Binary Black Holes, Gravitational Waves, and Numerical Relativity

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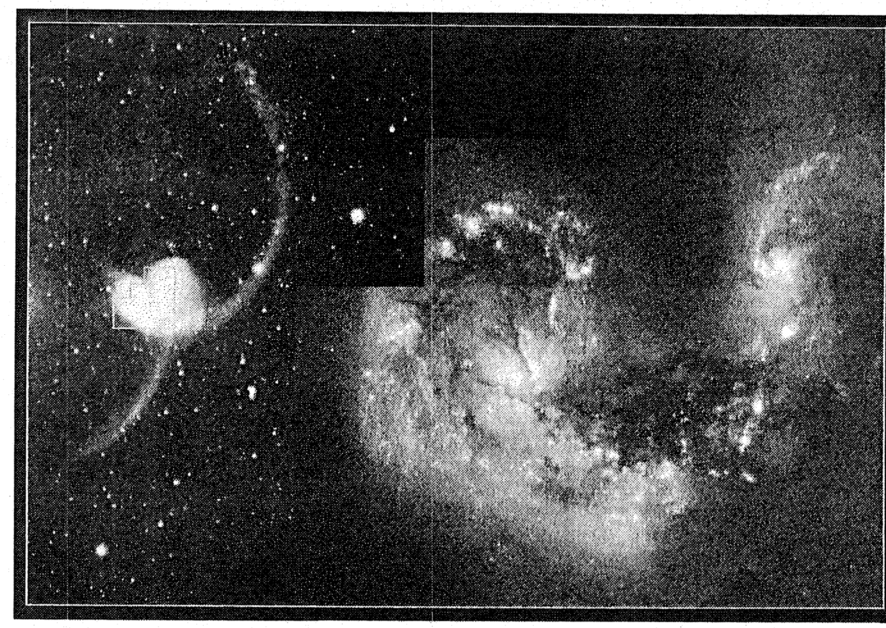
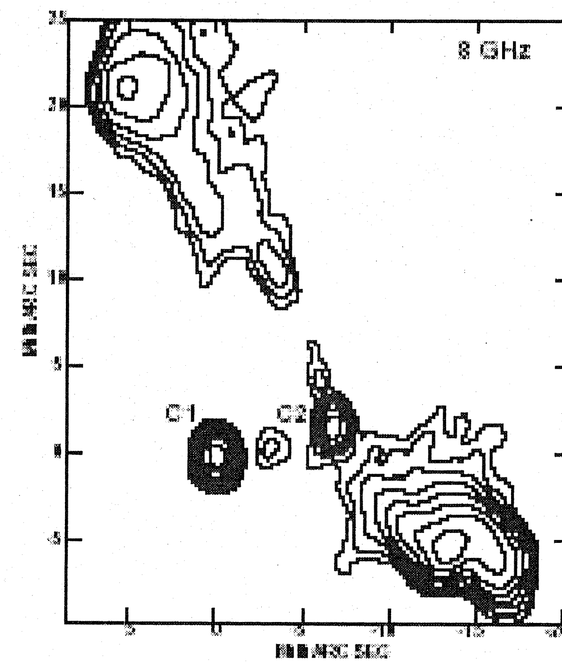
MBH binaries….

- MBHs are found at the centers of most galaxies
- Most galaxies merge one or more times

  → *MBH binaries*
- MBH mergers trace galaxy mergers
- MBH mergers are strong sources of gravitational waves
- These GWs are detectable by LISA out to z ~ 10 or more
- Expect ~ several events/year, or more (possibly more...)
- Observing these GWs can probe early stages of structure formation





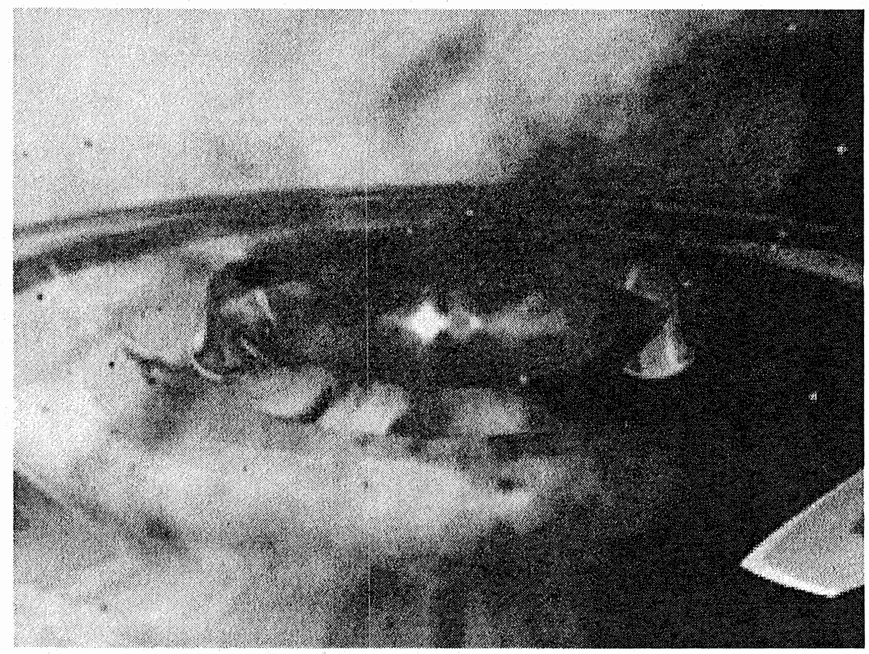
(Rodriguez, et al. ApJ, astro-ph/0604042, VLBA)

A Different Type of Astronomical Messenger *Gravitational Waves . . .*

- Ripples in spacetime curvature
- Travel at velocity $v = c$
- Generated by masses with time changing quadrupolar moments
- Carry energy and momentum
- Interact weakly with matter

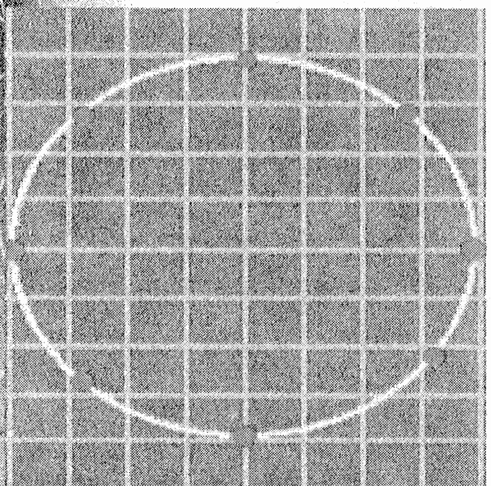
→ ***carry info about deep, hidden regions in the universe***

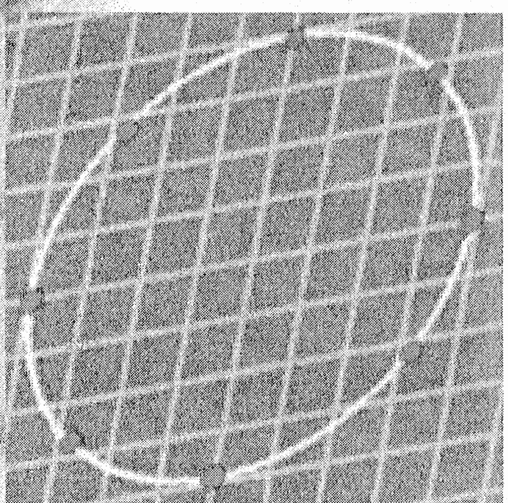
- First *indirect* detection of GWs: Hulse-Taylor binary pulsar PSR 1913+16
  - **Orbital period decay agrees with GR to within the observational errors of < 1%**
  - **Nobel Prize 1993**



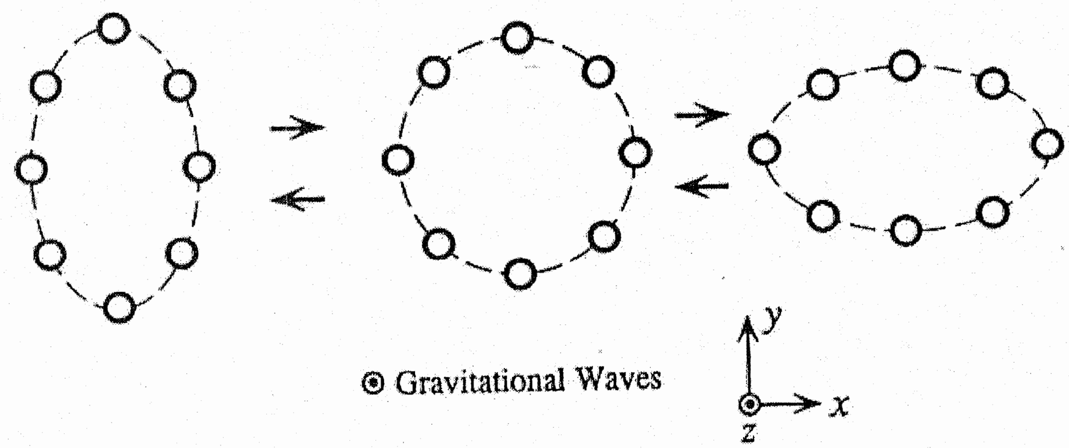
- The *direct* detection of gravitational waves will open a fundamental new window on the universe...

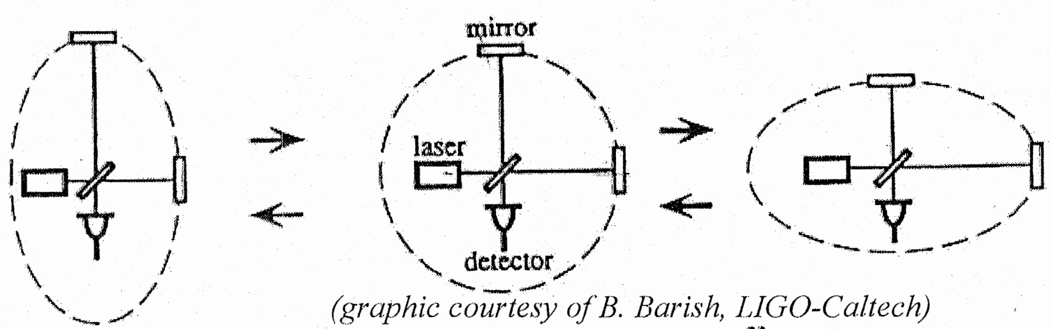
Detecting gravitational waves. . .





- Detector of length scale L
- A passing gravitational wave distorts detector via 2 polarization states, h_+ and h_x
- Measure strain amplitude $h(t) = \_L/L$
- Source waveforms scale as $h(t) \sim 1/r$

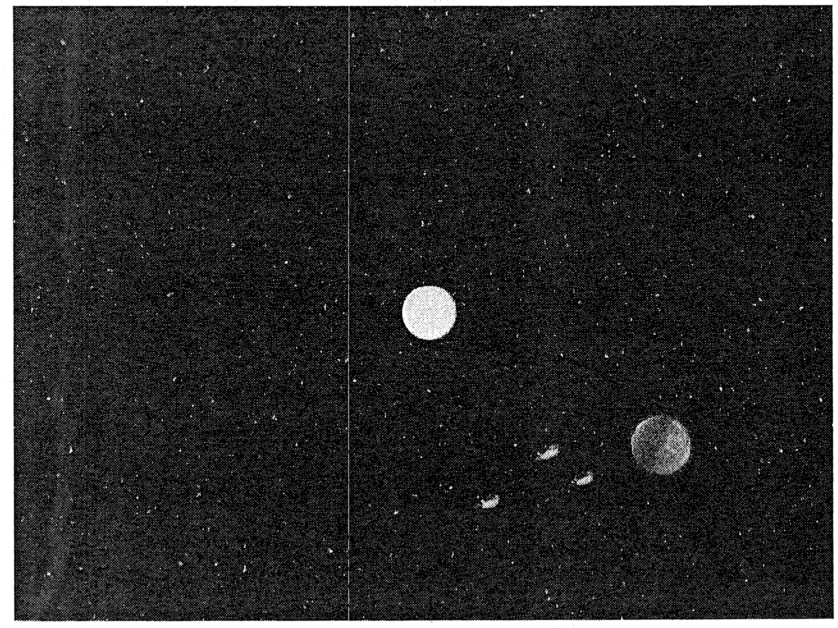


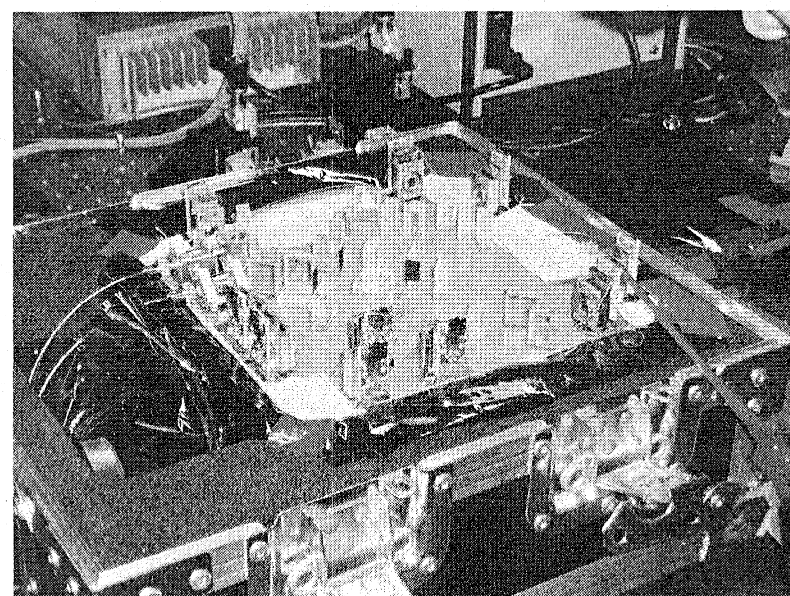


(graphic courtesy of B. Barish, LIGO-Caltech)

LISA: Laser Interferometric Space Antenna

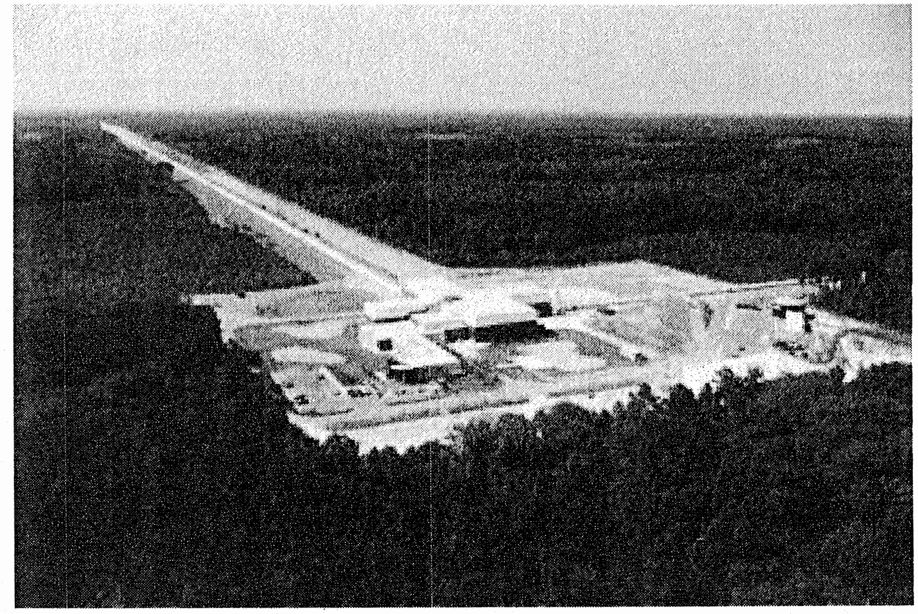
- NASA/ESA collaboration
- GSFC & JPL partnership
- detect *low frequency* GW
  10^{-4} Hz $\leq$ $f_{GW} \leq 1$ Hz
- typical sources: MBH/MBH binaries, galactic compact binaries, extreme mass ratio binaries…
- 3 spacecraft in equilateral triangle
  - orbits Sun at 1 AU
  - 20^{o} behind Earth in its orbit
- arm length $L = 5 \times 10^{6}$ km
- optical transponders receive & re-transmit phase locked light
- precision measurements:
  strain amplitude $h = \_L/L < 10^{-20}$



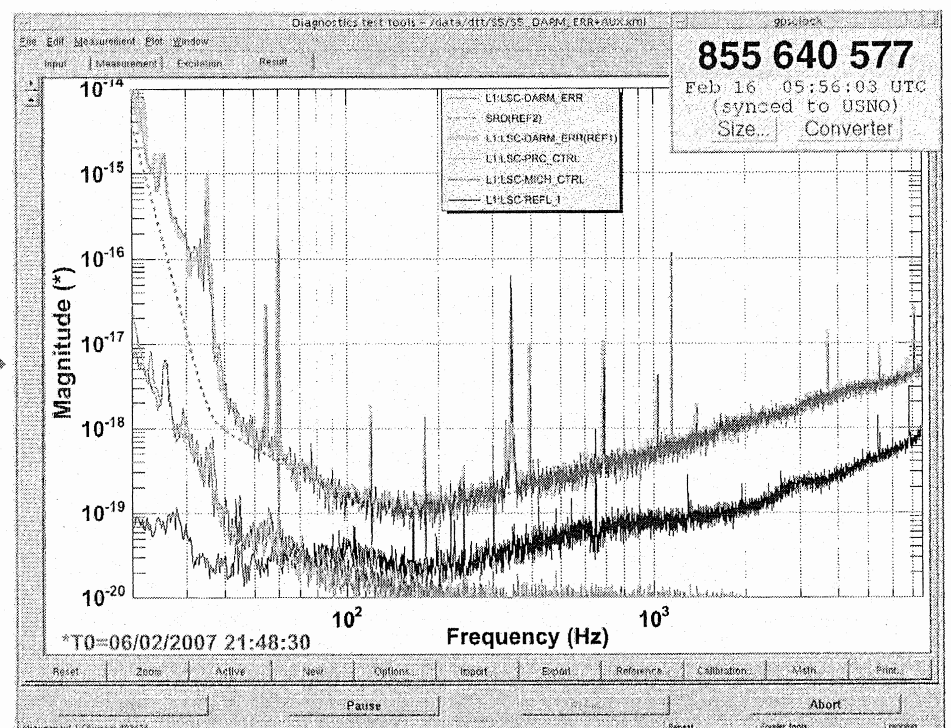


Ground-based detectors . . .

- detect high frequency GW
  10 Hz $\leq$ f_{GW} $\leq 10^4$ Hz
- kilometer-scale arms
  - LIGO: Hanford, WA, and Livingston, LA; $L = 4$ km
  - VIRGO: PISA, $L = 3$ km
  - GEO600: Hannover $L = 600$ m
- Typical sources: NS/NS, NS/BH, BH/BH, stellar collapse...

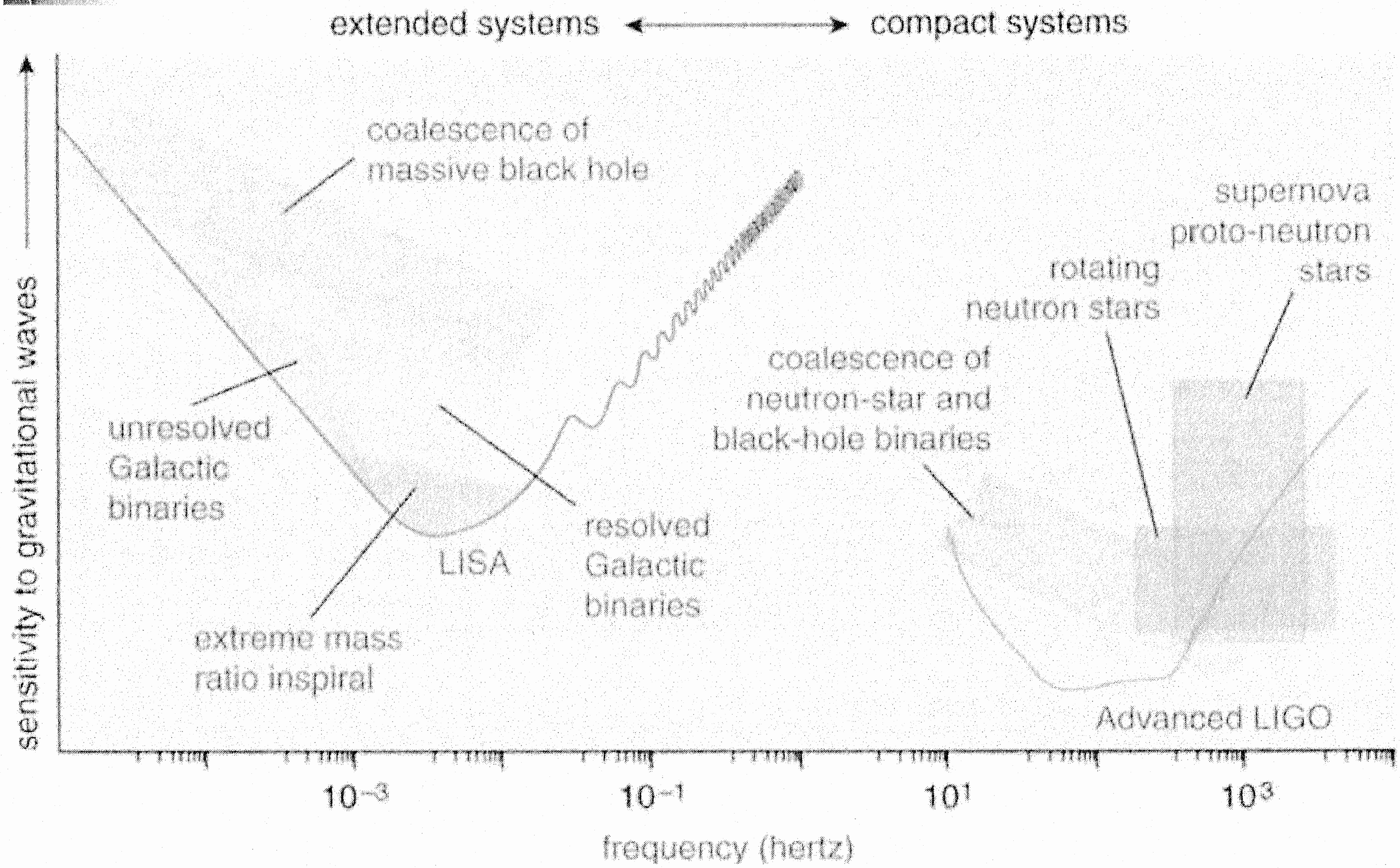


- LIGO/GEO currently in year-long science data-taking run....
- Current sensitivity of detector →
- Available in read-only mode daily at http://ilog.ligo-la.caltech.edu/ (see the Detector Group Log, Figure of Merit 4)

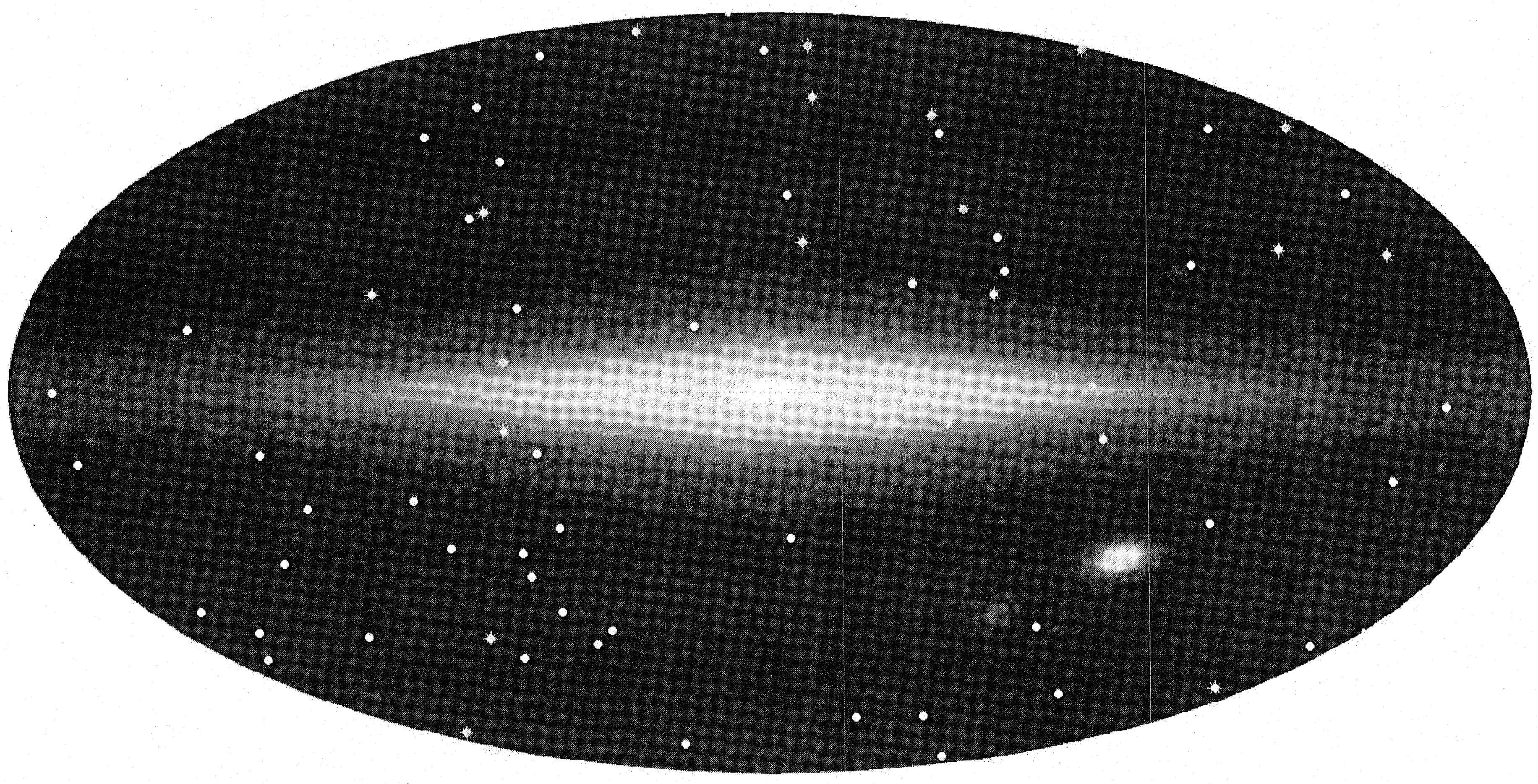


Gravitational Wave Spectrum…

- **Complementary observations, different frequencies & sources…**



Simulation of the GW sky in the LISA band….

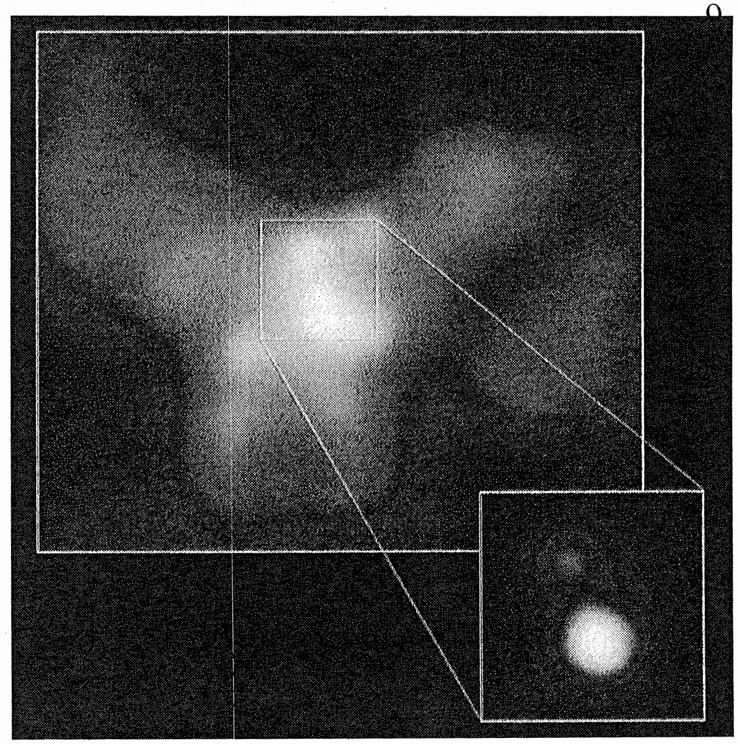


http://www.lisa-science.org/resources/talks-articles/science

Ground-based detectors will also see NS and stellar BH binary coalescences, supernovae…

MBH mergers…

- Final merger of MBHs occurs in the arena of very strong gravity
- Gravitational waves encode the dynamics of massive objects
- Observing GWs allows ***direct*** confrontation of GR w/ observations
- MBH mergers are strong GW sources



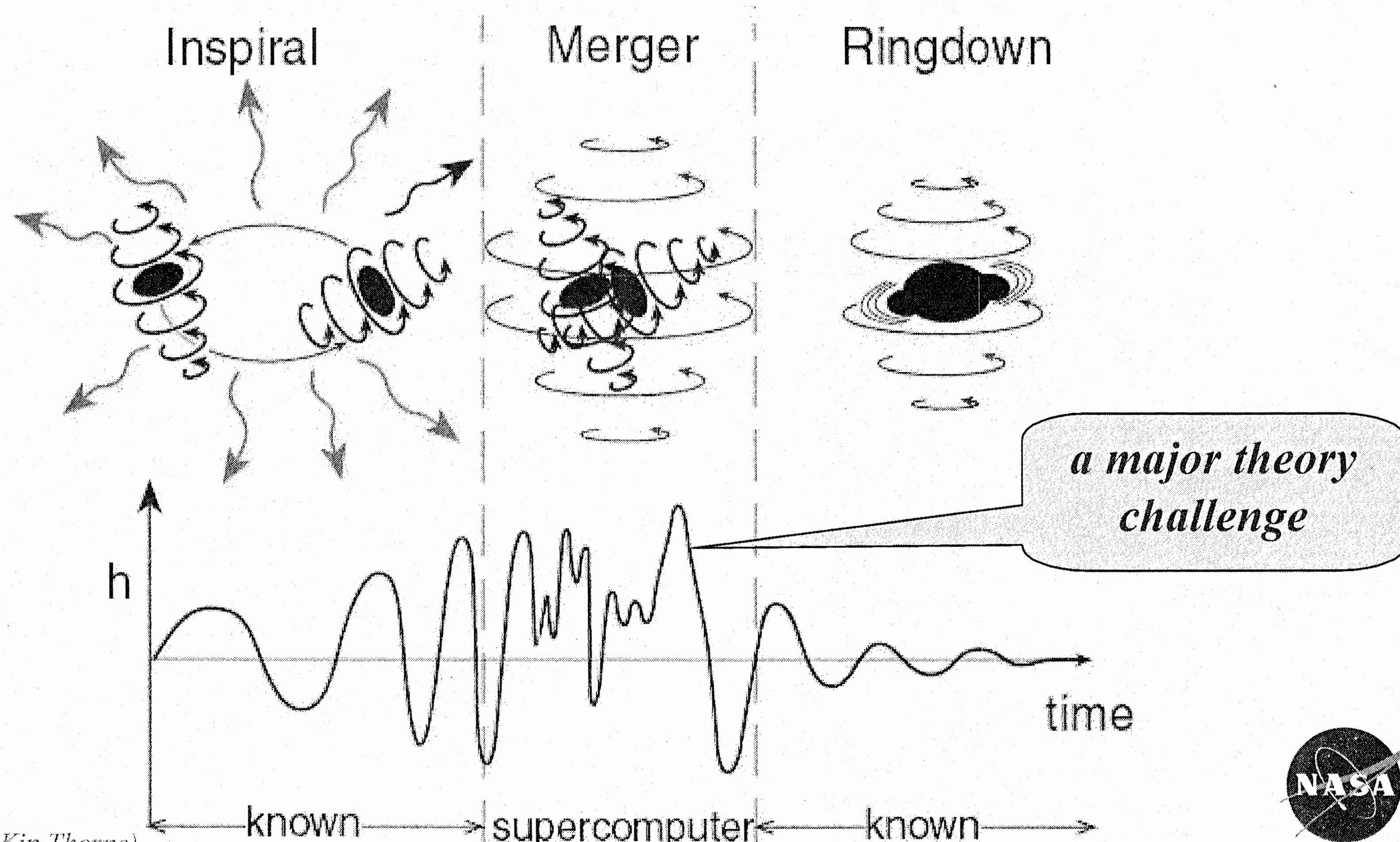
(NASA/CXC/MPE/S.Komossa et al.)

- ***LISA can confront GR with observations in the dynamical, strong field regime…***

  …. if we know the merger waveforms
- When MBHs are spinning, and/or $m_1 \neq m_2$, the GW emission is asymmetric → recoil kick
- If this kick is large enough, it could eject the merged remnant from the host structure… and ***affect the rates of merger events***
- MBH mergers could produce ***interesting spin dynamics***

GWs from final merger of black hole binary...

- **Strong-field merger is brightest GW source, luminosity ~ $10^{23}L_{SUN}$**
- **Requires *numerical relativity* to calculate dynamics & waveforms**
- **Waveforms scale w/ masses, spins → apply to ground-based & LISA**

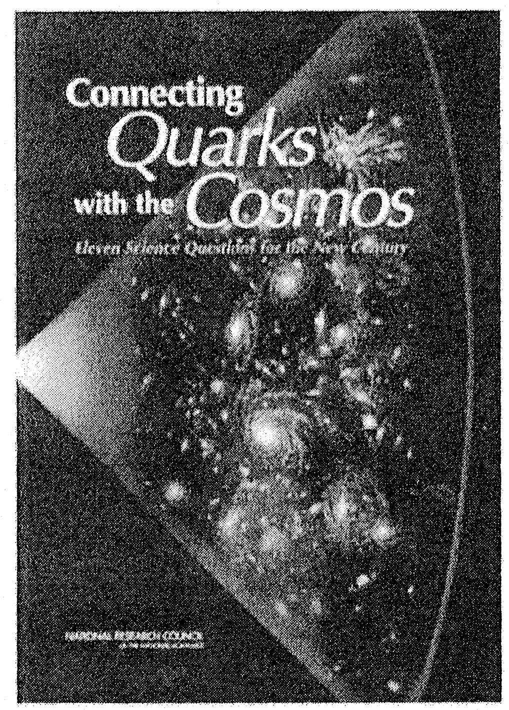


(graphic courtesy of Kip Thorne)

A major theory challenge….

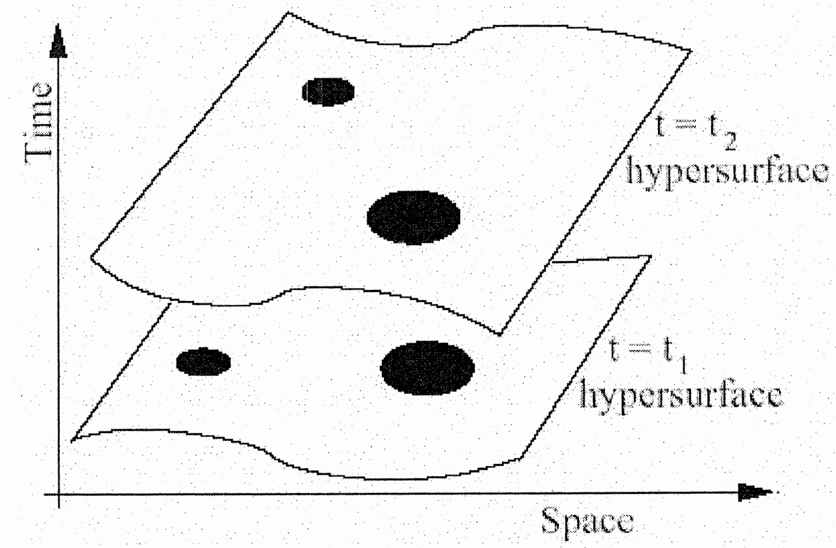
"Nearly as difficult as building these *(gravitational wave)* observatories, however, is the task of **computing the gravitational waveforms** that are expected when two black holes merge. This is a **major challenge** in computational general relativity and one that will stretch computational hardware and software to the limits. However, a bonus is that the **waveforms will be quite unique to general relativity**, and if they are reproduced observationally, scientists will have performed a **highly sensitive test of gravity in the strong-field regime**."

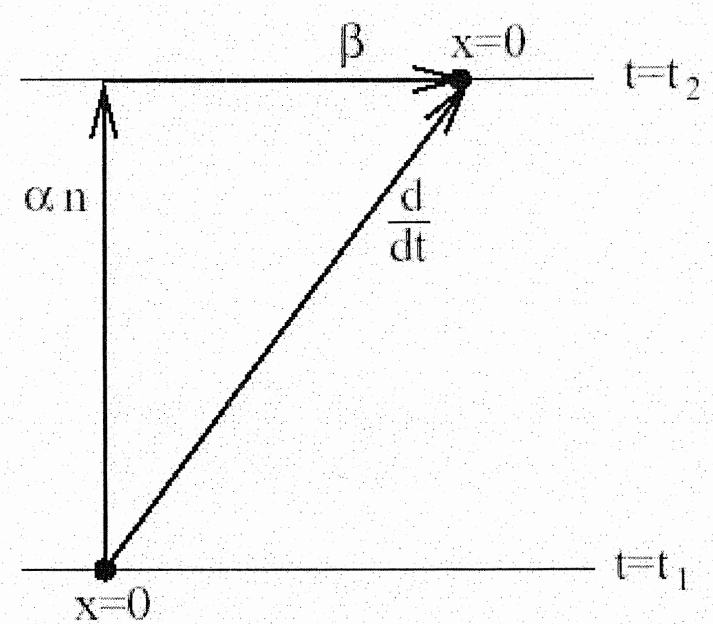
-- "What are the Limits of Physical Law?" in ***Connecting Quarks with the Cosmos: Eleven Science Questions for the New Century*** (Board on Physics and Astronomy, National Academies, 2003), p. 118.



Numerical Relativity....

- Solve Einstein eqns numerically
- Spacetime sliced into 3-D *t* = constant hypersurfaces
- Einstein's eqns split into 2 sets:
  - Constraint equations
  - Evolution equations
- Constrained initial data at t = 0
- Evolve forward in time, from one slice to the next
- Typically solve 17 or more nonlinear, coupled PDEs
- Coordinate or gauge conditions: relate coordinates on neighboring slices
  - lapse function _, shift vector _^{i}





A Brief History of BBH simulations....

- 1964: Hahn & Lindquist: try to evolve collision of 2 "wormholes"
- 1970s: Smarr and Eppley: head-on collision of 2 BHs, extract GWs
  - Pioneering efforts on supercomputers at Livermore Natl Lab
- 1990s: LIGO moves ahead & work on BBH problem starts up again..
  - Work on 2-D head-on collisions at NCSA
  - NSF Grand Challenge: multi-institution, multi-year effort in 3-D

→ *This is <u>really</u> difficult! Instabilities, issues in formalisms, etc...*

  - Diaspora: multiple efforts (AEI, UT-Austin, PSU, Cornell...)
  - Difficulties proliferate, instabilities arise, codes crash....
  - *"Numerical relativity is impossible..."*
- 2000s: LIGO/GEO/VIRGO and LISA spur more development
  - New groups arise: Caltech, UT-Brownsville, LSU, NASA/GSFC...
- Since 2004.....
  - Breakthroughs & rapid progress throughout community
  - Orbits, waveforms, and astrophysical applications....

Issues and ingredients for success...

- **Formulations of the Einstein equations**
  - **fully 2nd order, fully 1st order, mixed 1st and 2nd order PDEs**
  - **which variables to use?**
  - **incorporate constraints into evoln eqns? solve constraints?**
- **Coordinate conditions:**
  - **lapse function _ – "singularity avoiding" time slicing**
  - **shift vector _^{i} – keep coordinates from falling into black holes...**
- **Constrained initial data to approximate astrophysical binary**
  - **start on approx quasi-circular orbits**
  - **inward radial velocity...**
- **How to handle the black holes:**
  - **excision? punctures?**
  - **comoving coordinates? move the black holes?**
- **Variable grid resolution to handle multiple scales:**
  - **_$_{GW}$ ~ (10 – 100)M**
  - **c = G = 1 → 1 M ~ 5 x 10^{-6} (M/M$_{Sun}$) sec ~ 1.5 (M/M$_{Sun}$) km**
  - **finite differences w/ mesh refinement; spectral methods**

The 1st complete BBH orbit…

- **Bruegmann, Tichy, & Jansen, PRL, 92, 211101 (2004), gr-qc/0312112**
- **equal mass, nonspinning BHs**
- **Represent BHs as "punctures":**

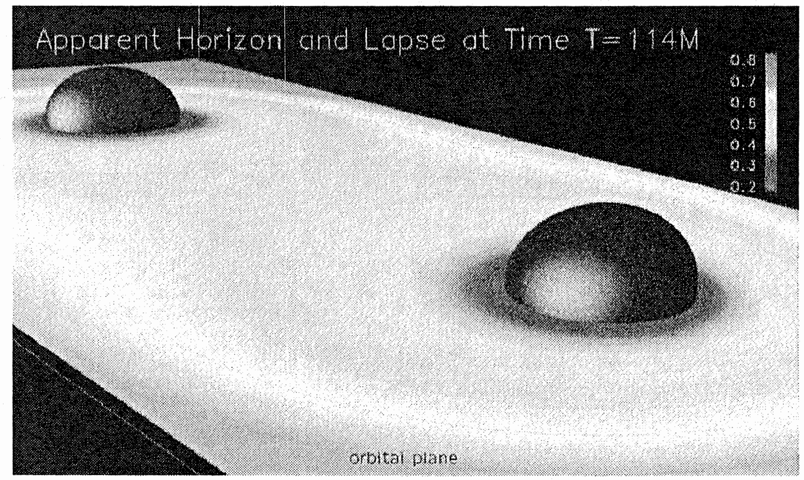
$$g_{ij} = \psi^4 \delta_{ij} \qquad \psi = \psi_{BL} + u$$

$$\psi_{BL} = 1 + \sum_{n=1}^{2} m_n / 2 | r - r_n |$$

- **Handle singular _BL analytically; evolve only nonsingular *u***

  → fix the BH punctures in the grid
- **Use comoving shift vector _**
- **Conformal formalism**
  - g_{ij}, $A_{ij} \sim \partial_t g_{ij}$
  - **1st order time, 2nd order space**

- **Traditional numerical relativity techniques**
- **Excise BHS at late times**

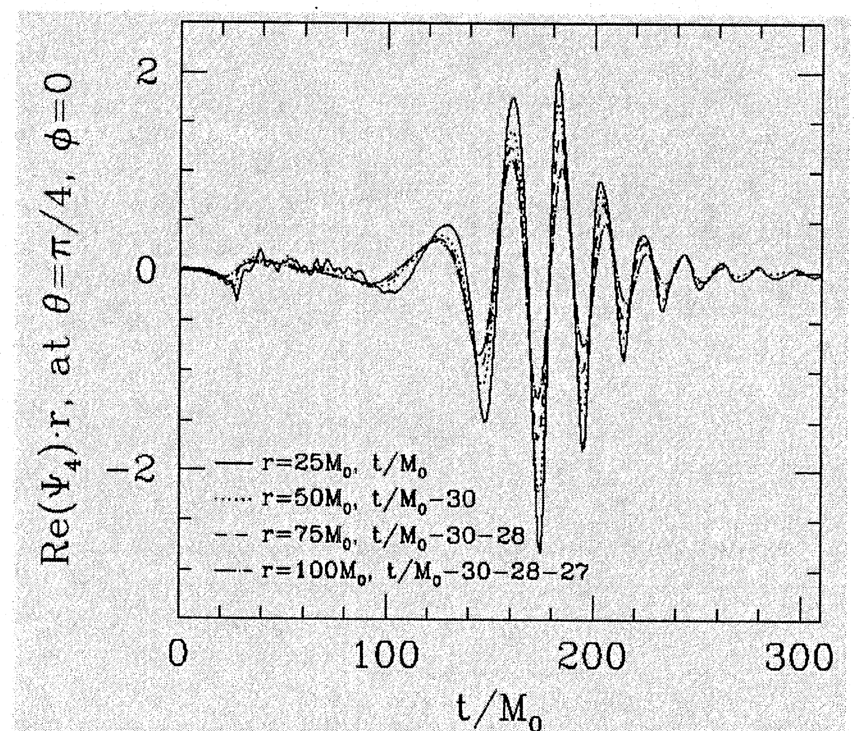


- **Ran for ~ (125 – 150)*M* and BHs completed ~ 1 orbit**
- **Crashed before BHs merge**
- **Not accurate enough to be able to extract GWs**

The 1st orbit, merger, & ringdown…

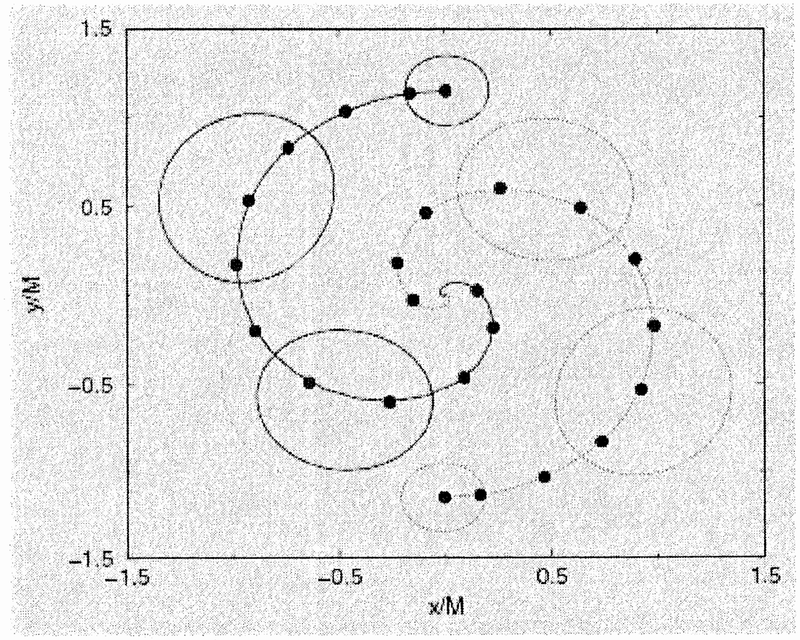
- Pretorius, PRL, 95, 121101 (2005) gr-qc/0507014
- Different formalism: based on "generalized harmonic coords"
  - metric g_{ij} is basic variable
  - 2^{nd} order in space & time
- Excised BHs move through grid
- AMR: high resolution around BHs, tracks BHs as they move
- "Compactified" outer boundary: edge of grid at spatial infinity
- Equal mass, nonspinning BHs
- Start with 2 "blobs" of scalar field that collapse to BHs, then complete ~ 1 orbit

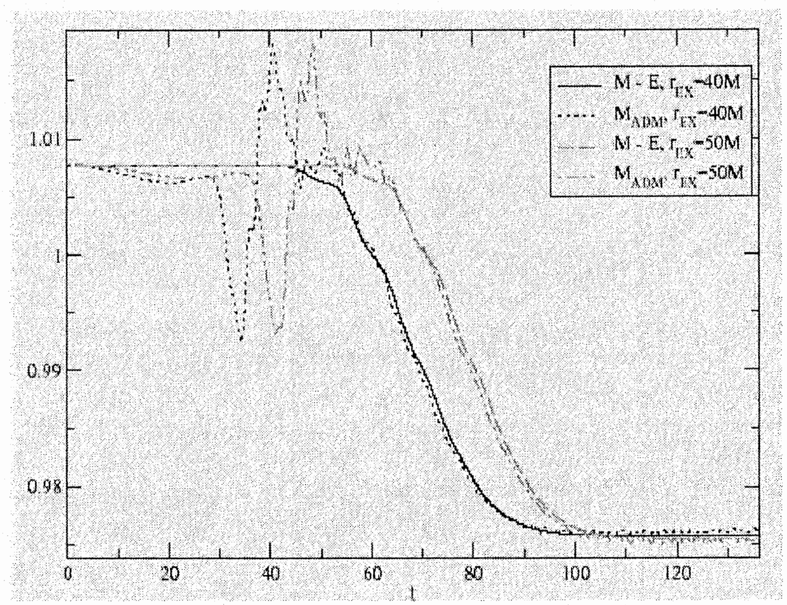
- Indiv BH mass M_0 ($M \sim 2M_0$)
- Show waveforms extracted at different radii (scaled)
- Re(_4) ~ d^2/dt^2 (h_+)



A new idea: "moving puncture BHs"

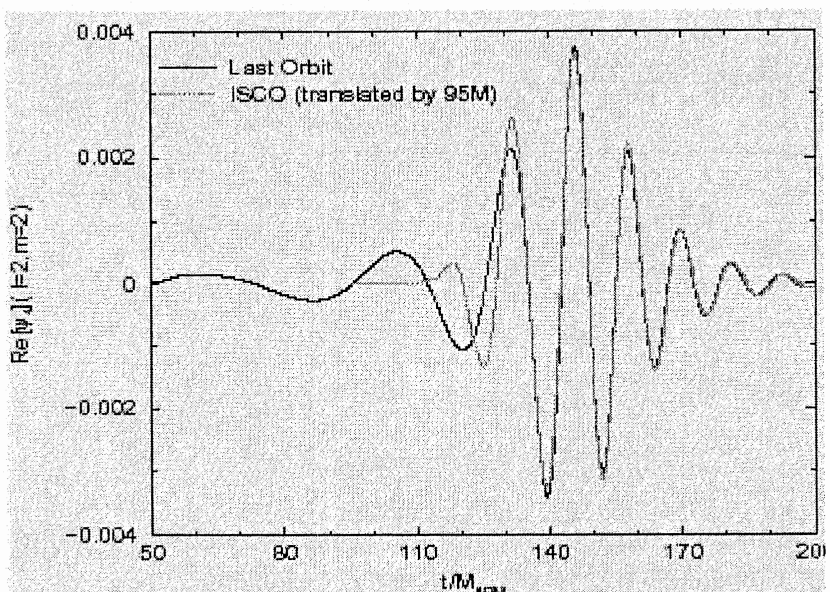
- Allow puncture BHs to move across grid w/out excision
- Simultaneous, independent discovery by UTB & GSFC groups:
  - Campanelli, et al., PRL, 96, 111101 (2006), gr-qc/0511048
  - Baker, et al., PRL, 96, 111102 (2006), gr-qc/0511103
- Do not split off singular part _BL
  - Regularize near puncture
  - New conditions for _ & _i
- Uses conformal formalism
- Enables long duration, accurate simulations





A powerful new idea….that spread rapidly

- Developed w/in "traditional" numerical relativity approach:
  - Conformal formalism, BHs represented as punctures
- A simple, powerful new idea: allow the punctures to move
- Requires novel coordinate conditions:
  - Van Meter, et al., "How to move a puncture black hole without excision...," PRD 73 (2006) 124011 2006), gr-qc/0605030
- UTB, GSFC moved ahead rapidly, quickly able to do multiple orbits
- Moving punctures quickly adopted by other groups:
  - PSU, AEI/LSU, FAU/Jena…
  - At April 2006 APS meeting, a full session devoted to BBH mergers w/ moving punctures!
  - Summer 2006: method adopted by most of community

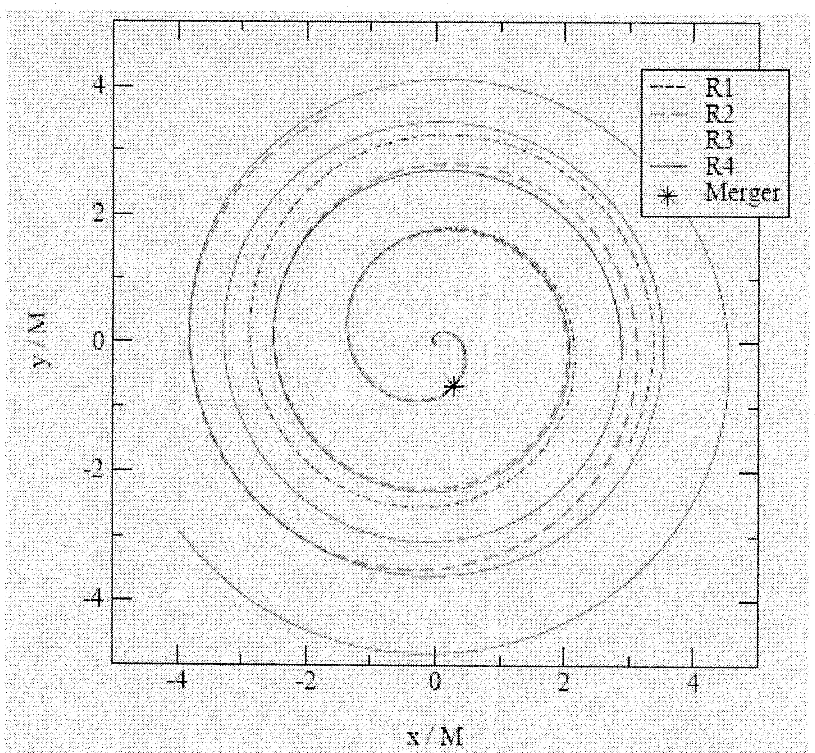


Campanelli, et al., PRD, 73, 061501 (2006), gr-qc/06010901

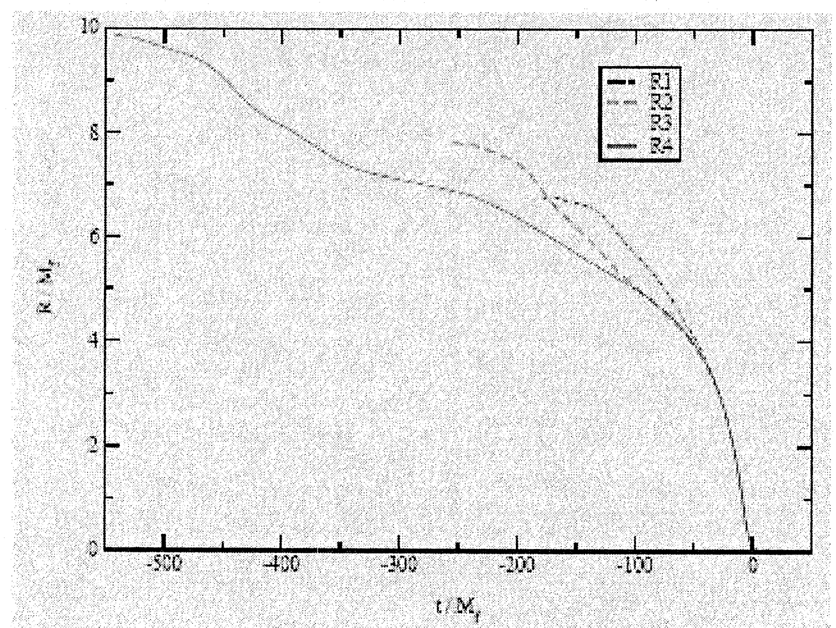
Revealing universal behavior…

- Baker, al., PRD, 73, 104002 (2006), gr-qc/0602026
- Long duration simulations of moving punctures with AMR
- Equal mass, nonspinning BHs
- Run several cases, starting from successively wider separations
- BH orbits lock on to universal trajectory ~ one orbit before merger

BH trajectories (only 1 BH shown)

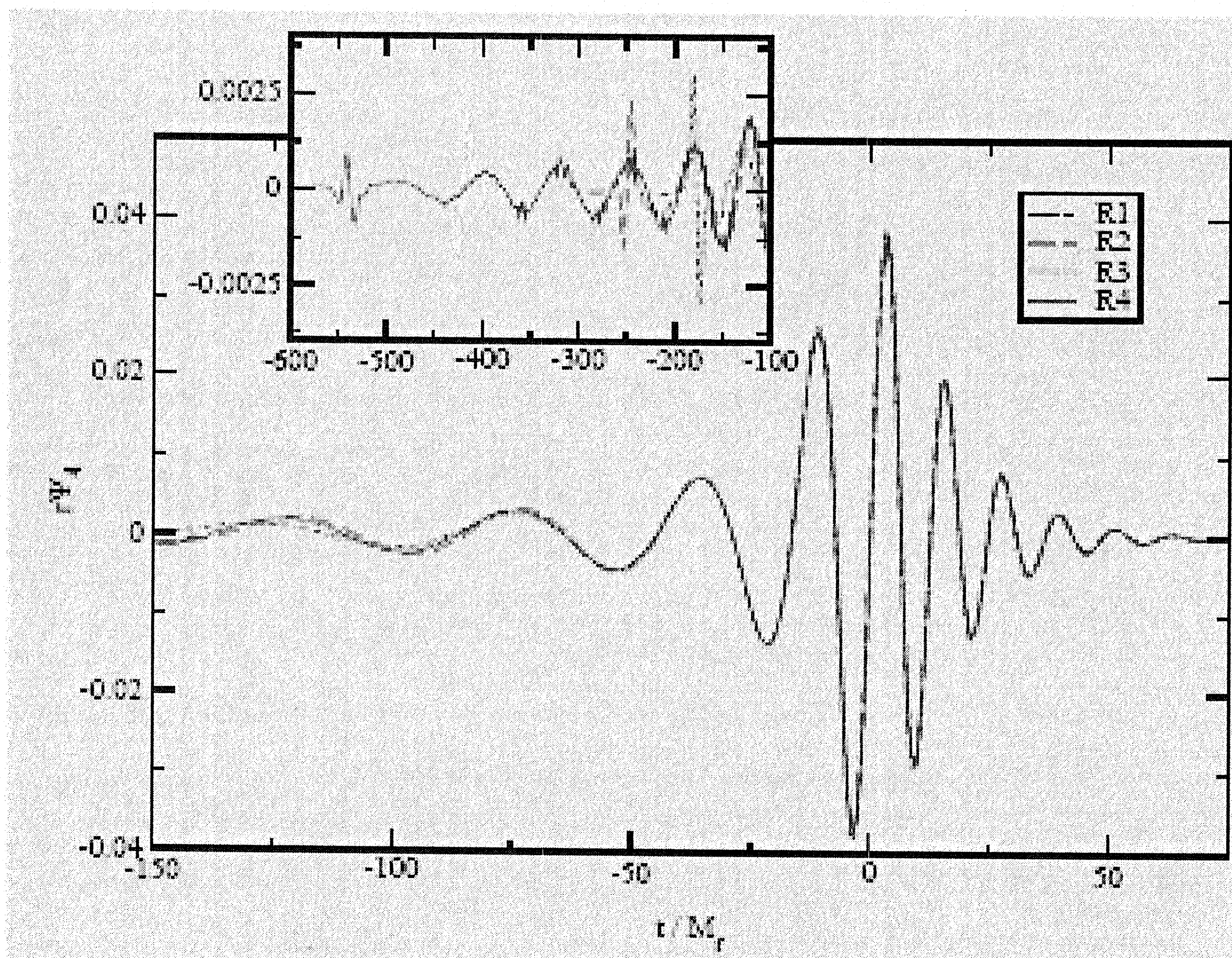


BH separation vs. time

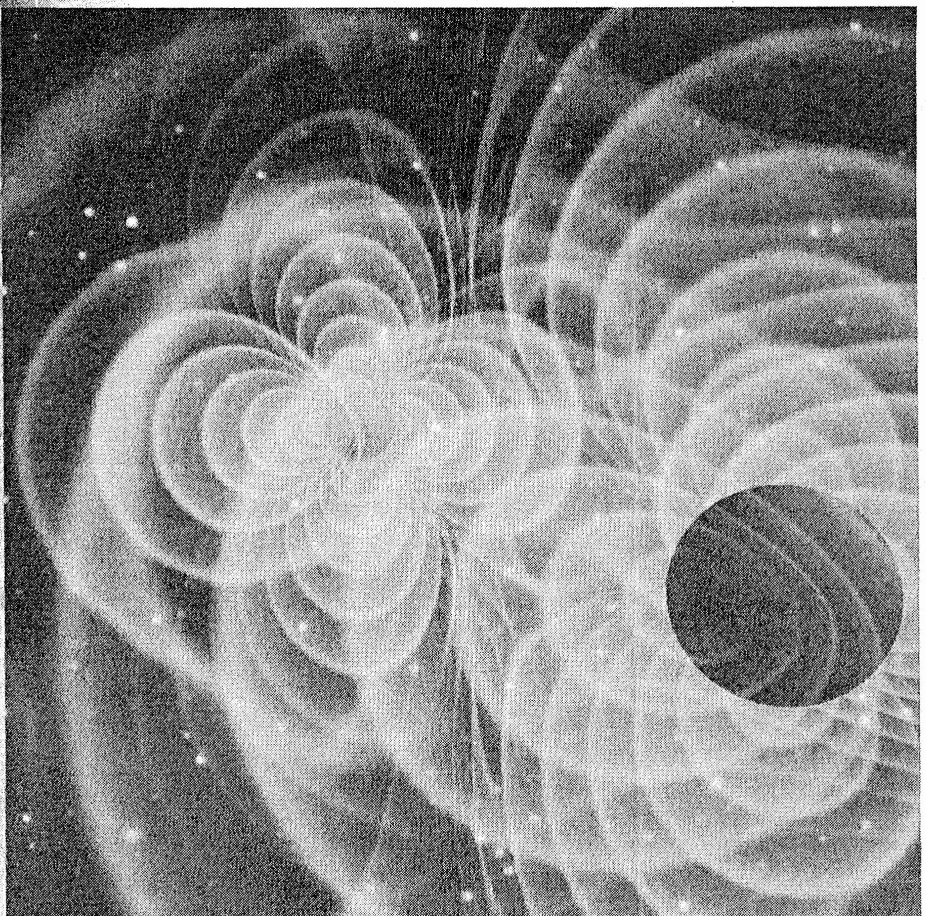


Universal waveform....

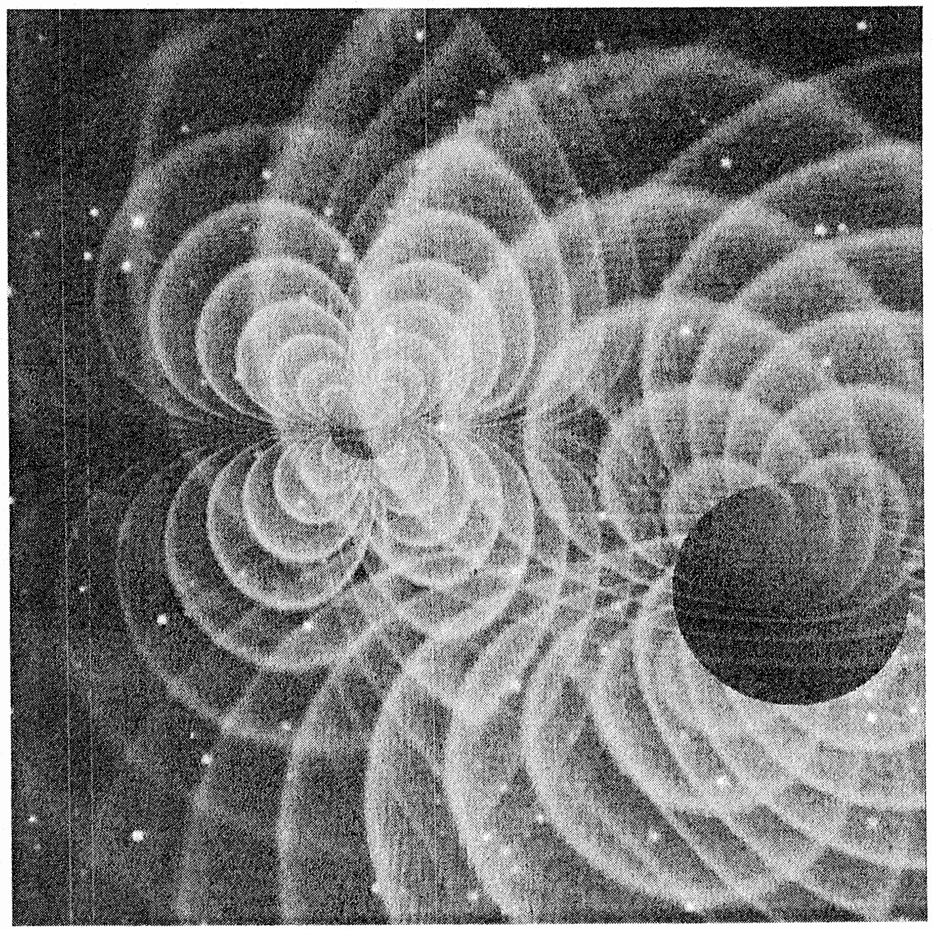
- Universal dynamics produces universal waveform....
- All runs agree to within < 1% for final orbit, merger & ringdown



Binary Black Holes: The Movies



$\mathrm{Re}[\ _{4}\ ] \sim d^2/dt^2\ h_+$

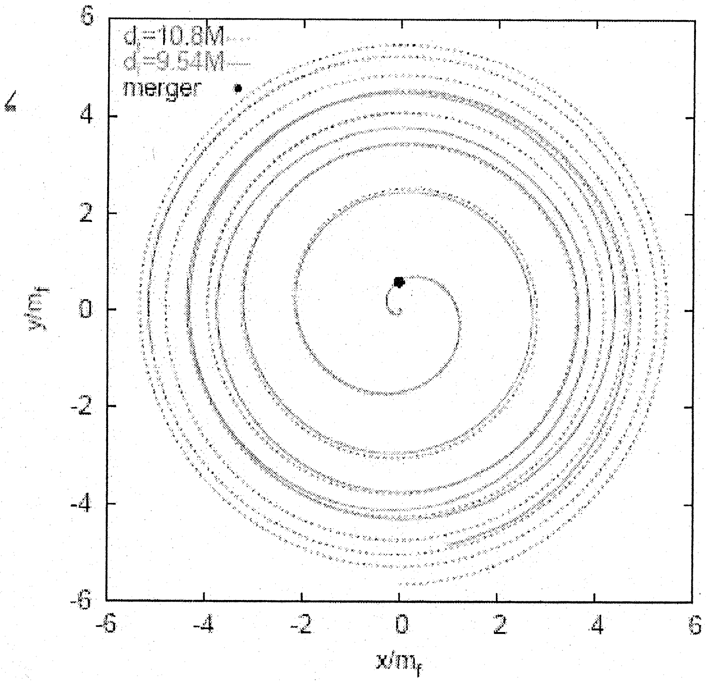


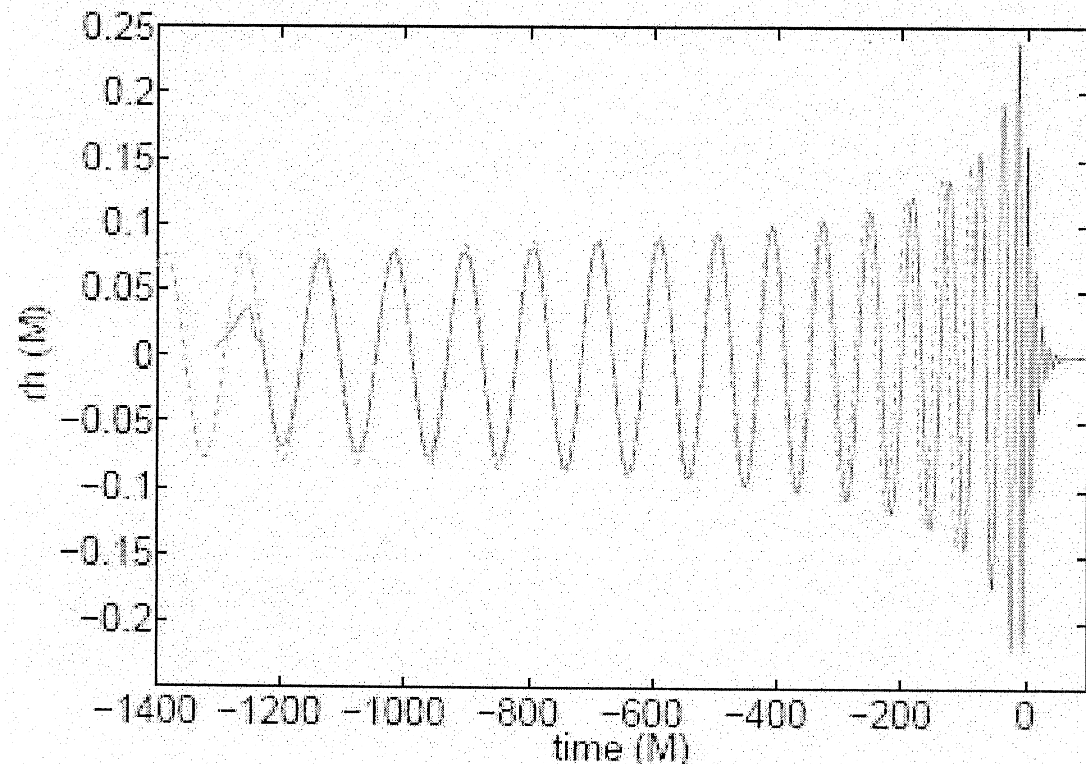
$\mathrm{Im}[\ _{4}\ ] \sim d^2/dt^2\ h_x$

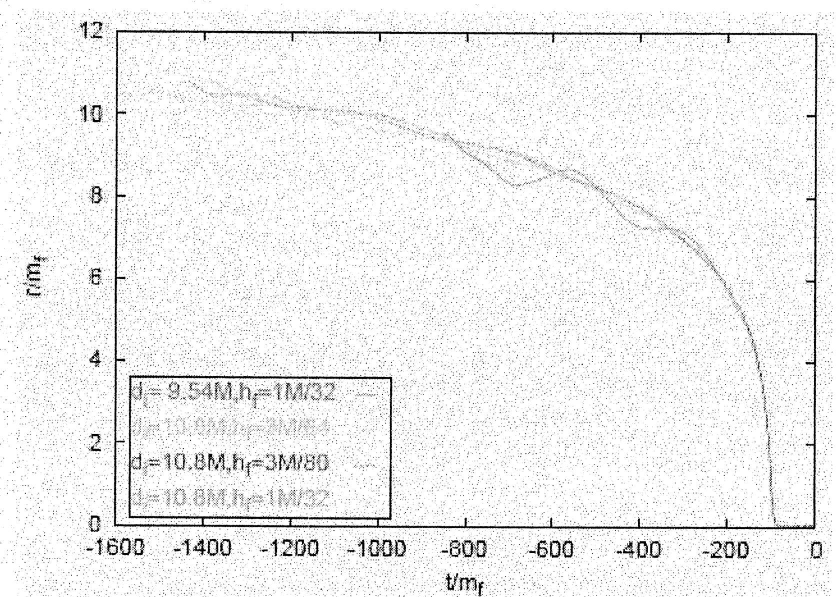
(Visualizations by Chris Henze, NASA/Ames)

Longer runs, starting in late inspiral...

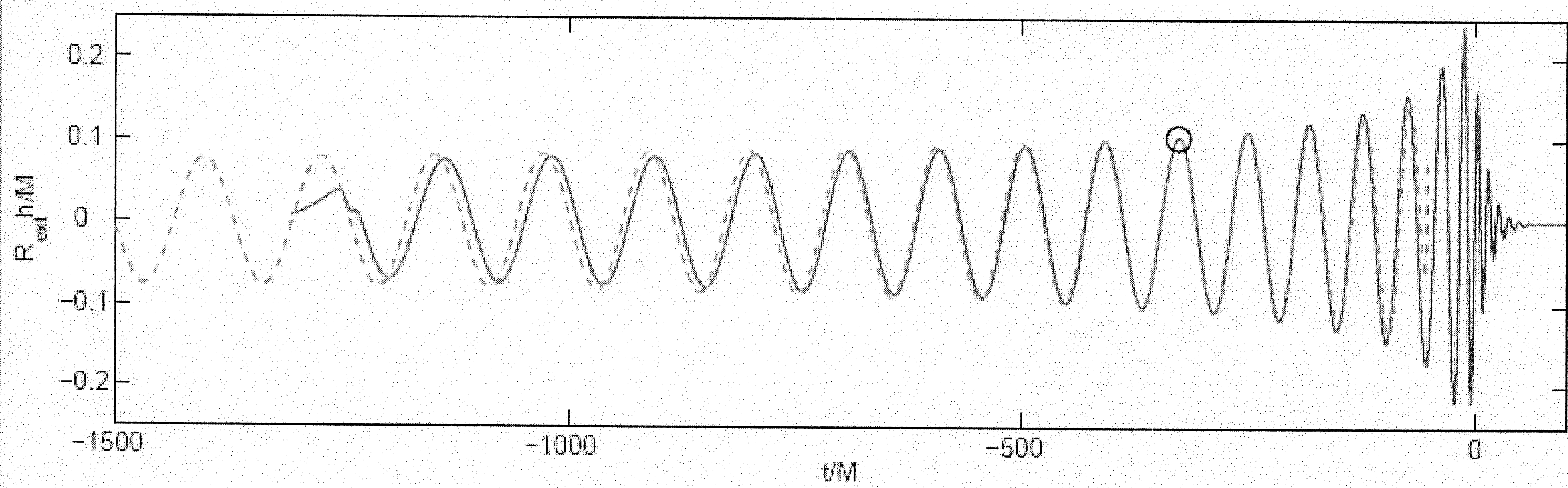
- Baker, et al., gr-qc/0612117; gr-qc/061202*
- Evolve ~ 1200M and ~ 7 orbits before merging
- Lower initial eccentricity e ~ 0.008
- Validation of 3.5 PN in late inspiral







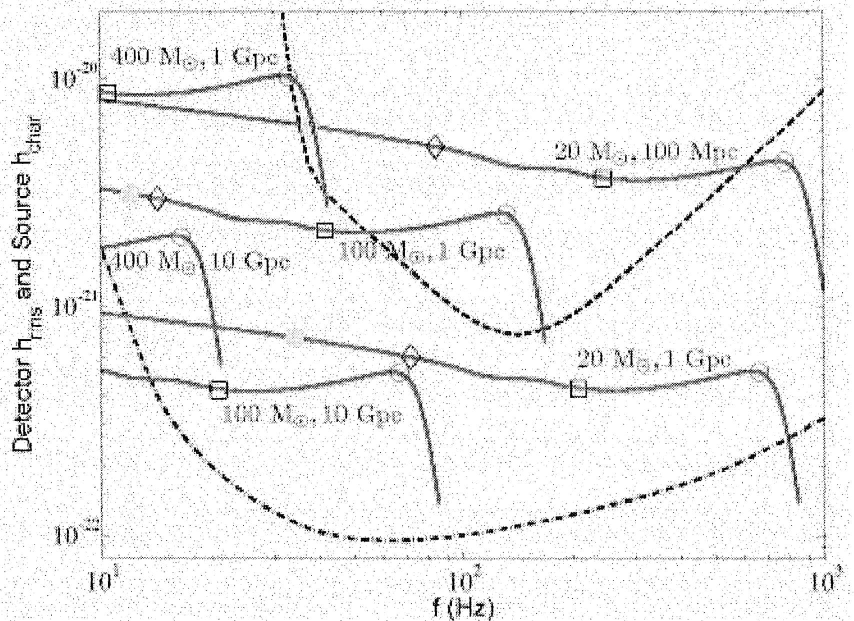
"Observing" the mergers…

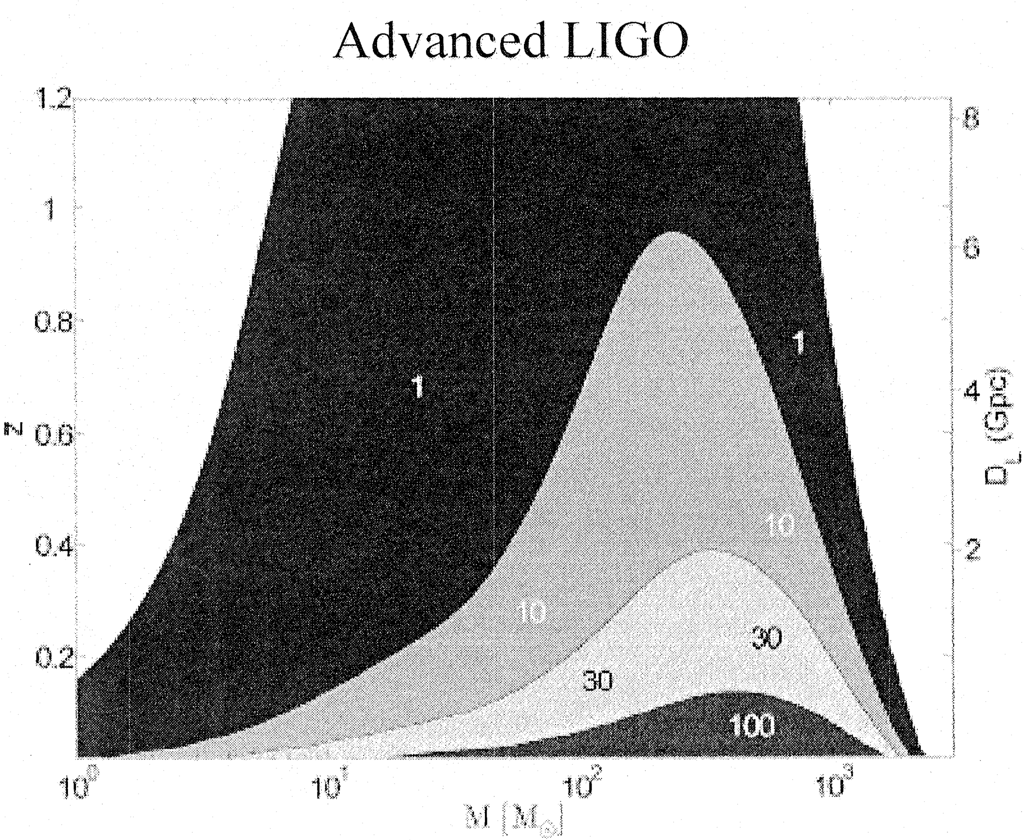


- **Baker, et al., gr-qc/0612117**
- **Make composite waveform**
- **Compare sensitivity, SNR for current and future detectors**

Observing BH mergers w/ LIGO…

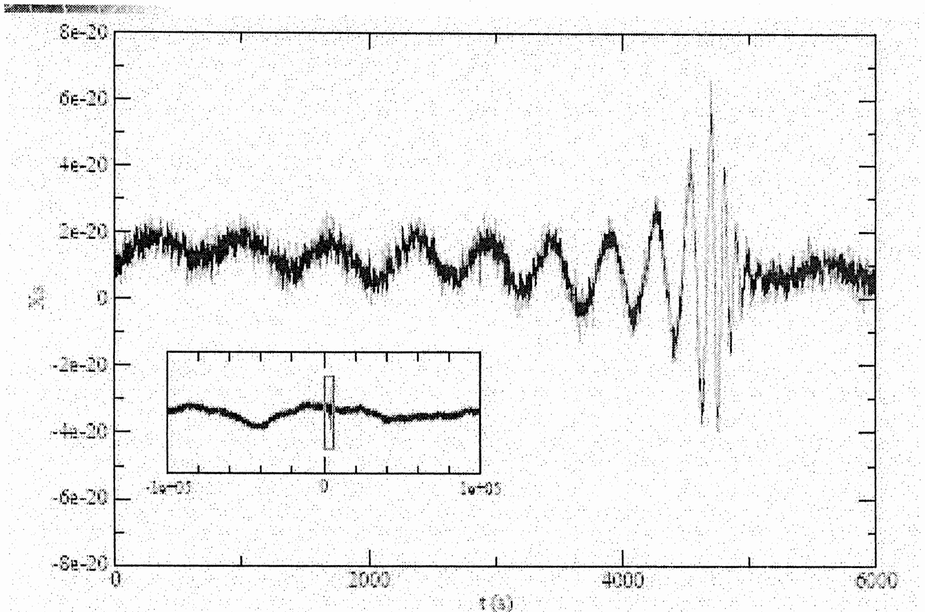
- Note these results are for equal mass, non-spinning BHs
- Unequal masses, spins will alter these results…

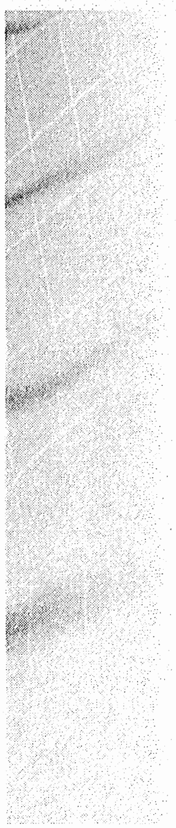


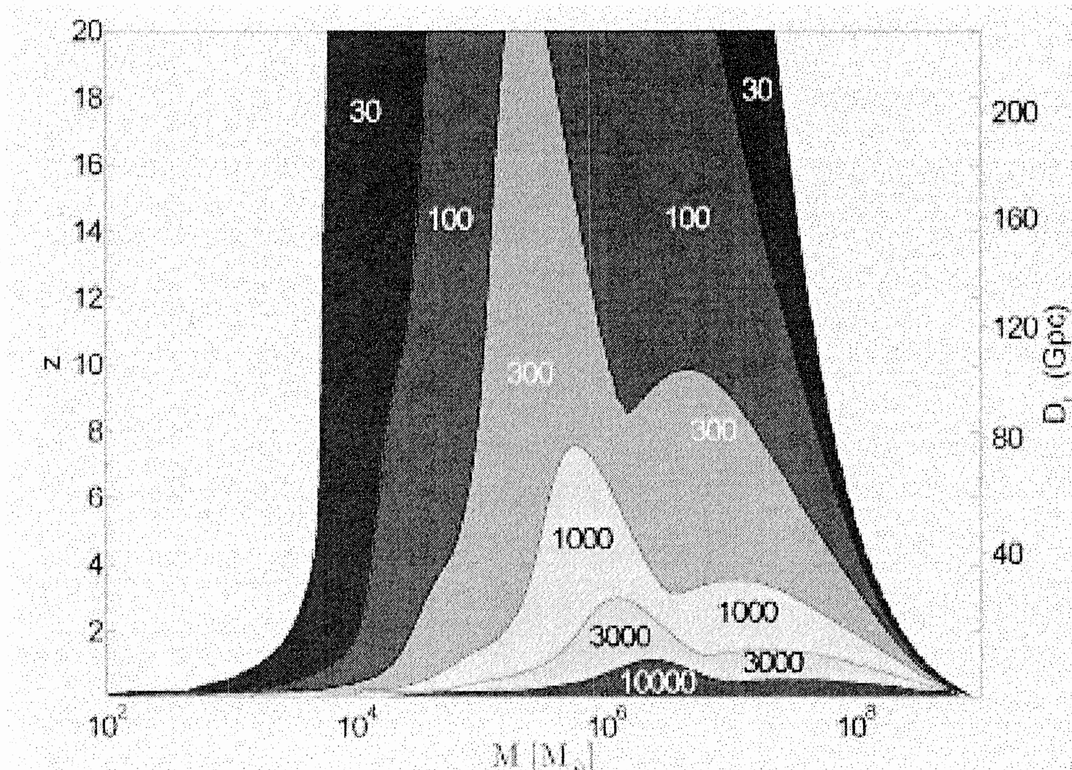


Observing MBH mergers with LISA….

Baker, et al., gr-qc/0612117

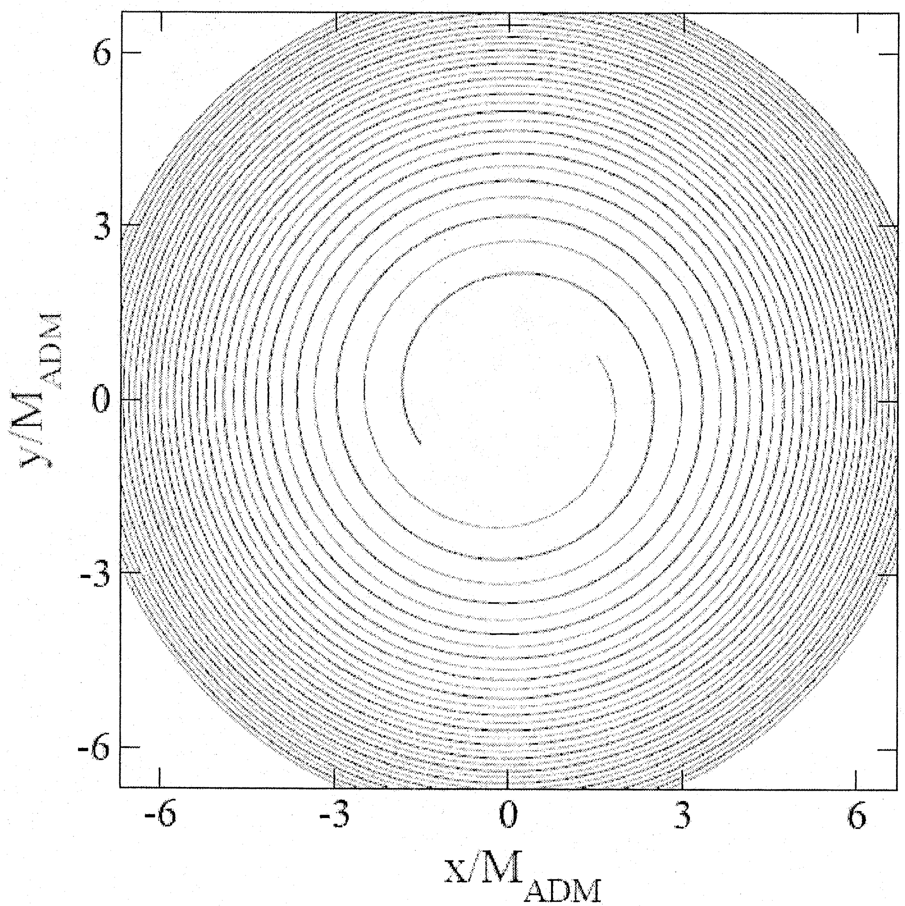






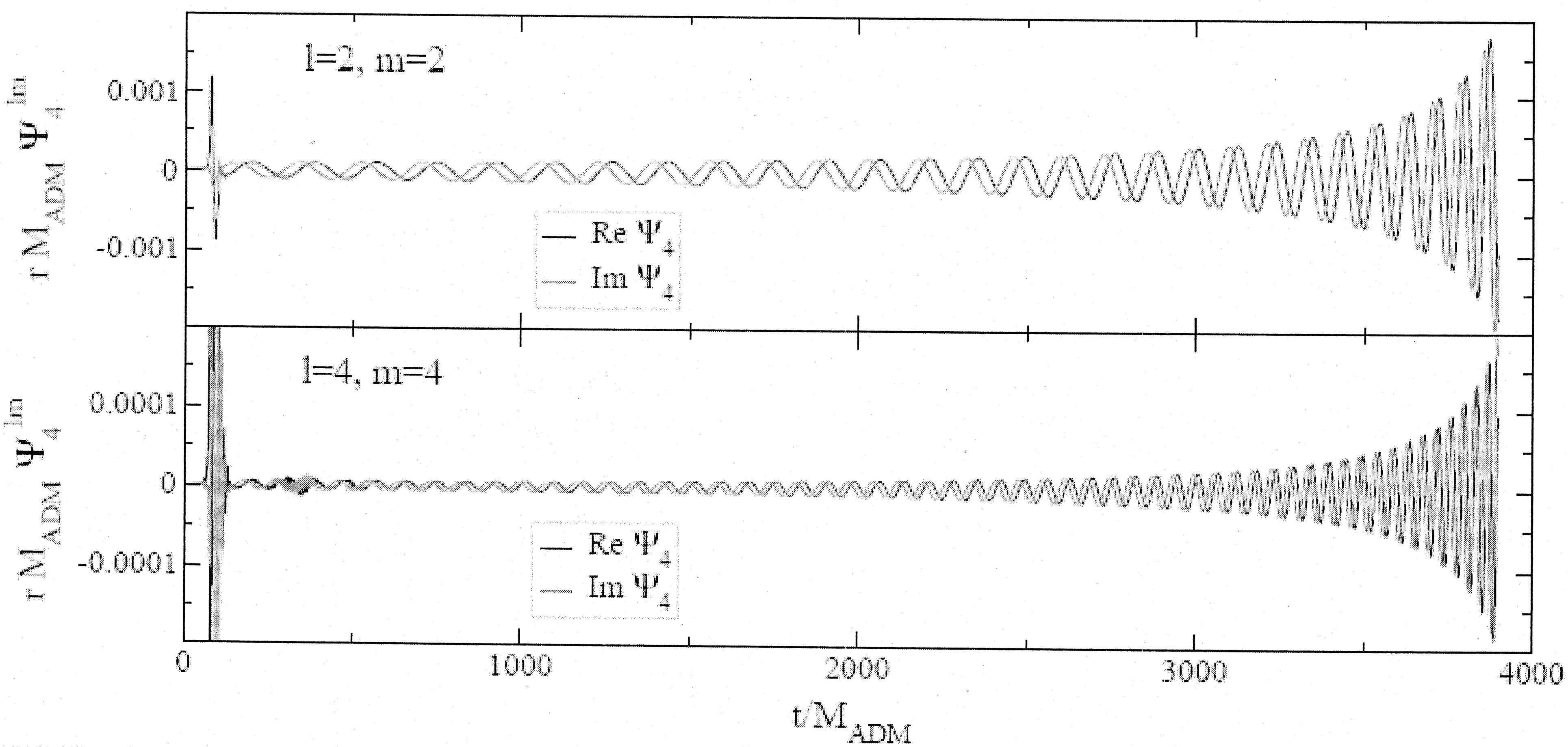
New results for inspiral regime…

- Caltech/Cornell collaboration
- Use 1st order form of generalized harmonic formalism
- Multi-domain spectral code – very rapid convergence
- BHs are excised
- Rotating coordinates
- Evolve ~ 15 orbits of inspiral
- Need to re-grid to handle merger and ringdown – work in progress



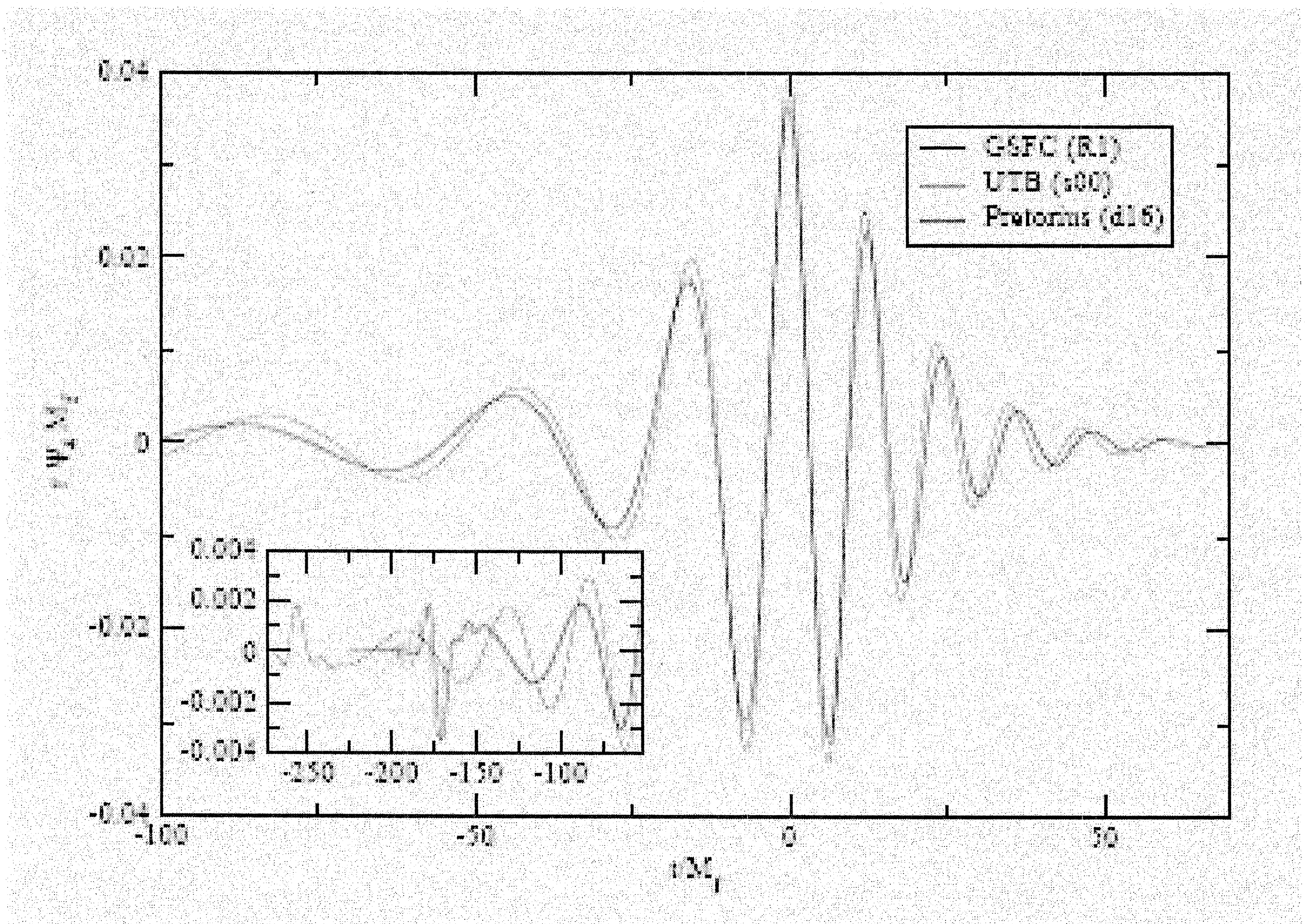
Long wavetrains...

- Evolve for nearly 4000M
- Very low phase errors...< 0.1 radians over 15 orbits
- confirm results for accuracy of PN in during inspiral



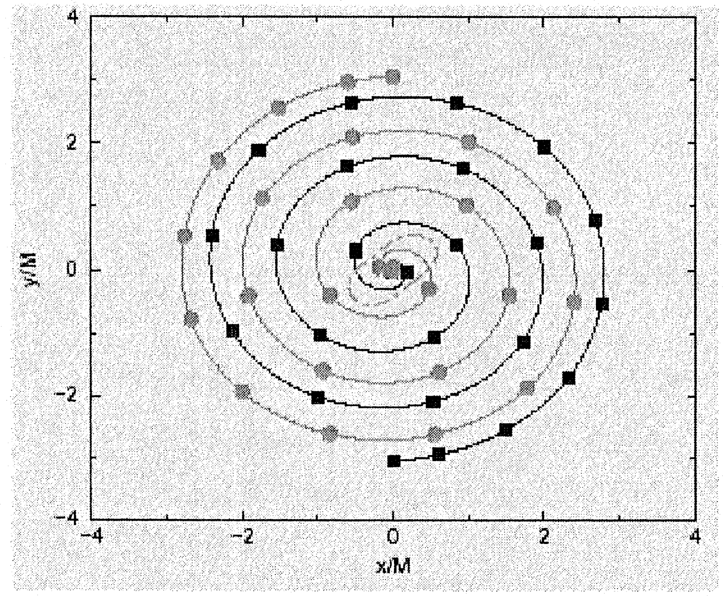
Comparison of gravitational waveforms…

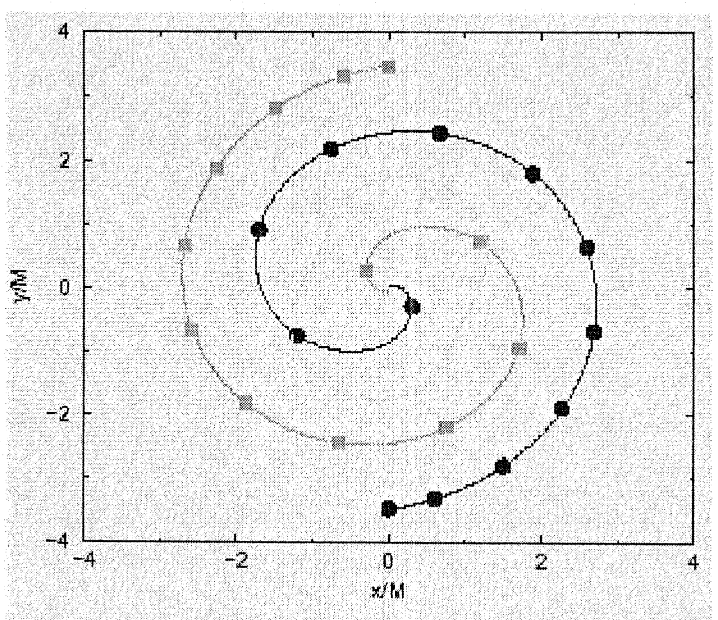
- **Baker, Campanelli, Pretorius, Zlochower, gr-qc/0701016**
- Compare GWs from equal mass, nonspinning case
- 3 different , independently-written codes

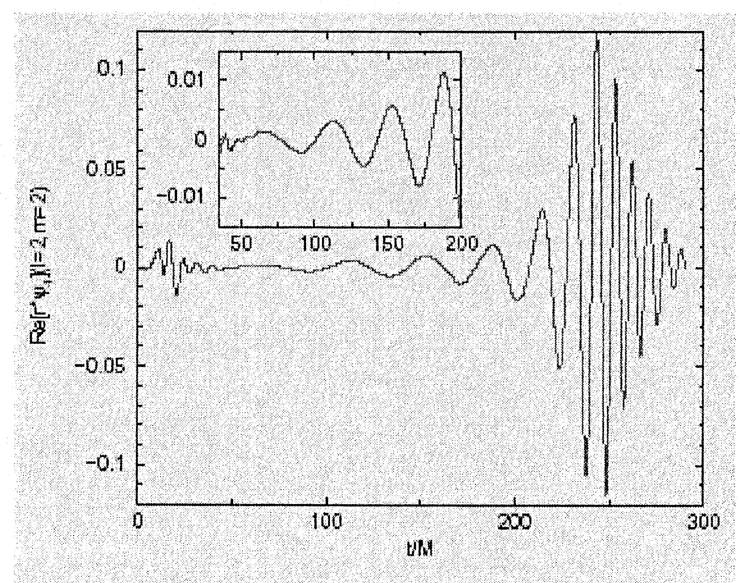


Evolutions of equal mass BHs with spin…

- Campanelli, et al., Phys.Rev. D74 (2006) 041501 (gr-qc/0604012)
- Moving punctures; 1st BBHs with spin
- Equal masses, each with a = 0.75 m
- Initially M_ = 0.05 → $T_{orbital}$ ~ 125M
- Anti/aligned → attractive/repulsive
- Final a=0.9M (aligned), a=0.44M (anti)

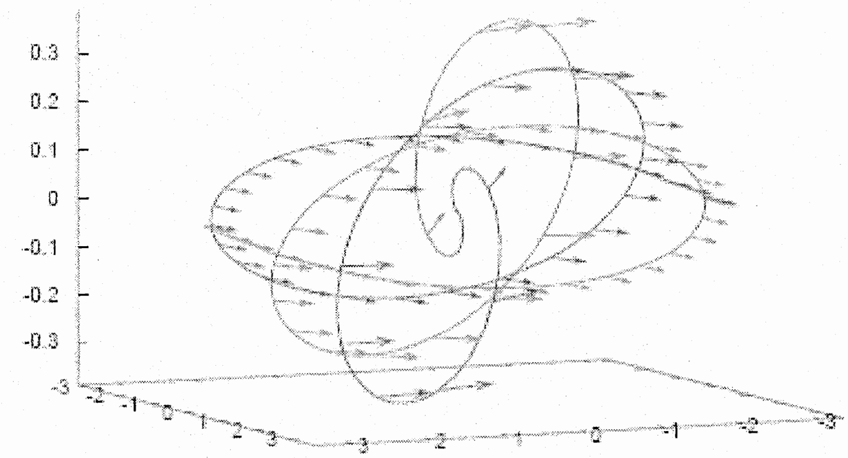


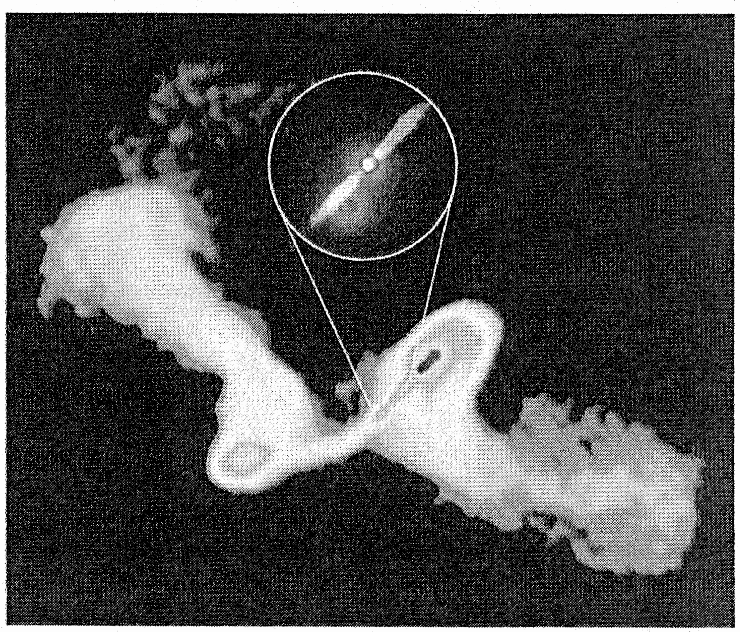


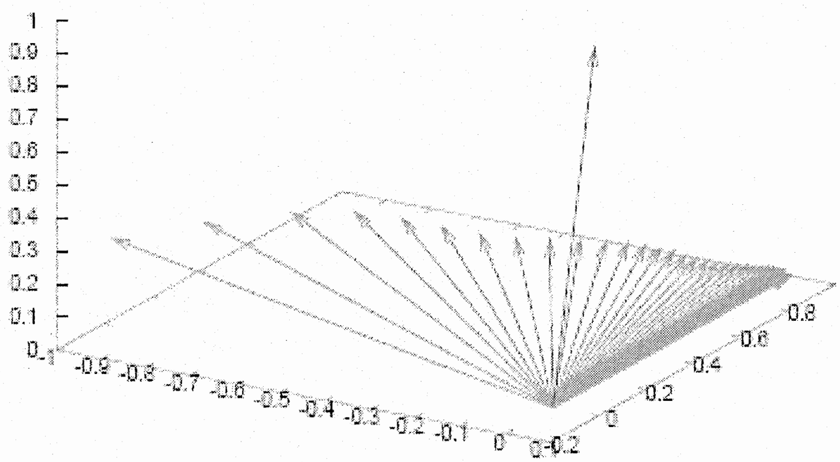


A spin flip… Campanelli, et al., gr-qc/0612076

- Equal masses and spins
- Parallel spins in orbital plane
- Spins precess by > 90 deg →
- Final spin of remnant "flips" by ~ 72 deg from initial spins
- Also evolve w/ parallel spins at 45 deg to orbital plane
- Remnant BH has spin flipped by ~ 34 deg from initial spins

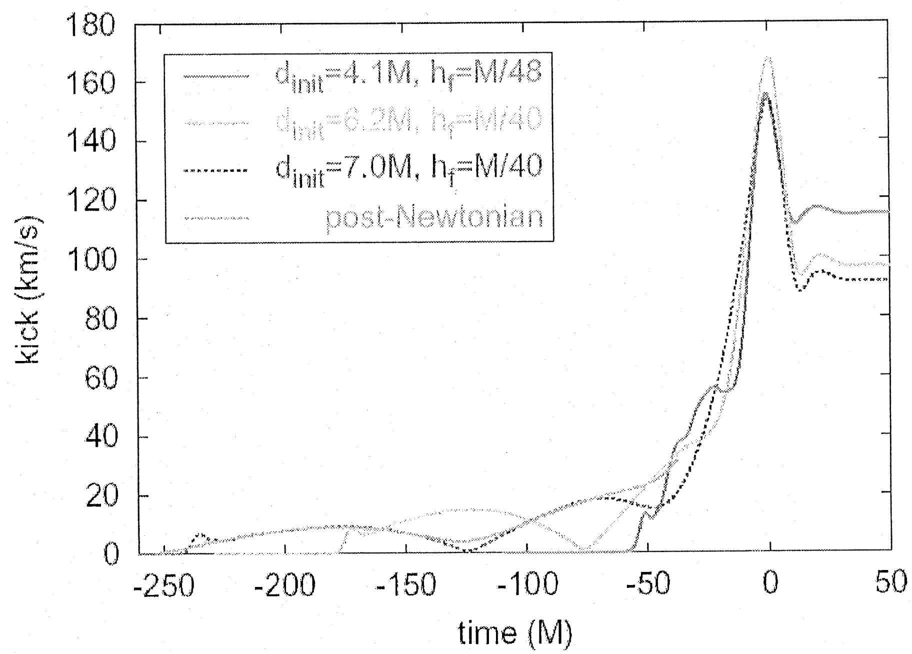






Unequal mass BBH mergers...

- When $m_1 \neq m_2$, the GW emission is asymmetric
- GWs carry momentum, so merged remnant BH suffers recoil 'kick'
- Most of recoil occurs in strong gravity → numerical relativity
- Baker, et al., ApJL, 653, L93 (2006) astro-ph/0603204:
  - **$q = m_1/m_2 = 0.67$**
  - **widest separation run completes ~ 2.5 orbits before merger**
  - **agrees w/ PN over most of 1st orbit to better than 1%**
  - **Overall, report kick values in the range v_{kick} = (86 – 97)km/s**



- Gonzales, et al, gr-qc/0610154
  - **Ran series of runs, w/ mass ratios in the range $0.253 \leq q \leq 1$**
  - **Find max kick V_{max} = 175.7±11 km/s for q = 0.36 ± 0.03**

Recoiling from mergers of spinning BHs...

- Astrophysical BHs are spinning…how will this impact the kicks?
- Many new results…
  - **Herrmann, et al., gr-qc/0701143**
    - q = 1, spins anti/aligned with orbital angular momentum
    - a/m = 0.2, 0.4, 0.6, 0.8 → v_{kick} up to ~ 400 km/s
  - **Koppitz, et al., gr-qc/0701163**
    - q ~ 1, a/m ~ 0.15, spins anti/aligned → v_{kick} up to ~ 250 km/s
  - **Campanelli, et al., gr-qc/0701164**
    - q = 0.5, spinning larger BH a/m = 0.885 with spin at -45 deg to orbital plane, orbits nonspinning smaller BH → v_{kick} ~ 454 km/s
  - **Gonzalez, et al., gr-qc/0702052**
    - q = 1, a/m ~ 0.8, spins in orbital plane, oppositely directed, chosen to maximize kick → get v_{kick} ~ 2500 km/s !!
  - **Baker, et al., astro-ph/0702390**
    - model v_{kick} for spins aligned/anti to within ~ 10%

→ ***interesting parameter space…more studies to come***

Current status of BBH merger simulations...

- Impressive recent progress on a broad front: many research groups, different codes, methods…
- Equal mass, nonspinning BBHs: several groups are now evolving for several orbits, followed by the plunge, merger, and ringdown
- There is general agreement on the ***simple waveform shape*** and that
  - total GW energy emitted in last few cycles

    _E ~ (0.035 – 0.04)M *(depends on the number of orbits)*
  - final BH has spin a ~ 0.7M
- Long runs now possible…~ 7 orbits before merger
- Applications to GW data analysis are beginning
- ***Explosion*** of work on nonequal mass and spinning BH mergers and the resulting kicks
  - Interesting parameter space
  - Important astrophysical applications…

The emerging picture….