MUON CATALYZED FUSION

Edward A. G. Armour
School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK

ABSTRACT

Muon catalyzed fusion is a process in which a negatively charged muon combines with two nuclei of isotopes of hydrogen, e.g. a proton and a deuteron or a deuteron and a triton, to form a muonic molecular ion in which the binding is so tight that nuclear fusion occurs. The muon is normally released after fusion has taken place and so can catalyze further fusions. As the muon has a mean lifetime of 2.2 microseconds, this is the maximum period over which a muon can participate in this process. This article gives an outline of the history of muon catalyzed fusion from 1947, when it was first realised that such a process might occur, to the present day. It includes a description of the contribution that Drachman has made to the theory of muon catalyzed fusion and the influence this has had on the author's research.

INTRODUCTION

I am very grateful to the organizers for the invitation to speak at this Symposium to mark the retirement of Aaron Temkin and Dick Drachman. It is a very great pleasure to be able to contribute to this occasion in this way.

The particle the muon that is central to muon catalyzed fusion was discovered by Anderson and his first graduate student, Neddermeyer, in 1936 when studying cosmic rays (ref. 1). In accordance with relativistic quantum field theory, there is a negatively charged muon, μ^-, and its positively charged antiparticle, μ^+, each having mass $207m_e$, where m_e is the mass of the electron. Each has a mean lifetime of 2.2 microseconds, before decaying through the weak interaction into an electron or a positron and a neutrino and an antineutrino. See, for example, ref. 2. The muon is a lepton, as it is unaffected by the strong interaction. It follows that it is a fermion with spin $\frac{1}{2}$. In what follows, I will assume that the muon being considered is negatively charged.

If the electron in a hydrogen atom (H) is replaced by a muon, the result is a hydrogen-like atom $p\mu$ called muonium. The reduced mass of this atom is $186m_e$. Thus its bohr radius is $\frac{a_0}{186}$, where a_0 is the bohr radius of H. The binding energy of each of its bound states is $186\times$ (the corresponding value for the H atom). Muonium is thus very compact and strongly bound. The proximity of the positive charge on the proton and the negative charge on the muon make it similar to a neutron.

An electron can bind two protons to form a weakly bound ion, H_2^+, the hydrogen molecular ion. If the electron in this ion is replaced by a muon, the resulting ion in its ground state is very compact and strongly bound like muonium. As is to be expected, these properties remain if either or both protons are replaced by a deuteron (d) or a triton (t).

It has been known since the 1930s that the sun generates its energy by a fusion reaction in which hydrogen is converted into helium (see, for example, Bethe (ref. 3). However, this reaction requires a temperature of several million degrees to overcome the Coulombic repulsion between the protons involved in the reaction. The proton-proton fusion process is very much slower than those involving deuterons or tritons as it involves the weak interaction. Extensive research into using a fusion process of this type as a source of energy here on Earth has been going on for many years. However, even for the most favourable fusion process, $d+t$, very high temperatures are required. See, for example, ref. 4. This has made progress slow.

A natural question to ask is whether any way could be found of bringing about fusion that did not require very high temperatures. In 1947, Frank (ref. 5, 6) suggested that in the presence of protons and deuterons a slow muon might bind to a p to form $p\mu$, which as noted earlier is similar to a neutron. This could come close to a d and form $pd\mu$ which we have seen is tightly bound. Frank thought that it was sufficiently tightly bound that p-d fusion might occur. With luck, this would leave the muon free to catalyze further fusions, i.e. bring about p-d fusion while remaining unchanged at the end of the reaction.

Shortly afterwards Sakharov (ref. 7) discussed the possibility of energy production by such a process and later it was considered further by Zel'dovich (ref. 8). Both concluded pessimistically that a muon was only likely to be able to catalyze one or two fusions.

The first experimental observation of muon catalyzed fusion was made by Alvarez et al. in 1956 (ref. 9). Though unaware of the above theoretical speculations, they correctly interpreted tracks in their bubble chamber at Berkeley as representing p-d fusion catalyzed by a single muon. Bubble chamber tracks representing the path of a muon in a mixture containing hydrogen and a small amount of deuterium that catalyzes two p-d fusions are shown in Figure 1 in ref. 10.

Upon reading about Alvarez's exciting discovery in the New York Times, Jackson (ref. 11) proceeded to make an analysis of energy production. Like Sakharov and Zel'dovich, he came to a pessimistic conclusion: very few fusions could be catalyzed by a single muon.

The scene now shifted, after a pause, to the Soviet Joint Institute for Nuclear Research (JINR) at Dubna, north of Moscow. Interest in muon catalyzed fusion revived when experiments on d-d fusion catalyzed by a muon carried out by Dzhelepov et al. (ref. 12) at Dubna in 1966 revealed a strong and unexpected temperature dependence in the rate of formation of $dd\mu$. This was a very exciting discovery. It strongly suggested that a resonant process was involved and held out the possibility of a large increase in the fusion rate under suitable conditions.

It was not long before a form was suggested for this resonant process. In 1967, Vesman (ref. 13) proposed that the formation of $dd\mu$ could occur by the reaction

$$d\mu + \mathrm{D}_2 \rightarrow [(dd\mu)^* dee] \tag{1}$$

where the species on the right-hand side is a muonic molecular complex in which $dd\mu$, in a weakly bound excited state, forms one of the nuclei. The energy lost by the system through the formation of the weakly bound state goes into exciting the vibrational and rotational states of the muonic molecular complex. Due to the quantised structure of these states, this will be a resonant process and the formation rate will be very sensitive to the kinetic energy of the $d\mu$. This accounted qualitatively for the temperature dependence observed by Dzhelepov et al.

Such a mechanism depends crucially on the existence of a weakly bound state with binding energy less than the 4.5 eV dissociation energy of D_2. The problem was taken up by two Russian physicists, Gershtein and Ponomarev. Gershtein had earlier been involved in work on muon catalyzed fusion with Zel'dovich (ref. 14). Ponomarev was his graduate student.

After ten years' work, Gershtein and Ponomarev and their group (refs. 15,16) were indeed able to confirm the existence of a weakly bound state of $dd\mu$. They showed that it had rotational and 'vibrational' quantum numbers $(J, v) = (1, 1)$ and binding energy $\sim$2 eV.

The Born–Oppenheimer approximation does not work well for $dd\mu$ as the ratio of the masses of the muon and the deuteron is too large. The method Ponomarev and his group used to remove this difficulty was the adiabatic representation method (ref. 17) in which the energy and wave function of the state were calculated using a large basis set of the form $\left\{\Psi_i(\boldsymbol{r})\frac{1}{R}\chi_i(\boldsymbol{R})\right\}$ where $\boldsymbol{r}$ is the position vector of the muon with respect to the geometric center of the nuclei as origin, $\boldsymbol{R}$ is the internuclear vector, $\Psi_i(\boldsymbol{r})$ is a Born–Oppenheimer wave function for the muon and

$\chi_i(\boldsymbol{R})/R$ is a nuclear wave function. The $\{\chi_i(\boldsymbol{R})\}$ were determined by solving close-coupling type integro-differential equations.

Gershtein and Ponomarev were also able to show by this method that a corresponding weakly bound state of $dt\mu$ exists with binding energy $\sim$1 eV. In addition, they were able to show that rapid formation of $dt\mu$ by the Vesman mechanism would enable a muon to catalyze $\sim$100 d-t fusions (ref. 15). The rapid formation rate of $dt\mu$ was confirmed experimentally by Bystritsky et al. (ref. 18) in 1980.

GROWTH OF INTEREST IN MUON CATALYZED FUSION

Initially at Dubna and later at the Kurchatov Institute in Moscow, Ponomarev built up an extensive group of very able Russian physicists working on the problem of muon catalyzed fusion. The very promising advances described above took place at a time when the Cold War made direct contacts with Russian scientists difficult. However, the results were available in the literature, as can be seen from the list of references. They appeared in English after a slight delay, but greater intimacy could be achieved by a reader with a knowledge of Russian.

In 1979 the accelerator at Dubna that had been used in the experiments was shut down for refurbishment. The experimental lead was taken up by collaborations at Los Alamos, led by Jones, at the Paul Scherrer Institute (PSI) in Switzerland led by Petitjean and at the University of Tokyo and KEK in Japan led by Nagamine.

On the theoretical side, the work of Gershtein and Ponomarev became widely known. Outside the Soviet Union, expertise had accumulated for many years on calculations on few-body Coulombic systems using the Rayleigh–Ritz variational method. It was possible that this method might give more accurate results for the binding energies of states of $dd\mu$ and $dt\mu$ than the adiabatic representation method (ref. 17).

The first such calculation in the West was carried out by Bhatia and Drachman (ref. 19). I consider that it was very much in character for Drachman to be the first in this way. I can think of a number of reasons why muon catalyzed fusion appealed to him. The Vesman mechanism (ref. 13), as substantiated by Gershtein and Ponomarev, was a very interesting and exciting process involving an unusual particle, a muon, and the formation of a muonic molecular complex associated with resonant behaviour. Muon catalyzed fusion was a novel topic to an extent that few topics are. It transcended more than national boundaries in the best spirit of science. Drachman's knowledge of Russian made it possible for him to get up-to-date earlier on new developments in Soviet journals and go further and read items such as untranslated reports written by Ponomarev's group. Finally, muon catalyzed fusion just might be a viable way of bringing about fusion at room temperature that could be used as a commercial source of energy.

Bhatia and Drachman (ref. 19) carried out calculations of binding energies of all states of the muonic molecular ions $pp\mu$, $pd\mu$, $pt\mu$, $dd\mu$, $dt\mu$ and $tt\mu$, except the states of $dd\mu$, $dt\mu$ and $tt\mu$ that have $J > 1$. They used the Rayleigh–Ritz variational method with up to 440 basis functions $\{\psi_i\}$ containing Hylleraas-type functions, i.e. functions that are linear in one or more interparticle distances.

For $J = 0$,

$$\psi_i = \mathrm{e}^{-(\gamma r_1 + \delta r_2)} r_1^l r_2^m r_{12}^n \mathrm{D}_0^{0+} \quad (l, m, n \geq 0),$$

where

$$r_1, r_2 = \text{distances of the muon from the nucleons,}$$
$$r_{12} = \text{distance between the nucleons,}$$

and D_0^{0+} is a rotational harmonic (ref. 20).

For $J = 1$, the rotational harmonics D_1^{1+} and D_1^{1-} and functions of the angle between the directions of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ were used to include the unit of angular momentum.

In some cases, the calculated binding energy proved to be rapidly convergent and lower energies were obtained than in any previous calculations. For the key weakly bound states, however, they obtained binding energies consistent with previous results, but significantly smaller in magnitude.

Bhatia and Drachman pointed out that similar results were obtained by Frolov and Efros (ref. 21) at the Kurchatov Institute in Moscow using up to 375 basis functions $\{\chi_i\}$ of the form

$$\chi_i = \exp[-\alpha_{1i}r_1 - \alpha_{2i}r_2 - \alpha_{12i}r_{12}]\Omega_i^J(\boldsymbol{r}_1, \boldsymbol{r}_2),$$

where $\Omega_i^J(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is a function of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ appropriate to the angular momentum J of the state under consideration. (This work was later published in ref. 22.)

The calculations were followed by many subsequent calculations by the Rayleigh–Ritz variational method using various types of basis set.

For example, Hu (ref. 23) and Szalewicz et al. (ref. 24) used Hylleraas-type basis functions similar to Bhatia and Drachman (ref. 19). Aissing and Monkhorst (ref. 25) relied on exponential basis functions similar to Frolov and Efros (refs 21,22) with many non-linear parameters. Kamimura (ref. 26) showed the effectiveness of using basis functions similar to Frolov and Efros but with exponentials in the form of Gaussians. Vinitsky et al. (ref. 27) and Hara and Ishihara (ref. 28) used Hylleraas-type basis functions with the muon part of the functions expressed in terms of prolate spheroidal coordinates. These calculations gave highly accurate results.

THE RESONANT REACTION THAT LEADS TO FUSION

The most favourable muonic molecular ion for muon catalyzed fusion is $dt\mu$. For example, the d-t fusion rate for this ion is more than 1000 times larger than for any other ion. Also, it produces nearly the highest energy per fusion (17.6 MeV). See, for example, ref. 29.

In the resonant reaction

$$t\mu + D_2 \rightarrow [(dt\mu)_{11}dee] \tag{2}$$

$dt\mu$ is formed in its $J = 1$, $v = 1$ state. However, rapid fusion only takes place if $dt\mu$ is in a state with $J = 0$. It is thus important to know the binding energies and wave functions of the various states of $dt\mu$ below the very weakly bound $(1, 1)$ state. These binding energies are shown in Figure 1 which is taken from a revue article on muon catalyzed fusion by Bhatia and Drachman (ref. 30).

The binding energies in this figure are calculated including only the Coulombic interaction. The resonant formation rate of the complex in reaction (2) is very sensitive to the energy of the $(1, 1)$ state of $dt\mu$. Small corrections due to relativistic, QED, hyperfine and other effects have been calculated. See, for example, refs. 29, 2 and 31.

Reaction (2) is incomplete without an indication of what results from the formation of the $[(dt\mu)_{11}dee]$ complex. With inclusion of the most important decay products, it becomes

$$t\mu + D_2 \underset{\text{back decay}}{\rightleftarrows} [(dt\mu)_{11}dee] \xrightarrow[\text{decay}]{\text{Auger}} [(dt\mu)_{Jv}de]^+ + e, \tag{3}$$

where $(J, v) = (0, 1)$, $(2, 0)$, $(1, 0)$ or $(0, 0)$. If the total angular momentum of the $t\mu + D_2$ is taken to be zero for simplicity, the cross section $\sigma_r(E)$ for this resonant process is determined by the Breit–Wigner formula (ref. 32).

$$\sigma_r(E) = \frac{\pi}{k^2} \frac{\Gamma_e \Gamma_a}{(E - E_r)^2 + \frac{1}{4}(\Gamma_e + \Gamma_a)^2}, \tag{4}$$

where

$$
\begin{aligned}
E &= \text{the energy of the system,}\\
E_r &= \text{the energy of the resonant state,}\\
k &= \text{wave number of the relative motion of } t\mu \text{ and } D_2,\\
\Gamma_e &= \text{partial width for back decay,}\\
\Gamma_a &= \text{partial width for Auger decay.}
\end{aligned}
$$

It follows from this that the deexcitation rate of $[(dt\mu_{11})dee] = \frac{\Gamma_a}{\hbar}$. It is very important that this is sufficiently fast that the $[(dt\mu)_{11}dee]$ does not simply decay back into $t\mu + D_2$ (ref. 33).

Bhatia et al. (ref. 34) carried out a calculation of the deexcitation rate of the $(1,1)$ state of $dt\mu$ in $[(dt\mu)_{11}dee]$ into all states of lower energy. In $[(dt\mu)_{11}dee]$, the d, t and μ are close together. To a good approximation, they can be looked upon as forming a nucleus of charge $+1$ in a molecular complex, similar to D_2, but with one d nucleus replaced by a fictitious particle with mass equal to that of $dt\mu$.

To be able to carry out their calculations, Bhatia et al. simplified this to a $dt\mu$ in its $(1,1)$ state acting as the nucleus for a hydrogen-like atom in its ground state. They took the initial and final wave functions ψ_i and ψ_f, to be of the form

$$
\begin{aligned}
\psi_i &= \psi_i(\boldsymbol{r}_1, \boldsymbol{r}_2)\psi_{1s}(\boldsymbol{r}),\\
\psi_f &= \psi_f(\boldsymbol{r}_1, \boldsymbol{r}_2)F_k(\boldsymbol{r}),
\end{aligned}
$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of the d and μ, respectively, with respect to the t as origin and

$\boldsymbol{r}$ = position vector of the electron with respect to the center of mass of $dt\mu$ as origin.

$\psi_i(\boldsymbol{r}_1, \boldsymbol{r}_2)$ and $\psi_f(\boldsymbol{r}_1, \boldsymbol{r}_2)$ are the initial and final state wave functions for the $dt\mu$. They were calculated using Hylleraas-type basis functions in a similar way to the calculation described in ref. 19.

$\psi_{1s}(\boldsymbol{r})$ is the $1s$ initial state of the electron and $F_k(\boldsymbol{r})$ is its final continuum state.

The Auger decay of the $dt\mu$ in its $(1,1)$ state is brought about by the finite size of the $dt\mu$. If the d, t and μ were all at the same point, the system would indeed be equivalent to a hydrogen atom in its ground state. However, this is only approximately the case; a finite size correction must be included. This is of the form of a multipole expansion

$$V = \frac{e^2\hat{\boldsymbol{r}}}{r^2} \cdot [(m_1 - 1)\boldsymbol{r}_1 + (m_2 + 1)\boldsymbol{r}_2] + \ldots \tag{5}$$

where e is the charge on the proton,

$$m_1 = \frac{m_d}{M}, \quad m_2 = \frac{m_\mu}{M} \quad (M = m_d + m_t + m_\mu)$$

and m_d, for example, is the mass of the deuteron.

The V can be treated as a perturbation using Fermi's Golden Rule. See, for example, ref. 35. The results Bhatia et al. obtained by including the first, dipole term in (5) are given in Table 1. Note that the deexcitation rate to the $(0,1)$ state with zero angular momentum is much larger than the other deexcitation rates.

The results obtained in some other calculations for the key $(1,1) \rightarrow (0,1)$ deexcitation rate are given in Table 2. The calculation in which I was involved (ref. 38) differs from the others in

that it allows for the molecular structure of the $[(dt\mu)_{11}dee]$ complex. It can be seen that allowing for this does not significantly affect the deexcitation rate.

The circumstances that led to this calculation are of interest. In December 1987, I visited the atomic physics group at Goddard Space Flight Center to give a seminar on calculations I was carrying out on positron-hydrogen-molecule scattering. Drachman talked to me about the calculations that he was carrying out with Bhatia and Chatterjee (ref. 34) on the deexcitation rate of the $[(dt\mu)_{11}dee]$ complex. Knowing that I had come into positron physics from theoretical chemistry, he encouraged me to see if I could take the molecular nature of the complex into account when calculating the deexcitation rate. As I pointed out earlier, this was not allowed for in ref. 34. He gave me a translation that he had made of a report by two members of Ponomarev's group, Men'shikov and Faifman, on the treatment of $[(dt\mu)_{11}dee]$ as a molecular complex.

I was sufficiently interested to put my graduate student, Lewis, onto the problem of how to take into account the molecular nature of $[(dt\mu)_{11}dee]$ when calculating the deexcitation rate. It was not long before I came to appreciate the special features of muon catalyzed fusion that had attracted Drachman to it.

Lewis and I collaborated with Hara at RIKEN in Japan to allow for the fact that $[(dt\mu)dee]$ and $[(dt\mu)de]^+$ in reaction (3) should be treated as D_2-like and D_2^+-like molecules, respectively, with one nucleus replaced by a particle of mass equal to that of $dt\mu$ (ref. 38).

Quite soon I was in contact with Ponomarev. I visited him and his group at the Kurchatov Institute in Moscow in 1990 and gave a seminar on my work. The Cold War had ended and contacts with Russian scientists were multiplying to make up for the long period of separation.

With my postdoc, Harston, I went on to collaborate with Faifman and Strizh on the calculation of the rate of the reaction (3) that leads to the formation of $dt\mu$ in a state with zero angular momentum. Men'shikov and Faifman (ref. 32) considered that the inequalities $\Gamma_e \ll \Gamma_a \ll \epsilon$, where ϵ is the kinetic energy of the relative motion of $t\mu$ and D_2, held for Γ_a and Γ_e in equation (4). Under these conditions, $\sigma_r(E)$ is effectively the cross section for the formation of the complex (ref. 39). Also,

$$\sigma_r \approx \frac{2\pi^2}{k^2}\Gamma_e\delta(E - E_r). \tag{6}$$

Γ_e is determined by the matrix element $\langle\chi_f|\bar{V}|\chi_i\rangle$, where χ_i and χ_f are the initial state $t\mu + D_2$ wave function and the wave function for the $[(dt\mu)_{11}dee]$ complex, respectively. $\bar{V}$ is the potential that brings about the formation of the complex, i.e. the interaction between the $t\mu$ and D_2. Men'shikov and Faifman showed that it could more conveniently be taken to be the analogue of V in equation (5), allowing for the molecular structure of the complex.

By skillful use of mathematics (ref. 40), Faifman et al. (ref. 41) were able to evaluate $\langle\chi_f|\bar{V}|\chi_i\rangle$, with $\bar{V}$ the leading dipole term in its multipole expansion. During visits to Nottingham by Faifman and Strizh, this was extended to include the next term which is the quadrupole term (ref. 42). It was found that including the quadrupole term reduced the magnitude of the peak reaction rates by between 20% and 30%.

In fact, this treatment does not allow fully for the molecular nature of the resonant process in which $dt\mu$ is formed in a state with $J = 0$ in which fusion can take place rapidly. Cohen at Los Alamos, who has made many contributions to the theory of muon catalyzed fusion over the years, encouraged me to apply the methods of quantum reactive scattering to reaction (3). This reaction has some similarities with the chemical reaction

$$H + D_2 \rightarrow H + D_2 \text{ or } HD + D. \tag{7}$$

This and other chemical reactions have been extensively studied by these methods. See, for example, ref. 43.

However, comparison of reactions (3) and (7) shows that (3) has a special feature not present in (7), namely the Auger decay process which leads to the loss of the electron. Reaction (7) corresponds quite closely to the much slower side reaction

$$t\mu + \mathrm{D}_2 \leftrightarrows [(dt\mu)_{11}ee] \rightarrow [(dt\mu)_{Jv}e] + \mathrm{D} \tag{8}$$

where $(J, v) = (0,1)$, $(2,0)$, $(1,0)$ or $(0,0)$.

This reaction proceeds through the *same* resonances as reaction (3). In this case, the resonant complex decays forward into the muonic molecule $[(dt\mu)_{Jv}e]$ and a D atom or backward into $t\mu + \mathrm{D}_2$.

One of the methods that has been applied to reaction (7) is the method of Pack and Parker (ref. 44). They use adiabatically adjusting, principal axes hyperspherical coordinates (APH) in their calculations. These are elegant coordinates that transform smoothly between different channels such as $\mathrm{H} + \mathrm{D}_2$ and $\mathrm{HD} + \mathrm{D}$. Together with Pack, my postdoc Zeman and I applied this method to reaction (8) (refs. 45–47).

Unfortunately, there was no easy way of including the Auger decay channel directly in our treatment. However, as pointed out by Men'shikov and Faifman (ref. 32), the coupling between the resonant channels is small as the lifetime of the resonant complex is much longer than the time the complex takes to complete a vibration. Thus the various decay processes operate essentially independently. This made it possible for us to obtain what we expect to be accurate values for Γ_e and Γ_n, the partial widths for back decay and for the decay of the complex into $[(dt\mu)_{Jv}e] + \mathrm{D}$, by analysing our results for the cross section for the resonant reaction (8) using the Breit–Wigner formula.

Owing to the complexity of the calculation, we were only able to consider scattering states with zero total angular momentum. As the $dt\mu$ is in its $(1,1)$ state in the complex, this meant that the fictitious diatomic molecule in the complex, with one nucleus $dt\mu$, had to be in a state with angular momentum equal to 1. Care had to be taken to make sure that all important corrections to the Born–Oppenheimer potential were included in calculating the potential energy surface for the reaction.

An interesting picture of the surface function eigenvalues of the hyperradius ρ is given in Figures 3 and 4 in ref. 46. In Figure 3, the bottom four curves correspond asymptotically to the open channel $(0,1)$, $(2,0)$, $(1,0)$ and $(0,0)$ states of $[(dt\mu)_{Jv}e]$. Figure 4 highlights the curve that corresponds to the closed channel $(1,1)$ state asymptotically among curves that correspond in this region to the various D_2 states.

It was found, as expected, that the reaction proceeded only through the resonances. The resonant states were found to be the vibrational states of the complex with vibrational quantum number $v_c = 3$ and 4. The center of the $v_c = 2$ resonance was calculated to be just slightly below threshold.

Somewhat to our surprise, we found that our calculated value of the back decay rate (ref. 47) was much larger than the value obtained for it by Lane (ref. 33) and comparable with the calculated values of the Auger decay rate in Table 2. Further work is necessary to resolve this discrepancy.

It was a great pleasure for me to be able to give Drachman a copy of this treatment of the resonant process that leads to fusion (ref. 46). It showed what had resulted from his encouragement to me to consider the molecular nature of the complex in reaction (3).

MUON CATALYZED FUSION AS A SOURCE OF ENERGY

I have not said anything so far about muon catalyzed fusion as a possible source of energy. To consider this, we must look at the full muon catalyzed fusion cycle. This begins with the projection of a muon into a mixture of deuterium and tritium at room temperature. The sequence of events is then as shown in Figure 2.

What I have not considered up until now is how the cycle continues after fusion takes place. The muon is a lepton and is thus unaffected by the strong interaction that brings about the d-t fusion

$$d + t \rightarrow \alpha + n. \tag{9}$$

However, the muon is negatively charged and may combine with the α particle to form a bound state. This process is referred to as 'sticking'. If the muon remains bound in this way, it is lost to the cycle.

However, the $\alpha\mu$ is emitted with a kinetic energy of 3.5 MeV (ref. 30). As it slows down in the medium, it undergoes many collisions which may result in the stripping of the muon from the α particle. In addition, the muon can become available through transfer reactions that result in $t\mu$ or $d\mu$ and a free α particle.

We can define an effective sticking probability, w_s, by

$$w_s = w_s^0(1 - R), \tag{10}$$

where w_s^0 is the initial sticking probability and R is the muon reactivation coefficient that measures the extent to which stripping occurs. w_s^0 has been calculated by several methods: variational (refs. 49–51), adiabatic (ref. 52), adiabatic hyperspherical (ref. 53) and Monte Carlo (ref. 54). Detailed calculations have been carried out of R by Cohen (ref. 55), Markushin (ref. 56) and Rafelski et al. (ref. 57).

If we consider the rate values given in Figure 2, the muonic atom formation rate λ_a is much larger than either λ_{dt}, the rate of transfer of a muon from $d\mu$ in its ground $1s$ state to $t\mu$ or $\lambda_{dt\mu}$, the resonant formation rate of $dt\mu$. Consequently, the rate of the muon catalysis cycle λ_c is practically independent of λ_a. For the same reason, it is practically independent of the $dt\mu$ deexcitation rate (see Table 1) and the fusion rate.

λ_c is then given by

$$\frac{1}{\lambda_c} \approx \frac{c_d q_{1s}}{\lambda_{dt} c_t} + \frac{1}{\lambda_{dt\mu} c_d}, \tag{11}$$

where c_d and c_t are the concentrations of deuterium and tritium, respectively, in the mixture ($c_d + c_t = 1$). See refs. 29, 48 and 58. q_{1s} is the probability that the $d\mu$ reaches its ground state. The rate of transfer of the muon from d to t is much slower for $d\mu$ in this state. The first term on the right-hand side of equation (11) is the time the muon spends as $d\mu$ waiting to transfer to t and the second term is the time it spends as $t\mu$ waiting to form the $dt\mu$ molecule.

If sticking did not occur the average number X_c of cycles catalyzed by one muon would be λ_c/λ_0, where $\frac{1}{\lambda_0} = 2.2 \times 10^{-6}$ sec is the mean lifetime of the muon. Sticking modifies it to

$$X_c = \frac{1}{\frac{\lambda_0}{\lambda_c} + w_s}. \tag{12}$$

Theory and experiment indicate that $w_s \approx 0.5\%$ (refs. 48, 58, 59). If $w_s = 0.43\%$, as in Figure 2, it follows from equation (12) that X_c cannot be larger than 232.

With the *estimated* cost of ~ 8 GeV to produce a muon (ref. 60) and a maximum energy release of 17.6 MeV per cycle, break even would be achieved if $X_c \approx 450$. A considerably higher X_c value, at least 900, would be required for muon catalyzed fusion to be a commercially viable source of energy. $X_c = 150$ is currently attainable. Possible ways of making X_c as high as 900 by increasing the density of the deuterium and tritium mixture and decreasing w_s by increasing R are outlined in ref. 61. See equation (10).

14.1 MeV of the 17.6 MeV generated by the *d-t* fusion reaction (9) is carried away by the neutron. The present maximum X_c value of ~ 150 would be adequate for the production of energy by using neutrons obtained in this way to convert ^{238}U into plutonium fuel for conventional fission reactors (refs. 29, 60, 30, 62). Unfortunately, this method of energy production would not be as 'clean' as using the muon catalyzed fusion cycle directly.

The greatly expanded interest in muon catalyzed fusion in the 1980s was followed by a gradual decline in interest with the realisation that it was unlikely to be a viable source of energy. At the beginning of the 90s, the Advanced Energy Projects (AEP) Division of the US Department of Energy withdrew funding from experiments on muon catalyzed fusion at Los Alamos. The AEP Division considered that the research had been successful as it had shown that muon catalyzed fusion was not a viable source of energy.[1]

Experimental work continued at the Paul Scherrer Institute (PSI) in Switzerland (ref. 63, 64) and at Dubna and elsewhere in Russia (ref. 65). In the late 90s and early 2000s, a considerable impetus was given to experimental work on muon catalyzed fusion by Nagamine and his Japanese group from RIKEN who carried out experiments using the pulsed muon beam at the Rutherford Appleton Laboratory (RAL) in the UK (refs. 59, 61, 66) and also by the international collaboration based on Marshall's group at TRIUMF in Canada (refs. 67, 68). This essentially brings us to the present day.

The reader who wishes to obtain an overall view of muon catalyzed fusion should choose from the review articles in refs. 2, 10, 29, 30, 48, 58, 63, 69 and 70. The most recent, detailed overview is given in the Proceedings of the International RIKEN Conference on Muon Catalyzed Fusion and Related Atoms (MuCF01) at Shimoda in Japan in April, 2001 (ref. 71).

CONCLUSION

Today with the maximum number of fusions attained per cycle still only about 150, it is difficult to argue that muon catalyzed fusion should be considered seriously as a possible energy source. However, if Nagamine (ref. 61) or anyone else were able to increase this number to ~ 900, the situation would change, but this seems unlikely.

This is unfortunate but over the years the study of muon catalyzed fusion has given many physicists, not least Dick Drachman and myself, many moments of real pleasure. Nobody knows why physical phenomena can be described using mathematics (ref. 72) but to apply it successfully to an elegant physical problem is an experience to be savoured.

I am very grateful to Dick Drachman for introducing me to muon catalyzed fusion and for many valuable discussions with him over the years about it, positron physics and other subjects. It has always been a great pleasure for me to visit Goddard Space Flight Center and discuss my work with him, Aaron Temkin and Anand Bhatia. I wish Dick and Aaron all the best in their retirement.

[1] James Cohen, private communication, 2005.

Table 1 Calculated values for the deexcitation rates (ref. 34)

Transition	Energy difference ΔE (eV)	Deexcitation rate ($sec^{-}1$)
$11 \rightarrow 01$	34.1745	6.44×10^{11}
$10 \rightarrow 00$	86.6682	3.31×10^{10}
$11 \rightarrow 20$	101.88	8.56×10^{10}
$20 \rightarrow 10$	129.93	5.12×10^{10}
$01 \rightarrow 10$	197.6370	3.32×10^{10}
$11 \rightarrow 00$	318.4797	1.57×10^{9}

Table 2 Other calculations of the $(1,1) \rightarrow (0,1)$ deexcitation rate

Calculation	Deexcitation rate $11 \rightarrow 01$ (sec^{-1})
Vinitsky, Ponomarev and Faifman (ref. 36)	11.4×10^{11}
Scrinzi and Szalewicz (ref. 37)	10.20×10^{11}
Armour, Lewis and Hara (ref. 38)	8.63×10^{11}
Bhatia, Drachman and Chatterjee (ref. 34)	6.44×10^{11}

Figure 1 Energy level diagram and dipole Auger (or internal conversion) transitions f $dt\mu$ molecular ion (ref. 30). Binding energies for $J = 0$ and $J = 1$ are from ref. 25 and for ref. 26. Units are eV.

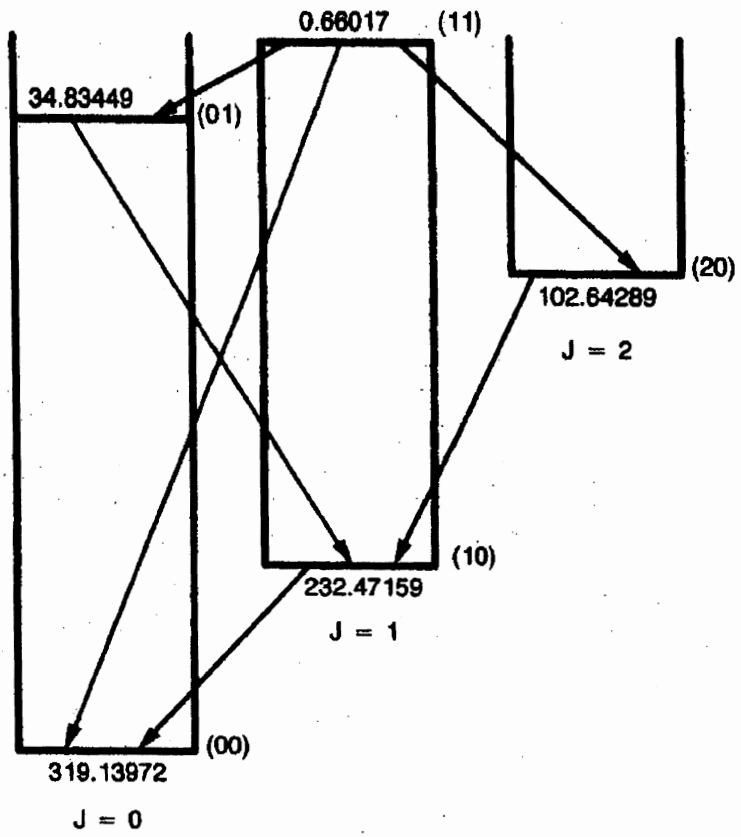
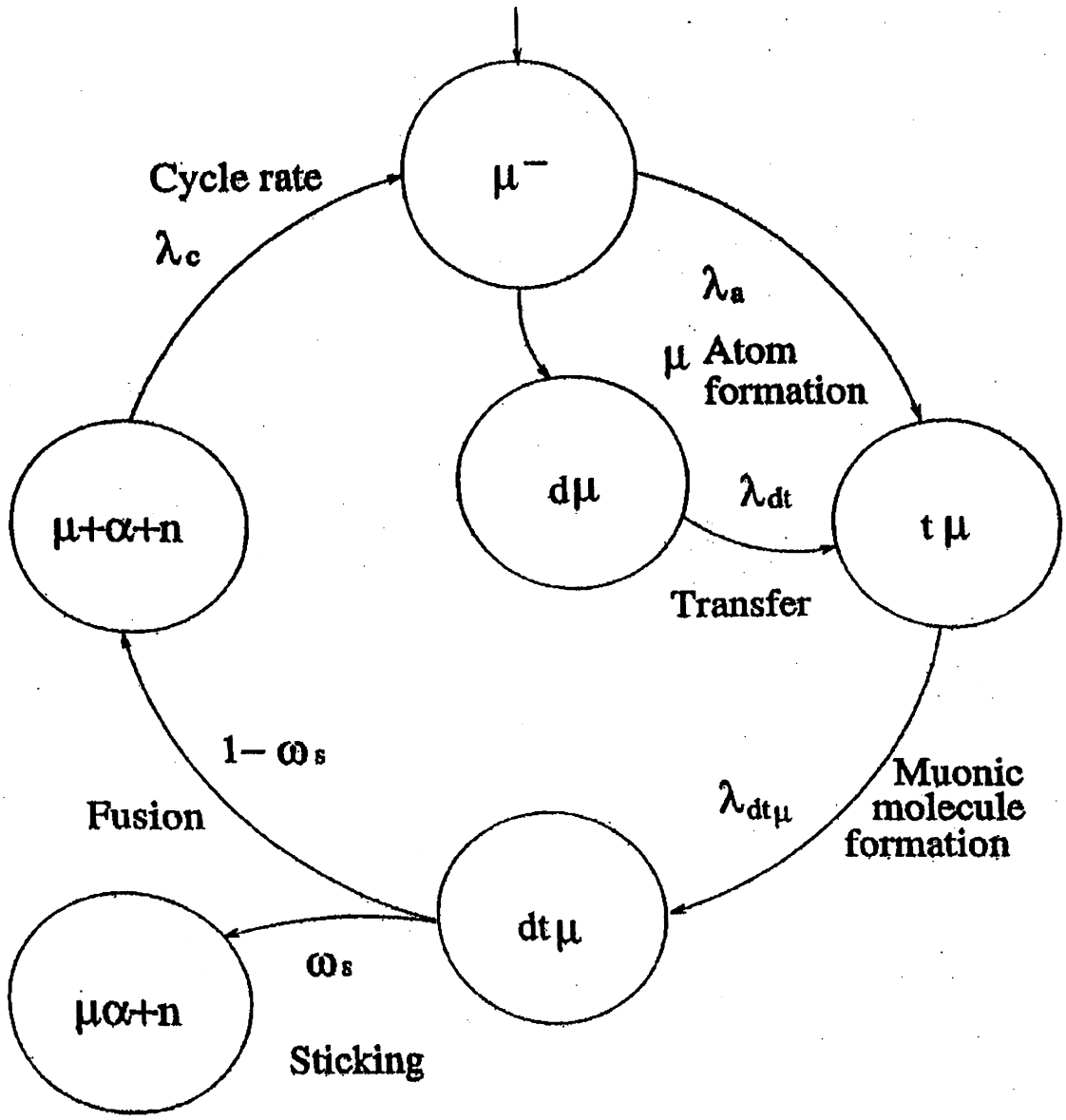


Figure 2 The principal muon catalysis fusion cycle in a deuterium and tritium mixture (ref. 29). Side chains involving d-d and t-t fusion are not shown. λ_a = muonic atom formation rate $\approx 4 \times 10^{12}\, s^{-1}$; λ_{dt} = muon transfer rate from $(d\mu)_{1s}$ to $(t\mu)_{1s} \approx 3 \times 10^8\, s^{-1}$; $\lambda_{dt\mu}$ = resonant formation rate of $dt\mu \approx 4 \times 10^8\, \mathrm{sec}^{-1}$; ω_s = effective sticking probability $\approx 0.43\%$; λ_c = cycle rate. Values (ref. 48) are for $T = 300\mathrm{K}$ and liquid hydrogen density (4.25×10^{28} atoms m^{-3}).



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