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RELATIVE VELOCITY AS A METRIC FOR PROBABILITY OF COLLISION CALCULATIONS*

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ABSTRACT

Collision risk assessment metrics, such as the probability of collision calculation, are based largely on assumptions about the interaction of two objects during their close approach. Specifically, the approach to probabilistic risk assessment can be performed more easily if the relative trajectories of the two close approach objects are assumed to be linear during the encounter. It is shown in this analysis that one factor in determining linearity is the relative velocity of the two encountering bodies, in that the assumption of linearity breaks down at low relative approach velocities. The first part of this analysis is the determination of the relative velocity threshold below which the assumption of linearity becomes invalid. The second part is a statistical study of conjunction interactions between representative asset spacecraft and the associated debris field environment to determine the likelihood of encountering a low relative velocity close approach. This analysis is performed for both the LEO and GEO orbit regimes. Both parts comment on the resulting effects to collision risk assessment operations.

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INTRODUCTION

The Flight Dynamics Analysis Branch (FDAB) at the Goddard Space Flight Center (GSFC) provides routine orbital conjunction threat assessment for NASA assets, including the Earth Science Constellation (ESC) and the Tracking Data and Relay Satellite System (TDRSS). Conjunctions occur when a close approach is predicted between two orbiting objects within a specified region of interest. Conjunctions between NASA assets and any space object in the US Strategic Command (USSTRATCOM) Special Perturbation catalog are processed and analyzed by the GSFC Conjunction Assessment (CA) team. The Probability of Collision (Pc) is one of several metrics that are monitored in determining the risk level of a given conjunction.

The Pc is routinely calculated using a two-dimensional (2-D) analytic method and a Monte Carlo numerical method. A three-dimensional (3-D) analytical method for calculating the Pc exists and is currently used on a case-by-case basis. The 3-D method is not used routinely as it is more computationally complex and the processing time is an order of magnitude or more longer than the 2-D method. One of the goals of this analysis is to determine whether the 2-D Pc calculation is sufficient for routine CA operational support and when it is appropriate to use the 3-D Pc calculation.

The 2-D analytical method is the widely known reduction of the 3-D problem detailed by Alfriend and Akella[1]. A key assumption in this reduction is that the relative velocity between the two objects is high, resulting in a short encounter duration. This enables one to treat the motion between the two objects as rectilinear as opposed to curvilinear. This reduction in complexity is what significantly reduces the computation time between the two methods, which is important in routine operations.

The 3-D analytical method currently implemented was developed by McKinley[2] and does not make the rectilinear relative motion assumption. Typically, this method is not required because most encounters seen operationally are nearly instantaneous resulting in rectilinear motion. There are cases however, in which the rectilinear assumption is not valid, thus the characteristics of curvilinear relative motion must be examined as well.

Part One of this analysis examines historical operational data to validate the current assumption that most of the close approach encounters observed operationally are rectilinear as opposed to curvilinear. A trade space study is performed to determine at what relative velocity the rectilinear relative motion assumption for encounters breaks down and requires use of the 3-D method.

Part Two of the analysis is a statistical study of the likelihood of encountering a conjunction with a relative velocity below this threshold.

In both Part One and Two, both the low Earth orbit (LEO) and geosynchronous Earth orbit (GEO) orbit regimes will be analyzed. The LEO orbital regime is represented by the Earth Science Constellation, which reside in sun-synchronous orbits with mean equatorial altitudes around 705 km. The GEO orbital regime is represented by the TDRS system.

ANALYSIS

PART ONE: DETERMINATION OF RELATIVE VELOCITY THRESHOLD

Historical operational data was examined to determine how often low relative velocity encounters occur. From June of 2005 through June of 2007 3,680 conjunction events were processed for 11 asset spacecraft in the ESC operating in LEO. For the ESC missions, a conjunction event is defined as any object traversing a .5 km Radial x 5 km In-Track x 5 km Cross-Track ellipsoid centered on the asset. The lowest observed relative velocity for any event was 73 m/sec and the Pc for this event was zero. McKinley[2] demonstrated for a particular LEO case that the 2-D method compares well with the 3-D method for relative velocities on the order of 13 m/sec and above. This is well below any of the observed relative velocities. Nearly 40% (1,459 out of 3,680) of the events had relative velocities of 14,500 m/sec or greater suggesting they were rectilinear relative motion encounters. These encounters are essentially instantaneous because the two objects are traveling in nearly opposite directions. Figure 1 shows the distribution of relative velocities for all the operational events examined.

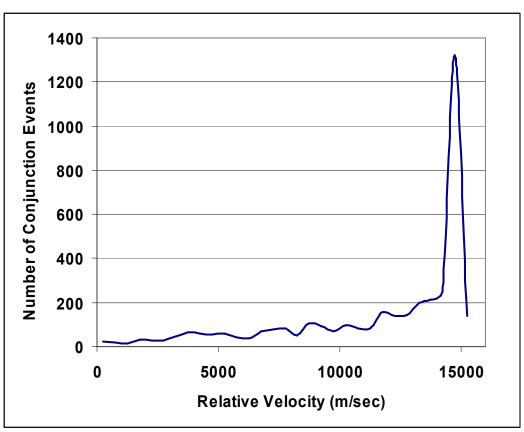

Figure 1: Relative velocity distribution of ESC Operational Data

Only 24 events (0.7%) contained relative velocities less than 500 m/sec. Of those 24 events, only one had a 2-D Pc greater than 1.0e-10. For this case, the 2-D Pc was 2.95e-4 and the 3-D Pc was 2.76e-4, a difference of less than 7%. This difference in Pc is not considered a significant difference in operationally assessing risk. The relative velocity for this conjunction was 240 m/s.

It is clear from examination of historical operational data involving ESC constellation assets that low relative velocity encounters rarely occur. Only 1 of 3,680 events had a relative velocity less than 500m/sec *and* a Pc greater than 1.0e-10.

In addition to examining the ESC events, TDRS events were also examined. The TDRS satellites operate in the GEO orbit regime. A TDRS conjunction event consists of any object that is closer than a 5 km stand-off distance to any of the TDRS satellites. There are far fewer occurrences of TDRS events because of the sparse population of debris at GEO compared with sun-synchronous LEO. Of the roughly 12,000 objects in the catalog, only 900 (7.5%) are GEO. GEO conjunction events occur, on average, once a month, as opposed to several LEO events per day.

Twelve operational TDRS conjunction events have been examined thus far. The lowest relative velocity observed was 12 m/sec, but the associated Pc for this case was 0. The Pc for all twelve events was less than 1.0e-10, mainly because the miss distances were fairly large compared to the uncertainty in the states. The combination of large miss distances and smaller state covariance will generally result in a Pc of 0. In order to calculate a viable Pc using the 2-D and 3-D methods, the covariance values for these twelve cases were scaled by a factor of between 3 and 15. The resulting Pc for each method compared very well for all twelve cases. The largest difference between the 2-D and 3-D Pc was just over 7%.

Examination of operational data for both LEO and GEO regimes indicates that there have been no observed events that warranted the 3-D Pc calculation. We can conclude, then, that the current methodology of computing the Pc using the 2-D method has been sufficient for all observed events thus far and the 2-D method is well suited for routine operational use. All operational data is continually monitored in order to identify any low relative velocity cases that would warrant the use of the 3-D Pc calculation.

While the examination of operational data shows that the 2-D Pc calculation is sufficient for observed events, it does not rule out the possibility that an event will occur that necessitates the 3-D Pc calculation. A closer examination of cases where the relative velocity is much less than the lowest observed operational value is warranted.

For this part of the study, the generation of well-defined encounter geometries was used for LEO and GEO cases. These encounter geometries were created by varying miss distance and relative velocity. The encounter geometries simulate close approaches with relative velocities on the order of 100 m/sec and below. The goal was to determine at what values of relative velocity the 2-D and 3-D methods begin to diverge. The results below show that it is not just relative velocity that drives this transition, but also the combined covariance relative to the miss distance.

Once the encounter geometries were generated, the Pc was calculated using both the 2-D and 3-D methods. The results were compared in order to determine when they begin to diverge. In this case, divergence is loosely defined as an order of magnitude difference. Curvilinear motion was "modeled" several ways. The first way was by keeping the relative velocity constant while traversing different sigma levels of the combined covariance. The second approach was by varying the relative velocity while traversing a constant combined covariance region.

Input states for the encounter geometries were generated by starting with two spacecraft (the "asset" and the "object") with identical orbits, and offsetting the object spacecraft by a specified position and velocity. Cases were generated for various miss distances and relative velocities. The state uncertainty for each object is summed to form a combined covariance ellipsoid[5]. For this study, a spherical covariance was used for each object. The 1-sigma combined covariance was set to 100 m for the first run and then varied to achieve n-sigma cases by scaling the covariance in subsequent runs. As the combined covariance increases, the encounter duration changes. Encounter duration describes the amount of time a secondary object is passing through a n-sigma combined covariance ellipsoid that is centered around the asset as described in Figure 2.

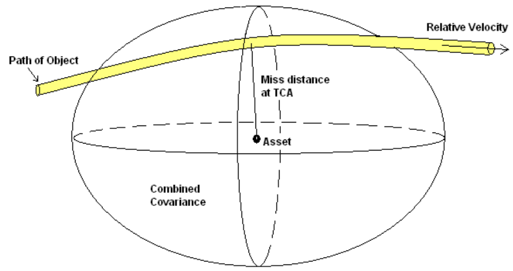

Figure 2: Encounter Geometry between the asset and object.

The duration of the encounter will generally depend on three factors: relative velocity, miss distance and the volume of the combined covariance. The 3-D Pc will "accumulate" at different rates based on the encounter duration. Figure 3 shows an example of how changing one of these parameters affects the encounter duration. For this particular example, the combined covariance is increased while the miss distance and relative velocity are held constant, thus increasing the encounter duration. The time it takes for the total Pc to accumulate is 60 seconds for a combined covariance of 300m while it takes nearly 10 times that (572 seconds) for a combined covariance of 3000m. Similar comparisons can be made by varying the relative velocity and miss distance.

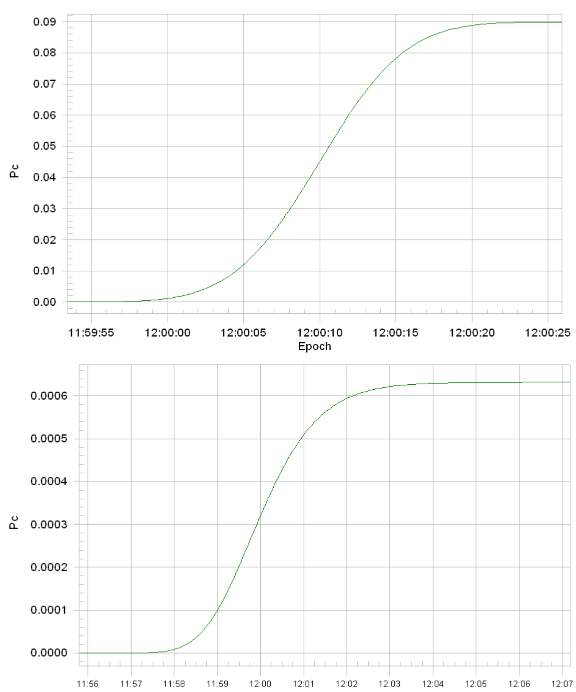

Figure 3: Pc Accumulation time for 300m (top) and 3000m (bottom) combined covariance cases. Both cases have the same relative velocity and miss distance.

Ninety cases were examined for each orbit regime (LEO and GEO) using various relative velocities and combined covariance volumes. All combinations of relative velocity and combined covariance volumes were analyzed for miss distances of 100 m, 500 m, and 1 km. Figure 4 shows the resulting percent difference in Pc calculations between the 2-D and 3-D method as a function of the combined covariance and relative velocity for a 100 m miss distance. The combination of low relative velocity and high covariance results in large Pc differences. Pc differences for relative velocities greater

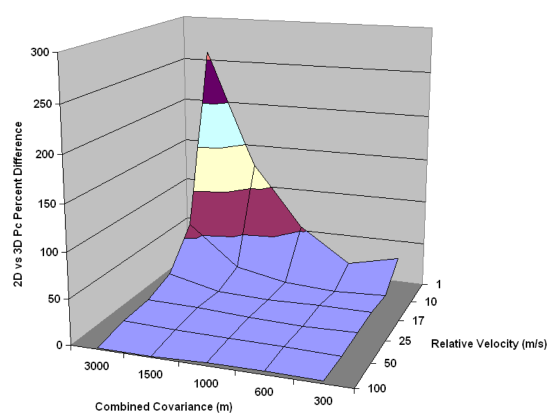

Figure 4: 2-D vs. 3-D Pc Sensitivity to Combined Covariance and Relative Velocity for the 100 m miss distance scenario

than 1 m/sec agree to within 10% for any of the combined covariance values used, while the 1 m/sec case has large differences as the combined covariance increases. Figure 5 shows the results for the 500 meter miss distance case. The differences in Pc calculations are less than 5% for relative velocities greater than 10 m/sec. These differences

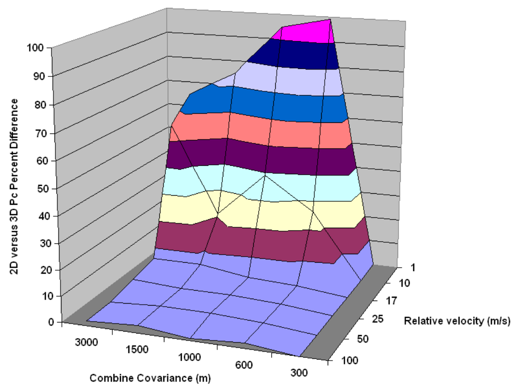

Figure 5: 2-D vs. 3-D Pc Sensitivity to Combined Covariance and Relative Velocity for the 500 m miss distance scenario

increase dramatically for relative velocities below 10 m/sec and appear to be somewhat independent of the combined covariance. Results for the 1 km miss distance were similar in that relative velocities below 10 m/sec yielded large Pc differences while relative velocities greater than 10 m/sec resulted in differences of 10% or less.

Percent differences between the two methods for relative velocities above the 10 m/sec range appear to be largely unaffected by miss distance and combined covariance. Effects of the combined covariance and miss distance can be more easily seen at the 10 m/sec and below threshold.

It is clear from the results presented here that relative velocity, combined covariance, and miss distance all contribute to the encounter duration. Percent differences between the 2-D and 3-D methods were plotted versus encounter duration (Figure 6). The results show a fairly linear relationship. Figure 6 shows that encounter durations below approximately 500 seconds result in differences in Pc of less than 30% and while this seems like a large number, when evaluating risk based on Pc values, a 30% difference is largely insignificant.

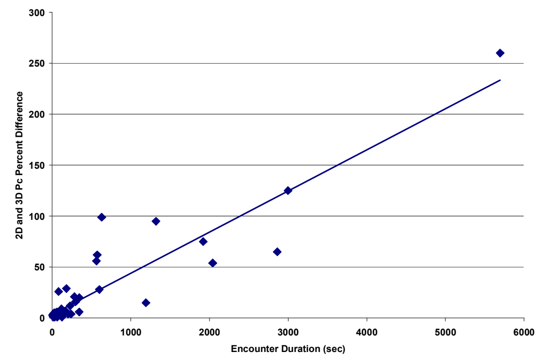

Figure 6: Percent Difference of 2D and 3D Pc Calculations as a Function of Encounter Duration

It is important to note that the results were identical for both LEO and GEO cases with the same conjunction characteristics since calculation of Pc is based on relative motion.

Based on the results of this analysis, the breakdown of the 2-D method generally occurred at relative velocities of 10 m/sec and below. It has been shown however, that depending on the miss distance and covariance, the 2-D method may still be sufficient for relative velocities as low as 1 m/sec. A good metric for determining when the 2-D method for calculating the Pc breaks down is the encounter duration. If the encounter duration is less than 500 seconds then the 2-D method can be used without any real loss of accuracy in terms of operational evaluation. For longer encounter durations (> 500 sec) there is a more significant loss of accuracy and the 3-D method may be warranted.

PART TWO: STATISTICAL EXAMINATION OF LOW RELATIVE VELOCITY LIKELIHOOD

This section describes the analytic approach to determining the likelihood of encountering a close approach that violates the criteria for the Pc metric as described in Part One. The approach taken was a Monte Carlo simulation and statistical data analysis. The Monte Carlo simulation iteratively generated a random secondary object state which was compared to a representative asset state. Both states were propagated to the Time of Closest Approach (TCA), and the relative velocity between the two objects was calculated. The results from the simulation were then compiled and statistically examined to determine the likelihood of an occurrence of a low relative velocity conjunction. The probability of a random secondary object conjuncting with the representative asset and the probability that the conjunction is of low relative velocity is calculated using conditional probability theory. This simulation and corresponding calculations were performed for a sun-synchronous, low earth orbit representative asset and repeated with a geosynchronous asset.

The first step of this analysis was to capture a snapshot of the space object environment. This was accomplished by using publicly available two line element (TLE) sets published in USSTRATCOM's General Perturbation (GP) Catalog. At the time of this analysis, there were 11,861 such space objects. The next step was to characterize the distribution of space objects through histograms for the six Keplerian orbital elements. Then a secondary object state was generated by randomly selecting a value for each of these six elements according to the distributions characterized by these histograms. The TCA was then calculated for the two orbiting bodies. Finally, the two states can be propagated to this epoch and the relative velocity can be calculated. This process was iterated to generate a statistically meaningful number of representative secondary object states.

The probability space must first be examined to find the likelihood that a randomly generated secondary object state has a low relative velocity conjunction. In the entire probability space, there exist two events of concern in this analysis: the probability of a conjunction occurring between two objects and the probability of such a conjunction occurring within the low relative velocity threshold.

Let P(A) be the probability of a random secondary object in the space debris field population possessing a potential for conjunction with the representative asset state. Let P(B) be the probability of an encounter between two orbiting space objects being of low relative velocity. P(B|A), therefore, is the probability that given a conjunction has already occurred between the secondary object and the asset, the conjunction is below the low relative velocity threshold.

P(A) can be quickly calculated from the number of states that passed the geometry filter and the total number of iterations performed. The geometry filter checked the apogee and perigee of the generated secondary object against the corresponding values for the representative asset to ensure they are in similar orbit regimes. The probability P(A) is calculated from the relative frequency definition of event probability. The low earth orbit simulation yielded 94,128 valid secondary object states that passed the geometry filter from the 598,790 total iterations:

$$P_{A,LEO} \approx \frac{N_{ConjunctiveStates}}{N_{Iterations}} = \frac{94{,}128}{598{,}790} \approx 0.1572$$

For a representative geosynchronous asset, the probability is calculated analogously as:

$$P_{A,GEO} \approx \frac{N_{ConjunctiveStates}}{N_{Iterations}} = \frac{9{,}425}{535{,}787} \approx 0.0176$$

These values are consistent with the results observed by Demarest[6] using the GP catalog. To find the likelihood of a random secondary object having a low relative velocity conjunction with the representative asset state P(B|A), the statistical survey of relative velocities simulated must first be observed. The results are shown in Figure 7 and Figure 8 for the sun-synchronous, low earth orbit asset and geosynchronous asset, respectively.

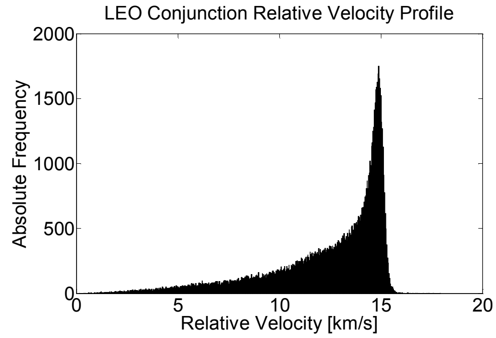

Figure 7: Distribution of relative velocity of potential conjunctions between representative sun-synchronous, LEO asset and random secondary object state

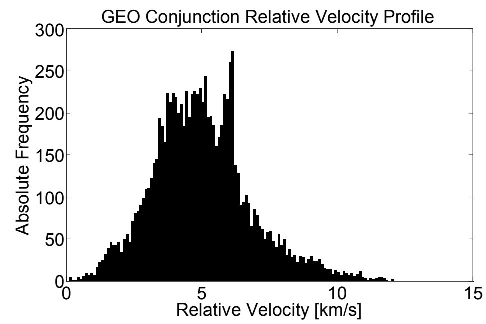

Figure 8: Distribution of relative velocity of potential conjunctions between representative GEO asset and random secondary object state

Since P(B|A) is the probability of occurrence of a conjunction at or below a certain relative velocity threshold, it is more useful to convert this density function to a cumulative probability distribution function (CDF), which is shown in Figure 9 and Figure 10.

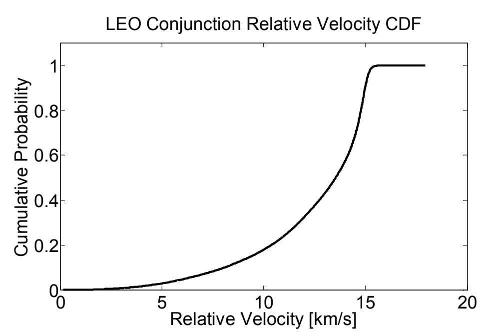

Figure 9: Relative velocity CDF of potential conjunctions between representative sun-synchronous, LEO asset and random secondary object state

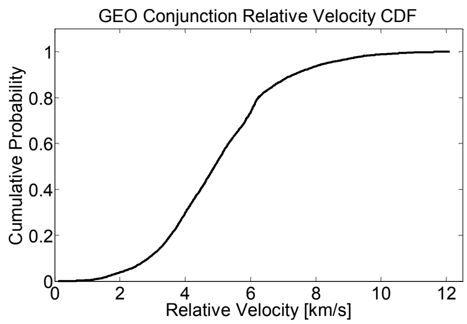

Figure 10: Relative velocity CDF of potential conjunctions between representative GEO asset and random secondary object state

As with any analytical model, it is important to validate it with any empirical knowledge of the modeled system. The GSFC CA Team is provided routine conjunction information for many robotic, sun-synchronous, low earth orbit missions. This empirical data can be directly compared to the analytical model previously described as shown in Figure 11.

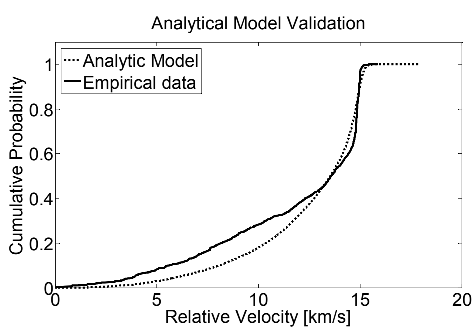

Figure 11: Analytic model validation by comparison to GSFC CA empirical data

Although the analytic model does accurately represent and model the overall behavioral distribution of conjunction relative velocities, there are minor, local discrepancies. There are several possible reasons for this difference. The analytic model only uses a single representative state, in this case the Terra spacecraft; whereas, the empirical data is for all Earth Science Constellation member missions with conjunction assessment operations. There are currently 11 such missions – all with slightly different orbits than the representative asset state chosen. In addition, the debris population snapshot was taken from the USSTRATCOM GP catalog, which only includes space objects that have been identified and publicly catalogued; whereas, the GSFC CA empirical data also included "AnalystSats." AnalystSats are space debris objects not yet identified and publicly catalogued, but currently being tracking by the Space Surveillance Network (SSN). Lastly, the analytic model does not consider correlations between the orbital elements.

These discrepancies discussed, however, do not affect the results of the analysis. The comparison of the analytic model and empirical data is used to highlight that a representative asset is sufficient to model the likelihood of encountering a low relative velocity conjunction. It demonstrates the characteristics of the representative asset conjunction interaction with the space object environment and it also shows the relative velocity profile is similar for all sun-synchronous LEO spacecraft.

From the simulated data, the lowest relative velocity case observed had a value of 130 m/s. However, the threshold for low relative velocity conjunctions has been shown to be approximately 10 m/s from Part One. An exponential curve can be fit to the cumulative probability function along the path where the first derivative is increasing. This exponential curve fit takes the form:

$$P(B \mid A)_{LEO} = P(X \leq x_0) = 0.0053e^{0.3405x_0}$$

$$P(B \mid A)_{GEO} = P(X \le x_0) = 0.0062e^{2.7339x_0}$$

where x_0 is the relative velocity in km/s.

Applying the 10 m/s low relative velocity threshold, the probability that, given a conjunction between the random secondary object and the representative asset has occurred, the conjunction probability of low relative velocity is:

$$P(X \le 0.001 km/s)_{LEO} = 0.0053e^{0.3405x_0} = 0.0053e^{0.3405 \cdot 0.001} \approx 0.0053$$

$$P(X \le 0.001 km/s)_{GEO} = 0.0062e^{2.7339x_0} = 0.0062e^{2.7339 \cdot 0.001} \approx 0.0062$$

As previously mentioned, this is the conditional probability for a low relative velocity conjunction given that a conjunction has already occurred. The event of interest for this anaylsis seeks the probability that a conjunction between the random secondary object and representative asset occurs *and* that conjunction is a low relative velocity encounter. This event is the intersection of events A and B, or P(AB).

From conditional probability theory, this calculation is straightforward:

$$P(A \cap B) = P(AB) = P(B \mid A)P(A)$$

where:

- P(A) is the probability that the random secondary object state conjuncts with the representative asset state
- P(B|A) is the probability that, given a conjunction has occurred, the conjunction is a below the low relative velocity conjunction threshold
- P(AB) is the probability that the random secondary object state conjuncts with the representative asset state *and* the conjunction is below the low relative velocity conjunction threshold

After substituting the constituent probabilities previously calculated, the likelihood of a random secondary object from the entire debris field population conjuncting with a representative asset can be calculated. For the low earth orbit asset, with a relative velocity less than or equal to 10 m/s, the conditional probability is determined to be 0.084 %. The conditional probability for a geosynchronous asset is calculated to be 0.011%. Both results demonstrate that encountering a conjunction that is of low relative velocity is on the order of 1 in 1000 to 1 in 10,000. This is consistent with the operationally observed frequency of about 1 in 4000 previously discussed.

CONCLUSIONS

The goal of this analysis was to determine whether low relative velocity cases occur for any of the supported assets and what the likelihood is of encountering such a case. An additional goal was to define a metric to help determine where the breakdown occurs between the 2-D and 3-D methods.

Results from processing operational data show that the 2-D method for calculating the Pc has been sufficient for all observed events in both the LEO and GEO regimes. Only one event occurred with a relative velocity less than 500 m/sec *and* a Pc greater than 1e-10 from the nearly four thousand events processed.

Results of the trade space study indicate that curvilinear relative motion during an encounter is dependant on a combination of relative velocity, miss distance, and combined covariance. Breakdown of the 2-D method was shown to occur around the 10 m/sec relative velocity range.

Using the 10 m/s relative velocity threshold determined in Part One, the second part examined the likelihood of this phenomenon occurring in routine CA operations. From the statistical analysis, it was shown that the probability of occurrence of this event is around 0.1% for the LEO and around 0.01% for the GEO orbit regime, indicating that a low relative velocity conjunction event is unlikely.

The results of this analysis show that conjunctions that warrant the 3-D PC calculation have not been observed for supported assets in either the LEO or GEO regimes. Furthermore, the probability of this occurring has been shown to be less than a tenth of a percent. Therefore, routine conjunction assessment operations does not need to include the calculation of the Pc using the 3-D method. The encounter duration may be the key metric in being able to quickly determine whether the 3-D method is necessary. Results suggest that encounter durations of longer than 500 seconds may warrant use of the 3-D method.

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