Statistical Properties of SEE Rate Calculation in the Limits of Large and Small Event Counts

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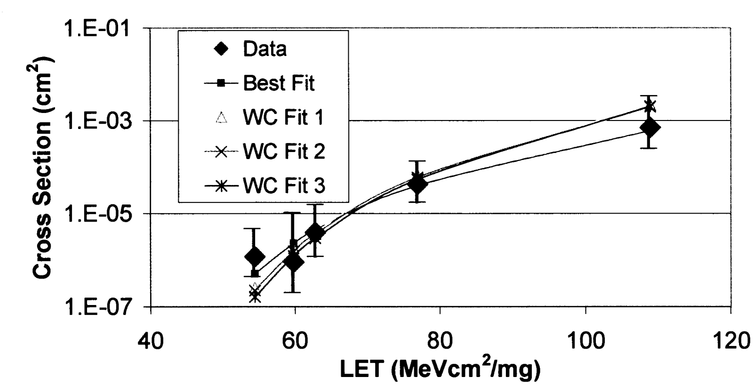
Motivation for a Statistical Approach

- Goal of SEE rate calculation is to bound the SEE rate
  - by how much?
- Estimated rate depends on the fit to the σ vs. LET curve
  - Should the fit be a "Best" fit or a conservative worst-case fit
  - Does the answer depend on error bars



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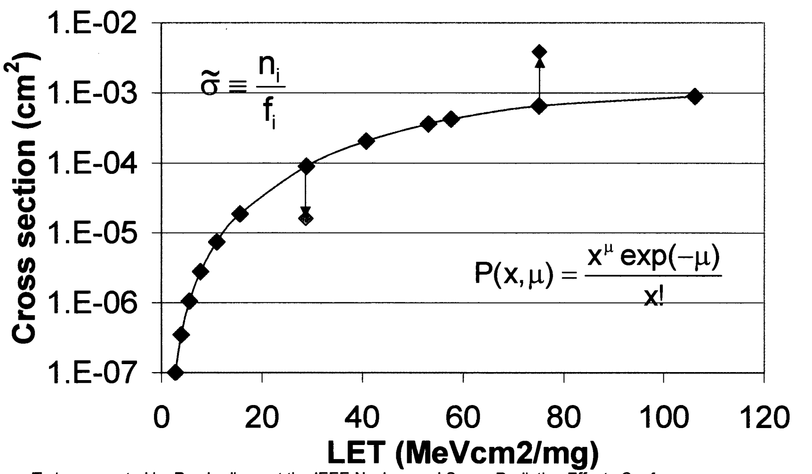
Outline

- Understanding errors on SEE cross sections
- Methodology: Maximum Likelihood and confidence Contours
- Tests with Simulated data
- Applications

Cross sections and Errors

$$\sigma(LET_i) = \sigma_{lim} \times (1 - \exp(-((LET_i - LET_0)/w)^s)$$



Maximum Likelihood and Confidence

- Likelihood $\mathcal{L}$ is the product of the probabilities of all our data
  - Probabilities of our observations are driven by Poisson fluctuations, so

$$\mathcal{L} = \prod_{i=1}^{n} P(x_i, \mu) = F_i \times \sigma_{lim} \times \left[1 - \exp\left\{-\left(\frac{LET - LET_0}{W}\right)^s\right\}\right]$$

  - Values of (LET_0, σ_{lim}, w and s) that maximize $\mathcal{L}$ give a best fit to data
    - Must be done numerically
  - Advantage of Maximum Likelihood is that the ratio of the likelihood for different parameter values gives a measure of their relative probability

$$\log(\mathcal{L}(\{LET_0^*, \sigma_{lim}^*, w^*, s^*\})/\mathcal{L}(\{LET_0, \sigma_{lim}, w, s\}_{bf})) \geq -0.5\chi^2(P,k)$$

    - This allows construction of confidence contours around our best-fit values
- The highest SEE rate for parameters within a confidence contour will be the WC rate consistent with our data at that confidence level.
  - Use Figure of merit (FOM) to select parameter sets $FOM = C \times \sigma_{lim} / LET_{0.25}^2$

Example with Simulated Data

Sample Data: Generated with LET_0=26, σ_{lim},=3.13×10^{-4},w=70 and s=2.5

LET_{EFF}	7.8	11.03	15.6	28.8	40.73	53.1	57.6	75.1	106.2
Cross Section	0.0E+00	0.0E+00	0.0E+00	1.0E-07	6.3E-06	2.8E-05	4.0E-05	1.1E-04	2.4E-04
Events Observed	0	0	0	1	50	100	100	100	100
Effective Fluence	1.0E+07	1.0E+07	1.0E+07	1.0E+07	8.0E+06	3.6E+06	2.5E+06	9.5E+05	4.2E+05

LET_0 / σ_{lim}	20	21	22	23	24	25	26	27	28
2.13E-04	-20.41	-15.29	-12.74	-12.82	-15.57	-21.10	-29.51	-41.02	-56.28
2.63E-04	-33.21	-21.23	-12.18	[redacted]	[redacted]	[redacted]	[redacted]	-13.17	-23.93
3.13E-04	-62.62	-43.77	-28.22	-16.01	-7.19	[redacted]	[redacted]	[redacted]	-8.19
3.63E-04	-103.68	-77.97	-55.92	-37.57	-22.96	-12.15	[redacted]	[redacted]	[redacted]
4.13E-04	-153.39	-120.81	-92.26	-67.77	-47.37	-31.11	-19.05	-11.37	-8.65
4.63E-04	-209.75	-170.30	-135.26	-104.63	-78.44	-56.73	-39.56	-27.08	-19.86

Example with Simulated Data (Cont'd)

Slice in w-s for best fit cross section and onset LET

W \ S	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
60	-88.4	-67.1	-49.6	-35.7	-24.8	-16.7	-11.2	-7.9	-6.7
62	-71.1	-51.7	-36.2	-24.1	-15.2	-9.0			
64	-56.5	-39.0	-25.4	-15.2	-8.2				
66	-44.2	-28.6	-16.9	-8.6					-8.1
68	-34.1	-20.3	-10.5						-12.8
70	-25.8	-13.9			0.0			-11.1	-19.1
72	-19.2	-9.2					-9.5	-17.2	-26.9
74	-14.1					-7.9	-15.2	-24.6	-36.1
76	-10.4					-13.1	-22.1	-33.2	-46.3
78	-7.9				-11.0	-19.3	-30.0	-42.8	-57.5
80				-9.2	-16.6	-26.6	-38.9	-53.3	-69.6
82			-7.9	-14.0	-23.1	-34.7	-48.6	-64.6	-82.4

Example with Simulated Data (Cont'd)

LET_{EFF}	7.8	11.03	15.6	28.8	40.73	53.1	57.6	75.1	106.2
Cross Section	0.0E+00	0.0E+00	0.0E+00	1.0E-07	6.3E-06	2.8E-05	4.0E-05	1.1E-04	2.4E-04
Events Observed	0	0	0	1	2	4	4	4	4
Effective Fluence	1.0E+07	1.0E+07	1.0E+07	1.0E+07	8.0E+06	3.6E+06	2.5E+06	9.5E+05	4.2E+05

σ_{lim} \ LET_0	20	21	22	23	24	25	26	27	28
1.13E-04	-6.94						-7.22	-8.89	-11.71
2.13E-04	-8.89								
3.13E-04	-15.57	-10.74	-6.85						
4.13E-04	-24.29	-17.52	-11.96	-7.55					
5.13E-04	-34.16	-25.45	-18.22	-12.40	-7.92				
6.13E-04	-44.76	-34.12	-25.22	-17.98	-12.32	-8.16			

Example with Simulated Data (Cont'd)

W \ S	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1
45	-43.2	-28.5	-19.4	-13.6	-9.9	-7.6							
50	-34.0	-21.1	-13.2	-8.4									
55	-27.0	-15.6	-8.9										
60	-21.6	-11.5											-7.4
65	-17.4	-8.5										-7.9	-9.8
70	-14.1				0.0						-8.2	-10.3	-12.6
75	-11.5									-8.2	-10.5	-13.0	-15.6
80	-9.5								-8.0	-10.5	-13.1	-15.9	-18.7
85	-8.0							-7.4	-10.1	-12.8	-15.8	-18.8	-21.9
90	-6.8						-6.7	-9.4	-12.2	-15.3	-18.5	-21.8	-25.1
95							-8.4	-11.3	-14.5	-17.8	-21.2	-24.8	-28.4
100						-7.2	-10.1	-13.3	-16.7	-20.3	-24.0	-27.7	-31.6
105						-8.7	-11.9	-15.4	-19.0	-22.8	-26.7	-30.7	-34.7
110					-7.1	-10.2	-13.7	-17.4	-21.3	-25.3	-29.4	-33.6	-37.8
115					-8.4	-11.7	-15.5	-19.4	-23.5	-27.7	-32.0	-36.4	-40.9
120					-9.7	-13.3	-17.2	-21.4	-25.7	-30.1	-34.6	-39.2	-43.8
125				-7.6	-11.0	-14.8	-19.0	-23.4	-27.9	-32.5	-37.2	-41.9	-46.7
130				-8.6	-12.3	-16.4	-20.7	-25.3	-30.0	-34.8	-39.7	-44.6	-49.6
135				-9.7	-13.6	-17.9	-22.4	-27.2	-32.1	-37.0	-42.1	-47.2	-52.3
140			-7.3	-10.8	-14.9	-19.4	-24.1	-29.0	-34.1	-39.2	-44.4	-49.7	-55.0

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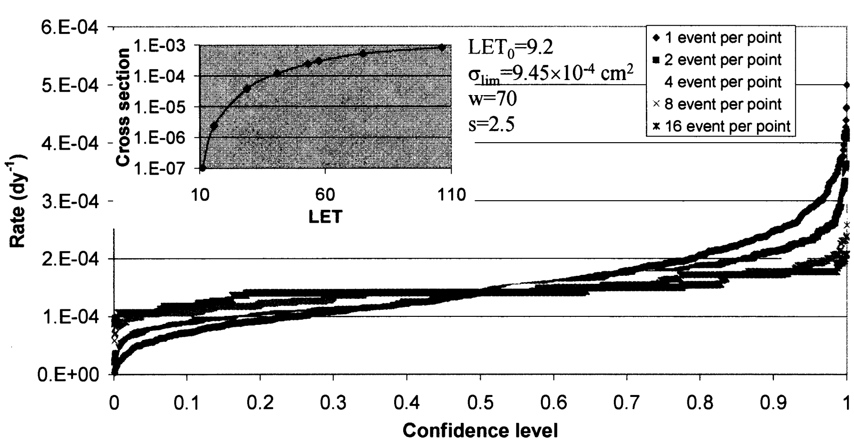
SEE Rate Confidence Contour vs. Event Count



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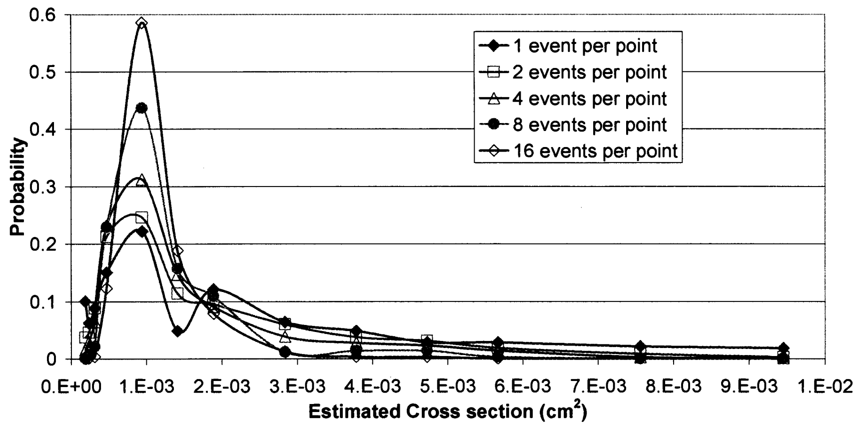
How Event Count affects Estimation



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Applications

- Determination of part-to-part and lot-to-lot variability
  - Poisson errors sometimes give an impression of variability
  - Variability is established at significance 1-α if the contours for CL= α do not overlap
- Test planning—although we don't know a priori how a test will perform, we can simulate over a range of possible distributions to determine the worst-case dependence of rate error on event count
  - Can be used to ensure efficient allocation of test time and resources
    - What ions to use?
    - How much time for each ion?
    - For destructive errors, how many parts to sacrifice at each ion?
  - Estimate derating factors as a function of # of events and confidence level.
- Dividing the data set in post-processing
  - Often events of interest to the application emerge only in post-processing
    - Example: Some SEFI may pose system-level problems, while others do not
    - Example: Only transients larger than a minimum amplitude and longer than a certain duration may pose a threat

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Example: Ultra-long Transients for OP293

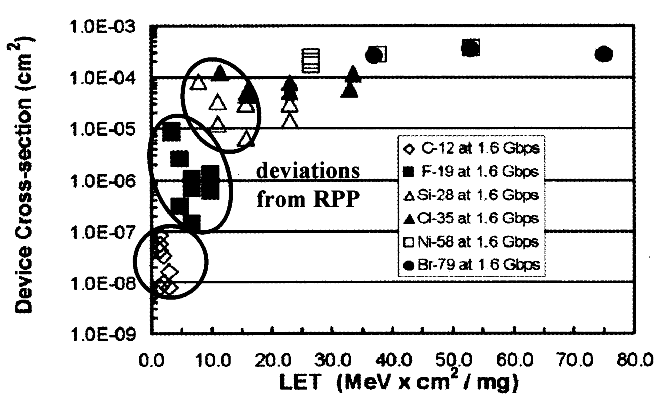
- Testing on Analog Devices OP293 revealed transients up to 1 ms long
  - Boards were already populated and there was no easy replacement
  - Circuit analysis revealed concern only for transients >120 μs and >2V

LET_{EFF} ($MeVcm^2/mg$)	Total transients	Transients (>120 ms, >2 V)	Cross Section (cm^2)
19.5	751	1	1×10^{-7}
28	525	3	3×10^{-7}
43	679	5	5×10^{-7}
49.5	1303	0	0
87.5	477	2	2×10^{-7}

- Testing on Analog Devices OP293 revealed transients up to 1 ms long
  - Boards were already populated and there was no easy replacement
  - Circuit analysis revealed concern only for transients >120 μs and >2V

Possible Extensions of the Method

- This work dealt with Poisson errors, where mean σ follows a Weibull
  - Method can also be extended to model other error including systematic errors



- If we can model it, we can apply this method
  - Errors on fluence, deviations from RPP, Weibull form for σ, and so on.

Conclusions

- Maximum Likelihood methods allow us to bound the SEE rate for a given confidence level IF we can model the errors on σ vs. LET curve
  - If event counts are high systematic errors will dominate
  - Otherwise errors are Poisson
- Allows unambiguous determination of a bounding rate for a given confidence level and data set.
- Applications include
  - Determination of part-to-part and lot-to-lot variability
  - Test Planning
  - Determination of bounding rates for marginal data (e.g. divided data sets)
- Technique can be generalized to include any error we can model
  - Other random errors (including part-to-part variations)
  - Systematic errors if we can characterize and model them

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