NASA/TM—2008-215246

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Prepared for the
22nd Annual Technical Conference
sponsored by the American Society for Composites
Seattle, Washington, September 17–19, 2007

National Aeronautics and
Space Administration

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July 2008

This work was sponsored by the Fundamental Aeronautics Program at the NASA Glenn Research Center.

Level of Review: This material has been technically reviewed by technical management.

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Probabilistic Usage of the Multi-Factor Interaction Model

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Abstract

A Multi-Factor Interaction Model (MFIM) is used to predict the insulating foam mass expulsion during the ascending of a space vehicle. The exponents in the MFIM are evaluated by an available approach which consists of least squares and an optimization algorithm. These results were subsequently used to probabilistically evaluate the effects of the uncertainties in each participating factor in the mass expulsion. The probabilistic results show that the surface temperature dominates at high probabilities and the pressure which causes the mass expulsion at low probability.

Introduction

The MFIM was conceived and developed at NASA Glenn Research Center (GRC) to represent complex material behavior. In this section background on how the MFIM model was formed and how it evolved to this point is provided.

The simulation of complex material behavior resulting from the interaction of several factors (such as temperature, nonlinear material due to high stress, time dependence, fatigue, etc.) has been mainly performed by a multiplicative factor-specific representation. For example, entire text books are devoted to plasticity, creep, fatigue and high strain rate to mention only a few. Investigators have derived equations that describe material behavior for each factor-specific effect. Suppose we visualize that the material behavior is a continuum represented by some surface. Then, we can think of some representation which describes that surface which is inclusive of all participating factors that affect material behavior either singly or interactively in various combinations. To that end, research has been an ongoing activity at GRC for about thirty years. It started with a primitive form of the multi-factor-interaction model MFIM representation for describing complex composite behavior in polymer matrix composites (ref. 1). It was extended to metal matrix composites (ref. 2) and continued to be evolving during the National Aerospace Plane and the High Speed Research Programs (ref. 3). The result of all this research is the development of the MFIM to represent complex material point behavior by a single equation (refs. 4 and 5). The development of this equation starts with the premise that, if we are to quantify the range of factors affecting material point properties, we need a description of point behavior (refs. 6 and 7). In this context, it is reasonable to consider that behavior constitutes an *n*-dimensional space (Point Behavior Space (PBS)) where each point on that surface represents a specific aspect of complex behavior. It is further reasonable to assume that PBS can be described by an assumed interpolation function. One convenient interpolation function is a polynomial of product form because mutual interactions among different factors can be represented by the overall product, and includes those cross products which are present in common algebraic polynomials. In this investigation, PBS is assumed to be described by the MFIM shown in following equation:

$$\frac{W}{W_0} = \left[1 - \frac{x_1}{x_{1f}}\right]^{ex1} \left[1 - \frac{x_2}{x_{2f}}\right]^{ex2} \left[1 - \frac{x3}{x_{3f}}\right]^{ex3} \ldots \left[1 - \frac{x_n}{x_{nf}}\right]^{exn} \qquad (1)$$

where $\frac{W}{W_0}$ is the ratio of predicted foam divot weight to some arbitrary divot weight; $\frac{x_1}{x_{1f}}$ is the ratio of factor (design variable that is known to influence the divot weight) to some arbitrary final condition; $ex1$ is an exponent which can be set to some default value (say 0.5), and n is the total number of variables. The factor final condition x_f has to be set to a value that is larger that the maximum value of the selected factor (i.e., $x_f > x_1$) (ref. 8). Note as well that the factors are normalized so that the model can represent anything that a user wants it to represent. Note also that the exponent is different for each factor. The exponents are selected so that the model represents some data. The only restriction is that the exponents must satisfy the initial and final conditions for each factor. The final condition can be an intermediate point in cases where the surface may require it. For the prediction of foam mass ejection (divot weight), typical factors include the dimensions of the void such as depth and diameter, void geometric location, foam height above the void, foam surface temperature and others. Results predicted by MFIM are compared to those obtained by test thereby verifying its accuracy. The block and logic flow diagram for the MFIM deterministic and probabilistic evaluation is shown in figure 1.

Testing for Data Acquisition

The simulation results presented in this section pertain to predicting the weight of foam loss for a thermal vacuum test. The objective here is to replicate the test numerically using the multi factor interaction model. The divot weight calculation results presented in this report are based on the schematic of the physical variables depicted. As shown in figure 2, the void is assumed to be right on the substrate surface. The total foam thickness is basically comprised of two components: void height and foam height over the void. The aspect ratio L_1/L_2 indicates the type of void. For example, voids with aspect ratios $L_1/L_2 > 2.5$, are treated as slot voids. On the other hand, voids with aspect ratios $L_1/L_2 < 2.5$, are treated as cylindrical voids. For slot type voids, the critical void dimension is L_2 while L_1 is the critical dimension for cylindrical voids. The criterion for defining the type of void was provided by the test program. Divot data obtained from the thermal vacuum test are discussed next.

Thermal vacuum testing panels were aimed to develop an empirically relationship for the void size which will produce a divot for regions of the tank not susceptible to cryo-ingestion and cryo-pumping type environments. For the test data supplied to GRC, notched cylindrical voids were placed at the substrate of the panels. Note that the voids are introduced into foam panels and then foam panels with voids are bonded to a substrate. During testing foam surface is heated using quartz lamps and a radiator plate to match the highest heating rate experienced in the flange area during flight. Tests took place in vacuum chamber to match pressure profile during flight. Pressure inside the void and time to divot and mass of divot were recorded during testing. Limited test data supplied to GRC are shown in table I. In the next few sections, the simulation of the divot as carried out by MFIM is described. Note that the divot weight measured after each test is listed in the last column to the right of the table. The lowest weight recorded for this set of data was 0.00044 lb while the highest weight was 0.145 lb. The debris allowable set by the test program was 0.038 lb.

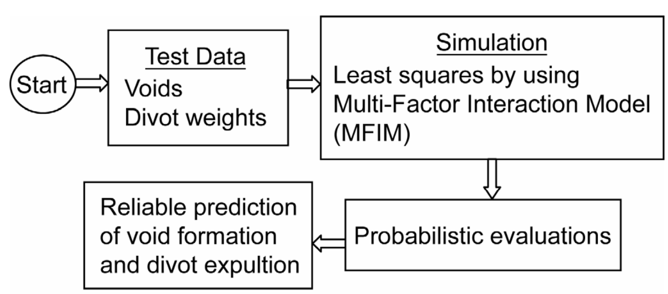

Figure 1. Block diagram for using the multi-factor interaction model.

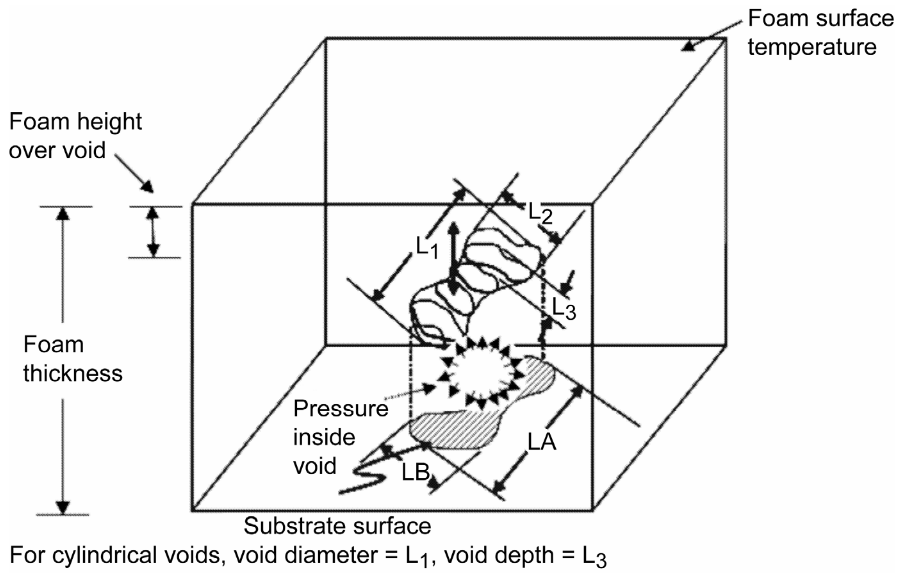

Figure 2. Schematic of foam physical variables that influence the divot process.

TABLE I.—DIVOT WEIGHTS FROM THERMAL VACUUM TEST
[Cylindrical voids]

Void diameter (VD), in.	Void height (VH), in.	Foam over void (FH), in.	Foam surface temperature (FST), °F	Pressure inside void (PR), psi	Time to fail (T), sec	Divot weight (W), lb
1.0	0.5	0.25	183.49	12	57	0.00044
0.5	0.5	0.25	352.07	10	73	0.00022
1.0	1.0	0.25	171.85	11	52	0.00044
1.0	0.5	0.5	547.09	12	86	0.00132
0.5	0.5	0.5	645.00	12	123	0.00044
1.0	1.0	0.5	519.62	11	84	0.00154
0.5	1.0	0.5	645.00	15	108	0.00044
3.125	2.0	2.0	412.19	12.35	77	0.10318
4.125	2.0	2.0	219.86	11.5	64	0.14506

Simulation of Divot Weight by MFIM (Deterministic)

The MFIM was used to simulate the foam divot based on cylindrical voids as performed by the thermal vacuum test for the test data that are listed in table I. As described earlier, the MFIM model is exponent based and takes into consideration the interaction of the various factors that would impact the response of interest (in this case it is the foam divot weight). The MFIM in expanded form is shown in eq. (2). The initial values of the exponents shown in eq. (2) were calculated using the least squares method available in MATLAB software (The Mathworks, Inc.) and taking the log on both sides, as shown in eq. (3). The values of the exponents obtained are shown in table II. Values that are used in MFIM equation are normalized with respect to their corresponding reference value, calculated as 120 percent of the maximum value. The Microsoft Excel (Microsoft Corporation) solver optimization algorithm was then used along with the MFIM equation to evaluate the deterministic results that are summarized in table III. The optimization model in the Excel was defined as follows: The objective was to minimize the mass loss at reference condition W_0 of 0.038 lb. The design variables were taken to be the 6 variables listed in table III along with the initial values of the 6 exponents calculated from MATLAB. The constraints imposed such as not allowing the 6 design variables to change more than 0.05 of the test data. The default options were taken for the solver, i.e., maximum iterations was set to 100; precision of solution was set to 0.00001; the forward difference was used for the derivatives, and the quasi-Newton method for the search direction. The results summarized in table II and figure 3 show comparisons between those that were obtained by using MFIM and the respective test data in the same column side-by-side. The comparisons are satisfactory. The plots shown in figure 3 are consistent with the tabular comparisons.

$$\frac{W}{W_0} = \left[1 - \frac{VD}{VD_f}\right]^{ex1} \left[1 - \frac{VH}{VH_f}\right]^{ex2} \left[1 - \frac{FH}{FH_f}\right]^{ex3} \left[1 - \frac{FST}{FST_f}\right]^{ex4} \left[1 - \frac{PR}{PR_f}\right]^{ex5} \left[1 - \frac{t}{t_f}\right]^{ex6} \tag{2}$$

$$\begin{aligned} \log W - \log W_0 = ex1 \log\left[1 - \frac{VD}{VD_f}\right] + ex2 \log\left[1 - \frac{VH}{VH_f}\right] + ex3 \log\left[1 - \frac{FH}{FH_f}\right] \\ + ex4 \log\left[1 - \frac{FST}{FST_f}\right] + ex5 \log\left[1 - \frac{PR}{PR_f}\right] + ex6 \log\left[1 - \frac{t}{t_f}\right] \end{aligned} \tag{3}$$

Substituting these exponents in eq. (2) the MFIM is obtained:

$$\begin{aligned} \frac{W}{W_0} = \left[1 - \frac{VD}{VD_f}\right]^{-0.21} \left[1 - \frac{VH}{VH_f}\right]^{1.41} \left[1 - \frac{FH}{FH_f}\right]^{-4.82} \left[1 - \frac{FST}{FST_f}\right]^{-1.12} \\ \left[1 - \frac{PR}{PR_f}\right]^{3.49} \left[1 - \frac{t}{t_f}\right]^{2.30} \end{aligned} \tag{4}$$

TABLE II.—EXPONENTS FOR MFIM
[Powers in MFIM equation]

ex1	ex2	ex3	ex4	ex5	ex6
–0.2118	1.4095	–4.8207	–1.1231	3.4936	2.2966

TABLE III.—DETERMINISTIC RESULTS—MFIM PREDICTIONS COMPARED WITH DATA

Void diameter, in.		Void height, in.		Foam height, in.		Foam surface temperature, (°F)		Pressure inside void, psi		Time to fail, (sec)		Divot weight, (W), lb	
MFIM	Test	MFIM	Test	MFIM	Test	MFIM	Test	MFIM	Test	MFIM	Test	MFIM	Test
0.9965	1.00	0.5428	0.50	0.2227	0.25	183.4716	183.49	11.9744	12	56.9956	57	0.00043	0.00044
0.4776	0.50	0.5500	0.50	0.2000	0.25	352.0334	352.07	10.0500	10	73.0065	73	0.00097	0.00022
0.9981	1.00	1.0374	1.00	0.2000	0.25	171.8327	171.85	11.0073	11	51.9956	52	0.00049	0.00044
0.9850	1.00	0.5350	0.50	0.5500	0.50	547.1354	547.09	12.0385	12	85.9999	86	0.00110	0.00132
0.4965	0.50	0.5454	0.50	0.5500	0.50	645.0365	645.00	12.0310	12	122.9927	123	0.00025	0.00044
1.0055	1.00	0.9500	1.00	0.5500	0.50	519.6700	519.62	10.9500	11	83.9888	84	0.00131	0.00154
0.5007	0.50	0.9856	1.00	0.5366	0.50	645.0374	645.00	14.9826	15	107.9884	108	0.00004	0.00044
3.1205	3.13	1.9666	2.00	1.9500	2.00	412.1489	412.19	12.3195	12.35	76.9912	77	0.10317	0.10318
4.0800	4.13	2.0284	2.00	1.9500	2.00	219.8381	219.86	11.4615	11.50	63.9923	64	0.14506	0.14506

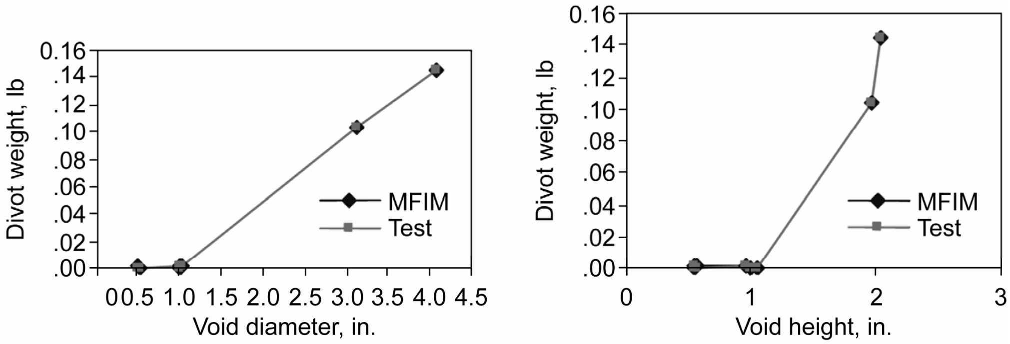

Figure 3. Comparison of results predicted by MFIM with test results for divot weight.

Probabilistic Evaluation Using the MFIM

The MFIM was used to perform the probabilistic evaluation. The mean divot weight obtained from the test data last column of table III is shown below:

$$\text{Mean Divot Weight} = \frac{4(0.00044)+0.0002+0.00132+0.00154+0.10318+0.14506}{9} = 0.0281 \text{ lb} \tag{5}$$

This mean divot weight is used in the probabilistic evaluation.

Simulation of Divot Weight by MFIM (Probabilistic)

The model shown in eq. (4) was used in the probabilistic analysis. The mean values for the six independent variables (void diameter and height, foam height above the void, foam temperature, pressure inside the void and time to failure) are listed in table IV. The complete input needed to run the probabilistic analysis code is listed in the same table. It includes the coefficient of variation and the probabilistic distribution type. The results from the probabilistic evaluation are summarized in table V where the starting vector lists the mean of the variables. The other columns list their respective values for 1/10,000 and 9999/10,000. Figure 4 shows in graphical form the cumulative distribution function of the divot weight. The range in the divot weight is about 0.012 lb. The 1/10,000 cumulative probability divot weight is 0.000105 lb and the 9999/10,000 cumulative probability divot weight is 0.012145 lb. The values of the independent variables at 1/10,000 and 9999/10,000 probabilities are tabulated in table V. Most of the divots would have values close to the mean. Very few divots have values close to 0.000105 and 0.012145 lb. The probability density function for the divot weight is plotted in figure 5. As shown in the plot, a 2.41 σ (standard deviation, s) can be achieved for the divot weight, increasing therefore, the predictive reliability of the computational simulation. One of many advantages of the probabilistic simulation is the fact that one can determine what kind of design would be required to produce a zero or near zero divot weight as demonstrated in this case. Since the equation of the cumulative distribution function is known, the values of the independent variables can be determined for the zero or near zero divot weight.

TABLE IV.—PROBABILISTIC MFIM EVALUATION OF DIVOT WEIGHT
[Thermal vacuum test with cylindrical voids]

Primitive variable	Mean	Coefficient of variation (%)	Distribution type
Void diameter (VD), in.	0.9999	5	Normal
Void height (VH), in.	0.5000	5	Normal
Foam height over void (FH), in.	0.5000	5	Normal
Foam surface temperature (FST), °F	547.0353	5	Normal
Pressure inside void (PR), psi	11.9988	5	Normal
Time to fail, sec	83.9914	5	Normal

TABLE V.—PROBABILISTIC MFIM EVALUATION OF DIVOT WEIGHT
[Thermal vacuum test with cylindrical voids]

Primitive variable	Starting vector	0.0001 probability	0.9999 probability
Void diameter (VD), in.	0.9999	0.9987	1.0019
Void height (VH), in.	0.5000	0.4835	0.5102
Foam height over void (FH), in.	0.5000	0.5092	0.4945
Foam surface temperature (FST), °F	547.0353	495.3470	640.2154
Pressure inside void (PR), psi	11.9988	13.7438	11.2284
Time to fail, sec	83.9914	90.7552	83.4058

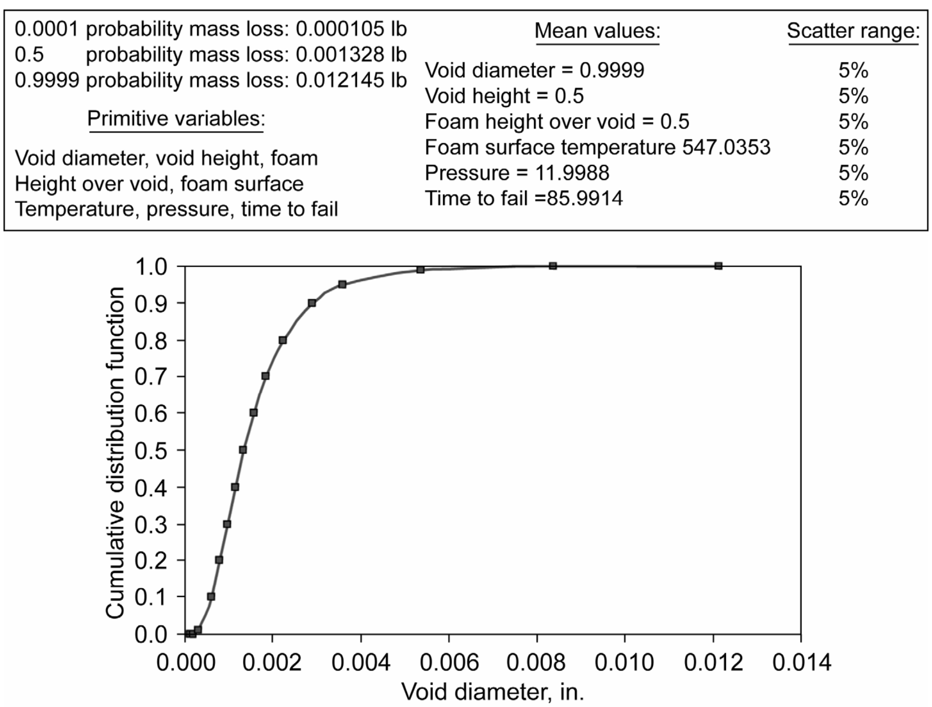

Figure 4. Cumulative distribution function of MFIM divot weight (thermal vacuum test with cylindrical voids).

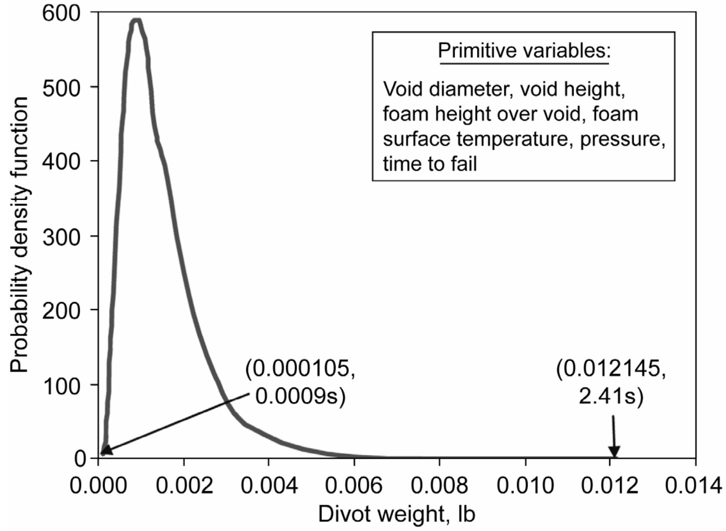

Figure 5. Probability density function of MFIM divot weight (thermal vacuum test with cylindrical voids).

An important byproduct of the probabilistic analysis is the probabilistic sensitivities. In deterministic analysis, the sensitivity is defined as the measured change in the performance (divot weight) due to a change in the design variables. In probabilistic analysis, the sensitivity measures the change in the probability relative to the change in a distribution variable (e.g., mean and standard deviation). The deterministic partial derivatives are multiplied by the ratio (σ/μ) where σ is the standard deviation and μ is the mean of the variable to obtain the probabilistic sensitivity. A very useful probability sensitivity analysis is the determination of the relative importance of the primitive variable (void diameter, etc.). This can be done by conducting several probabilistic analyses in which one of the primitive variables is treated as deterministic value (by reducing the standard deviation to near zero) for each analysis. With the resulting probability changes, the relative importance of the primitive variable is determined. The sensitivities do in fact provide first order information on the importance of the individual primitive variable. Further information on sensitivity analysis can be found in the literature. The Fast Probability Integrator (FPI) (ref. 7) in-house computer program (developed by Southwest Research Institute for GRC) has been used in the probabilistic analysis of divot weight.

The probabilistic sensitivities of the primitive variables to the divot weight are plotted in figure 6. Based on the assumed scatter in the primitive variables, the most influential variable is the foam surface temperature followed by the void pressure and the time to failure. Void height, void diameter and foam height above the void, have relatively insignificant influence. The sensitivities do vary from one probability to the other. For example, the relative importance of the foam height is decreased at 9999/10,000 probability as compared to that of the low probability. The computational simulation system that is applied here is basically a virtual laboratory. Its proper use is effective in reducing expensive testing. The relative importance of the primitive variables is a function of the exponents selected in MFIM, mean values, coefficient of variations, and distribution types. A change in the sensitivity level is possible if any of the aforementioned factors is modified. As mentioned earlier, the void diameter, void height, foam height above the void, foam surface temperature at time of divoting, pressure inside the void, and time to fail are all assumed to be independent variables. Checks performed showed no dependence was evident for these variables.

Simulation of Divot Weight by MFIM (Deterministic)

The exponents for the three factors (void diameter, void height, and foam height over the void) were determined using the same procedure as before. The MFIM model used in this simulation is as follows:

$$\frac{W}{W_0} = \left[1 - \frac{VD}{VD_f}\right]^{-0.282} \left[1 - \frac{VH}{VH_f}\right]^{-0.131} \left[1 - \frac{FH}{FH_f}\right]^{-1.189} \tag{6}$$

In this solution, the exact ratio for each factor as provided in the test has been used in the MFIM model. The final condition for each factor was calculated as 120 percent of the maximum value that was given in the test data. The reference weight W_0 was set to 0.0060 lb. The results from the MFIM simulation are presented in table VI. As shown in the table, the maximum absolute difference between the test and MFIM prediction is 0.0069 lb and the minimum absolute difference is 0.0. The divot weight results obtained from the MFIM simulation are compared to the test data in figure 7 for the void diameters and for the void height in figure 8. The use of MFIM replicated the test with reasonable accuracy.

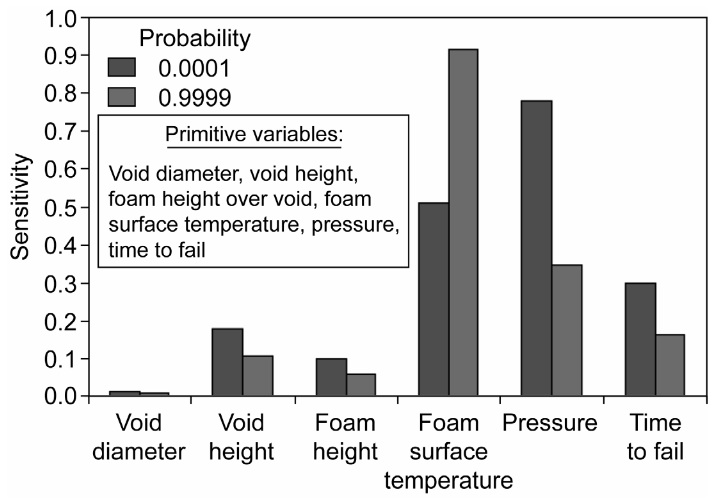

Figure 6. Probabilistic sensitivities of MFIM divot weight (thermal vacuum test with cylindrical voids).

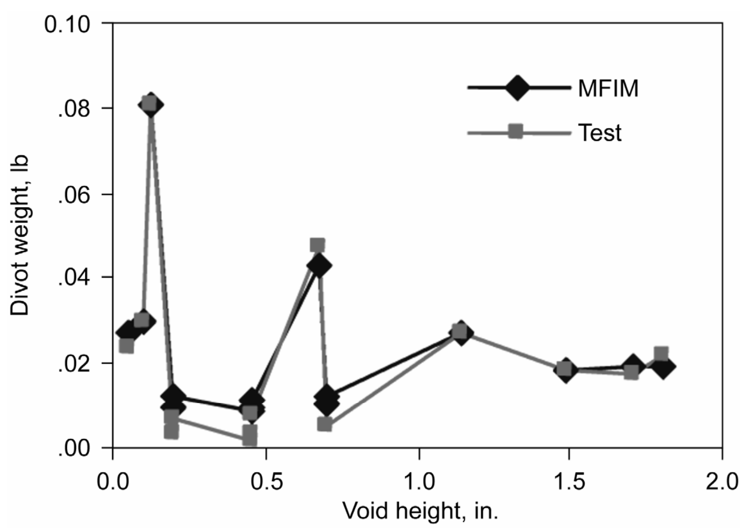

Figure 7. MFIM divot weight as a function of divot height (cylindrical voids–cryo ingestion test).

TABLE VI.—PROBABILISTIC RESULTS COMPARED WITH TEST DATA FROM CRYO INGESTION TESTS

Void diameter, in.		Void depth, in.		Foam over void, in.		Test divot weight, lb	MFIM divot weight, lb	Actual difference, lb (Test-MFIM)
MFIM	Test	MFIM	Test	MFIM	Test			
1.075	1.1250	0.4500	0.5000	0.4500	0.5000	0.0019	0.0088	–0.0069
1.575	1.6250	0.4500	0.5000	0.4500	0.5000	0.0034	0.0099	–0.0065
0.825	0.8750	0.2000	0.2500	0.7000	0.7500	0.0039	0.0093	–0.0054
1.075	1.1250	0.4500	0.5000	0.9500	1.0000	0.0081	0.0114	–0.0033
1.325	1.3750	0.7000	0.7500	0.7000	0.7500	0.0051	0.0107	–0.0056
1.825	1.8750	0.7000	0.7500	0.7000	0.7500	0.0055	0.0124	–0.0069
0.8250	0.8750	0.2000	0.2500	1.2000	1.2500	0.0072	0.0125	–0.0053
2.1170	2.1250	0.1253	0.1250	2.4844	2.5000	0.0810	0.0810	0.0000
2.1750	2.1250	0.6750	0.6250	2.0500	2.0000	0.0471	0.0426	0.0045
2.1750	2.1250	1.1408	1.1250	1.5500	1.5000	0.0272	0.0271	0.0001
1.8250	1.8750	1.7000	1.7500	1.2000	1.2500	0.0172	0.0195	–0.0023
1.4250	1.3750	1.8000	1.7500	1.3000	1.2500	0.0221	0.0192	0.0029
1.1104	1.1250	1.4837	1.5000	1.4528	1.5000	0.0182	0.0182	0.0000
1.0750	1.1250	0.0500	0.1000	2.0500	2.1000	0.0240	0.0276	–0.0036
1.3648	1.3750	0.0938	0.1000	2.0685	2.1000	0.0301	0.0301	0.0000

The values used in this part of the probabilistic evaluation are given in table VII. The probabilistic vectors for design 1/10,000 and 9999/10,000 are given in table VIII. The cumulative distribution function of the divot weight is shown in figure 9. The corresponding probability density curve is shown in figure 10. The respective probabilistic sensitivities are shown in figure 11. It can be seen in the summary of these results (tables and figures) that the probabilistic evaluation provides the most complete information.

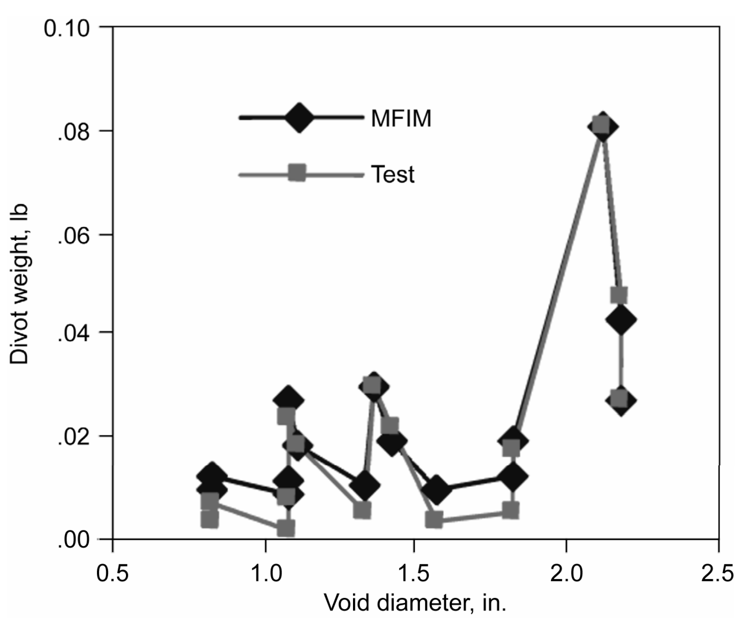

Figure 8. MFIM evaluation of divot weight (cylindrical voids–void ingestion test).

TABLE VII.—VARIABLE VALUES USED IN THE PROBABILISTIC EVALUATION

Primitive variable	Mean	Coefficient of variation (%)	Distribution type
Void diameter (VD), in.	1.1250	5	Normal
Void height (VH), in.	0.5000	5	Normal
Foam height over void (FH), in.	1.0000	5	Normal

TABLE VIII.—PROBABLE DESIGN VECTORS AT 1/10,000 AND 9999/10,000 PROBABILITIES—PROBABILISTIC MFIM EVALUATION OF DIVOT WEIGHT
[Cylindrical voids-cryo ingestion test]

Primitive variable	Starting vector	0.0001 probability	0.9999 probability
Void diameter (VD), in.	1.1250	0.9477	1.3152
Void height (VH), in.	0.5000	0.4543	0.5357
Foam height over void (FH), in.	1.0000	0.9620	1.0288

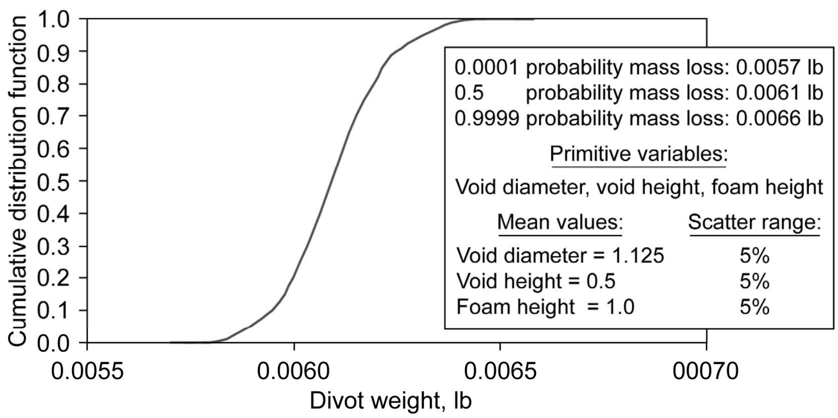

Figure 9. MFIM probabilistic cumulative distribution function of divot weight for the cryo ingestion test (cylindrical voids).

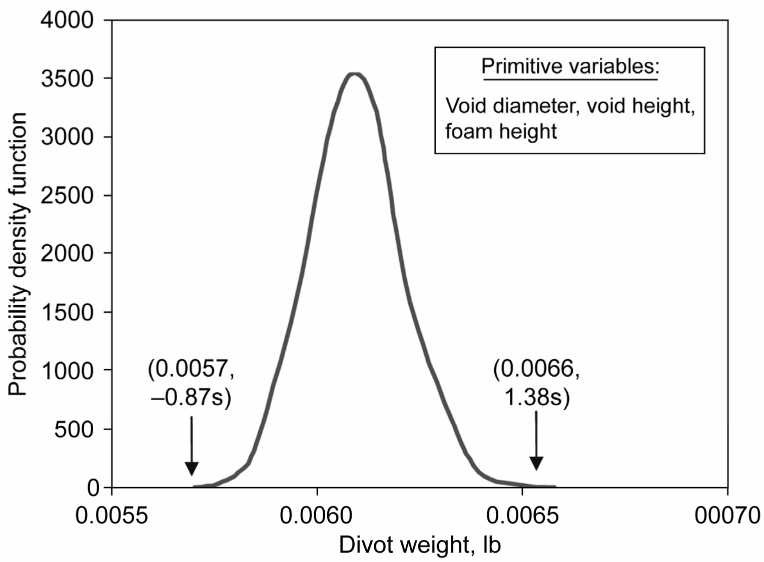

Figure 10. MFIM probability density function of divot weight for the cryo ingestion test (cylindrical voids).

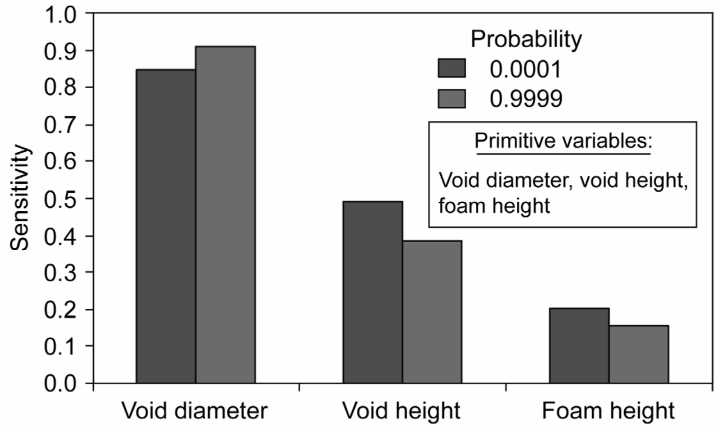

Figure 11. MFIM probabilistic sensitivities of divot weight for the cryo ingestion test (cylindrical voids).

Application of MFIM to Predict Foam Divot for Two Variables

One advantage of MFIM is that it can be an effective tool where little or no information exist about a particular process or behavior. The question that would arise at this stage is what type of foam divot weight one would expect if the two variables model was applied to component specific natural voids of the external tank. To demonstrate the effectiveness of MFIM, the reduced model shown in eq. (7) was put to use to hypothetically estimate foam divot weight based on existing voids in the region of question. The voids from experiment were grouped as cylindrical and slot type voids. The MFIM model of eq. (7) will address only the cylindrical voids.

$$\frac{W}{W_0} = \left[1 - \frac{VD}{VD_f}\right]^{-0.032} \left[1 - \frac{VH}{VH_f}\right]^{-0.091} \tag{7}$$

The exponents in the MFIM model were evaluated to be of (–0.032 and –0.091) based on the simulation of divot in the thermal vacuum test. The assumption here is that only two factors are present. Note that the maximum void diameter was around 0.9 in. and the maximum void height was around 0.3 in. The final condition VD_f and VH_f are the largest void diameter and void height. The preliminary calculations are summarized in table IX. The void diameter effect on the divot weight is shown in figure 12. The void height effect on the divot weight is depicted in figure 13. MFIM, unlike any other computational model, is capable of simulating very complex behavior of functional responses. That is evident in the plots presented in figures 12 and 13, where the response (divot weight) took on many fluctuating trends. The analysis presented is hypothetical. The MFIM calculated divot weight requires a reference value W_0 where it can be selected, for example, as a mean value of part specific historical divot weights. In this case, it was assigned a mean value of 0.0276 lb.

TABLE IX.—APPLICATION OF MFIM TO THE PRELIMINARY PREDICTION DIVOT WEIGHT IN THE LH_2 PAL RAMP OF ET94
[Cylindrical voids]

Void diameter, in.	Void height, in.	MFIM-divot weight, (W_0 = 0.0276 lb)
0.2500	0.0500	0.0284
0.28	0.1	0.0290
0.3	0.2	0.0309
0.3	0.03	0.0282
0.3	0.1	0.0290
0.35	0.05	0.0285
0.35	0.15	0.0299
0.35	0.05	0.0285
0.4	0.05	0.0286
0.4	0.1	0.0292
0.4	0.02	0.0283
0.4	0.1	0.0292
0.5	0.1	0.0294
0.6	0.29997	0.0659
0.7	0.1	0.0300
0.7	0.29997	0.0667
0.89991	0.15	0.0394

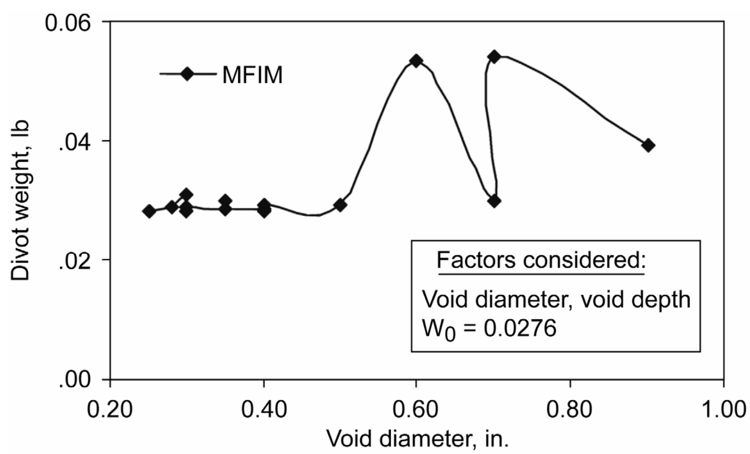

Figure 12. MFIM prediction of divot weight with void (cylindrical voids).

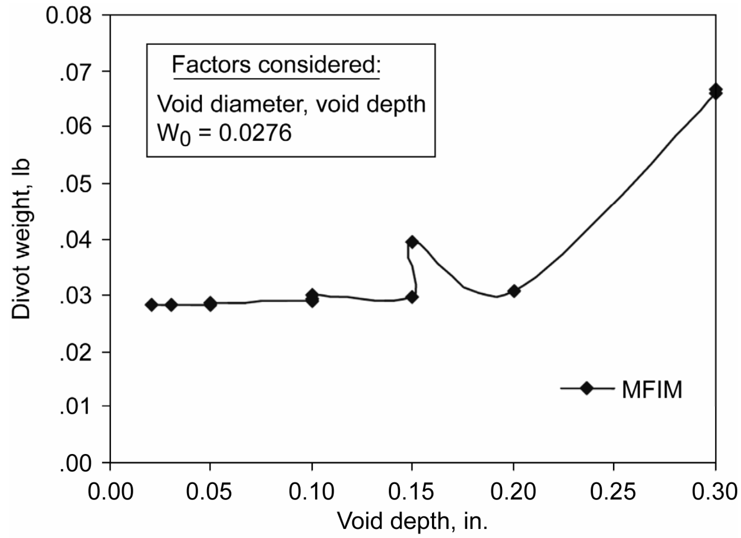

Figure 13. MFIM prediction of divot weight with void height (cylindrical voids).

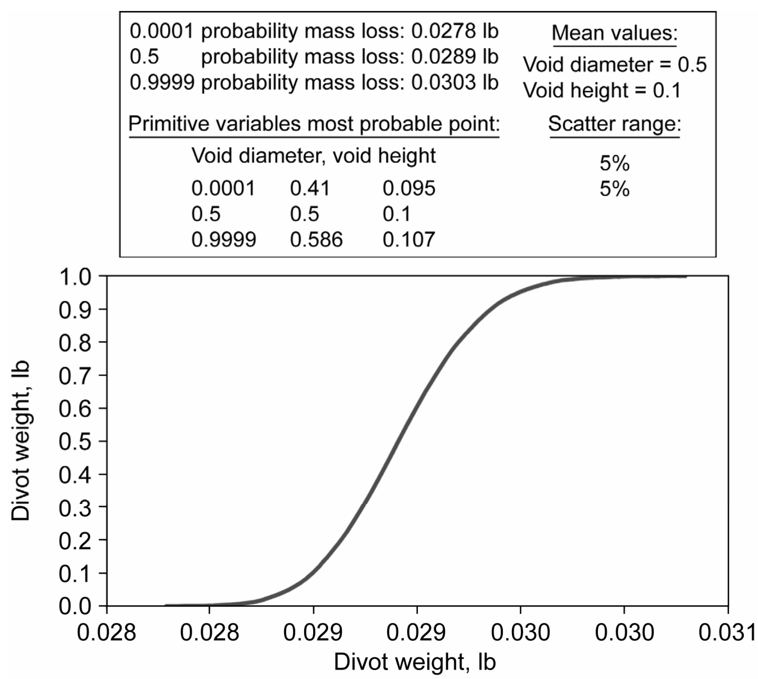

Figure 14. MFIM probabilistic cumulative distribution function of divot weight (cylindrical voids).

With the completion of the task of estimating the deterministic divot weight, it would be important to evaluate the probabilistic distribution and assess the influence of the foam void physical dimensions on the divot weight. The probabilistic evaluation of the divot weight for the assuming effects of thermal vacuum test is described herein. As in the case of the deterministic model, the probabilistic MFIM model consists of the same two factors: void diameter and void height. The mean values for the void diameter and void height are, respectively, 0.434 and 0.112. The standard deviations for the void diameter and void height are 0.11 and 0.03 in., respectively. The probabilistic distribution type for the two independent variables, void diameter and void height, is assumed to be Lognormal for computational convenience. The cumulative distribution function for the divot weight is shown in figure 14. The scatter in the divot weight is estimated to be around 0.007 lb. Based on the assumed uncertainties the divot weight is 0.0289 lb at a cumulative probability of 1/10,000 while it is 0.0296 lb at a cumulative probability of 9999/10,000. The cumulative distribution function presented in figure 14 indicates that the majority of the divots would have values close the mean. Very few divots would have weights under 0.023 lb and above 0.0298 lb. The probability density function (pdf) of the divot weight is presented in figure 15. The pdf analysis indicates that a scatter of 2.18 standard deviations can be achieved for the anticipated divot weight. The values of the void diameter and void height at the 1/10,000 and 9999/10,000 probabilities are tabulated in the insert in figure 14.

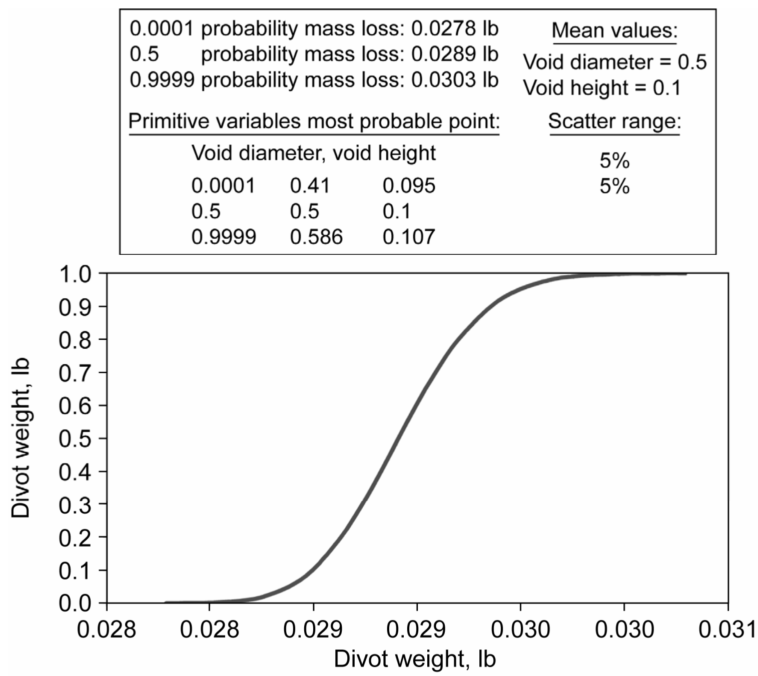

Figure 14. MFIM probabilistic cumulative distribution function of divot weight (cylindrical voids).

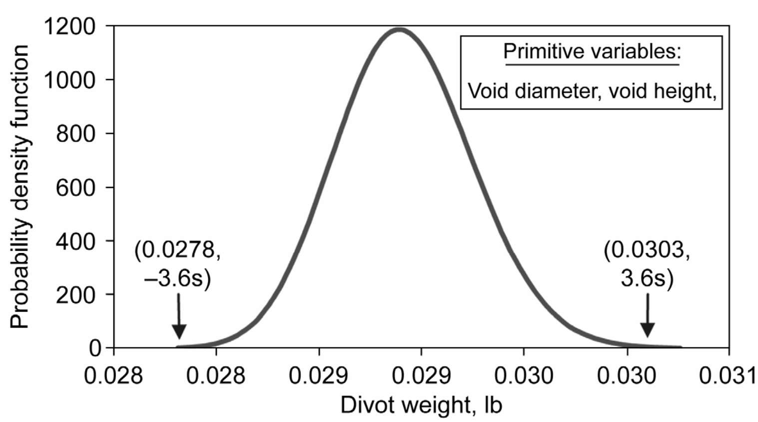

Figure 15. MFIM probability density function of divot weight (cylindrical voids).

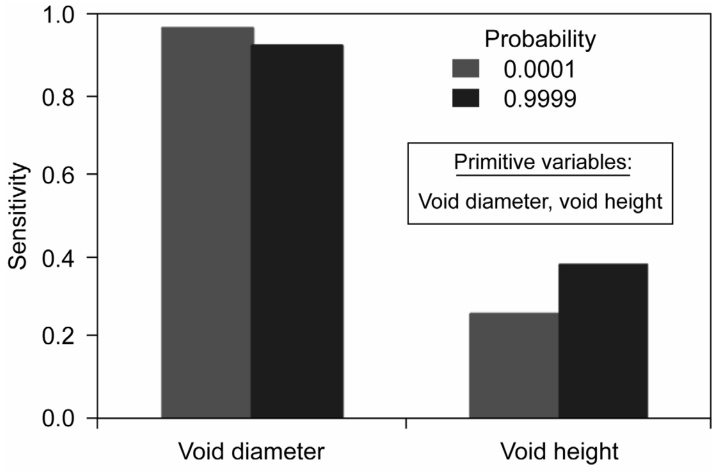

Figure 16. MFIM probabilistic sensitivities of divot weight for the two primitive variables.

An important byproduct of the probabilistic evaluation is the probabilistic sensitivities. Those are shown in figure 16. The sensitivity analysis indicates that the void diameter is at least four times more significant than the void height. Unlike traditional statistical analysis, the probabilistic analysis can yield the design vectors that would produce a specific divot weight and also can result in calculating the design vectors that would produce near zero divot weight. Additionally, the sensitivity analysis can set the stage for eliminating from the test matrix the variables that have minimum or no effect on the divot weight. That could cut the cost and time of running additional tests using variables that would not contribute to the divot or expulsion of foam. The major conclusions from predicting computationally the loss of foam are discussed in the next section.

Concluding Remarks

Work demonstrated the availability of specialized computational methods capable of simulating the foam divot weight as performed under several test programs. The relationship between defect size and divot weight is determined by applying the MFIM.

(1) The foam divot weight could be expressed as a function of the void physical dimensions such as the void diameter and the void height, though the void height was relatively insensitive in the simulation with respect to other variables such as foam thickness and foam height above the void.

(2) The difference between predicted and test divot weights was very reasonable.

(3) MFIM was shown to be useful as a predictive tool when limited data is available.

(4) Confidence in the MFIM is improved by arbitrarily using a portion of the test data to develop the model while using the remaining data for verification/ validation.

(5) Probabilistic evaluation of divot weight indicated that the dominant uncertainties are in the foam height above the void, the pressure inside the void, and foam surface temperature.

(6) The use of probabilistic methods is an effective way to assess the influence of various variables on the foam divot weight.

(7) MFIM probabilistic analysis can predict the design that would produce zero or near zero foam divot weight.

(8) Probabilistic evaluation of the MFIM shows that divot weight can be determined for $\pm 3\ \sigma$.

(9) Parametric curves show that continuous functions can be obtained by judicious applications of the MFIM.

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REPORT DOCUMENTATION PAGE						*Form Approved* *OMB No. 0704-0188*
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1. REPORT DATE *(DD-MM-YYYY)* 01-07-2008		**2. REPORT TYPE** Technical Memorandum				**3. DATES COVERED** *(From - To)*
4. TITLE AND SUBTITLE Probabilistic Usage of the Multi-Factor Interaction Model						**5a. CONTRACT NUMBER**
						5b. GRANT NUMBER
						5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Chamis, Christos, C.						**5d. PROJECT NUMBER**
						5e. TASK NUMBER
						5f. WORK UNIT NUMBER WBS 561581.02.08.03.15.03
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration John H. Glenn Research Center at Lewis Field Cleveland, Ohio 44135-3191						**8. PERFORMING ORGANIZATION REPORT NUMBER** E-16168-1
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001						**10. SPONSORING/MONITORS ACRONYM(S)** NASA
						11. SPONSORING/MONITORING REPORT NUMBER NASA/TM-2008-215246
12. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Categories: 05 and 09 Available electronically at http://gltrs.grc.nasa.gov This publication is available from the NASA Center for AeroSpace Information, 301-621-0390						
13. SUPPLEMENTARY NOTES						
14. ABSTRACT A Multi-Factor Interaction Model (MFIM) is used to predict the insulating foam mass expulsion during the ascending of a space vehicle. The exponents in the MFIM are evaluated by an available approach which consists of least squares and an optimization algorithm. These results were subsequently used to probabilistically evaluate the effects of the uncertainties in each participating factor in the mass expulsion. The probabilistic results show that the surface temperature dominates at high probabilities and the pressure which causes the mass expulsion at low probability.						
15. SUBJECT TERMS Probability; Numerical simulations; Divot weight; Sensitivities						
16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**		**19a. NAME OF RESPONSIBLE PERSON** STI Help Desk (email:help@sti.nasa.gov)
a. REPORT U	**b. ABSTRACT** U	**c. THIS PAGE** U	UU	22		**19b. TELEPHONE NUMBER** *(include area code)* 301-621-0390

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18