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MODELING KICKS FROM THE MERGER OF GENERIC BLACK-HOLE BINARIES

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ABSTRACT

Recent numerical relativistic results demonstrate that the merger of comparable-mass spinning black holes has a maximum "recoil kick" of up to $\sim 4000\,\mathrm{km\,s^{-1}}$. However the scaling of these recoil velocities with mass ratio is poorly understood. We present new runs showing that the maximum possible kick parallel to the orbital axis does not scale as $\sim \eta^2$ (where η is the symmetric mass ratio), as previously proposed, but is more consistent with $\sim \eta^3$. We discuss the effect of this dependence on galactic ejection scenarios and retention of intermediate-mass black holes in globular clusters.

Subject headings: black hole physics – galaxies: nuclei – gravitational waves — relativity

1. INTRODUCTION

Recently, numerical exploration of the radiative recoil "kick" of merging black holes has progressed considerably. In particular, efforts in this regard have led to suggested phenomenological formulae for the kick, largely based on post-Newtonian (PN) predictions such as that given by Kidder (1995), which have proved surprisingly successful. For example, Gonzalez et al. (2007) found that in cases of unequal masses ($q \equiv m_1/m_2 < 1$) and no spin, a simple modification of the PN formula originally found by Fitchett (1983) fits the numerical data quite well. For cases of spins parallel or antiparallel to the orbital angular momentum, a formula proposed by Baker et al. (2007), loosely based on PN calculations, is also consistent with numerical data. For spins with components perpendicular to the orbital angular momentum, a formula, again derived from PN calculations, has been proposed by Campanelli et al. (2007). This formula agrees well with numerical results for equal masses.

This last type of kick, which is parallel to the orbital angular momentum, is of particular interest because its computed magnitude can be very large (up to thousands of kilometers per second). However, unlike for the kicks that arise from unequal masses with no spins, or the in-plane kicks that arise from spins out of the orbital plane, the mass ratio dependence of these critical out-of-plane kicks has not been tested systematically by numerical experiments. In the current literature (specifically Campanelli et al. 2007), the dependence is drawn from the leading-order PN approximation. It is unclear whether this approximation is sufficient to predict the strong-field dynamics that presumably determines the kick. Indeed, hints of a deviation from this form are evident for mass ratio $q = 1/2$ in the runs of Lousto & Zlochower (2007). Therefore, although it can be shown that the angular dependence of the proposed formula is consistent with symmetry arguments, which are independent of the strong-field dynamics (Boyle et al. 2007; Boyle & Kesden 2007), the mass ratio dependence of this formula is currently not well justified.

Characterization of the dominant kick for unequal masses is especially important because, although the largest possible kicks would eject the remnant from any galaxy, for astrophysical applications it is the distribution of kick speeds that matters. For example, Bonning et al. (2007) find no evidence for quasars ejected from their hosts. If quasar activity is commonly induced by major galaxy mergers that lead to coalescence of supermassive black holes, the implications of this therefore depend in part on how frequently one expects a merger to allow ejection. In addition, the kick speed distribution has a major impact on the hierarchical growth of massive black holes at redshifts $z > 5$ (e.g., Volonteri 2007).

Here we investigate how the out-of-plane kick depends on the mass ratio, and find that, for mass ratios in the range $q = 1$ to $q = 1/3$, the kick drops off more rapidly with decreasing mass ratio than proposed by Campanelli et al. (2007). Specifically, we find that all current numerical data on kicks are well represented by

$$\vec{V}_{\rm recoil} = v_m\,\hat{e}_1 + v_\perp(\cos\xi\,\hat{e}_1 + \sin\xi\,\hat{e}_2) + v_\parallel\,\hat{e}_2, \quad (1)$$

$$v_m = A\eta^2\sqrt{1-4\eta}(1+B\eta), \quad (2)$$

$$v_\perp = H\frac{\eta^2}{(1+q)}\left(\alpha_2^\parallel - q\alpha_1^\parallel\right), \quad (3)$$

$$v_\parallel = \frac{K\eta^3}{(1+q)}\left[q\alpha_1^\perp\cos(\phi_1-\Phi_1) - \alpha_2^\perp\cos(\phi_2-\Phi_2)\right] \quad (4)$$

where $\eta \equiv q/(1+q)^2$ is the symmetric mass ratio, $\alpha_i^\parallel$ is the projection of the dimensionless spin vector $\vec{\alpha}_i = \vec{S}_i/m_i^2$ of black hole i along the orbital angular momentum, $\alpha_i^\perp$ is the magnitude of its projection, $\vec{\alpha}_i^\perp$, into the orbital plane, ϕ_i refers to the angle made by $\vec{\alpha}_i^\perp$ with respect to some reference angle in the orbital plane, and Φ_1 and Φ_2 are constants for a given mass ratio. Here, $A = 1.35\times10^4\,\mathrm{km\,s^{-1}}$, $B = -1.48$, $H = 7540\pm160\,\mathrm{km\,s^{-1}}$, $\xi = 215^\circ\pm5^\circ$, and $K = 2.4\pm0.4\times10^5\,\mathrm{km\,s^{-1}}$. This formula, similar in form to that given by Campanelli et al. (2007), synthesizes results from Gonzalez et al. (2007) for (2) and from Baker et al. (2007) for (1) and (3) [3]. For ξ and H we have fit available numerical data from Herrmann et al. (2007); Koppitz et al. (2007); Baker et al.

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[3] Note that in Baker et al. (2007), we used a simpler form for the zero-spin contribution, equivalent to (2) with $B = 0$.

TABLE 1
INITIAL CONFIGURATION AND FINAL KICK FOR EACH SIMULATION. $\phi_{1(2)}$ IS THE ANGLE MADE BY THE SPIN VECTOR OF HOLE 1(2) WITH THE VELOCITY VECTOR OF HOLE 1, AS SHOWN IN FIG. 1. NUMERICAL RESULTS FOR THE KICK COMPONENTS v_m (WHERE AVAILABLE) AND $v_{\|}$ ARE SHOWN. KICKS FOR EQUIVALENT SPINLESS RUNS ARE IN PARENTHESES.

q	$\phi_1(^o)$	$\phi_2(^o)$	v_m (km s^{-1})	$v_{\|}$ (km s^{-1})
1/1.1	0	180	24	-542
	315	135	24	-657
	270	90	25	-384
1/1.3	0	180	67	-386
	315	135	67	-525
	270	90	69	-348
1/1.5	60	240	92 (94)	-381
	0	180	95 (94)	-135
	315	135	91 (94)	168
	270	90	90 (94)	364
1/2	0	180	137 (140)	-37
	315	135	136 (140)	111
	270	90	136 (140)	193
	315	90	...	75
	0	90	...	-55
1/3	0	180	166	49
	315	135	166	48
	270	90	163	17

(2007). The qualitatively new part, the factor of η^3 in (4), replaces the factor of η^2 originally proposed by Campanelli et al. (2007), and is motivated by new numerical evolutions presented here. We give our methodology in § 2, and present our results, fits and possible analytical motivation, and astrophysical consequences in § 3.

2. INITIAL DATA AND METHODOLOGY

We simulated the inspiral and merger of a range of spinning black-hole binaries, with mass ratios in the range $1/1.1 \geq q \geq 1/3$. The initial configuration of momenta and spins is illustrated in Fig. 1. The parameters used in the numerical evolutions are presented in the first three columns of Table 1. For these evolutions, the smaller hole (m_1) has a dimensionless spin $|\vec{\alpha}_1| = 0.2$, while the larger hole's spin is $|\vec{\alpha}_2| = q^2|\vec{\alpha}_1|$. Both spin vectors initially lie in the orbital plane, at angles ϕ_1 and ϕ_2 to the initial velocity of hole 1 (see Fig. 1).

To perform our simulations, we employed the HAHNDOL evolution code, as described in Baker et al. (2007). The code's convergence properties are discussed in Baker et al. (2007) and in Baker et al. (2008). Initial separations were chosen to yield between one and four orbits prior to merger; the corresponding momenta were chosen, informed by PN theory (Damour et al. 2000), to minimize initial eccentricity.

3. RESULTS AND DISCUSSION

The recoil kicks resulting from the new simulations are given in the rightmost columns of Table 1. Now we consider the agreement of our data with the suggested formula of Campanelli et al. (2007):

$$v_{\|} = K\cos(\Theta - \Theta_0)\frac{\eta^2}{(1+q)}|\vec{\alpha}_2^{\perp} - q\vec{\alpha}_1^{\perp}|. \quad (5)$$

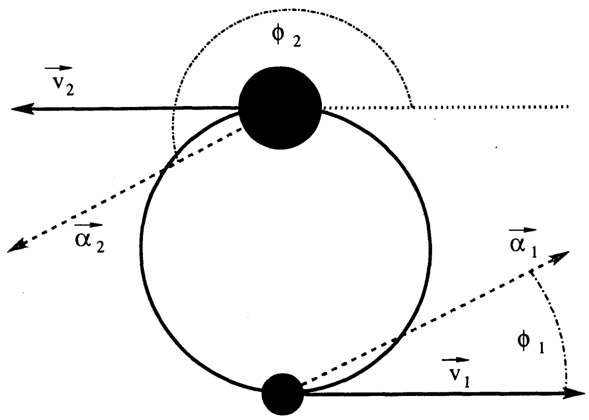

FIG. 1.— Configuration of black holes for all new simulations. The two holes' spins $\vec{\alpha}_{(1,2)}$ lie initially in the orbital plane, at angles ϕ_1 and ϕ_2 to $\vec{v}_1$, the smaller hole's initial velocity.

where Θ is the angle between $(q\vec{\alpha}_1^{\perp} - \vec{\alpha}_2^{\perp})$ and the separation vector, nominally just before merger, and Θ_0 is some constant for a given mass ratio. In practice, it is more convenient to consider Θ with respect to the initial separation vector, and absorb into Θ_0 the amount by which this angle precesses before merger. By this definition, then, Θ_0 depends on both the mass ratio and the initial separation. Also note there is a second angle implicit within the expression $|\vec{\alpha}_2^{\perp} - q\vec{\alpha}_1^{\perp}|$, which represents the angle between $\vec{\alpha}_1^{\perp}$ and $\vec{\alpha}_2^{\perp}$. Recognizing that $\cos(\Theta - \Theta_0)|\vec{\alpha}_2 - q\vec{\alpha}_1| = (\vec{\alpha}_2 - q\vec{\alpha}_1) \cdot \hat{n}$, where $\hat{n}$ is the initial unit separation vector, we can rewrite (5) as

$$v_{\|} = K\frac{\eta^2}{(1+q)}[q\alpha_1^{\perp}\cos(\phi_1 - \Phi_1) - \alpha_2^{\perp}\cos(\phi_2 - \Phi_2))] \quad (6)$$

where ϕ_i represents the initial angle of $\vec{\alpha}_i^{\perp}$, and Φ_1 and Φ_2 are parameters which depend on mass ratio and initial separation, interpretable as encoding both the spin orientations for the maximum kick as well as the amount of spin precession before merger. Here we keep the value of $K = 6.0 \times 10^4$, which was found to work well in the equal-mass case, and find the best fit for the parameters Φ_1 and Φ_2 for each mass ratio. The error of the best fit grows significantly with mass ratio (as seen in the column labeled by "$K\eta^2$" in Table 2), hence the mass-ratio-dependence of this formula is inaccurate. One might suppose that precession of the spins into the orbital plane could account for this. However, the v_m column in Table 1 shows that the in-plane kicks are close to those measured without spins (given in parentheses), hence this is not an explanation of the discrepancy in $v_{\|}$ from the η^2 scaling.

To conceive of other plausible candidates for the kick formula we begin with the spin expansion and symmetry arguments of Boyle et al. (2007); Boyle & Kesden (2007). For the spin configurations under consideration here,

$$v_{\|} = D(q)\alpha_1^{\perp}\cos(\phi_1 - \Phi(q)) - D(1/q)\alpha_2^{\perp}\cos(\phi_2 - \Phi(1/q)), \quad (7)$$

where D and Φ are some functions of mass ratio q, and we note that Φ must also depend on the initial separation. Further restricting ourselves to forms relatable to the factor of $\vec{S}_1/m_1 - \vec{S}_2/m_2$ appearing in PN calculations of the kick, which informed Campanelli et al. (2007); Lousto & Zlochower (2007) and has been numerically well-verified in the equal-mass case, we substitute $D(q) =$

TABLE 2
MAXIMUM PERCENT ERROR RESULTING FROM VARIOUS MODELS OF THE KICK, AS DISTINGUISHED BY OVERALL MASS-RATIO DEPENDENCE. SEE EQUATION (8).

q	$K\eta^2$	$K(a_2\eta^2 + a_4\eta^4)$	$K\eta^3$
1/1.1	0.22	0.23	0.20
1/1.3	0.75	0.80	0.78
1/1.5	1.26	1.31	1.28
1/2	15.57	2.46	1.41
1/3	39.37	10.81	1.82

$qC(\eta)/(1+q)$ to obtain:

$$v_{\|} = \frac{C(\eta)}{(1+q)} \left[q\alpha_1^{\perp} \cos(\phi_1 - \Phi_1) - \alpha_2^{\perp} \cos(\phi_2 - \Phi_2) \right], \quad (8)$$

where $\Phi_1 \equiv \Phi(q)$ and $\Phi_2 \equiv \Phi(1/q)$.

Regarding the overall scaling of $v_{\|}$, it is known to be related to the difference between the energy radiated in the $(l, m) = (2, 2)$ and $(2, -2)$ harmonics of the radiation (Brügmann et al. 2007). With no spin, these quantities are equal. With spin, we expect that $v_{\|} \sim E_{22(peak)}F$, where $E_{22(peak)}$ is the peak energy radiated in the $(2, 2)$ harmonic, and F represents the spin-dependent asymmetry between $E_{22(peak)}$ and $E_{2-2(peak)}$, i.e. $F \sim 1 - E_{2-2(peak)}/E_{22(peak)}$. For black holes with no spin it has been found that $E_{22(peak)} = a_2\eta^2 + a_4\eta^4$, where $a_2 = 0.0044$ and $a_4 = 0.0543$, gives a good fit to the numerical data. We do not expect spins orthogonal to the orbital angular momentum to change the scaling of the radiated energy significantly. If we further assume that the asymmetry factor F is independent of η, which finds some support in PN analysis since to leading order $\dot{P}_{\|}/\dot{E}$ is independent of η, then we are led to hypothesize that $v_{\|} \propto (a_2\eta^2 + a_4\eta^4)$.

In Table 2 we summarize the agreement of various kick formulas with the numerical data. For each formula, which has the form of Eq. (8), we found the best Φ_1 and Φ_2, per mass ratio, according to a least squares fit to the data given in Table 1. For each mass ratio, the resulting percent error is given for each model, maximized across initial angle. Referring to Eq. (8), the column headings $K\eta^2$, $K(a_2\eta^2 + a_4\eta^4)$ and $K\eta^3$ of Table 2 represent choices for $C(\eta)$ that were tested, where in each case K has been chosen so as to reproduce the value of the formula of Campanelli et al. (2007) in the equal-mass case. Note that $C(\eta) = K\eta^2$ gives exactly the formula of Campanelli et al. (2007). We see that the choice $C(\eta) = K(a_2\eta^2 + a_4\eta^4)$, for which we have strong-field heuristic justification, fits the data much more successfully than does $C(\eta) = K\eta^2$. However, a better empirical model was found to be $C(\eta) = K\eta^3$. For now we consider this our best fit, and leave open the interesting question of how to accurately relate this prefactor directly to E_{22}.

Our results affect the distribution of kick speeds given various assumptions about the spin parameters, spin orientations, and mass ratios involved in coalescences. This has particular application to the retention of the products of mergers of massive black holes in the current universe (e.g., Bonning et al. 2007) and electromagnetic signatures of kicks (e.g., Shields et al. 2007; Lippai et al. 2008), as well as coalescences in the early structure formation phase of redshift $z \sim 5 - 30$ (Merritt et al. 2004; Boylan-Kolchin et al. 2004; Haiman 2004; Madau & Quataert 2004; Yoo & Miralda-Escudé 2004; Volonteri & Perna 2005; Libeskind et al. 2006; Micic et al. 2006; Volonteri 2007), and for current-day mergers of intermediate-mass black holes (IMBHs), which might exist in dense stellar clusters (Taniguchi et al. 2000; Miller & Hamilton 2002b,a; Mouri & Taniguchi 2002b,a; Miller & Colbert 2004; Gültekin et al. 2004, 2006; O'Leary et al. 2006, 2007). Note that $q = 1$ to $q = 1/3$, is in the range of ratios expected for major mergers of galaxies, and as Sesana et al. (2004) show, this range is expected to account for most massive black hole mergers in the early $z > 10$ phase of black hole assembly.

Our new formula implies an important revision in our understanding of how easily IMBHs with $M \sim 10^2 - 10^3\ M_\odot$ are retained in globular clusters. A rich cluster has an escape speed $v_{\rm esc} \approx 50\,{\rm km\,s^{-1}}$ (Webbink 1985). Gültekin et al. (2006) showed that the Newtonian kicks involved in binary-single interactions are insufficient to reach this speed if the IMBH is at least $\sim 15 - 20$ times more massive than the objects with which it interacts. Using the Campanelli et al. (2007) formula, however, the maximum kick from gravitational radiation is $v_{\rm max} = 6 \times 10^4\,{\rm km\,s^{-1}}\eta^2$, implying that even IMBHs $30 - 35$ times more massive than the black holes with which they merge could get ejected. Holley-Bockelmann et al. (2007), focusing on cases in which stars lose little mass through their evolution and thus can leave behind stellar-mass black holes with masses $> 60 - 100\ M_\odot$, use this to argue that most IMBHs of even 1000 $M_\odot$ will be ejected from globulars. If instead stellar-mass black holes have masses $\sim 10\ M_\odot$, a mass of at least 400 $M_\odot$ would still be required to guarantee retention.

In contrast, our new formula suggests a maximum kick of $v_{\rm max} = 2.4 \times 10^5\,{\rm km\,s^{-1}}\eta^3$. Thus if $\eta < 0.06$, $v_{\rm max} < 50\,{\rm km\,s^{-1}}$. Therefore, an IMBH interacting with 10 $M_\odot$ black holes will stay in a rich globular if its initial mass is $M > 170\ M_\odot$, comparable to what is necessary for retention against Newtonian three-body kicks.

Our results also have implications for whether merged supermassive black holes stay in their host galaxies. The figure of merit is the fraction of kicks that exceed typical escape speeds from galactic centers (ranging from roughly 500 ${\rm km\,s^{-1}}$ for a small spiral to 2000 ${\rm km\,s^{-1}}$ for a giant elliptical), given assumptions about the distribution of spins and orbital orientations. The calculation of record for this purpose is that by Schnittman & Buonanno (2007), who used a kick formula based on effective one-body analysis and is different from that of Campanelli et al. (2007); this formula underestimates the highest kicks. Table 3 compares the fraction of kicks above 500 ${\rm km\,s^{-1}}$ and 1000 ${\rm km\,s^{-1}}$ using the Schnittman & Buonanno (2007) formula (an underestimate), the Campanelli et al. (2007) formula (an overestimate), and our results. It is clear that the Schnittman & Buonanno (2007) results were conservative: the fraction of large kicks is significantly higher than their estimate for comparable-mass mergers with plausible spins.

One consequence of the higher kicks is that retention of supermassive black holes after galactic major mergers

TABLE 3
FRACTION OF KICK SPEEDS ABOVE A GIVEN THRESHOLD, COMPARED WITH THE RESULTS OF SCHNITTMAN & BUONANNO (2007) (SB) AND CAMPANELLI ET AL. (2007) (CLZM). IN ALL CASES WE ASSUME AN ISOTROPIC DISTRIBUTION OF SPIN ORIENTATIONS.

Mass ratio and spin	Speed threshold	SB	CLZM	This work
$1/10 \le q \le 1$, $a_1 = a_2 = 0.9$	$v > 500\,\mathrm{km\,s^{-1}}$	$0.12(+0.06, -0.05)$	0.364 ± 0.0048	0.2283 ± 0.0014
	$v > 1000\,\mathrm{km\,s^{-1}}$	$0.027(+0.021, -0.014)$	0.127 ± 0.0034	0.085 ± 0.0008
$1/4 \le q \le 1$, $a_1 = a_2 = 0.9$	$v > 500\,\mathrm{km\,s^{-1}}$	$0.31(+0.13, -0.12)$	0.699 ± 0.0045	0.618 ± 0.0014
	$v > 1000\,\mathrm{km\,s^{-1}}$	$0.079(+0.062, -0.042)$	0.364 ± 0.0046	0.2547 ± 0.0013
$1/4 \le q \le 1$, $0 \le a_1, a_2 \le 1$	$v > 500\,\mathrm{km\,s^{-1}}$	$\cdots$	0.428 ± 0.0045	0.3484 ± 0.0015
	$v > 1000\,\mathrm{km\,s^{-1}}$	$\cdots$	0.142 ± 0.0034	0.0974 ± 0.0009

is even more challenging than previously thought, unless an astrophysical mechanism restricts the spin magnitudes (contrary to spin inferences from Fe Kα lines; see Iwasawa 1996; Fabian et al. 2002; Reynolds & Nowak 2003; Brenneman & Reynolds 2006) or the spins tend to align parallel to each other and to the orbital axis (Bogdanovic et al. 2007). Absent such a mechanism, one would expect tens of percent of merged galaxies to have no central black hole, in strong contradiction with observations (see Ferrarese & Ford 2005). Given that purely gravitational precession and radiation do not preferentially align spins in weak gravity (Schnittman 2004; Bogdanovic et al. 2007), nor are they expected to in strong gravity (A. Buonanno, private communication), alignment would have to come from external torques, e.g., by nuclear gas if there is a sufficient amount in the vicinity.

In conclusion, we have performed a systematic study of the mass ratio dependence of the out-of-plane kicks produced by the merger of spinning black holes. Our work shows that the Campanelli et al. (2007) candidate kick formula overestimates the out-of-plane kick systematically. However, we find that an additional factor of 4η agrees with our numerical results to within typical values of 1% for mass ratios between 1 and 1/3. This has considerable implications for black hole retention in early dark matter halos, galaxies, and globular clusters.

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