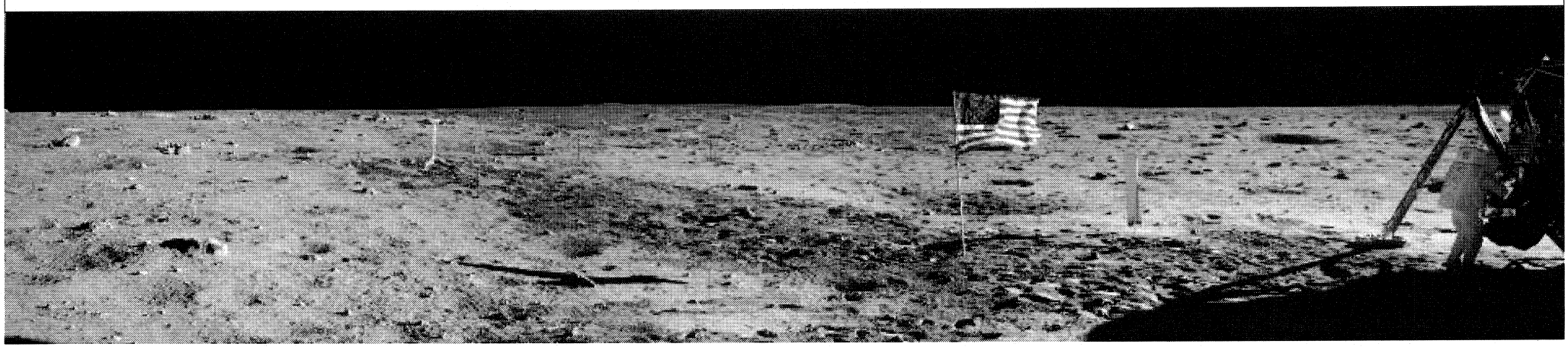
Lunar Frozen Orbits



Background

☾ The direction of NASA's ESMD for exploration architectures

☾ Focus on the need for minimal stationkeeping cost and constant (maximum) range for relay and communications to robotic and sortie locations

☾ Recent advances in lunar mission design experience and associated model updates from Lunar Prospector ('99) at 100 km and 30 km circular mean altitudes and Clementine ('94) in an elliptical 400 km by 3000 km altitude

☾ Previous (1963) and recent work on the interaction of perturbations that permit lunar frozen orbits (LFO).

- **Lidov, m.L., (1963), Ely and Lieb (2005),**
- **Ramanan and V. Adimurthy (2005), Park S.Y. and Junkins (1995)**
- **Elipe and Lara, (2003), Folta (1998, 1999 (post-LP mission results))**

☾ Global analysis at two orbit regimes: elliptical orbits and low lunar orbits

Basic Lunar Orbit Mechanics

A Few Observations From GSFC Operational Experience

Goddard Space Flight Center

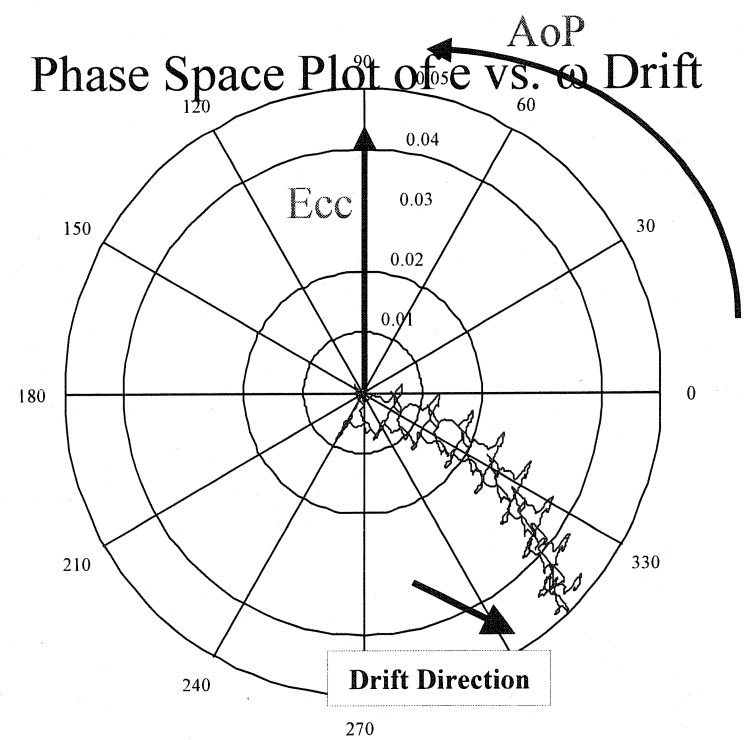
Lunar Orbit

- For Polar orbit, orbit plane is inertial fixed, no major precession in orbit node
  - Lunar Gravity (potential) is major perturbation below 500km altitude
  - Best available gravity model is from Lunar Prospector mission (deg and order of 165)
    - ✓ No data is available from far-side of moon (no tracking exist)
  - Potential causes eccentricity to increase, lowering periapsis, and argument of periapsis to drift
    - ✓ Lunar impact in ~ 4 weeks from 30km, ~ 6weeks from 50km, ~ 4months from 100km
  - Uncertainty in navigation accuracy (e.g. at 50km mean altitude, uncertainty is ~50m 1σ)
  - Inclination varies sinusoidally +/- 0.5 deg over several days with several deg secular drift per year

- Earth and Sun are major perturbations above 500km
  - Effects depend on orbit shape
    - ✓ Highly elliptical, e.g. 2000x10000km orbits impact within weeks
    - ✓ Have high rotation rate of line of apsides, e.g. 10deg over 1 month
    - ✓ Inclination changes are large

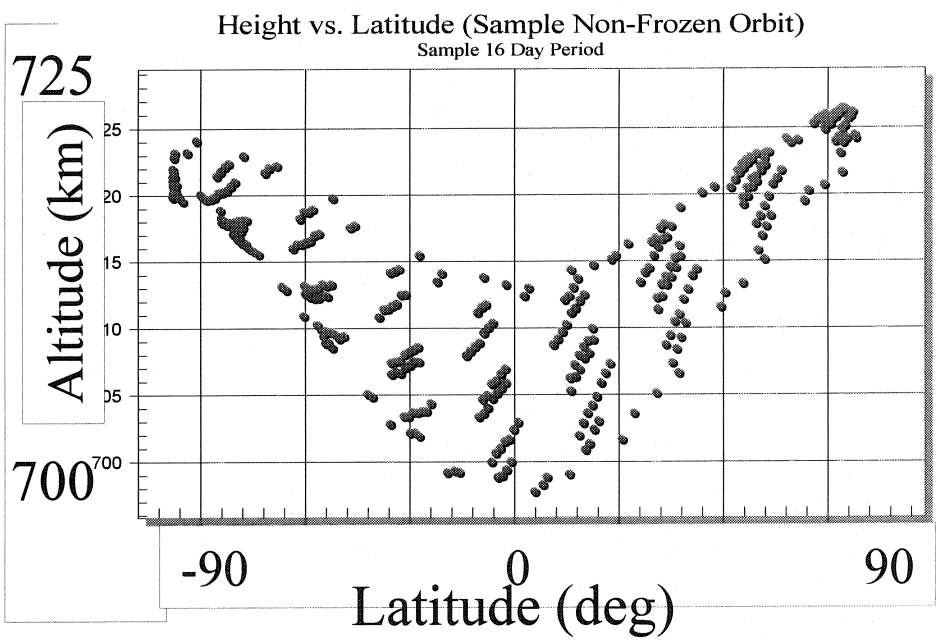
- Circular orbits are more stable but show inclination drift

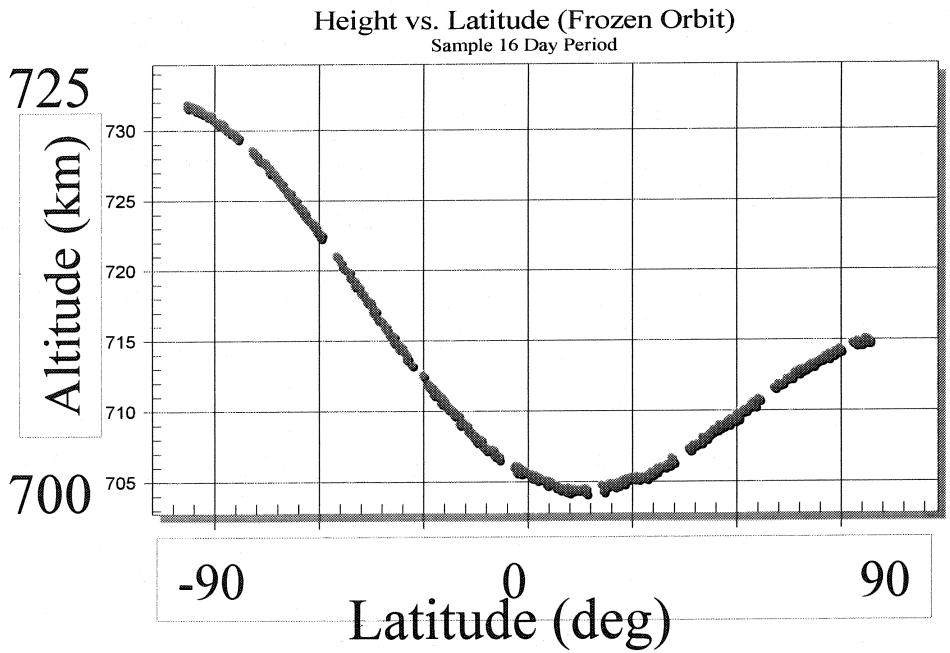
Phase Space Plot of e vs. ω Drift



Frozen Orbits – Earth, a Review

Useful for maintaining *a*, orbit altitude repeatability, and *e* & ω control





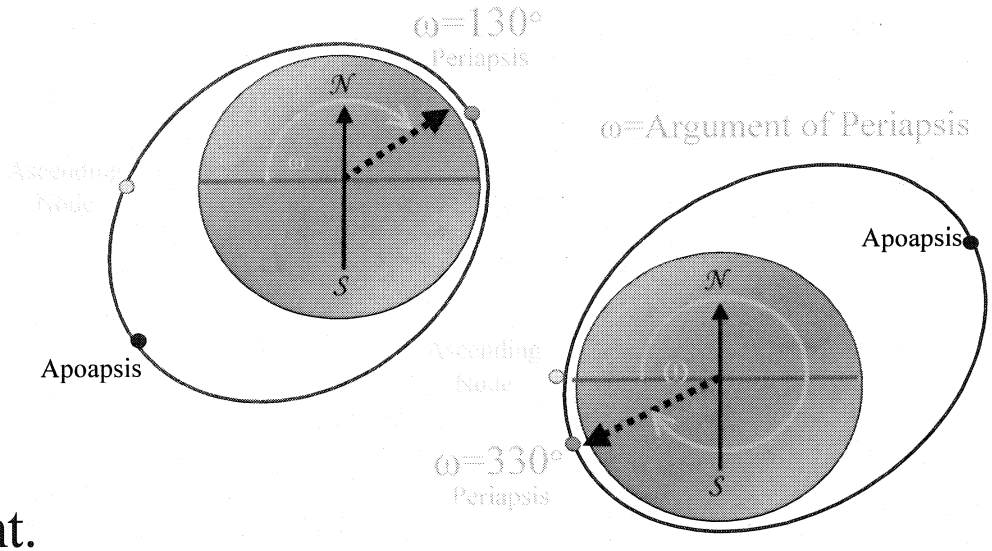
$$\frac{\partial \Omega}{\partial t} = -\frac{3}{2} J_2 \sqrt{\mu_\oplus} R_\oplus^{\,2} a^{-\frac{7}{2}} \left(1-e^2\right)^{-2} \cos i$$

$$\frac{\partial e}{\partial t} = -\frac{3}{2} J_3 \sqrt{\mu_\oplus} R_\oplus^{\,3} a^{-\frac{9}{2}} \left(1-e^2\right)^{-2} \sin i \left(1-\frac{5}{4}\sin^2 i\right) \cos \omega$$

$$\frac{\partial \omega}{\partial t} = 3 J_2 \sqrt{\mu_\oplus} R_\oplus^{\,2} a^{-\frac{7}{2}} \left(1-e^2\right)^{-2} \left(1-\frac{5}{4}\sin^2 i\right) \left\{1+\frac{J_3 R_\oplus}{2 J_2 a\left(1-e^2\right)} \left(\frac{\sin^2 i - e^2 \cos^2 i}{\sin i}\right) \frac{\sin \omega}{e}\right\}$$

General Lunar Orbit Properties

- Lunar Orbit is affected by
  - o Third-body accelerations
  - o Lunar gravity (potential model)
  - o Solar radiation pressure

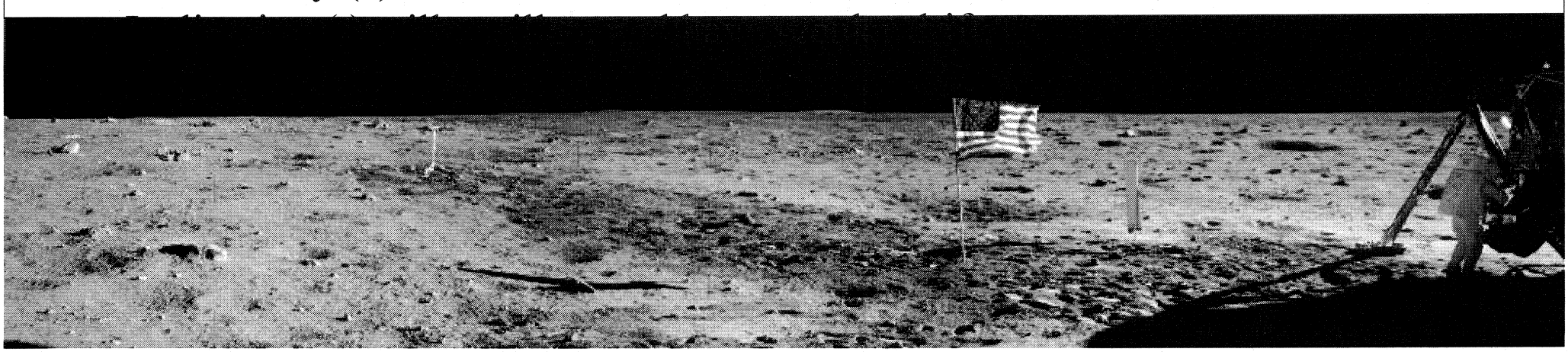


☾ Semi-major (a) axis remains constant.
- Minor changes to orbital velocity, but usually balanced.

☾ Ascending Node (Ω) remains inertially fixed for low lunar orbits at $i = 90^0$.
- Small torques on orbit angular momentum in the lunar equator frame as a result of Earth gravity.

☾ Argument of Periapsis (ω) will drift within the orbit plane.

☾ Eccentricity (e) will oscillate and have secular drift.



Planetary Equations and Forcing Functions (Lidov Reference)

- **Established the perturbation frame, and derived the 3rd body disturbance function averaged over one month (one apparent orbit of the Earth about the Moon) and one spacecraft orbit**

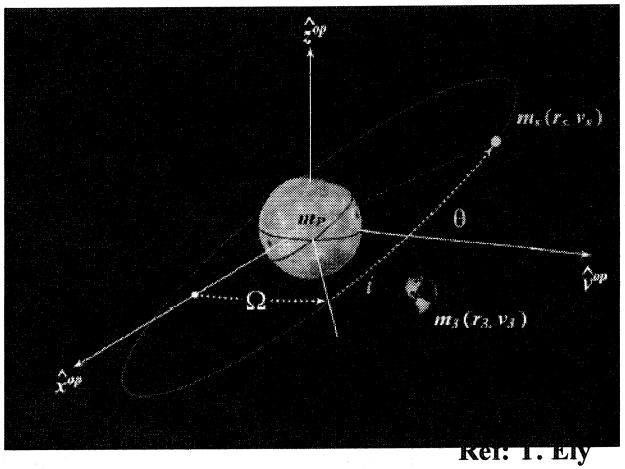
$$F_3 = \frac{1}{32} n_3^2 a^2 \left[\left(1+3\cos 2i\right)\left(2+3e^2\right) + 30e^2 \sin^2 i \cos 2\omega \right]$$

- **Applied to Lagrange Planetary Equations Yields:**

Semi-major axis: $$\frac{\partial a}{\partial t} = 0$$

Eccentricity: $$\frac{\partial e}{\partial t} = \frac{15}{8}\frac{n_3^2}{n} e\left(1-e^2\right)^{\frac{1}{2}} \sin^2 i \sin 2\omega$$

Inclination: $$\frac{\partial i}{\partial t} = -\frac{15}{16}\frac{n_3^2}{n}\frac{e^2}{\left(1-e^2\right)^{\frac{1}{2}}} \sin 2i \sin 2\omega$$



Ref: T. Ely

Ascending Node: $$\frac{\partial \Omega}{\partial t} = \frac{3}{8}\frac{n_3^2}{n}\frac{1}{\left(1-e^2\right)^{\frac{1}{2}}}\left[5e^2 \cos 2\omega - 3e^2 - 2 \right] \cos i$$

Arg. of Periapsis:

$$\frac{\partial \omega}{\partial t} = \frac{3}{16}\frac{n_3^2}{n}\frac{1}{\left(1-e^2\right)^{\frac{1}{2}}}\left[\left(3+2e^2+5\cos 2i\right)+5\left(1-2e^2-\cos 2i\right)\cos 2\omega\right]$$

Equations Applied to Elliptical Lunar Orbits

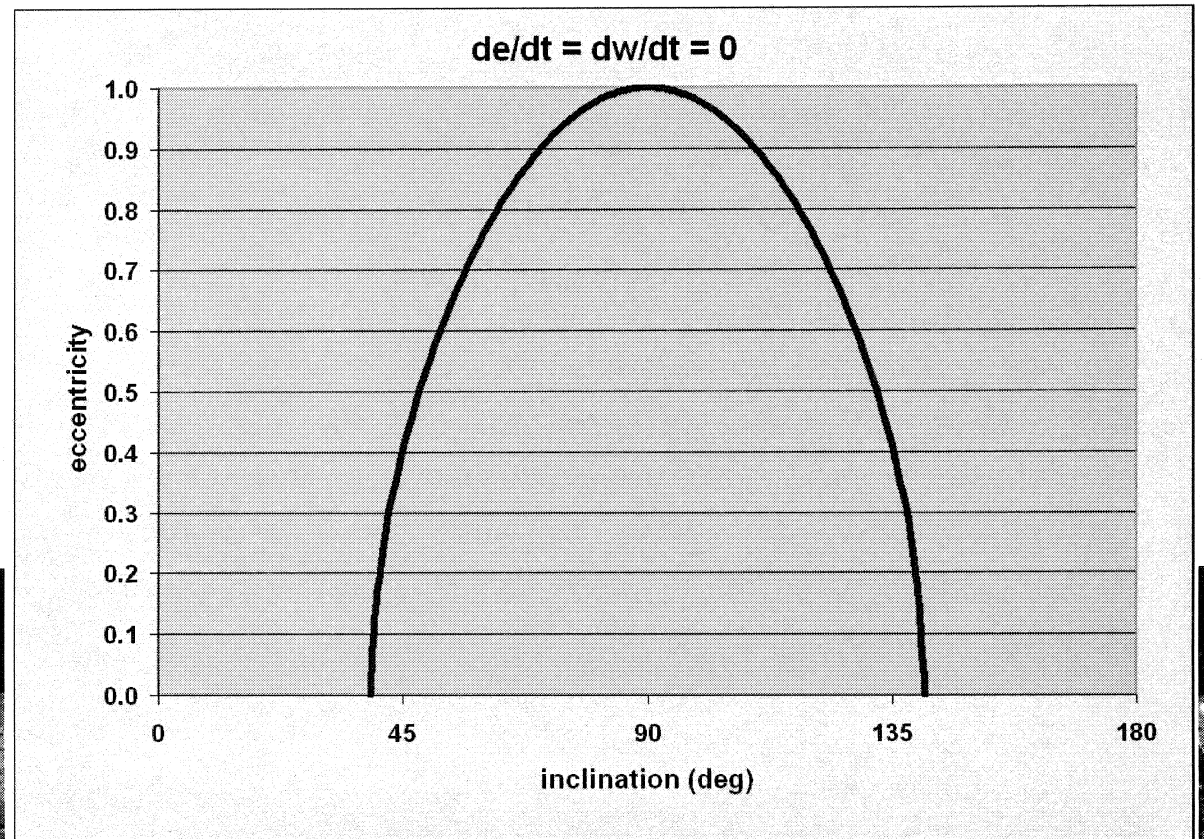
☾ There are no closed-form analytical solutions for frozen orbits below critical inclination of 39.23°

☾ For any inclinations between 39.23° through 140.77° (where ω = 90° or 270°), there exists an eccentricity which can be used to set both ω and *e* rates to zero; hence the frozen condition is satisfied.

Real solutions only when $i > 39.23°$

$\omega = 90^{o}, 270^{o},$

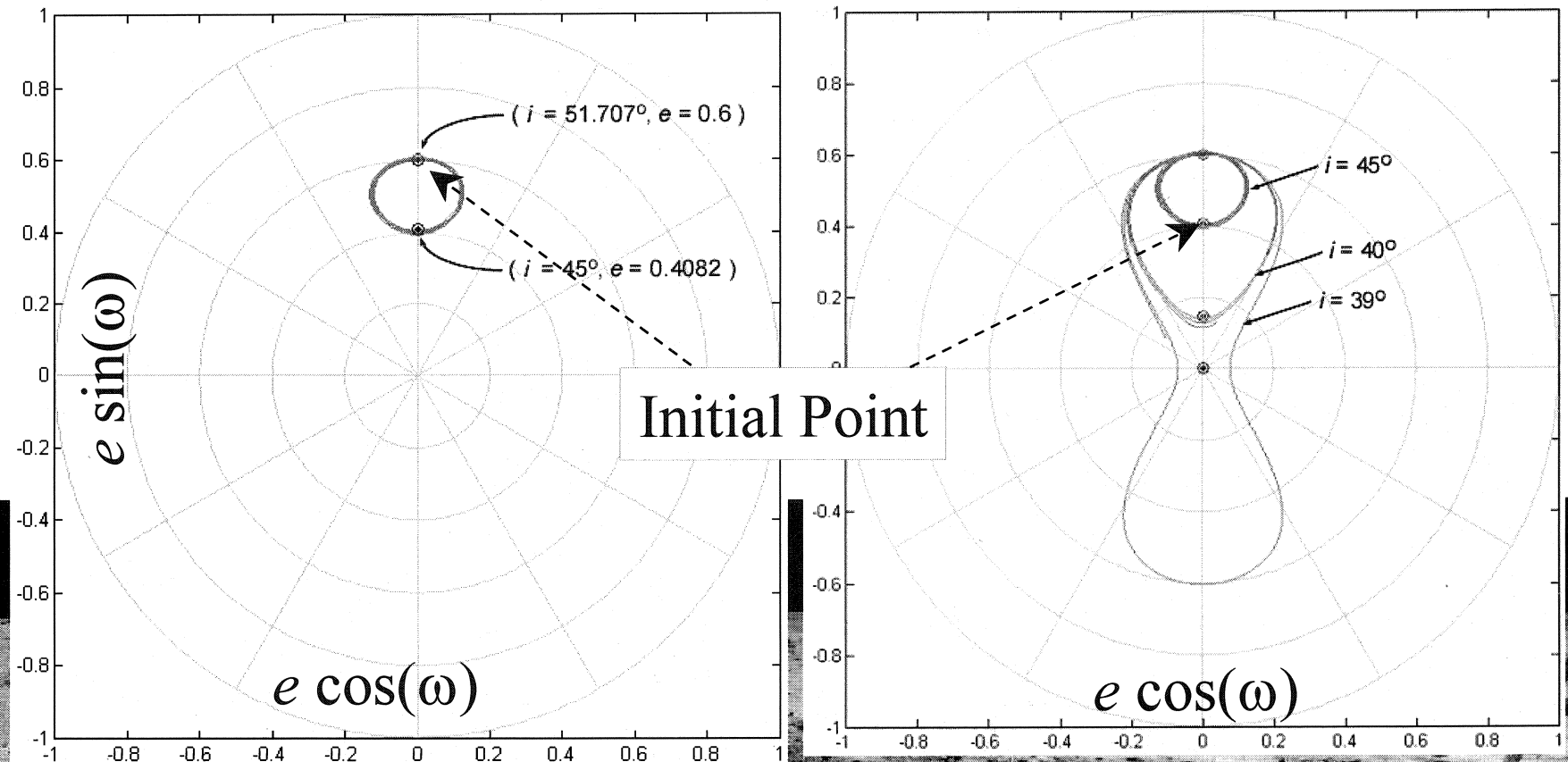
$$e = \left(1 - \frac{5}{3}\cos^2 i\right)^{\frac{1}{2}}$$



Evolution of *e* and ω

- For a given set of orbital parameters, we can use the relationship between eccentricity and inclination to find *e* for the given *i* and *i* for the given *e*.
- Eccentricity and ω vary between their frozen values for e given *i* and *i* given *e*.

$e = 0.6$	$\Rightarrow$	$i = 51.707^\circ$
$i = 45^\circ$	$\Rightarrow$	$e = 0.4082$



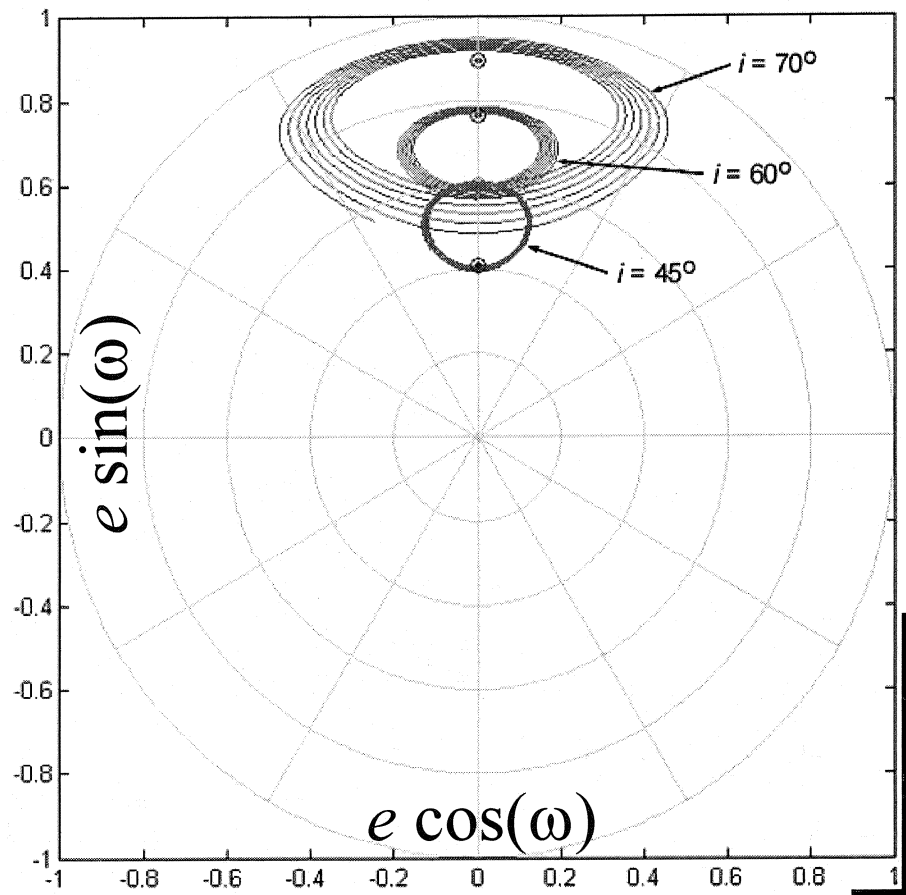
Evolution of *e* and ω

☾ Increasing inclination, the oscillatory loops growing smaller until an inflection point is reached and the loops begin to grow in the opposite direction, where the *e* given *i* point has now become the upper point and similar, if less stable oscillatory behavior is observed.

☾ Inflection inclination is defined as the inclination at which the *e* - *i* point converge to a single point.

☾ Inherent instability is observed in the outward spiral seen in the long-term orbit evolution

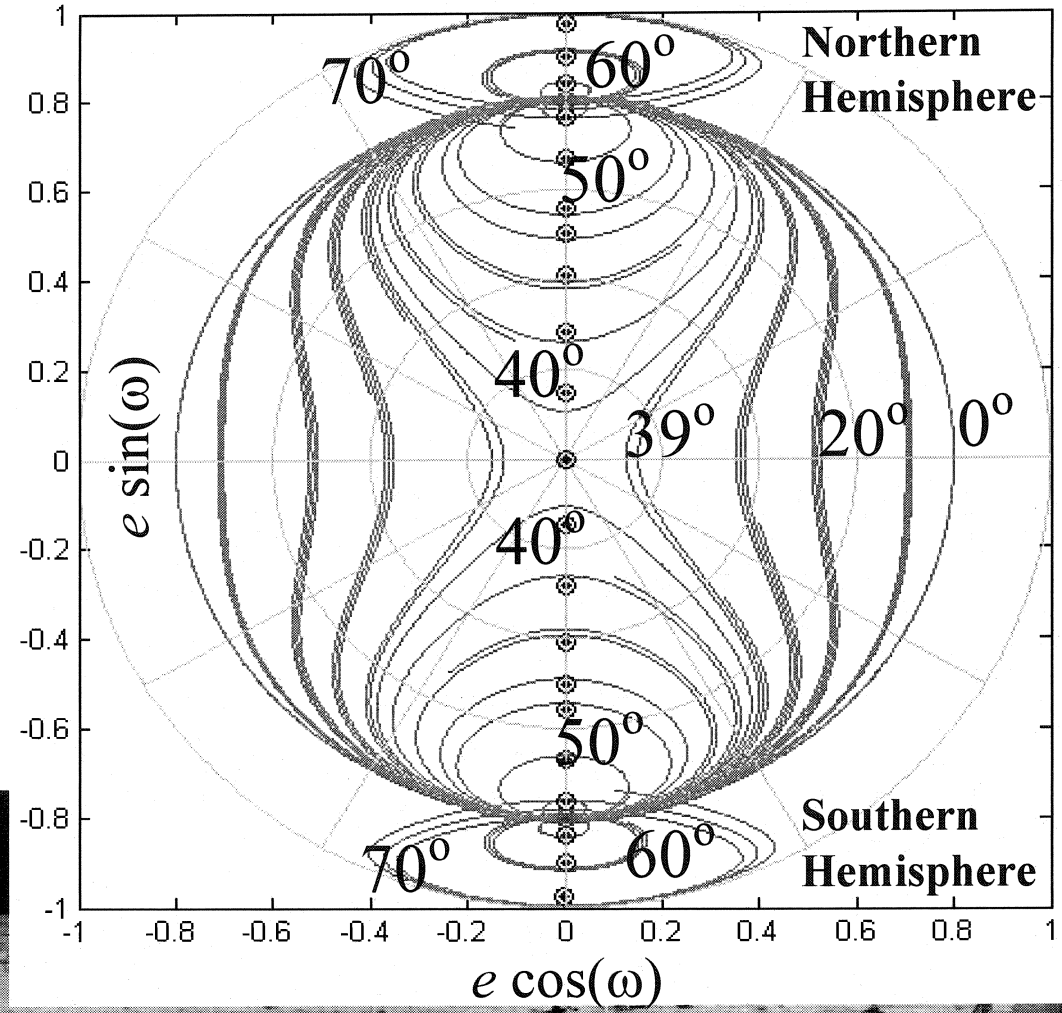
☾ The closer eccentricity and inclination conform to the frozen condition, the more stable the orbit becomes.



Global Frozen Conditions for Elliptical Lunar Orbits

- Using formulation, the evolution of e and ω can be derived for any i.
- Starting at $\omega = 270$ or $\omega = 90$, $(\cos(\omega) = 0)$, the orbit evolves along the i contour, with e increasing and decreasing.
- Some a and e combinations will not permit a full cycle

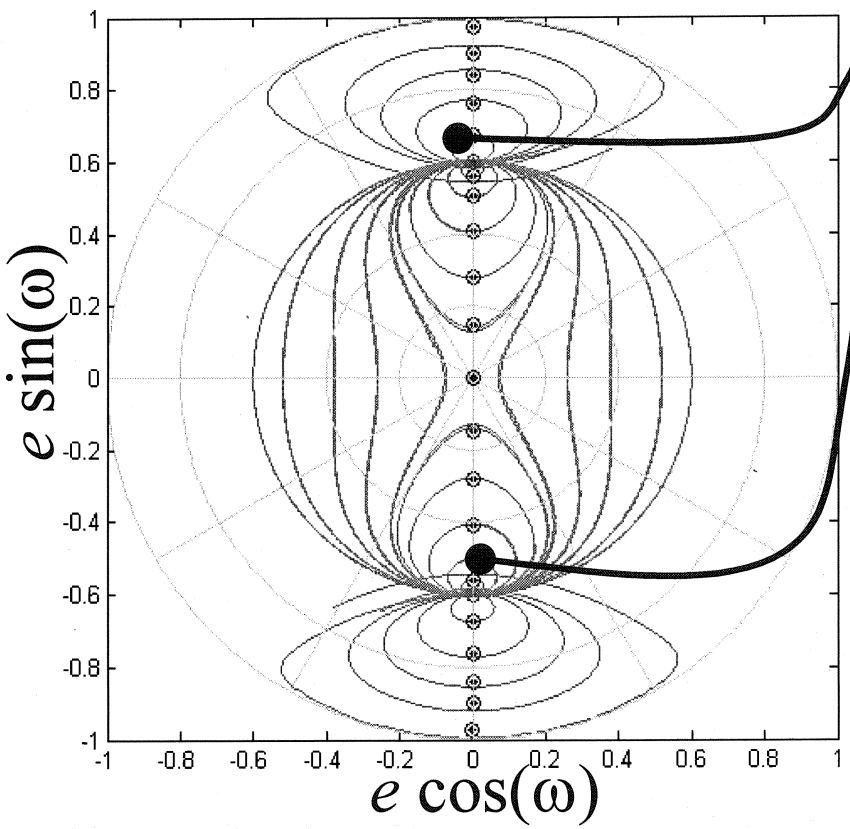
Evolution with $e = 0.6$ and various i

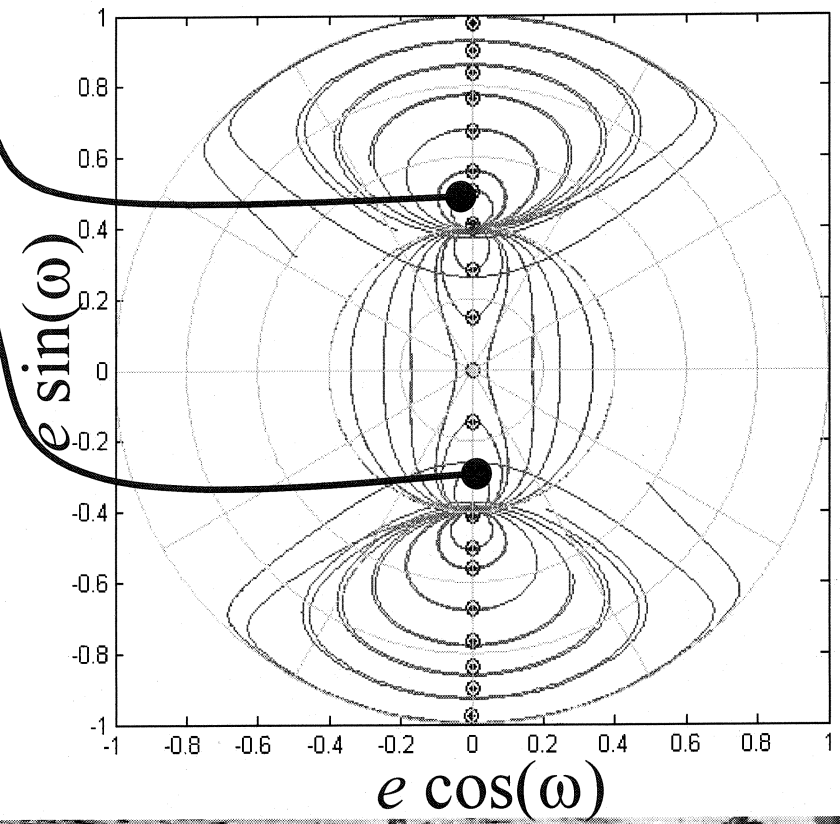


Evolution of *e* and ω
Effects of Variation in Initial e

- As *e* is reduced, inclination inflection point also changes
- Critical inclination is still the the same at 39.23°

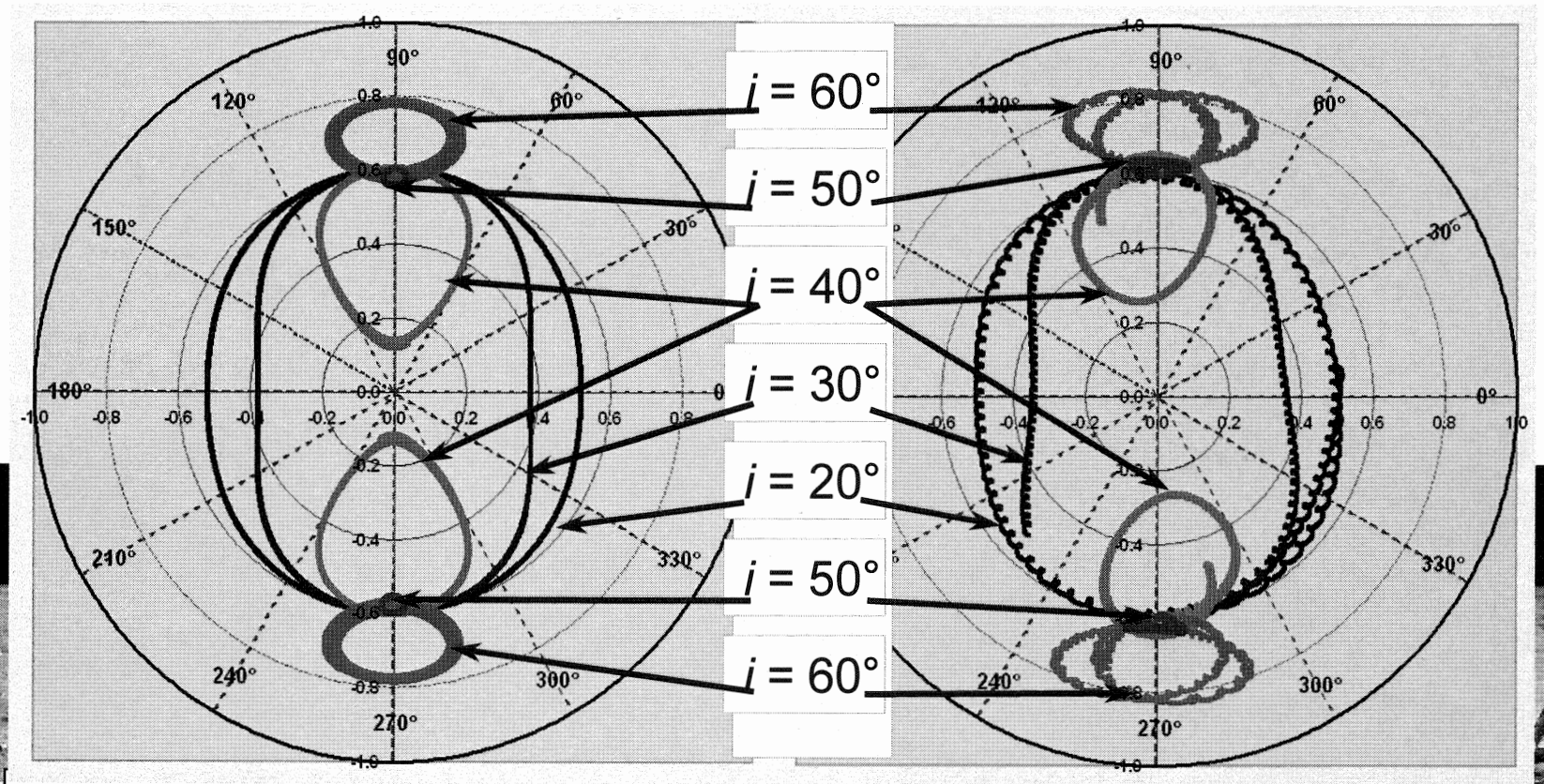
Inflection points





Evolution of *e* and ω
Numerical Verification

- Using full ephemeris and perturbation models with RK8/9 integration.
- Small loops due to changes in accelerations from lunar orbit eccentricity and orbital alignment WRT Earth.
- Shows general analytical form is preserved.

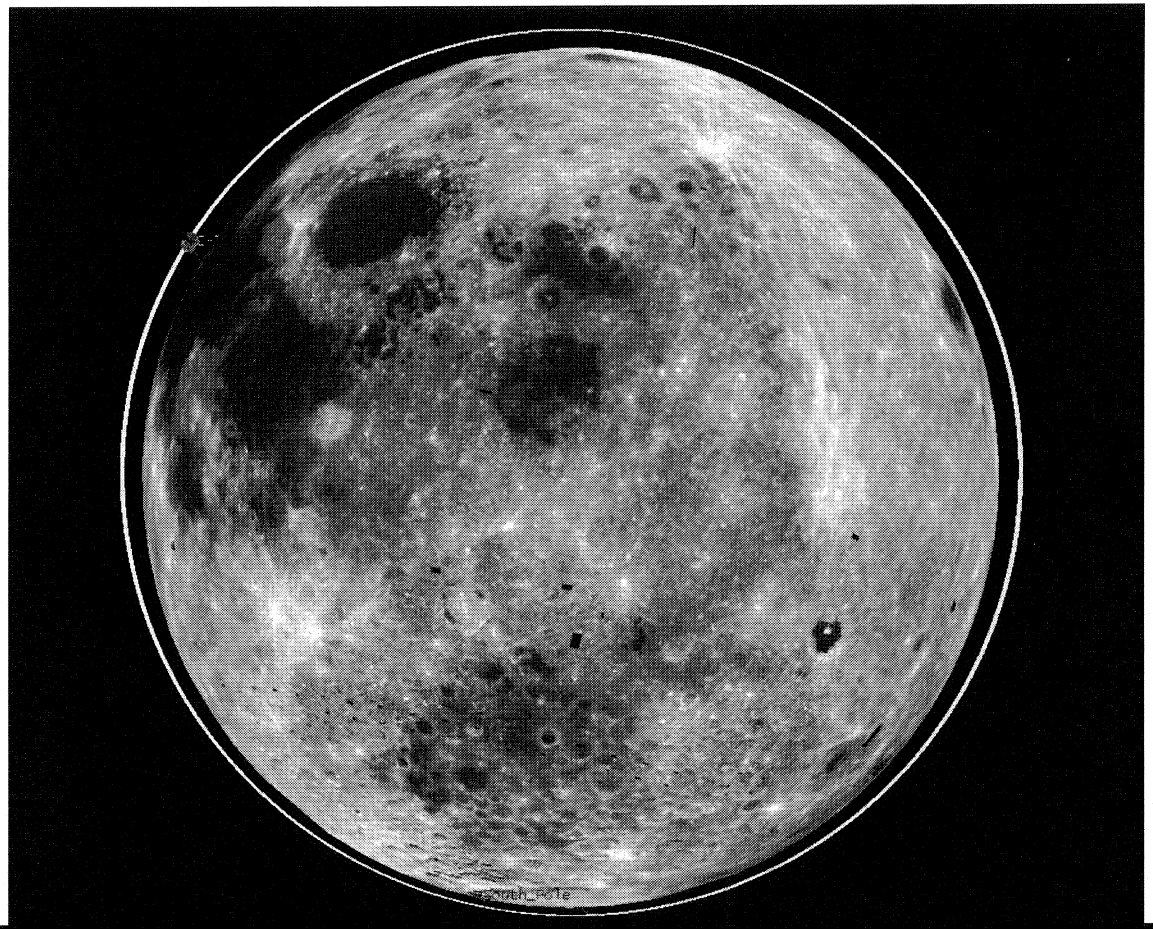


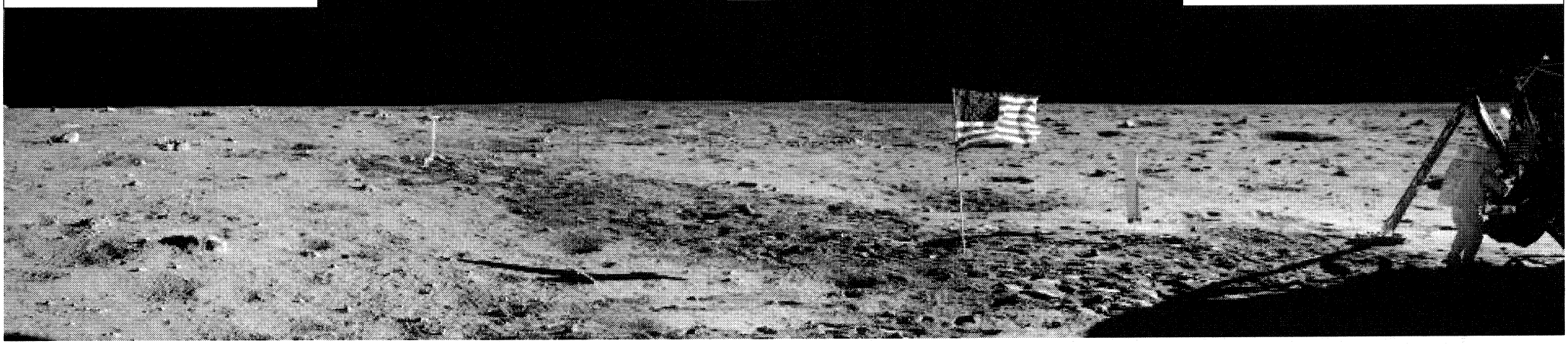
a: Analytical Solution

b: Numerical Solution

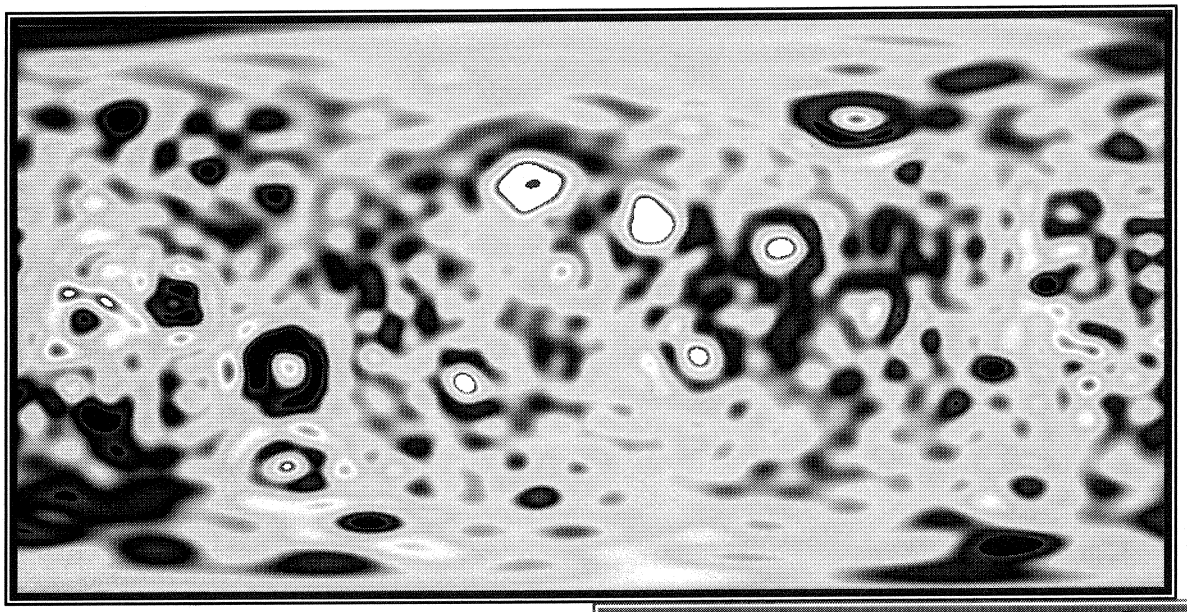
Lunar Frozen Orbit

Periapsis at south pole ~30km Apoapsis at north pole ~ 150 - 230km



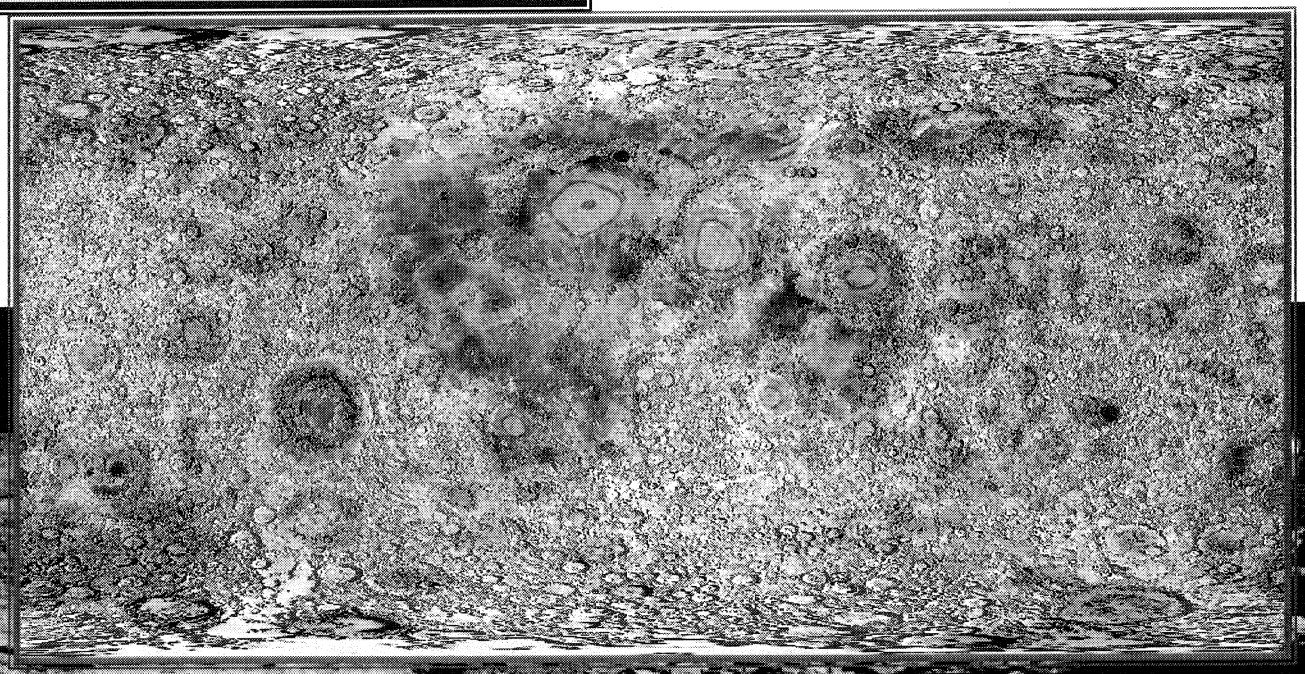


Lunar Gravity Accelerations From the 165 Degree and Order Potential Model



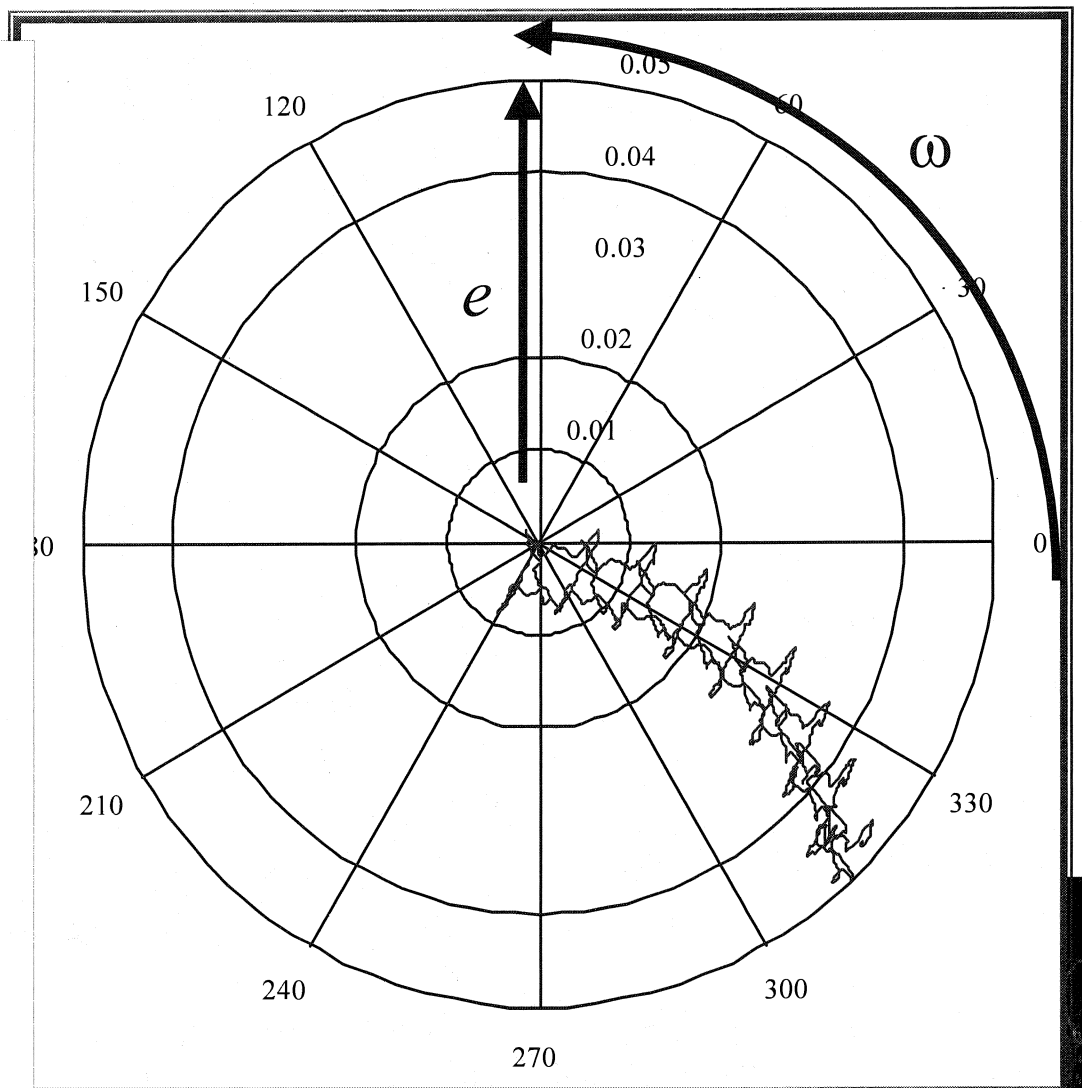
LP165 Lunar Potential Model Accelerations

LP165 Lunar Potential Model Accelerations Shown with Lunar



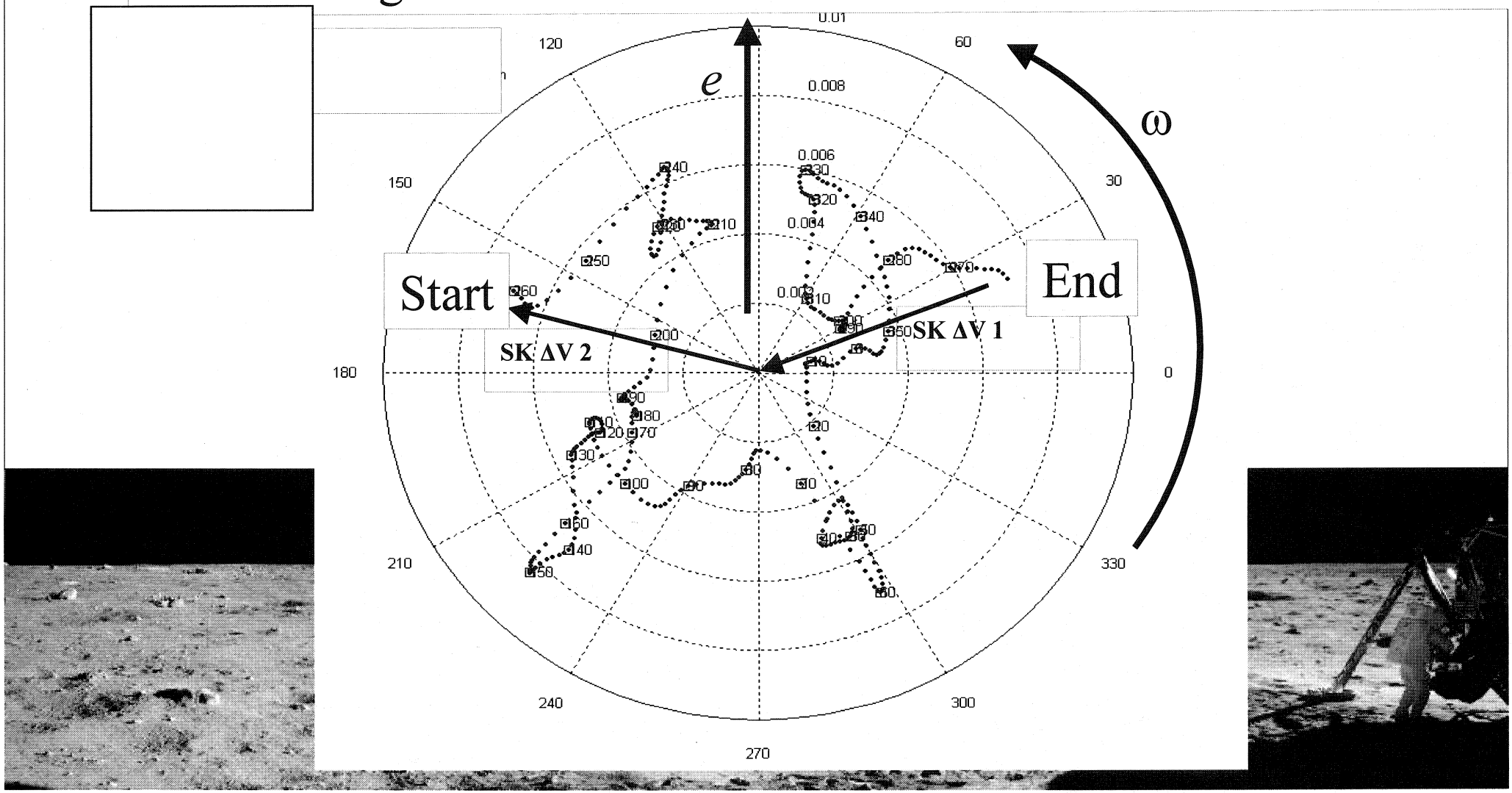
Low Lunar Orbit Polar Phase Plot
Initial Conditions of e = 0.0, i = 90°, a =1838 km

- Lunar non-spherical gravity creates a predictable pattern in e vs ω polar phase plot.
  - Assumes fixed Semi-Major Axis
- Pattern is repeatable every lunar sidereal period.
  - Pattern repeats and is continuous
  - Complete pattern generally moves left to right with side-by-side repeated sidereal patterns
- Pattern begins to warp outside of $e > 0.02$ circle.



Low Lunar Orbit Polar Phase Plot
$e = 0.0$, $i = 90^o$, $a = 1838$ km

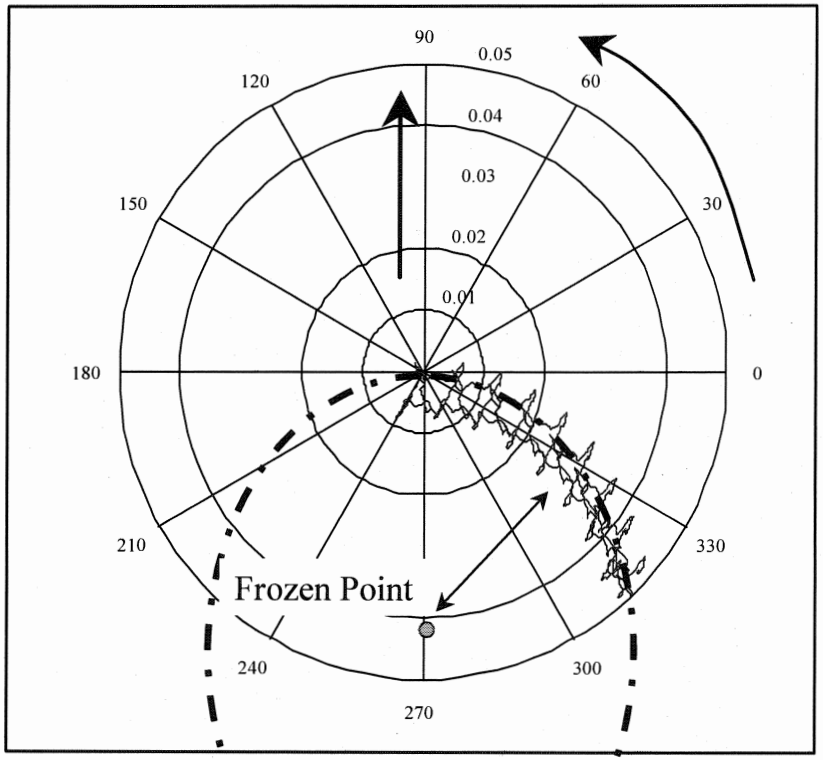
- Plot shows effect of lunar potential at 50 km mean altitude.
- Point every ascending node.
- Lunar longitude labeled.



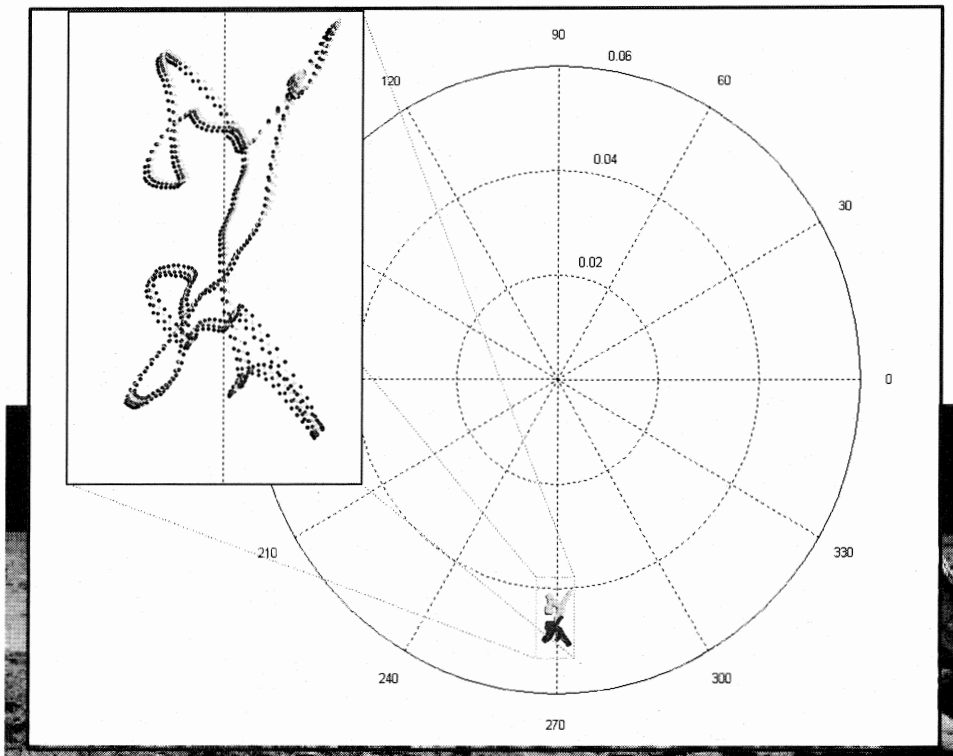
Low Lunar Orbits below 500 km altitudes

- Low lunar orbit dominated by lunar gravity potential.
- Gravity model defines the secular drift rates and the e and ω drift pattern.
- Lunar prospector operational data show a secular drift about a centered location.
- Use of simple differential corrector to vary initial conditions and target on minimal drift in e and ω.

Secular Drift with initial $e = 0.0$, $\omega = 0.0^{o}$, $i = 90^{o}$, $a = 1838$ km



Frozen conditions with initial $e = 0.043$, $\omega = 270^{o}$, $i = 90^{o}$, $a = 1861$ km

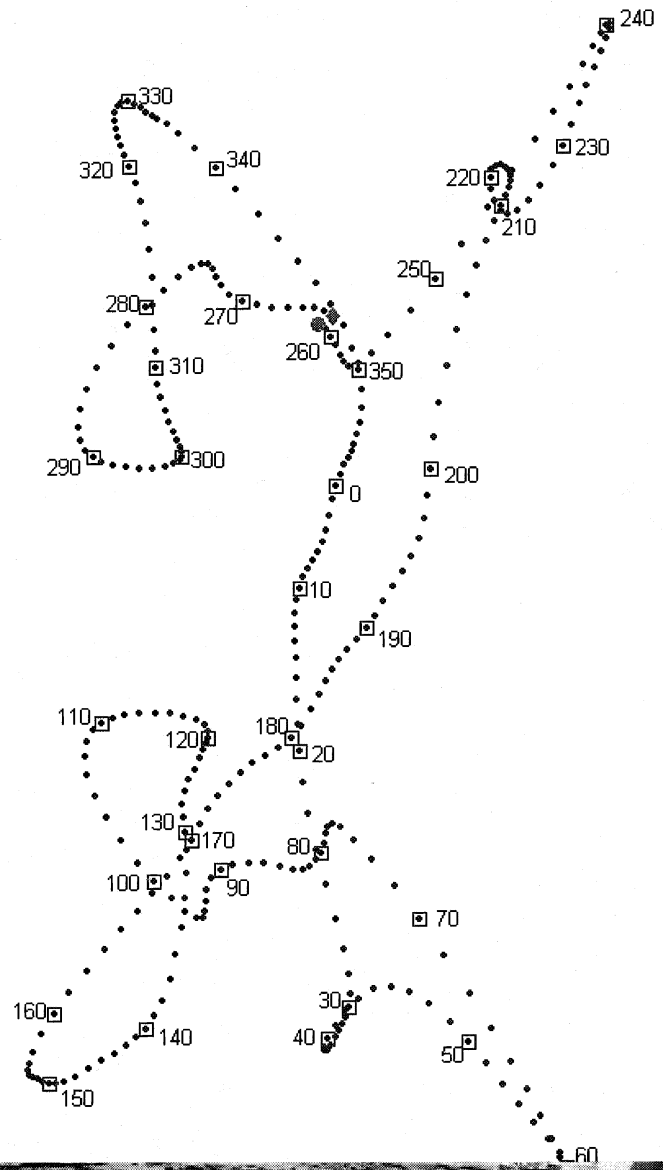


Repeatable Frozen Orbit
Phase Plot

No secular growth in *e* or ω because initial/final conditions match

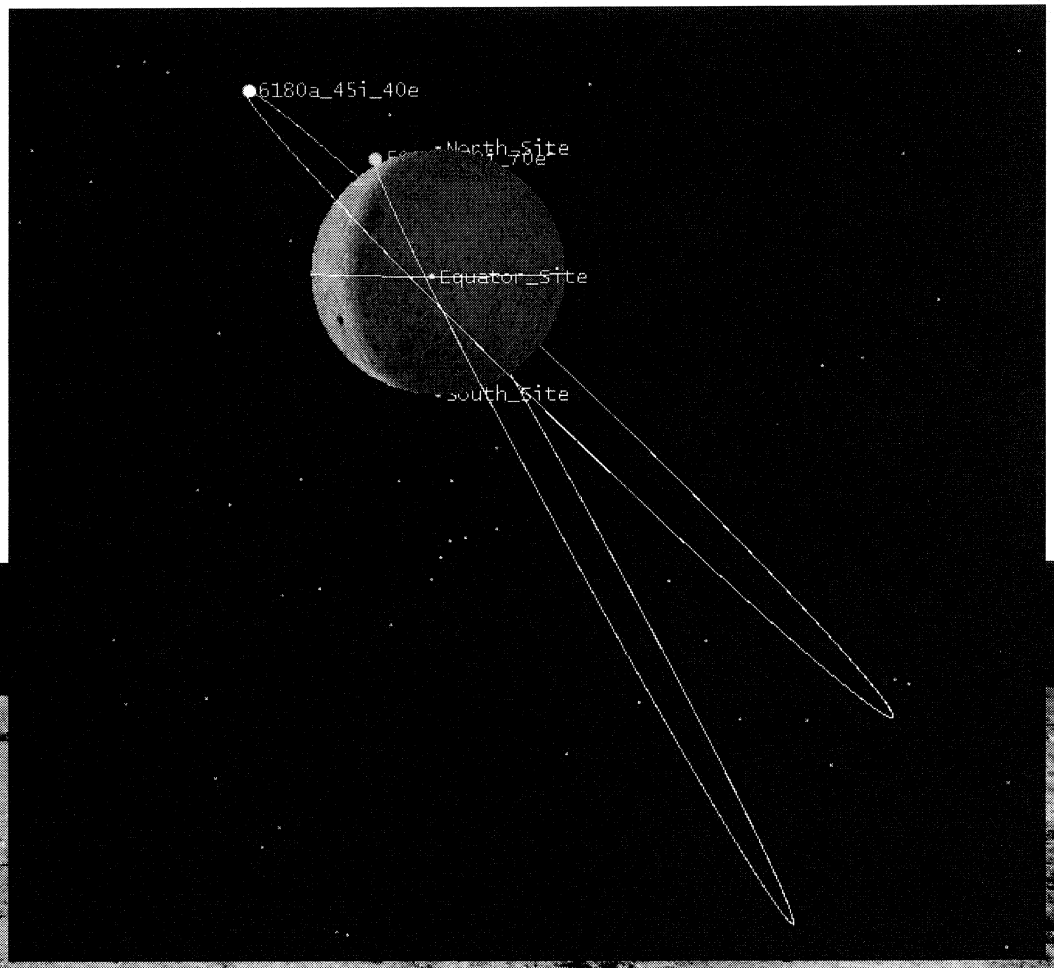
- Minimum eccentricity of 0.0408 equal to 45 x 197 km
- Maximum eccentricity of 0.0509 equal to 26 x 216 km

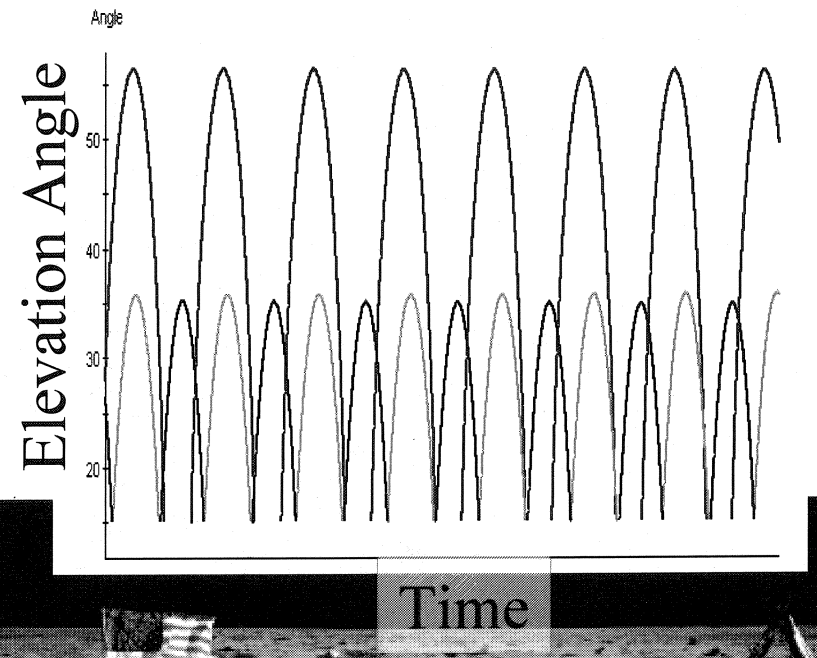
- Example Only– Your results



Lunar Elliptical Frozen Orbit Applications

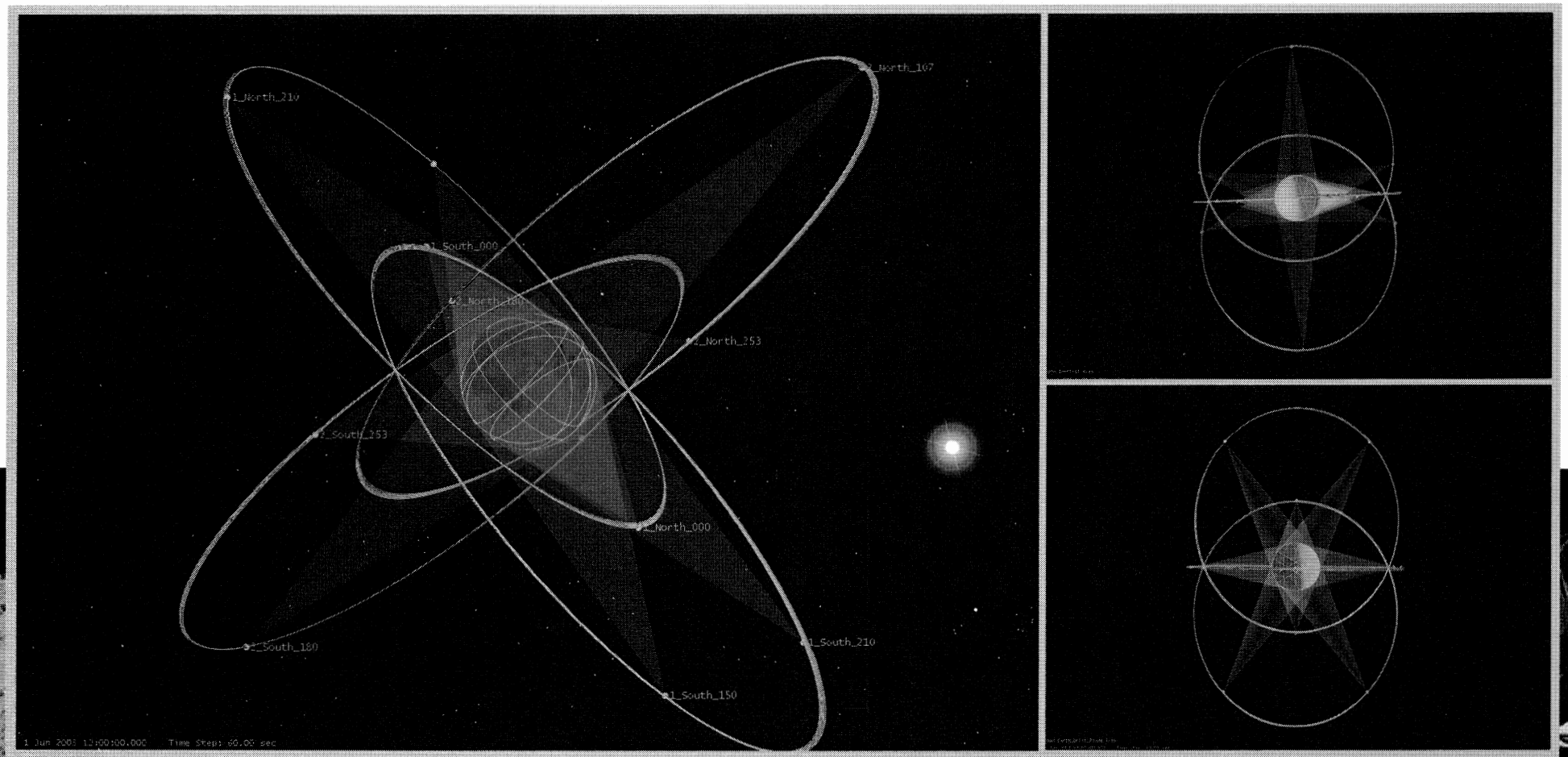
- Coverage to south pole with 62° inclination or two s/c in 45° inclination
- With i = 62° and a period of 12 hours, elevation angle near 60°, duration of 9 hours in a frozen orbit.
- With i = 45° and a period of 12 hours, elevation angle near 35°, duration of 6 hours, two s/c can provide continuous coverage in a frozen orbit.





Global Coverage using Frozen Orbits

- Constellation Parameters: 2 s/c per orbit plane, 18hr orbits (a~8049, e = 0.4082)
- Orbit planes at right angle to each other but at $i = 45^{o}$
- For continuous and simultaneous coverage, two orbit planes with $\omega = 90^{o}$ and 270°
- Nodes separated by 180°
- Utilize 8 or 12 spacecraft



Precision landing Impact of Navigation Errors

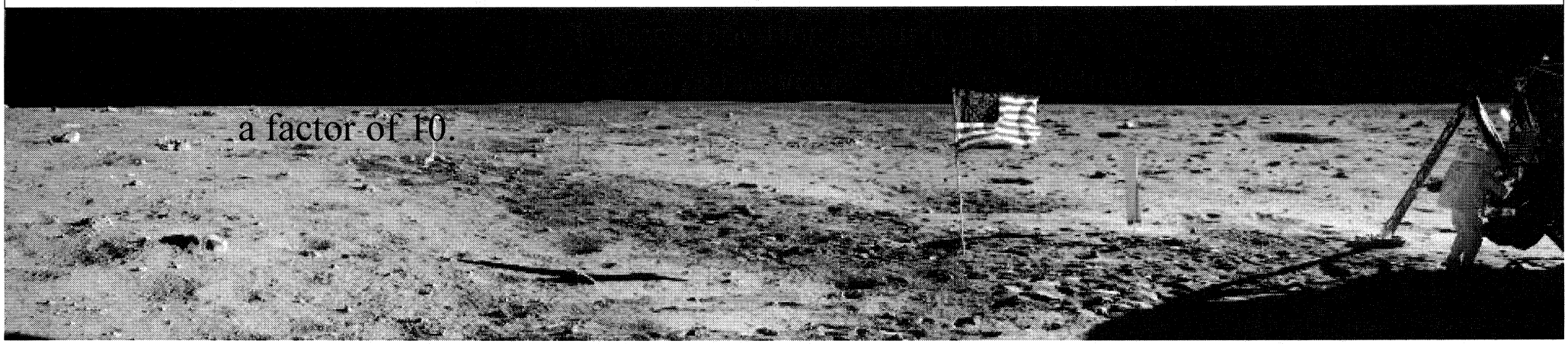
Analysis of initial navigation impact on landing location from a low lunar orbit

Anticipated 100km orbit navigation accuracy with current gravity model

- Long arc solutions (55hrs): 50m definitive and <60m predictive
- Short arc solutions (6 hrs): 400m definitive and <450m predictive

Assumed initial orbit navigation errors at start of descent:

- S1 = 25 m, 100 m, 100 m RIC (Radial, In-Track, Cross-Track) and no velocity error
- S2 = 50 m, 200 m, 200 m RIC and no velocity error
- S3 = 50 m, 200 m, 200 m RIC and 200 mm/s, 200 mm/s, 50 mm/s RIC
- 1,000 trial Monte-Carlo simulation propagated with the nominal steering law.
- Mean position error and standard deviation
  - S1 is 125 m and 65 m respectively
  - S2 with a mean of 253 m and a standard deviation of 134 m.



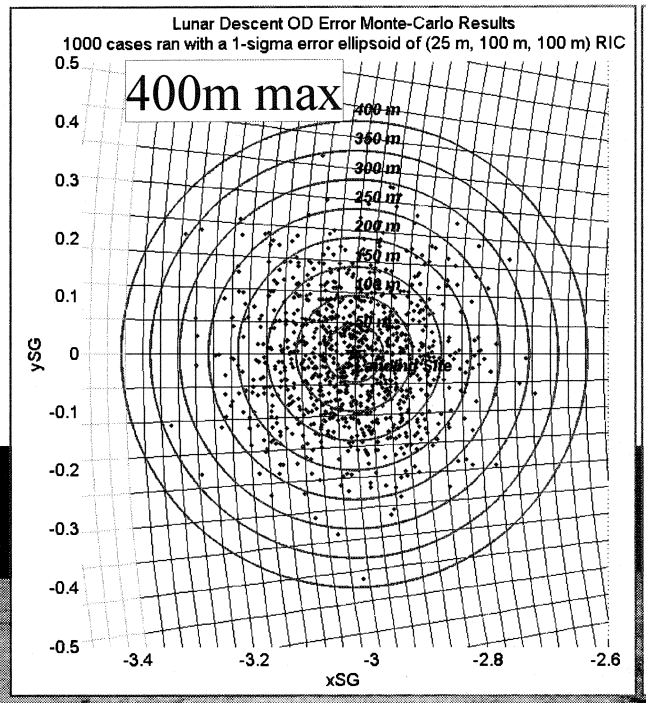
a factor of 10.

Precision landing Impact of Navigation Errors

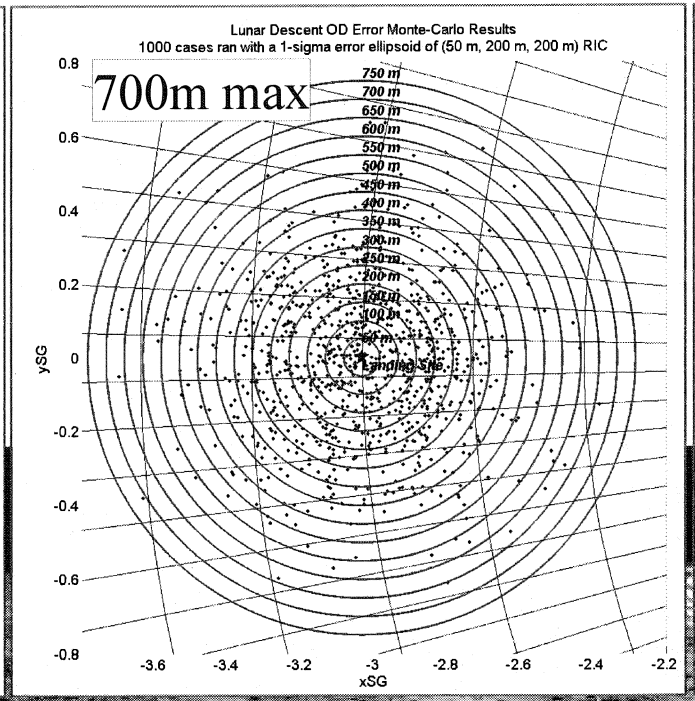
Results

- Figures show distribution of landing location on the moon surface for each 1000 Monte-Carlo cases.
- Each red circle represents a distance in meters from the landing site.
- The grid lines represent latitude and longitude line on the moon surface.

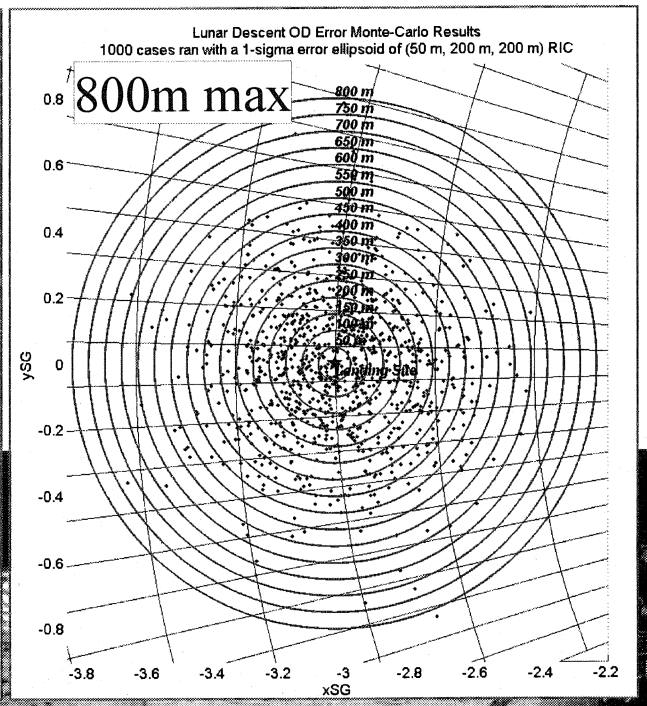
State error 25,100,100



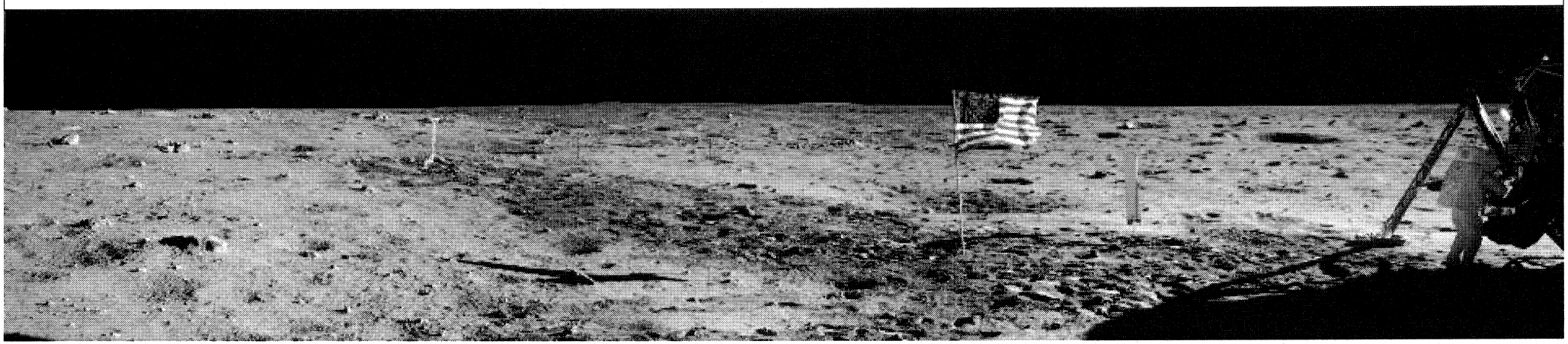
State error 50,200,200 w/ no vel error



State Error 50,200,200 w/ vel error



Navigation
Rendezvous and ProxOps



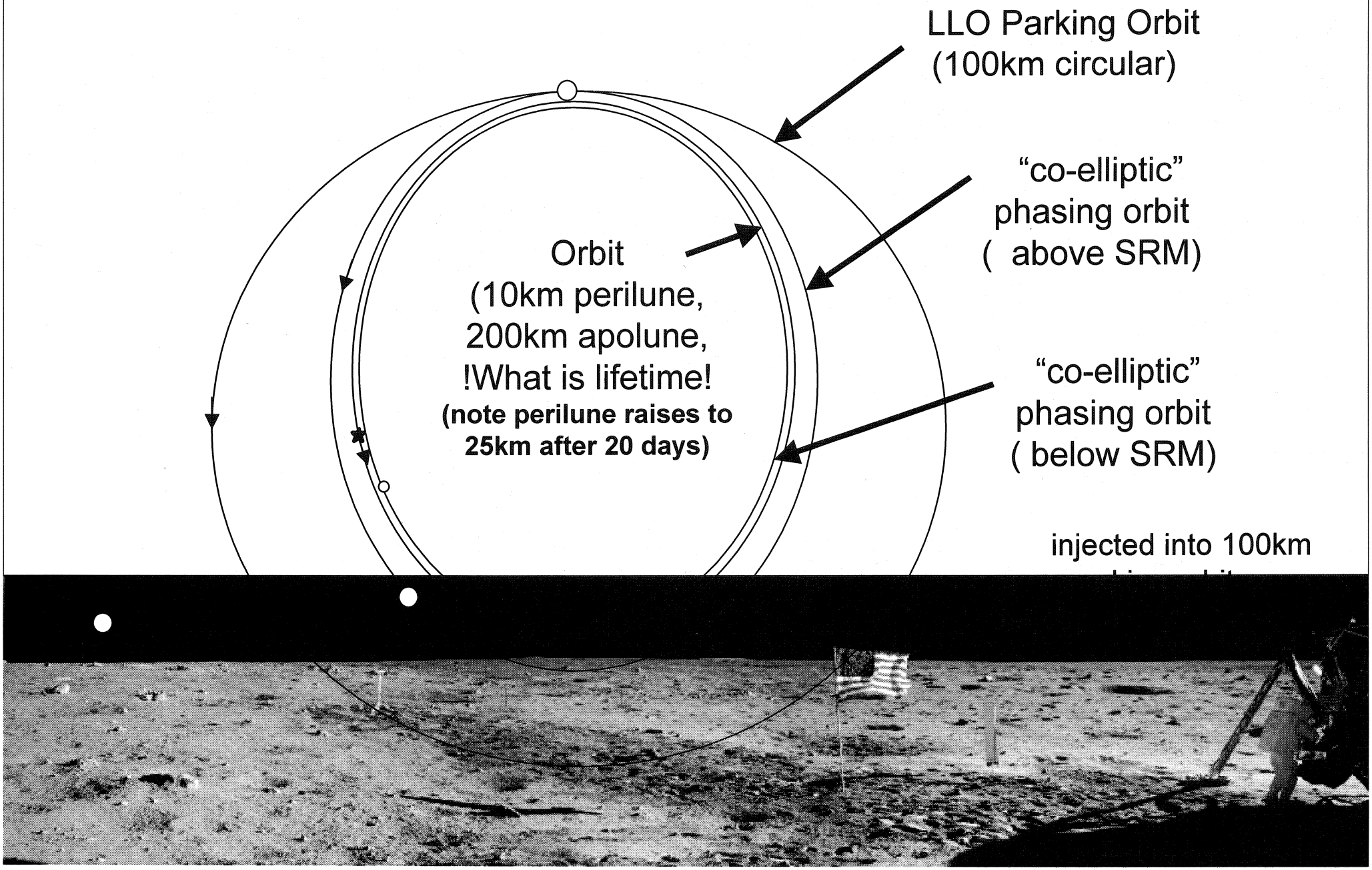
Navigation

Orbit Insertion and Pre-descent: All Architectures

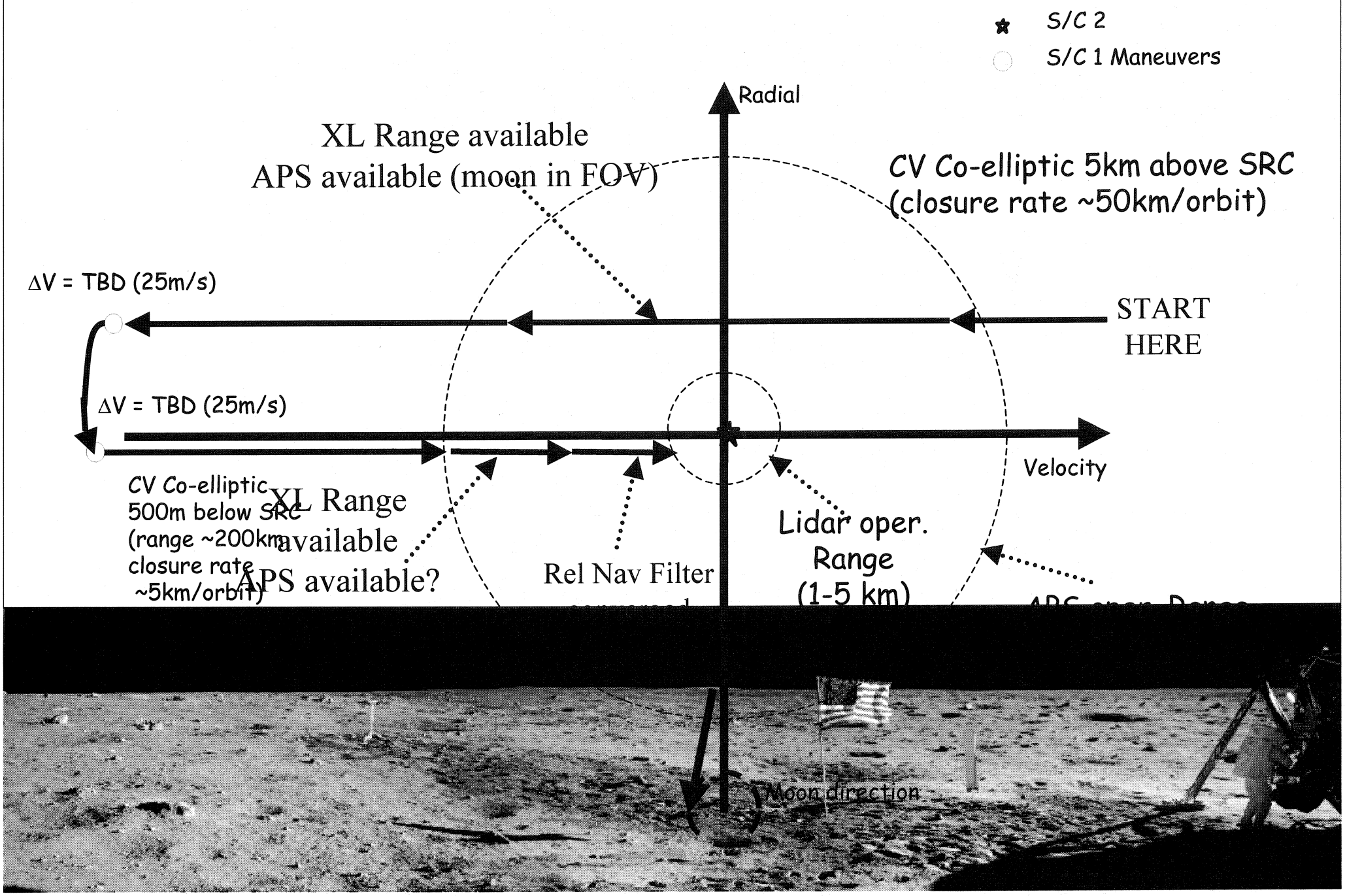
This navigation scenario taken from Operational experience of Lunar Prospector at 100km, 40km, and 30km circular orbit altitude and Clementine orbit of 400 x 3000km.

- Continuous range and Doppler tracking from 12 hours before insertion
- Doppler accuracy of a least 8mm/s
- North/South Stations when visible with convergence at 4 hours after start of solution using converged a-priori
- Continuous tracking of spacecraft through Lunar Insertion to descent phases
- Accuracy of ~1 km and ~ cm/s pre insertion
- Continuous tracking of a 55 hour data arc to provide convergence [note: short arc solutions are being analyzed]
- Accuracy of 50m and 1cm/s anticipated post insertion (based on reprocessing of Lunar Prospector data)
- Continuous tracking during and after first Hohmann transfer maneuver to lower periapsis for final descent phase and hand-over.
- Accuracy of predicted state for handover to final autonomous landing is 100m tbd and 2cm/s tbd.

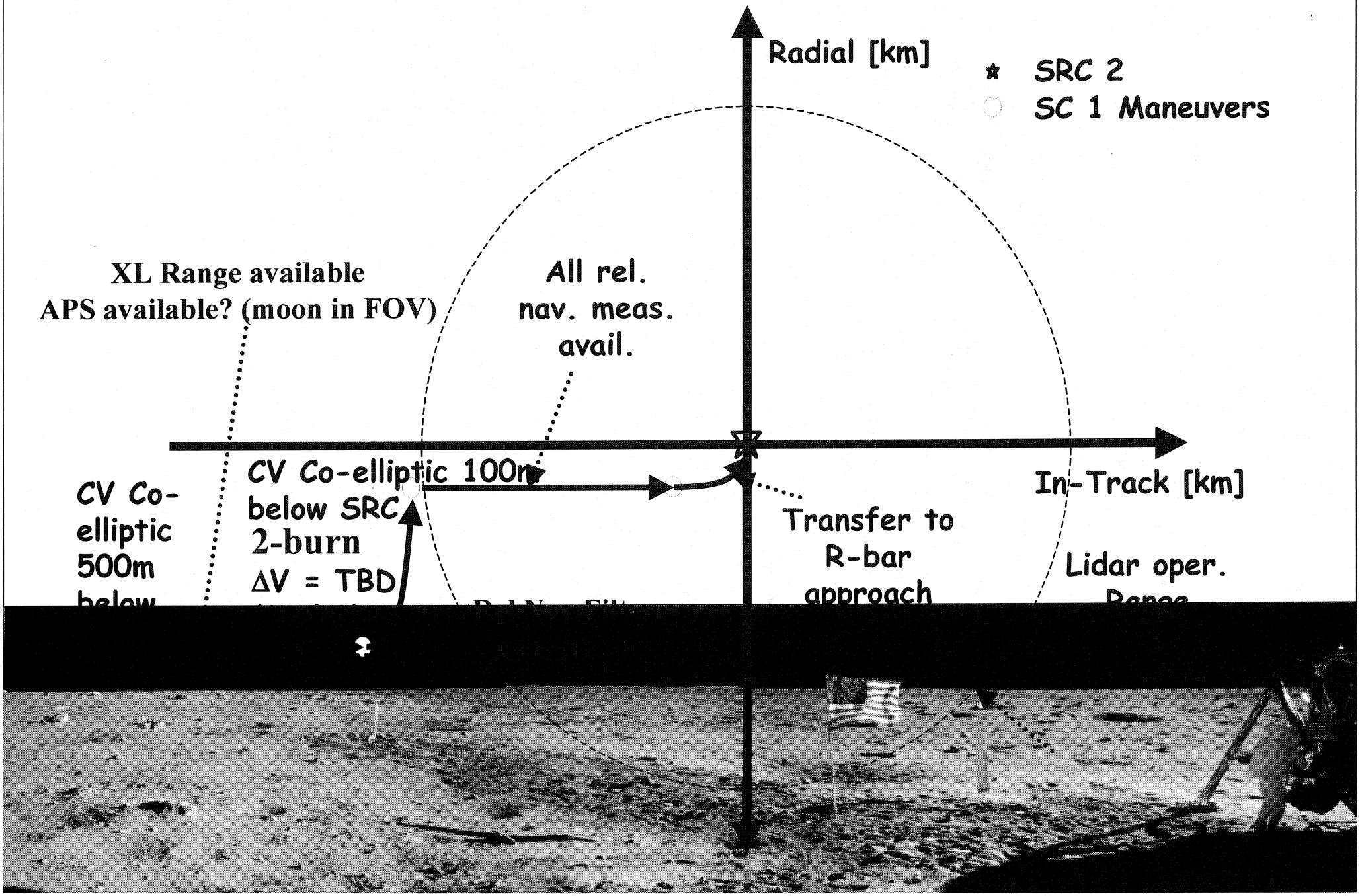
Rendezvous & Proximity Ops



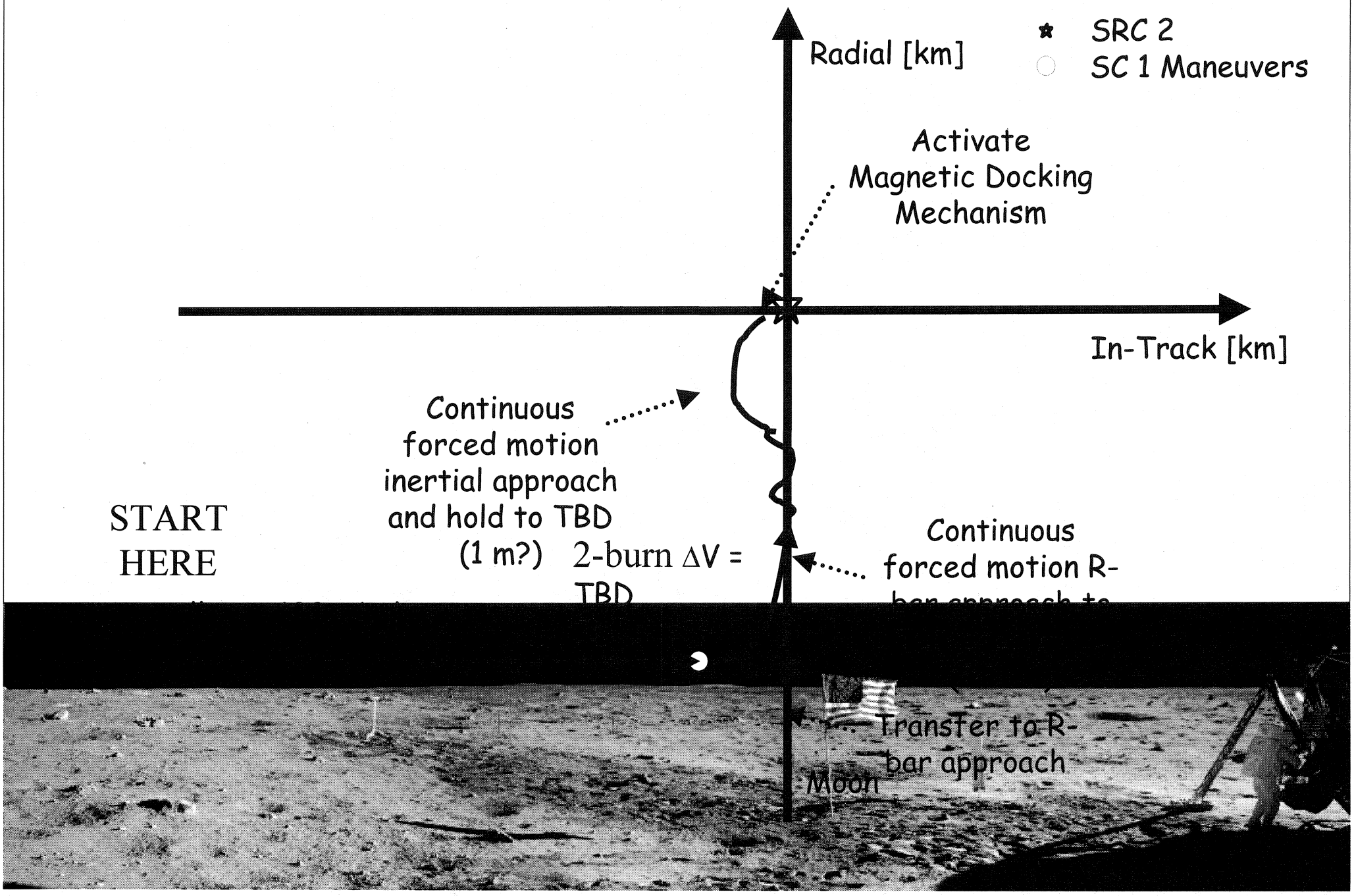
Rendezvous & Proximity Ops



Rendezvous & Proximity Ops



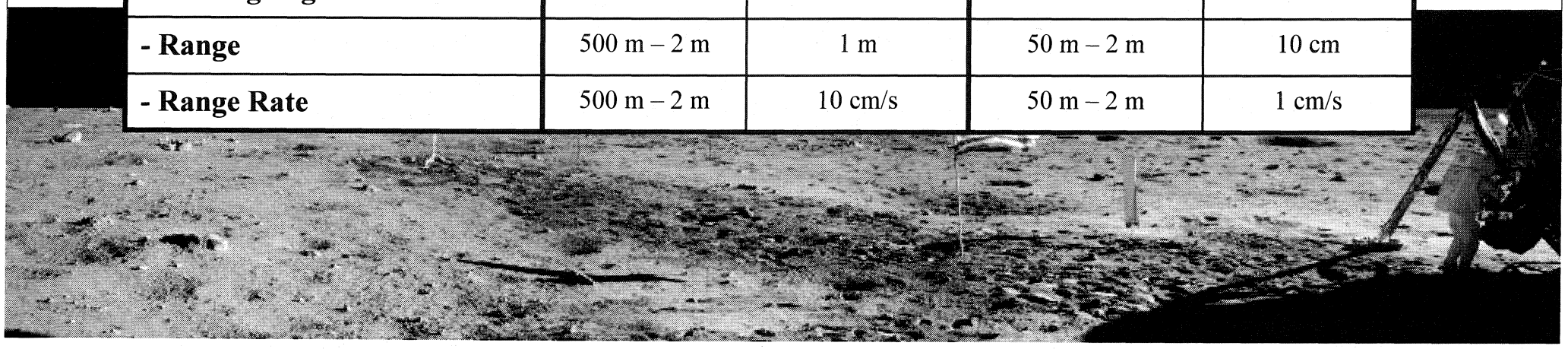
Rendezvous & Proximity Ops



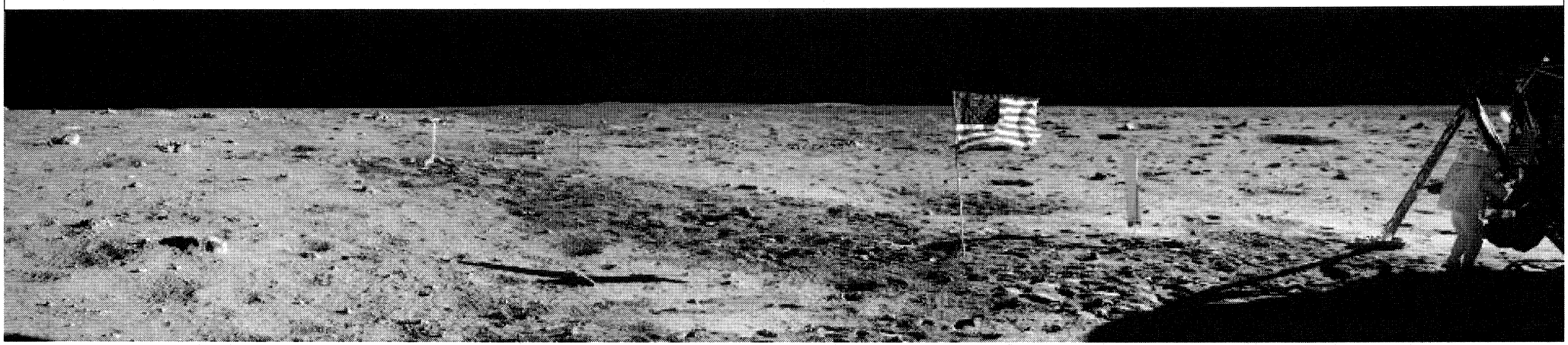
Possible Rendezvous Sensor Capabilities

Accuracies are estimates only

Sensor/Measurement	Requirement		Design Capability	
	Operational Range	**Accuracy**	**Operational Range**	**Accuracy**
Ground-based S-band ranging				
- Doppler	When in view	1 km OD	When ground signal received	6 Hz
- Range	When in view	1 km OD	When ground signal received	300-600 m
Relative S-band ranging				
- Doppler	10 km – 500 m	6 Hz (1 m/s)	20 Km – 250 m	0.6 Hz (10 cm/s)
- Range	10 km – 500 m	100 m	20 Km – 250 m	30-60 m
Optical tracking				
- Bearing angles	10 km – 2 m	1 arcmin	10 Km – 2 m	10 arcsec
- Range	500 m – 2 m	1 m	50 m – 2 m	10 cm
- Range Rate	500 m – 2 m	10 cm/s	50 m – 2 m	1 cm/s



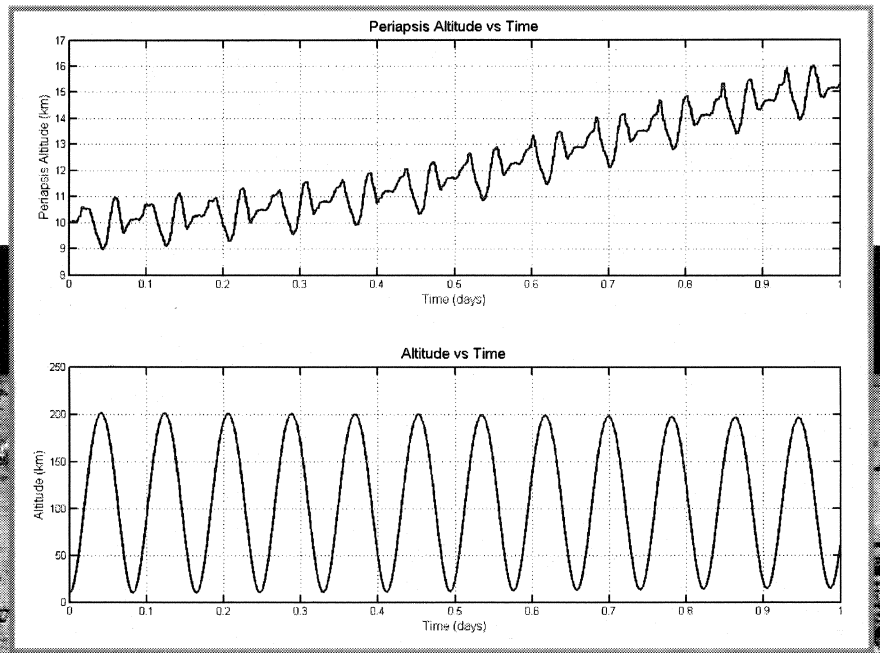
Launch and Orbit Raise



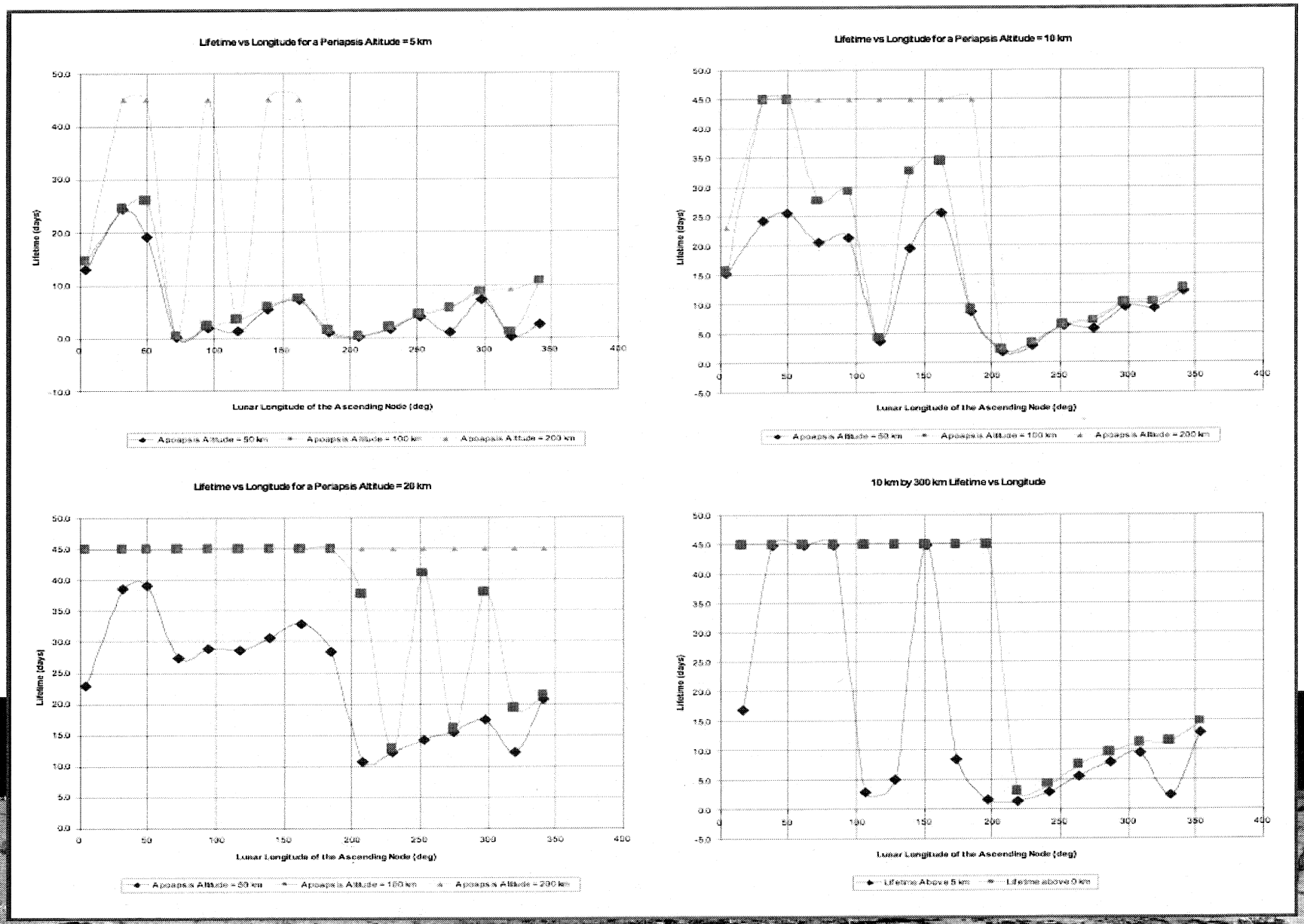
Using Lunar Potential to Raise Periapsis and Extend Lifetime

- The optimal orbit that results in greater than a 45 day lifetime, taking into account longitudes, overall altitudes and lifetimes and trying to minimize fuel used is
  - ✓Periapsis Altitude = 10 km
  - ✓Apoapsis Altitude = 200 km
  - ✓Lunar Launch Longitude = 40 degrees (0 - 180 range)
- From the first day the orbit shows an increasing instantaneous periapsis altitude (e decreases without ω drift)
- Sensitivity analysis performed on optimal orbit shows a +/- 1deg pitch tolerance and a 2% maneuver performance limit. Yields an attitude margins ~ 50%.

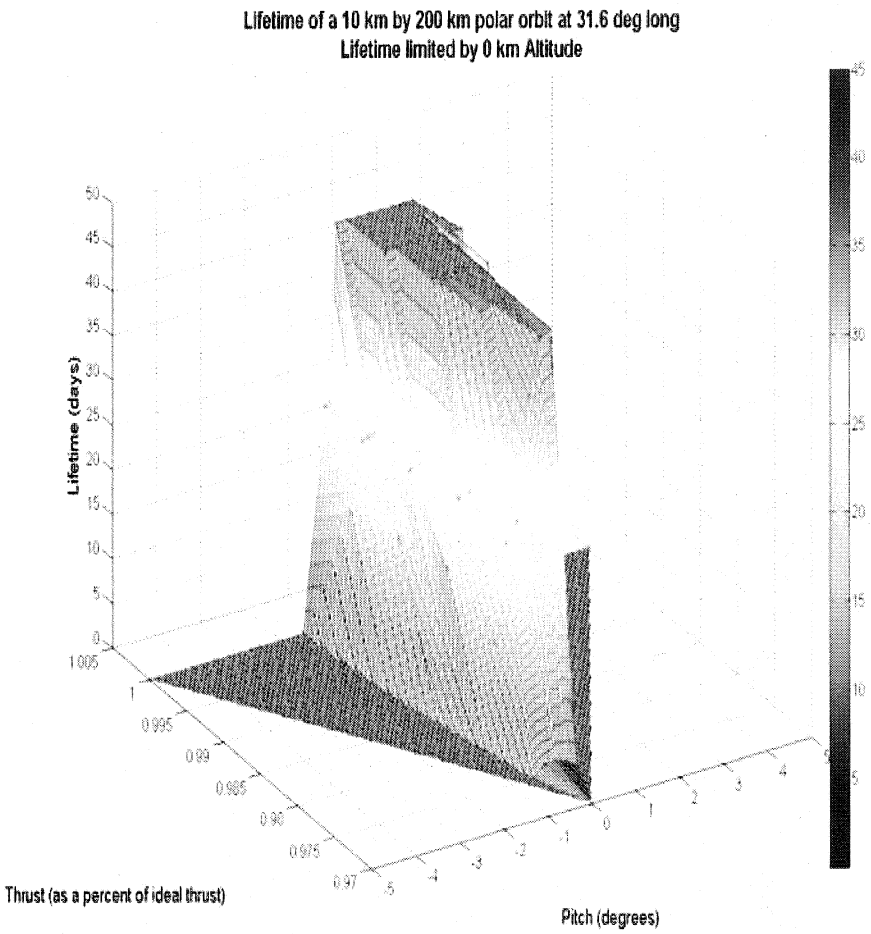
Increasing Periapsis Altitude over First Day

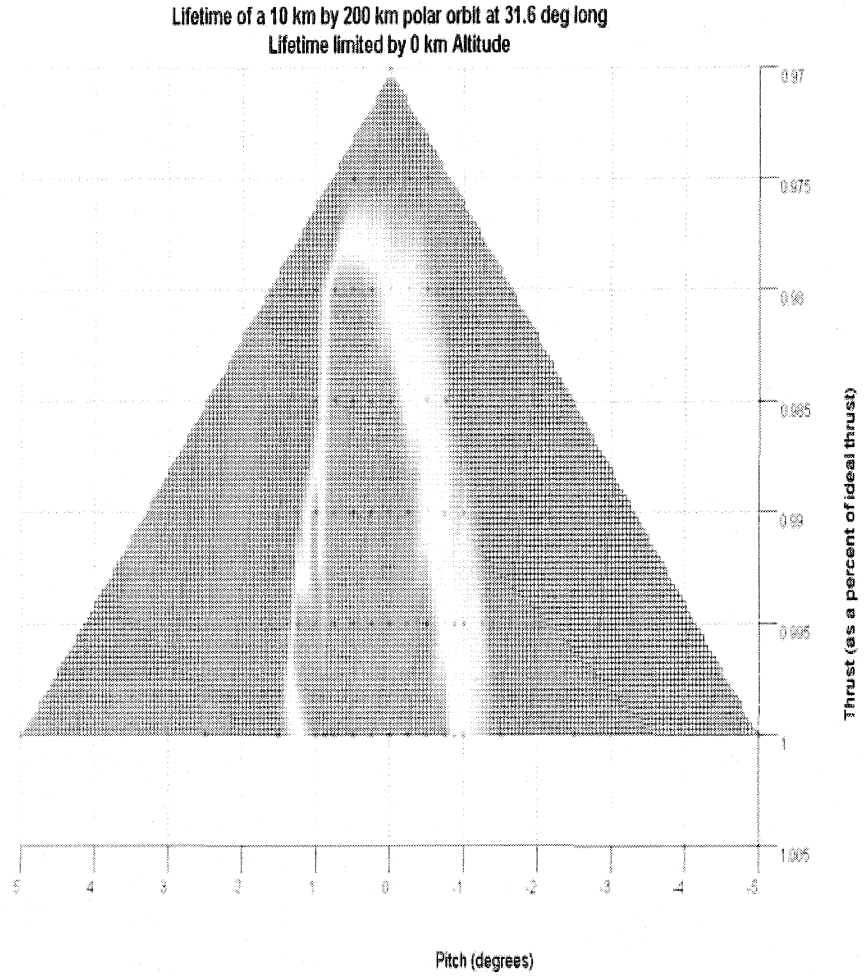


Longitude vs. Lifetime for Various Periapsis and Apoapsis Combinations



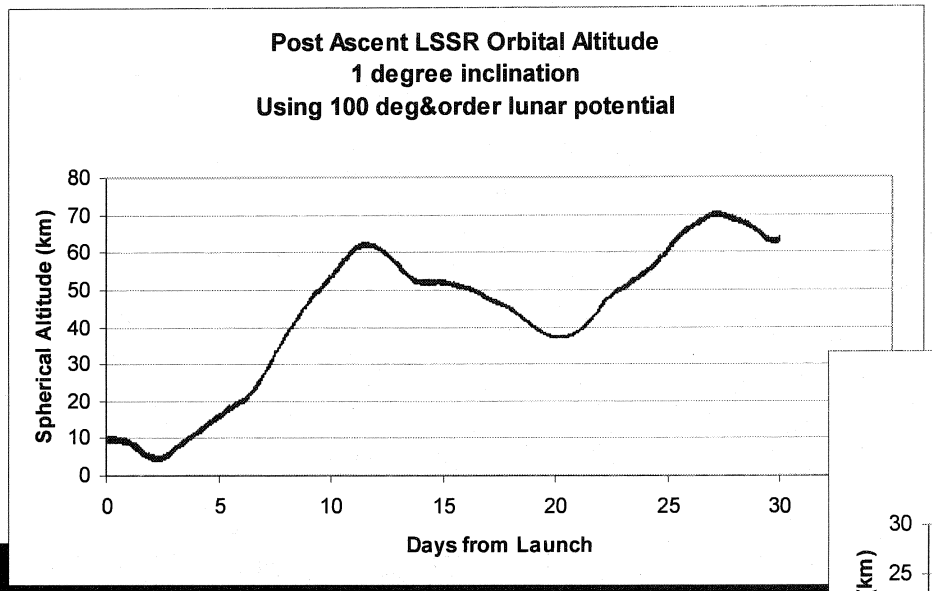
Lifetime for Various Thrust and Pitch Combinations

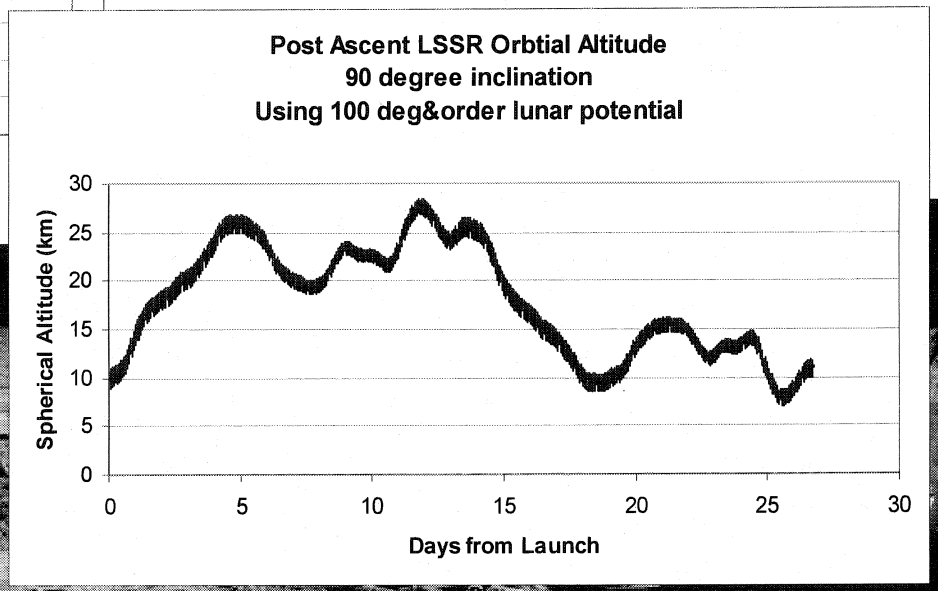




Post Ascent LSSR Orbit Spherical Altitude

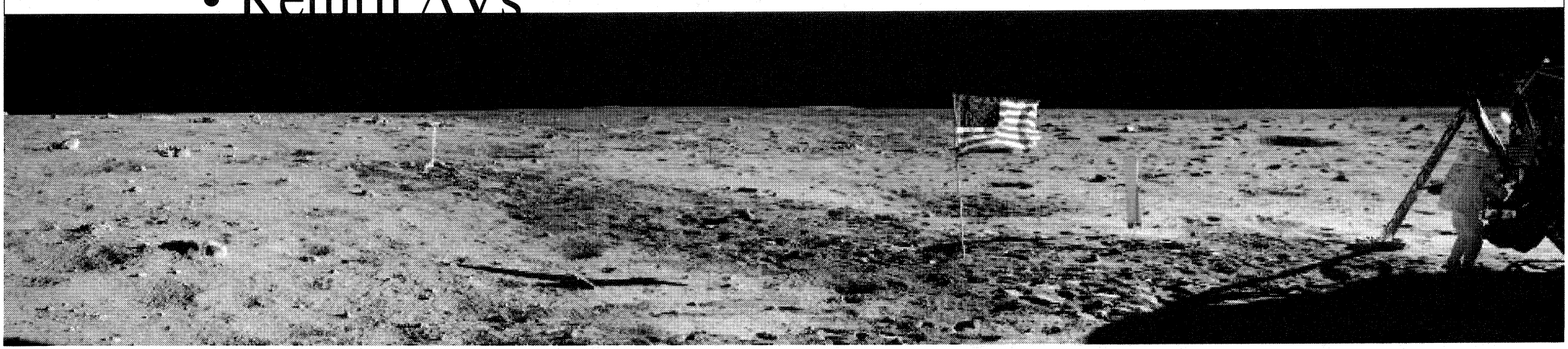
- Full Fidelity with Lunar Gravity (100deg & order)
- 30-day Simulation of 90-deg and 1-deg Inclinations
- Use of Natural Perturbations to Increase Perilune and Extend lifetime
- Based on GSFC Lunar Prospector Operational Support (30km orbit)





General Lunar Mission Design Slides – John move to where you need them:

- Direct vs. Weak Stability Transfer
- Launch Energy and Insertion ΔVs
- Return ΔVs



Robotic Lunar Missions – Successes and Failures
57 missions – 9 failures, 24 US Missions

Pioneer program
Pioneer 0 (USA, 1958) - failure - orbiter
Pioneer 1 (USA, 1958) - failure - orbiter
Pioneer 3 (USA, 1958) - failure - flyby
Pioneer 4 (USA, 1959) - partial success - flyby
Luna programme
See also: Lunokhod programme
Luna 1 (Soviet Union, 1959) - success - flyby
Luna 2 (Soviet Union, 1959) - success - impactor
Luna 3 (Soviet Union, 1959) - success - flyby
Luna 4 (Soviet Union, 1963) - partial failure - lander (became probe)
Luna 9 (Soviet Union, 1966) - success - lander
Luna 10 (Soviet Union, 1966) - success - orbiter
Luna 11 (Soviet Union, 1966) - success - orbiter
Luna 12 (Soviet Union, 1966-67) - success - orbiter
Luna 13 (Soviet Union, 1966) - success - lander
Luna 14 (Soviet Union, 1968) - success - orbiter
Luna 16 (Soviet Union, 1970) - success - sample return
Luna 17 (Soviet Union, 1970) - success - lander
Lunokhod 1 (Soviet Union, 1970-71) - success - rover
Luna 19 (Soviet Union, 1971-72) - success - orbiter
Luna 20 (Soviet Union, 1972) - success - lander
Luna 21 (Soviet Union, 1973) - success - lander
Lunokhod 2 (Soviet Union, 1973) - success - rover
Luna 22 (Soviet Union, 1974-75) - success - orbiter
Luna 24 (Soviet Union, 1976) - success - lander
Ranger program
Ranger 3 (USA, 1962) - failure - impactor
Ranger 4 (USA, 1962) - success - impactor
Ranger 5 (USA, 1962) - partial failure - impactor (became flyby)
Ranger 6 (USA, 1964) - failure - impactor
Ranger 7 (USA, 1964) - success - impactor
Ranger 8 (USA, 1964) - success - impactor
Ranger 9 (USA, 1964) - success - impactor

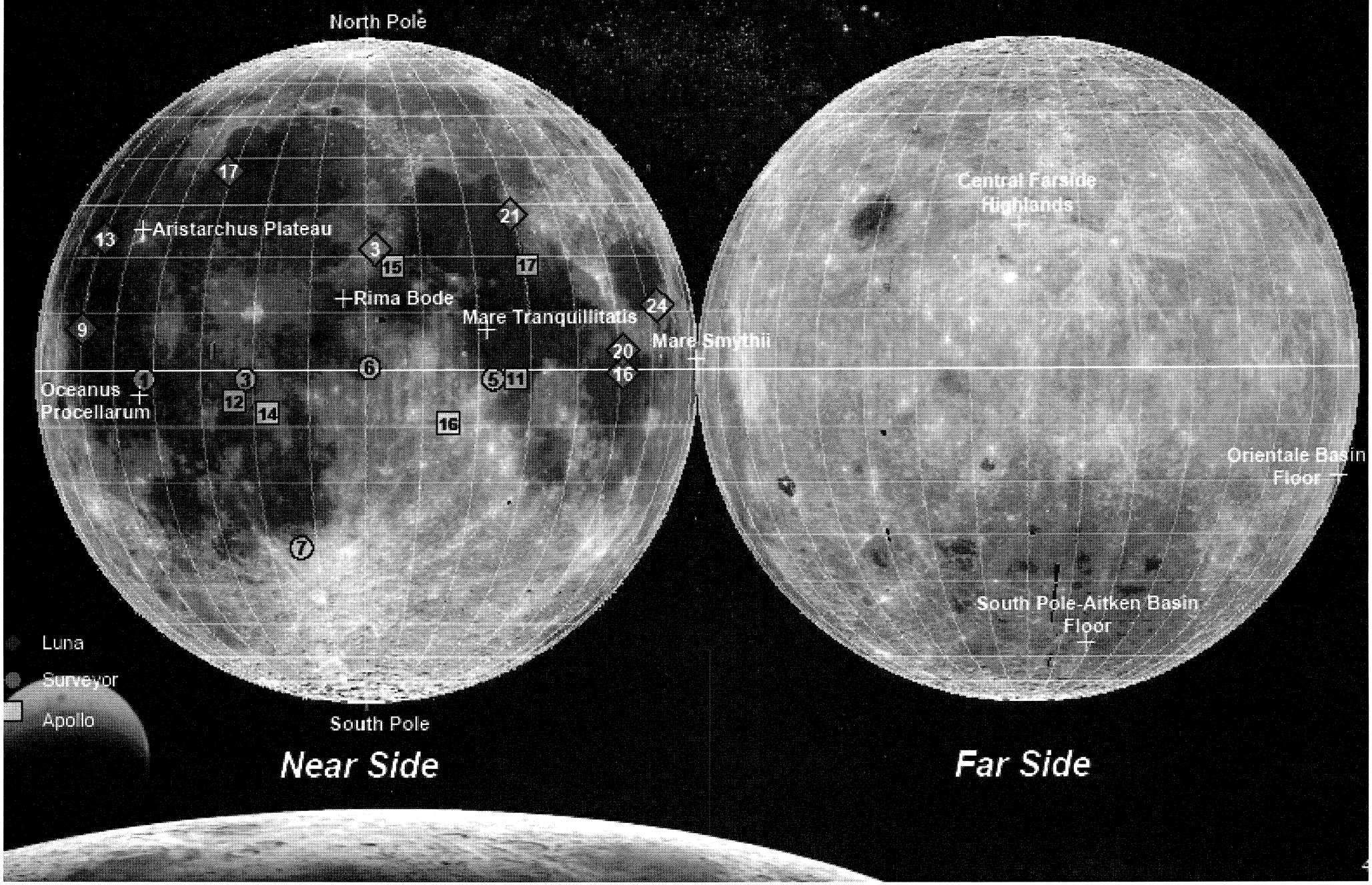
Zond program
Zond 3 (Soviet Union, 1965) - success - flyby
Zond 5 (Soviet Union, 1968) - success - flyby
Zond 6 (Soviet Union, 1968) - success - flyby
Zond 7 (Soviet Union, 1969) - success - flyby
Zond 8 (Soviet Union, 1970) - success - flyby
Surveyor program
Surveyor 1 (USA, 1966) - success - lander
Surveyor 2 (USA, 1966) - crashed - lander
Surveyor 3 (USA, 1967) - success - lander
Surveyor 4 (USA, 1967) - crashed - lander
Surveyor 5 (USA, 1967) - success - lander
Surveyor 6 (USA, 1967) - success - lander
Surveyor 7 (USA, 1968) - success - lander
Lunar Orbiter program
Lunar Orbiter 1 (USA, 1966) - success - orbiter
Lunar Orbiter 2 (USA, 1966-67) - success - orbiter
Lunar Orbiter 3 (USA, 1967) - success - orbiter
Lunar Orbiter 4 (USA, 1967) - success - orbiter
Lunar Orbiter 5 (USA, 1967-68) - success - orbiter
[Explorer 35
Explorer 35 (USA, 1967-73) - success - orbiter
Hiten
Hiten (Japan, 1990-93) - success - orbiter
Hagoromo (Japan, 1990) - failure - orbiter
Clementine
Clementine (USA, 1994) - success - orbiter
Lunar Prospector
Lunar Prospector (USA, 1998-99) - success - orbiter
SMART-1
SMART-1 (ESA, 2003-06) - success - orbiter

Selene (JAXA, 2007-) - success – orbiter

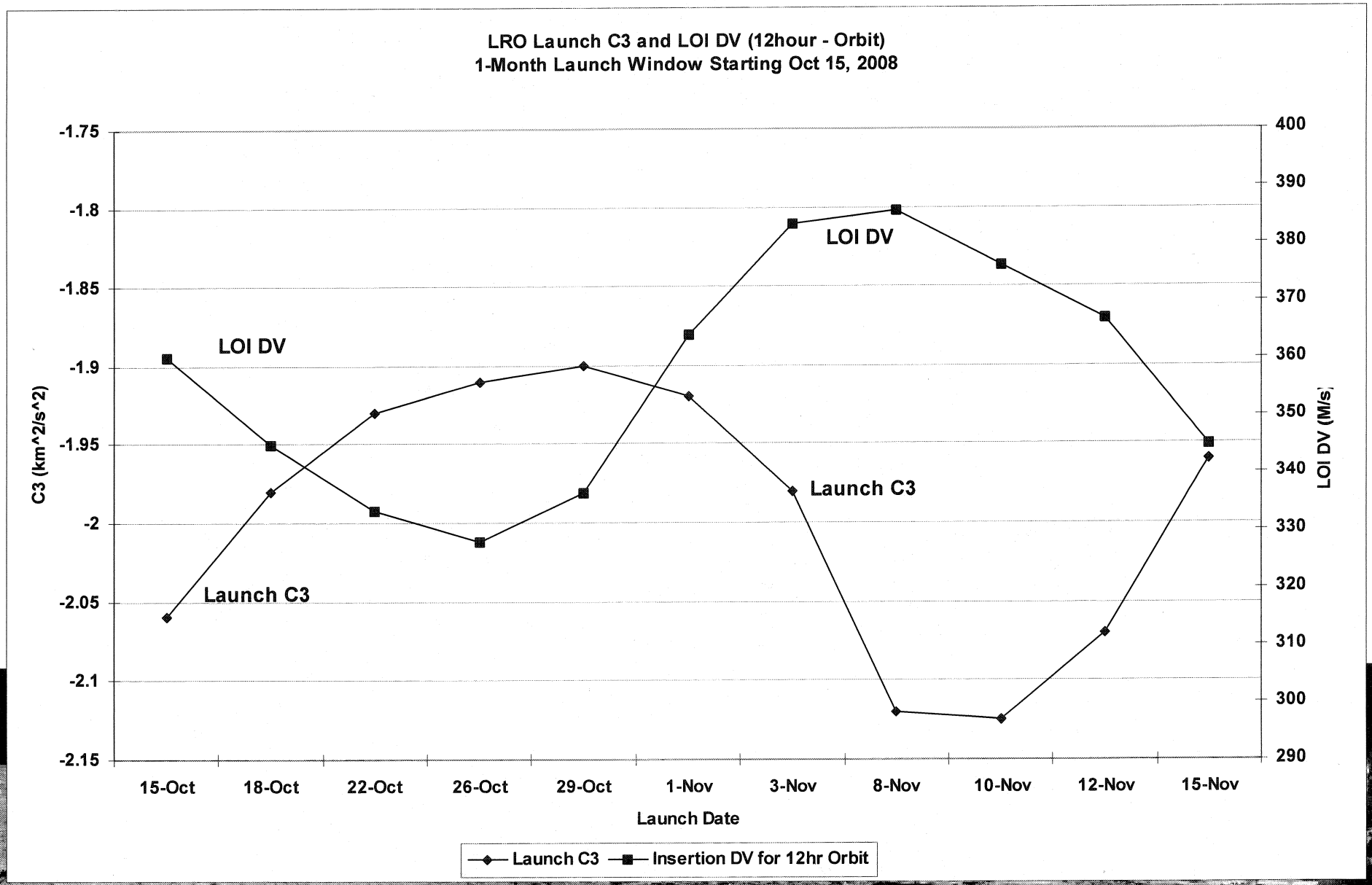
Chang'e (China, 2007-) - success – orbiter

LRO (USA, Launch 2008-) - - orbiter

High Priority Lunar Exploration Sites



Sample C3 and a Polar Lunar Orbit Insertion ΔV Variation over a Month

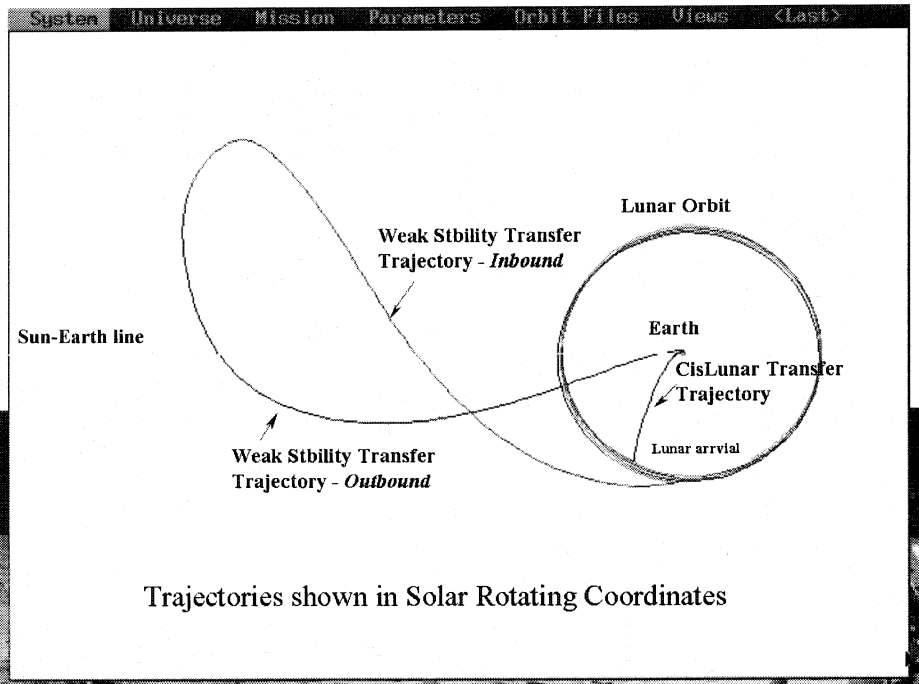


Sample Comparison of Direct and Weak Stability Boundary Transfers to a Low Lunar Orbit

	Direct Transfer	Weak Stbility Transfer
Launch Vehicle Injection (km/s)	3.13	3.20
Launch C3 (km^2/s^2)	-2.11	-0.70
Equivalent launch mass for C3 (kg)	1595	1545
Total Delta-V to attain mission orbit (m/s)	1017	889 / 985 with midcourse
Trip Time (days)	4.4	116
Max distance from Earth (million km)	0.4	1.54
Final Mass into Orbit (kg)	995	1023 / 978 with midcourse

Assumptions:

- ✓ Polar lunar mission orbit at 30km altitude
- ✓ Launch Vehicle is D2925H-9.5
- ✓ Final mass computed via rocket equation with starting launch mass, Isp=220sec, thrust = 1N, and ΔV as above



Trajectories shown in Solar Rotating Coordinates

- ➢ ΔV difference of 12.5% or 3.14% if WSB midcourse included
- ➢ Final mass difference of 2.7% or -1.7% if WSB midcourse included

Variation in Earth Return DVs

for Selected Inclinations of 0,30,60,& 90 degrees

- Figure show ΔV dependency on orbit plane angle to Earth (x-axis) and true anomaly (y-axis)
- Note: ΔVs are in ft/sec