussed. This result is shown in Figures 4 and 5, for a carrier frequency of $\omega_0 = 10^{14}\,Hz$, cavity of $1/\tau_c = 40MHz$ repetition rate, modulation frequency of $\omega_m = 10KHz$ (corresponding to 4,000 effective phase-shifts per modulation cycle), and a resonator quality factor of $Q = 3\times10^7$, corresponding to a resonance linewidth of 3.5 *MHz*.

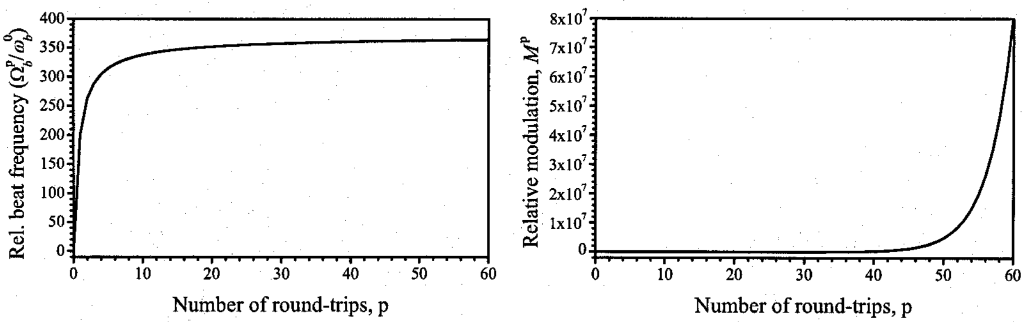

Figure 4. Evolution of the beat frequency for an over-coupled resonator with an initial group index $n_g^0 > 1$. The steady-state value is $\Omega_b^\infty = 360\omega_b^0$.

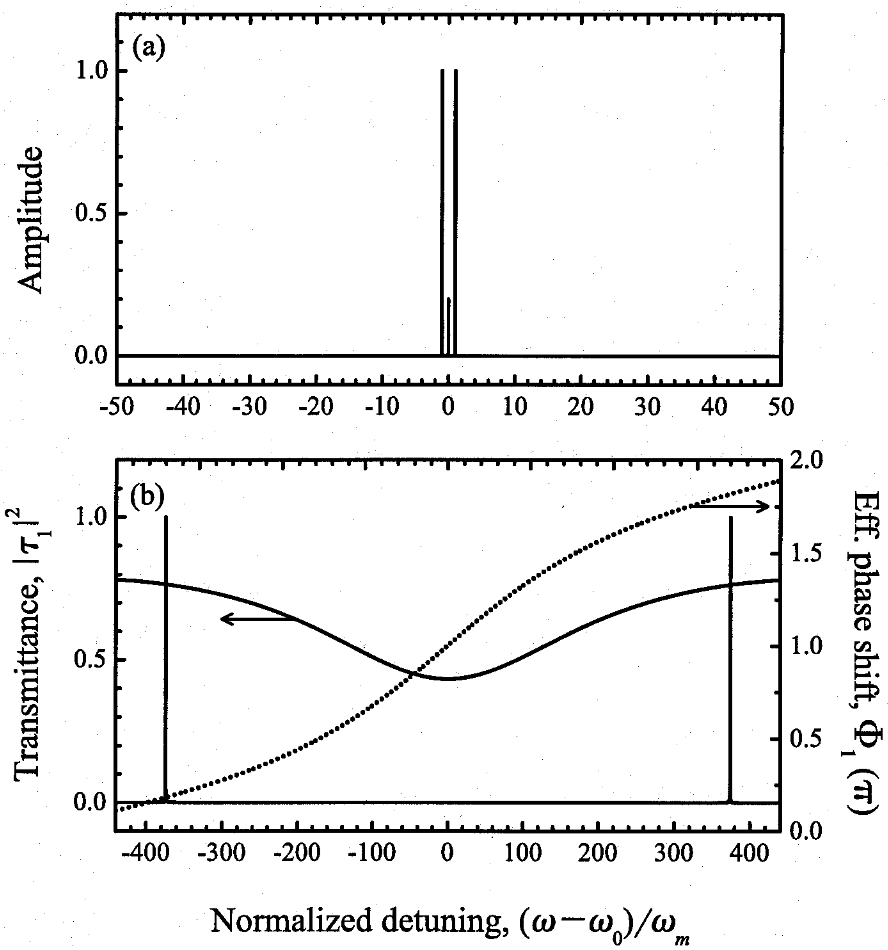

Figure 5. Enhancement of AM sidebands with $M > 1$. Both the beat-frequency and modulation increase. The carrier frequency is suppressed, resulting in a suppression of the dead-band. (TOP) Initial modulation frequency, and (BOTTOM) after steady-state is obtained. The effective phase shift and transmittance are also shown.

V. CONCLUSION

We find an enhancement in the sensitivity of a laser gyroscope to rotation for normal dispersion, while anomalous dispersion can be used to self-stabilize an optical cavity. Our results indicate that atomic media are of limited use for enhancing the laser gyro response. On the other hand, for optical resonators the dispersion reversal associated with critical coupling enables the amplitude and phase filters to work together, resulting in a substantial enhancement of the gyroscopic response.

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