Statistical Issues for Uncontrolled Reentry Hazards

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A number of statistical tools have been developed over the years for assessing the risk of reentering objects to human populations. These tools make use of the characteristics (e.g., mass, shape, size) of debris that are predicted by aerothermal models to survive reentry. The statistical tools use this information to compute the probability that one or more of the surviving debris might hit a person on the ground and cause one or more casualties.

The statistical portion of the analysis relies on a number of assumptions about how the debris footprint and the human population are distributed in latitude and longitude, and how to use that information to arrive at realistic risk numbers. This inevitably involves assumptions that simplify the problem and make it tractable, but it is often difficult to test the accuracy and applicability of these assumptions.

This paper looks at a number of these theoretical assumptions, examining the mathematical basis for the hazard calculations, and outlining the conditions under which the simplifying assumptions hold. In addition, this paper will also outline some new tools for assessing ground hazard risk in useful ways.

Also, this study is able to make use of a database of known uncontrolled reentry locations measured by the United States Department of Defense. By using data from objects that were in orbit more than 30 days before reentry, sufficient time is allowed for the orbital parameters to be randomized in the way the models are designed to compute. The predicted ground footprint distributions of these objects are based on the theory that their orbits behave basically like simple Kepler orbits. However, there are a number of factors - including the effects of gravitational harmonics, the effects of the Earth's equatorial bulge on the atmosphere, and the rotation of the Earth and atmosphere - that could cause them to diverge from simple Kepler orbit behavior and change the ground footprints. The measured latitude and longitude distributions of these objects provide data that can be directly compared with the predicted distributions, providing a fundamental empirical test of the model assumptions.

INTRODUCTION

One of the hazards of the space age is that objects in orbit eventually reenter the Earth's atmosphere. Often for large objects, some components tend to survive reentry and pose a hazard to persons on the ground. A whole science has developed around predicting what portions of a satellite survives the violent forces and heating of reentry. Such

aerothermal models as ORSAT and SCARAB[1] use detailed models of the shapes and material types of various components to predict what will and will not survive to reach the ground. From this information, a reentry "footprint" can be computed that can be used to determine risk on the ground.

For controlled reentries, the reentry target zone can be chosen to avoid populated areas on the Earth. For uncontrolled reentries, however, statistical tools must be used to map human distributions on the Earth under the spacecraft orbit. The exact time and location for uncontrolled reentries are notoriously difficult to predict with any accuracy. In the hours immediately before a reentry, it may be possible to narrow the possible groundtracks of the reentering object. But when the risk calculation is needed weeks or months before reentry, essentially any location on Earth over which the spacecraft flies is a potential reentry landing site. Nevertheless, it is possible to use even this vague information to compute meaningful risk statistics.

BASIC STATISTICAL TOOLS

Consider a piece of a reentering satellite that falls into a geographical region with area A that contains a number of individual human beings N. If the area of the surviving piece α is smaller than the person-area of a human being, then the risk is driven by the number of people in the region, not the size of the surviving debris piece. If, however, the surviving piece is large, then the risk increases with increasing size.

Assuming that the people in the geographic region are distributed randomly and that the piece falls at a random location within the region, then the probability becomes a simple binomial distribution – what is the probability that a given number of people will be under the piece when it falls versus those who will be elsewhere in the geographic region.

Typically, reentering debris objects are not especially large relative to the area of a geographical region. However, in the case of a reentering object containing a hazardous substance, one must consider the area over which the substance might be expected to spread and present a hazard to human beings. An "effective area" can be defined that represents the size of the hazardous zone, as if the debris were a giant pancake with that area. In such cases, the potential size of such a zone could get quite large. To simplify calculations, human beings in such cases can be approximated as points.

Figure 1 shows different ways of computing risk for a hypothetical risk problem where the effective area of the hazard zone is large compared to the size of human beings. The graph is plotted as a function of the ratio of the hazardous area α to the total geographical area of interest A. The number of humans N is set at 1000.

Historically, risk has been computed as the population density of human beings multiplied by the effective area of the hazard. This corresponds to N times the fraction α/A. However, this actually corresponds to the expected number (average) of people that might be within the hazard region. As can be seen in Figure 1, if the hazard area gets

large enough, this value can exceed "1". As such it is no longer a good probability measure. Instead, what is really needed is the probability that there will be one or more casualties.

Figure 1 shows several curves that represent the probability that exactly 1, 2, or 3 people will be within the hazardous zone. As the hazard area increases, each of these probabilities peaks, then begins to decline as more and more people are likely to be in the hazardous zone at the time of fall. Two composite curves are shown that give the probability of one or more casualties, and of ten or more casualties. Note that the probability of exactly one casualty peaks when the ratio $\alpha/A \sim 1/N$. As with all statistical questions, it is important to pose the right question before you compute the answer.

A simple approximation to the probability of one or more casualties can be computed using the Poisson relation

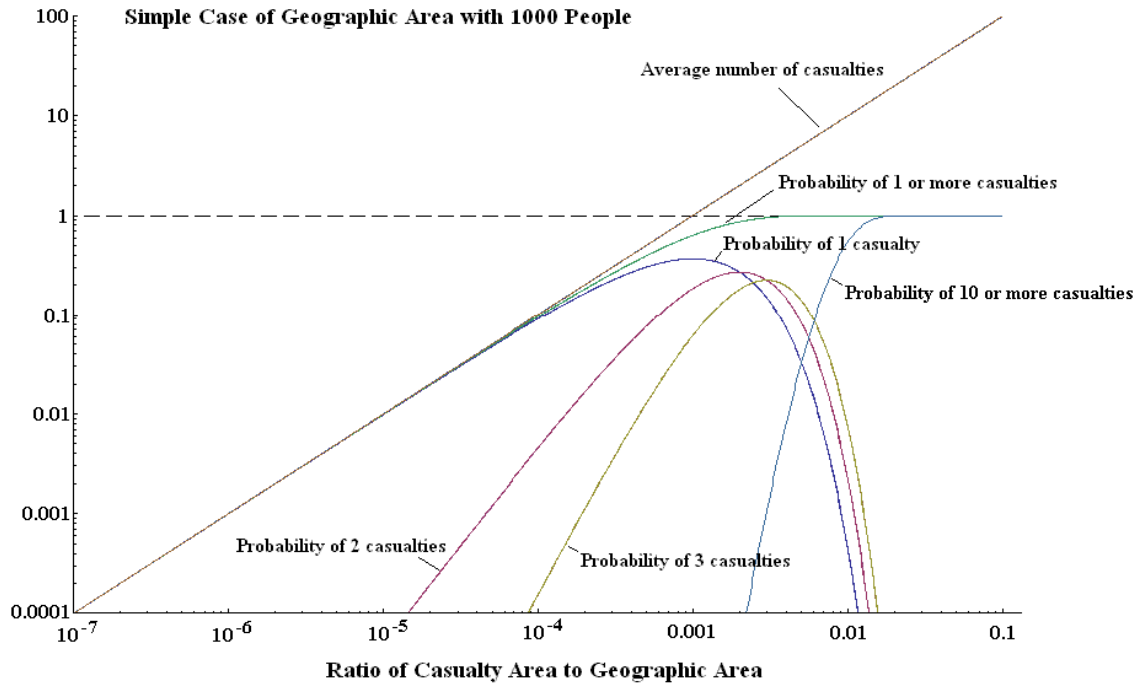$$(1) \qquad p_c = 1 - Exp\left(-N\,\frac{\alpha}{A}\right)$$

Figure 1 – The curves shown here represent the hypothetical reentry risks for a geographic region with a population N of 1000 people as a function of the ratio of the effective casualty area α (either the physical are of a reentering object or the "effective area" of a region of hazardous materials from the reentry) to the total area of the geographic region A (α is assumed to be much larger than the area of a typical human). As explained in the text, plotted is the expected (average) number of casualties, as well as the probability of exactly one, two, or three casualties. Also plotted is the probability of any casualties at all (the probability of one or more casualties) and the probability of ten or more casualties. Note that when the casualty area ratio is small, the expected number of casualties, the probability of a single casualty, and the probability of one or more casualties all converge.

RISK CALCULATIONS FOR ORBITS

Of course, orbiting objects do not orbit an Earth with homogeneously distributed populations. Approximately three quarters of the Earth's surface is covered by oceans, and the population on land is unevenly distributed.

For computing ground risks, NASA's Orbital Debris Program Office uses the databases from the Socioeconomic Data and Applications Center (SEDAC) at Columbia University[2]. In this paper, the data set used is the Gridded Population of the World, version 3 (GPWv3). This data set estimates the population in latitude/longitude grid positions on the Earth divided into 2.5×2.5 arc minute cells for reference years 1990-2015 in 5-year intervals. These cells are approximately 4.6 km long in the north-south direction. At the equator, the cells are also approximately 4.6 km wide in the east-west direction, but are narrower at more northerly and southerly latitudes.

As mentioned above, predicting reentry footprints weeks or months in advance is not possible with current computer models. Instead, planners are left predicting the likelihood of a reentering object ending up in a particular latitude/longitude bin. The rotation of the Earth and the precession of the ascending node of the orbit should result in a random distribution in longitude (see below for more on this assumption).

The important distribution should be that in latitude, which is a function of the orbit inclination. The fraction of time an ideal Kepler orbit spends over a latitude band between latitudes δ_1 and δ_2 is given by the equation

(2a) $\quad f(i,\delta_1,\delta_2) = g(i,\delta_2) - g(i,\delta_1)$

where

(2b) $\quad g(i,\delta) = +\dfrac{1}{2}$ $\qquad\qquad$ if $\ Sin(i) < Sin(\delta)$

(2c) $\quad g(i,\delta) = \dfrac{1}{\pi} ArcSin\left(\dfrac{Sin(\delta)}{Sin(i)}\right)$ $\quad$ if $\ -Sin(i) \leq Sin(\delta) \leq Sin(i)$

(2d) $\quad g(i,\delta) = -\dfrac{1}{2}$ $\qquad\qquad$ if $\ Sin(\delta) < -Sin(i)$

and i is the inclination and δ is the latitude. This distribution results in the orbit spending more time near the northernmost and southernmost latitudes, and less time near the equator. In addition, far northerly and southerly latitudes do not see risk from reentries except for orbits with near-polar inclinations.

Using these calculations and the gridded population of the world, the NASA Orbital Debris Program Office has computed average population density under orbits as a

function of year and inclination[3]. An example chart with two reference dates (showing the projected future growth of the Earth's population) is shown in Figure 2. This particular dataset is based on an earlier version of the global population database than the one described above. Note that sheltering by buildings or other structures is ignored for these calculations.

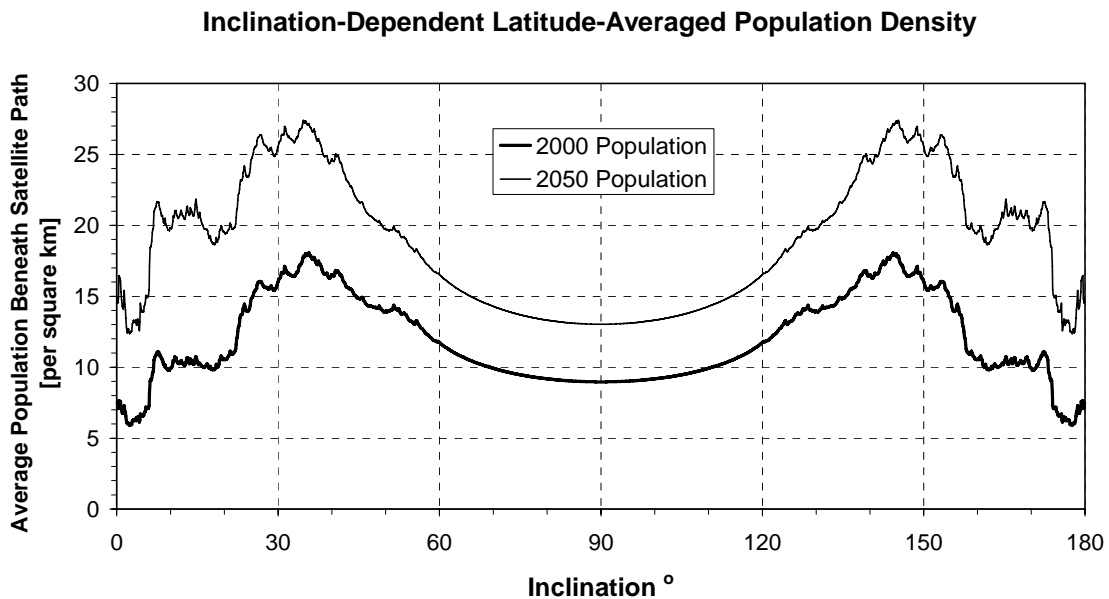**Inclination-Dependent Latitude-Averaged Population Density**

Figure 2 – This chart shows the average population beneath an orbit as a function of orbit inclination for two reference years (based on global population databases and future population predictions). The population is weighted by the time each orbit spends over various latitudes assuming the ideal Kepler orbit distributions outlined in the text. Note that sheltering by buildings is ignored in these calculations.

EMPIRICAL VALIDATION

The problem with this calculation is that it assumes that the orbiting objects are behaving like perfect Kepler orbits. There are several reasons to believe these assumptions might not be valid (for a description of the orbital effects described here, refer to a technical work on orbital mechanics such as Vallado's Fundamentals of Astrodynamics and Applications[4]).

First, satellites do not orbit a perfectly spherical Earth, but an oblate spheroid. The latitudinal variation in mass that is concentrated near the equator is reflected in the J_2 spherical harmonic term of the gravitational field. The effect of this term is to change the speed and azimuth of the orbit slightly as it passes over the equator relative to an ideal Kepler orbit.

Second, in addition to the radius of the Earth being somewhat greater at the equator, the atmosphere extends farther out into space near the equator relative to the center of the Earth. This has the potential effect of increasing the drag on the reentering object as it crosses the equator, perhaps resulting in a bias in the downrange reentry location as a function of latitude.

Third, the atmosphere rotates with the Earth, and the drag is a function of the relative velocity between the orbiting satellite and the local velocity of the atmosphere. Consequently, as the satellite crosses the equator, the relative motion of the satellite motion with respect to the atmosphere will be different than when the satellite is at the northernmost or southernmost regions of its orbit.

Any of these effects could potentially alter the ideal Kepler latitude distribution outlined in equation 2. To answer whether this distribution is applicable, we can use a statistical database of reentry locations and compare the longitudinal distribution to that predicted by theory.

The database used is a list of 47 satellites (rocket bodies and spacecraft) that reentered between 2003 and 2007. Each one was in a near-circular orbit, had been in orbit at least 30 days (to make sure the orbit nodes had time to be thoroughly "randomized"), and had an accurate latitude/longitude determination by the US Department of Defense. It should be possible to use statistical tests on the distribution of these data to see if they are consistent with theory.

The easiest parameter to test is the distribution in longitude, because it should be randomly distributed around the Earth. Because longitude "wraps around" the Earth, Kuiper's statistic is appropriate to use[5]. This statistic, similar to the more familiar Kolmogorov-Smirnov test, compares a cumulative data distribution to the corresponding cumulative theoretical distribution, and uses the maximum differences between the two as the statistic. The Kuiper's statistic is the sum of the maximum distance above and below the theoretical curve. Figure 3 shows the longitude comparison. The data are well within the 1-sigma limits for a random sample of 47 points from a uniform distribution. Therefore, to the limits of finite sampling, these reentries are completely consistent with random longitude of reentry, as expected.
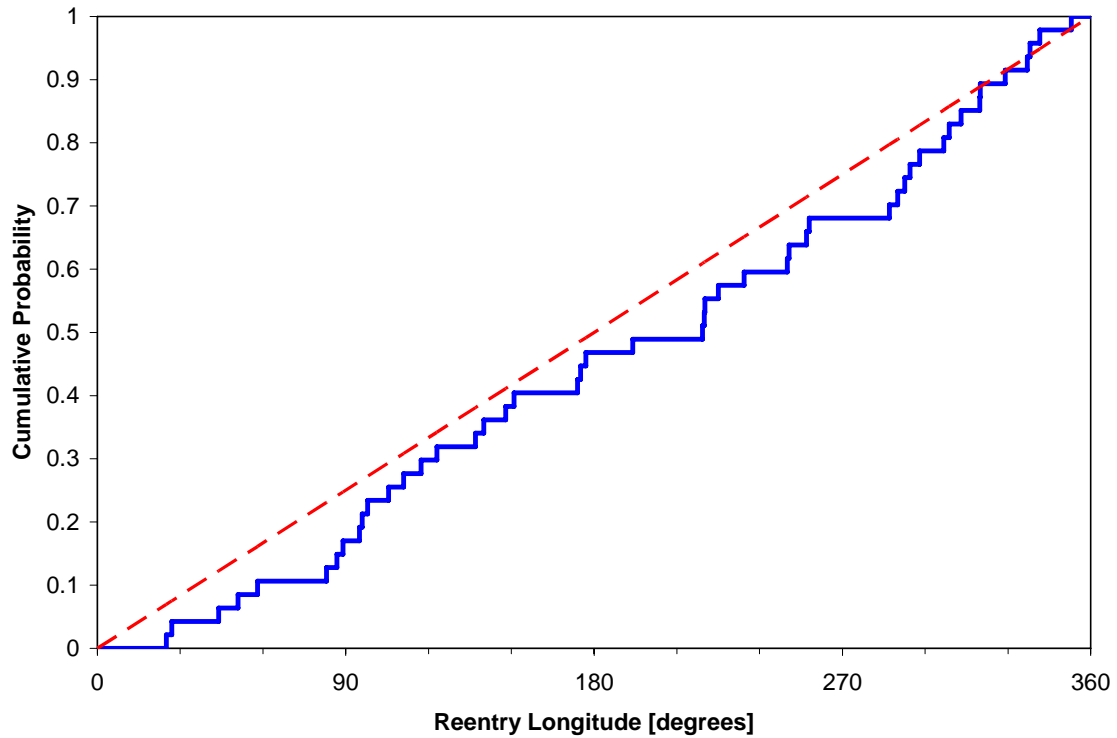
Figure 3 – The cumulative distribution in longitude of 47 random object reentries (solid curve) is compared to the theoretical uniform distribution (dashed line). The difference between these measured by the Kuiper's statistic is consistent with random data drawn from the uniform distribution.

Statistics on the latitude distribution are more problematic, because the reentering objects did not all have the same inclination. However, an alternative measure can be computed for each object that represents the integrated latitude distribution (using equation 2) between the southernmost point of the orbit (equal to the negative inclination) and the reentry latitude. If a random set of reentries are following the theoretical latitude distribution, then this integral value ought to be distributed uniformly. So the Kuiper's statistic can be computed by comparing the cumulative distribution of this alternative measure against a uniform distribution. Figure 4 shows this comparison. Although this data set shows some slight asymmetries, it is still within the 1-sigma range limits for a random sample of 47 points from a uniform distribution. Therefore to the limits of this finite reentry database, there appears to be no reason to reject the theoretical Kepler latitude/longitude distribution for uncontrolled reentries.
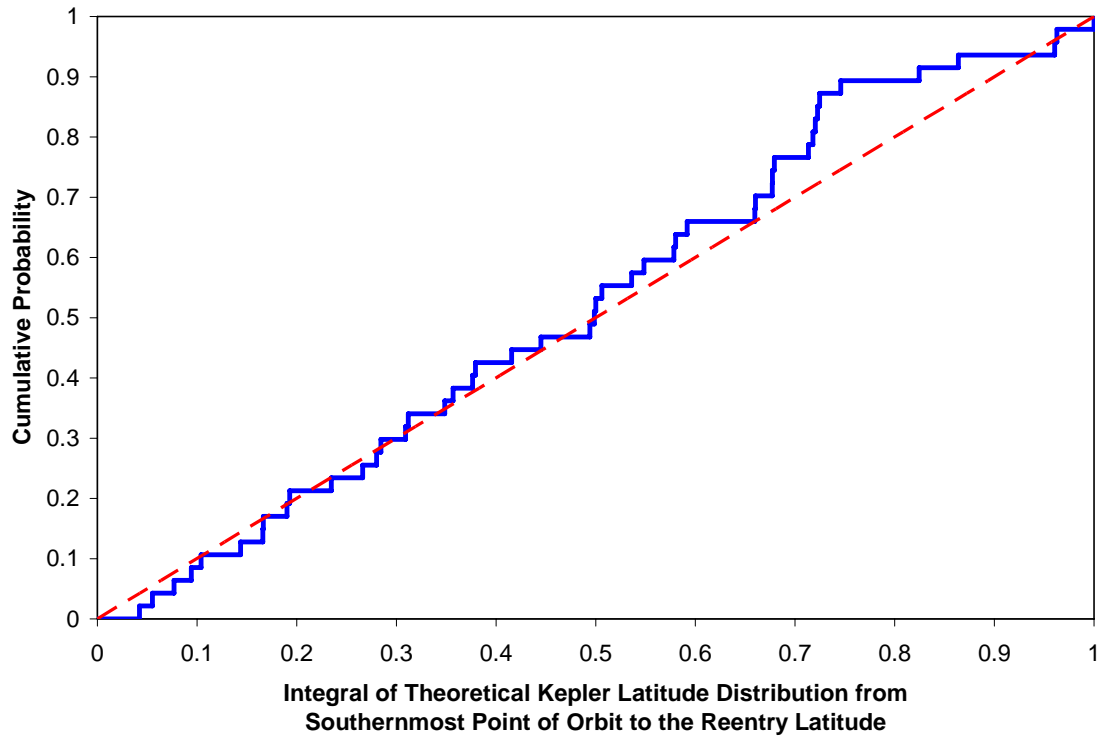
Figure 4 - The cumulative latitude distribution of 47 random object reentries (solid curve) is compared to the theoretical uniform distribution (dashed line).   The statistic shown here is the integral of the theoretical Kepler latitude distribution from the objects southernmost point (equal to the negative inclination) to the point of reentry.   The difference between these measured by the Kuiper's statistic is consistent with random data drawn from the uniform distribution. Therefore, the measured data is consistent with the theoretical inclination-dependent latitude distribution described in the text.

ALTERNATE RISK CALCULATION

Another way to discuss risk, especially with respect to reentries that might spread toxic substances, is to talk in terms of the probability of landing in a region with a particular population density or higher. Using the GPWv3 dataset and the latitude distributions described above, it is possible to compute this probability as a function of inclination. For simplification, the population within a GPWv3 2.5×2.5 arc minute cell is assumed to be randomly distributed within that cell. This should give a good approximation to distribution of concentrated urban areas beneath a particular orbit. Figure 5 shows this probability for typical satellite orbits. Also included are the population densities of several reference locations for comparison. Note that there is typically only a ~25% probability of falling on a region with any population at all due to the fact that approximately three quarters of the Earth's surface is covered in water.

For most orbits, there is a non-negligible chance that the reentering object will fall in an urban area where the spread of toxic material would be most dangerous.
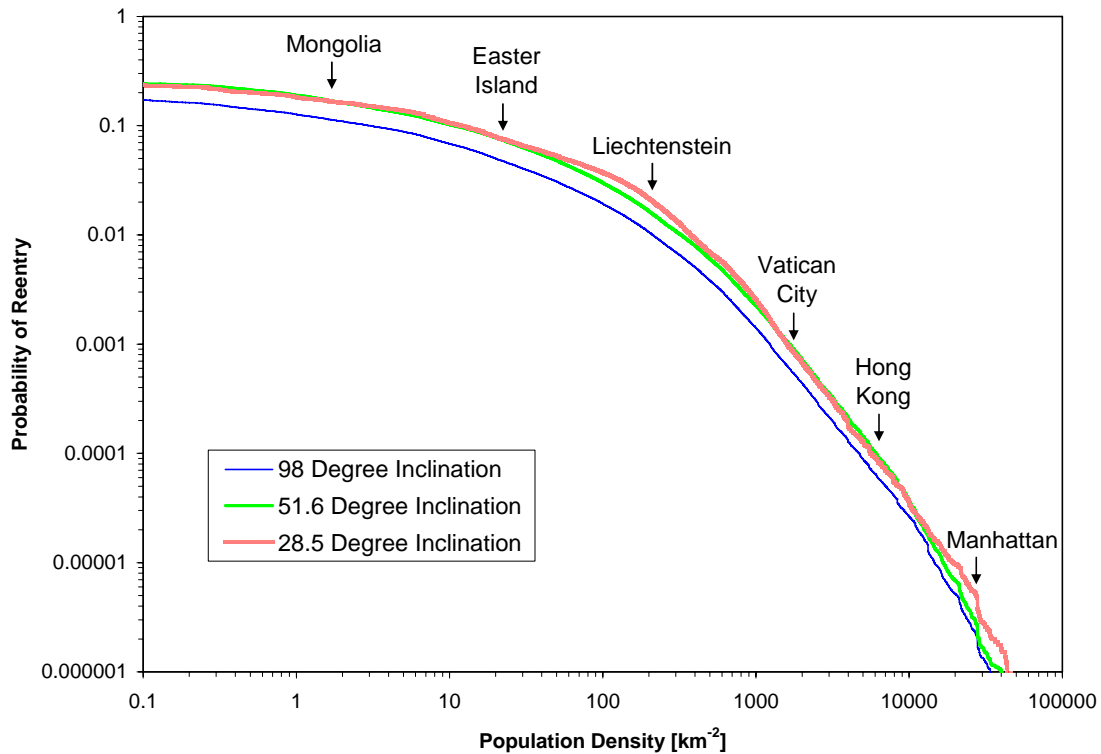
Figure 5 – This plot shows the probability that a reentering object will fall in a geographic region with a given population density or higher. The three curves are for three typical orbit inclinations. Note that the 98° inclination orbit probabilities are generally lower because such orbits spend a significant fraction of their time over sparsely populated polar regions. Also shown are reference population densities for a variety of geographic locations on the Earth.

CONCLUSIONS

In this paper I examined several of the assumptions used in computed ground risk from reentering objects. In extreme cases where the effective area of a reentering object is large (for instance due to the presence of toxic materials), it is important to compute the probability that there are one or more casualties, rather than relying on the expected casualty number. Also shown is an alternative method for assessing risk by computing the probability that a reentering object will fall in a densely populated region.

In this paper I also used measured reentry data to test the applicability of the ideal Kepler orbit approximation to the computation of ground risk. It was shown that the Kepler assumption is statistically consistent with the data so far available. It would be prudent to test this assumption in the future as more data becomes available.

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5. Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P., *Numerical Recipes in Fortran 77, The Art of Scientific Computing, Second Edition*, Cambridge University Press, 1992.