Gravitational radiation characteristics of nonspinning black-hole binaries

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Abstract. We present a detailed descriptive analysis of the gravitational radiation from binary mergers of non-spinning black holes, based on numerical relativity simulations of systems varying from equal-mass to a 6:1 mass ratio. Our analysis covers amplitude and phase characteristics of the radiation, suggesting a unified picture of the waveforms' dominant features in terms of an implicit rotating source, applying uniformly to the full wavetrain, from inspiral through ringdown. We construct a model of the late-stage frequency evolution that fits the $\ell = m$ modes, and identify late-time relationships between waveform frequency and amplitude. These relationships allow us to construct a predictive model for the late-time waveforms, an alternative to the common practice of modelling by a sum of quasinormal mode overtones. We demonstrate an application of this in a new effective-one-body-based analytic waveform model.

1. Outline

Since the discovery of the moving puncture recipe, numerical research into the merger of black-hole binaries (BHBs) has exploded [1, 2, 3, 4]. Moving past the canonical case of the equal-mass, nonspinning binary, assuming zero eccentricity, and taking advantage of the overall mass-scaling nature of the problem, BHBs occupy a seven-parameter space (mass ratio, spins of each hole). With LIGO being retooled as Enhanced LIGO, and LISA anticipated to fly in the next decade, it is imperative that we explore this parameter space as quickly and efficiently as possible, to supply the gravitational-wave detector community with the information they need for data analysis and detector tuning.

In this paper, we outline recent work investigating the one-dimensional parameter space of nonspinning binaries of varying mass ratio, with a focus on the relationships between harmonic modes and between mass ratios, somewhat complementary to work by [5]. The material in this brief article is a summary of numerical experiments and results more fully described in [6].

2. Numerical Methods

We evolved BHBs with mass ratios of 1:1, 2:1, 4:1 and 6:1 using the Goddard `Hahndol` code [7], which solves Einstein's equations on a Cartesian spatial grid, with adaptive mesh refinement supplied by the `Paramesh` infrastructure [8]. Initial data is of the "puncture" type [9], with momenta determined from a 2PN-accurate circular-orbit approximation [10], with constraints solved using the `AMRMG` multigrid elliptic solver [11]. Our evolution uses a moified form of the

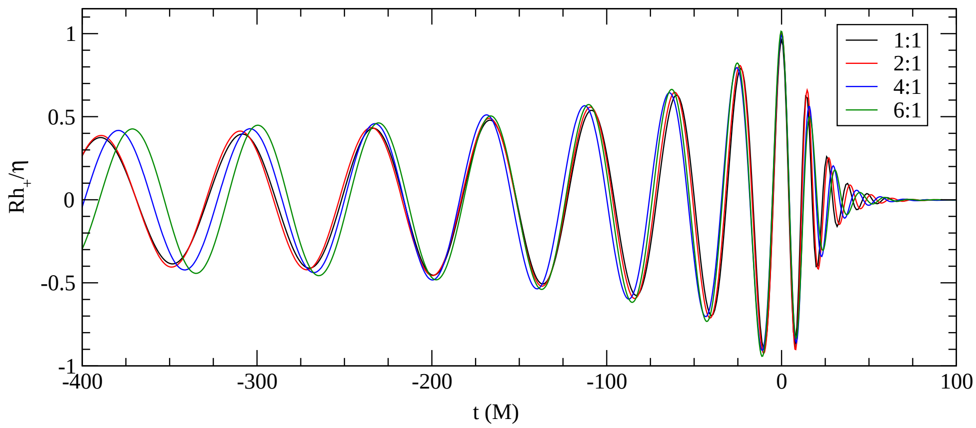

Figure 1. Gravitational-wave signals along polar axis from mass ratios 1:1, 2:1, 4:1 and 6:1. [Reprinted from [6].]

BSSN equations [12, 13], with 1+log slicing and Gamma-driver shift conditions appropriate for the "moving puncture" recipe [14].

3. Numerical Relativity Waveforms for Unequal-Mass BHBs

The first thing we note about the waveforms of the different mass ratios is that they show remarkable consistency, if viewed along the polar axis. Fig. 3 shows the "plus" polarisation of the waveform strain, h_+, as measured along the orbital axis, with amplitudes scaled by the *symmetric mass ratio* $\eta \equiv m_1 m_1/M^2$. The waveform shape is remarkably coherent across mass ratios from $200M$ before merger to the merger itself (taken here as the time of peak radiation amplitude). After the merger, the waveform is dominated by the fundamental quasinormal mode (QNM) ringdown frequency of the end-state hole, which depends strongly on the spin of the hole.

It is not surprising, perhaps, that the waveforms look so similar when they are all dominated by the quadrupole $(\ell, m) = (2, \pm 2)$ mode, which is insensitive to mass asymmetries. Viewed far off-axis, where the (2,2) mode no longer dominates, the signals can look very different. However we retain much simplicity if we decompose the signal into its harmonic modes:

$$h \equiv \sum_{\ell m} h_{\ell m} \; {}_{-2}Y^{\ell,m}(\theta, \varphi). \tag{1}$$

On performing this decomposition, we note that, after an initial pulse of high-frequency junk radiation, the strongest modes are circularly polarized. For instance, we can write the *strain-rate* – the first time-derivative of the waveform strain – in the form $\dot{h}_{\ell m} = A_{\ell m} e^{im\phi}$.

4. The Implicit Rotating Source Picture

Looking at gravitational-wave modes across harmonics reveals several general trends. For example, looking at the power emitted, mode by mode, we find that it is partitioned by modes according to leading-post-Newtonian (PN) order. The left panel of Fig. 2 shows the power for several leading modes for the 4:1 merger. The solid lines are the raw numerical data, while the dashed lines are the leading-order post-Newtonian predictions for the emitted power, as found in [15], calculated using the orbital frequency extracted from the (2,2) mode.

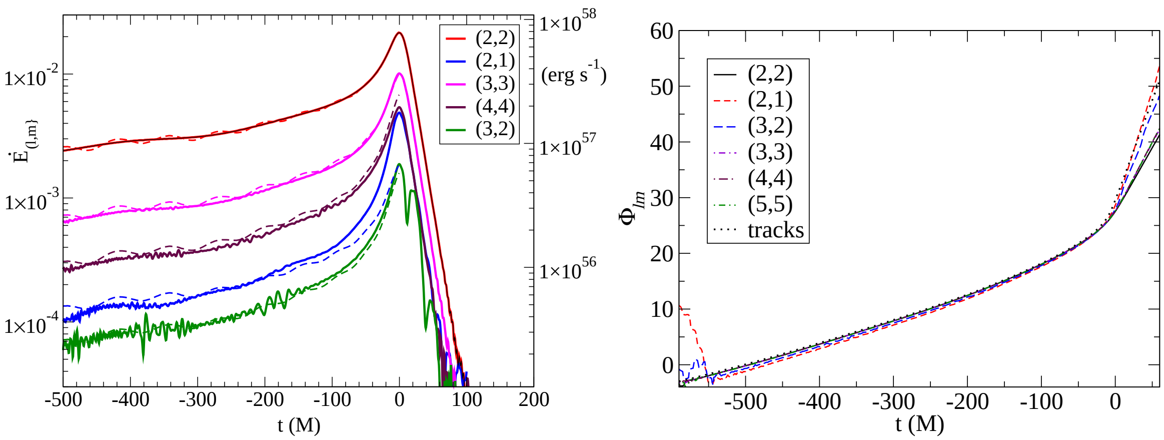

Figure 2. Left panel: Power in each mode for 4:1 mass ratio. Right panel: Rotational phase of each mode for 4:1 mass ratio. [Reprinted from [6].]

Similarly, we find that the phases of different modes are simply related through merger. While each mode accumulates phase at a different rate, we can define a "rotational phase" for the system:

$$\Phi_{\ell m} = \phi_{\ell m}/m + \text{ const.} \tag{2}$$

The right panel of Fig. 2 shows this rotational phase calculated for all the dominant modes of the 4:1 run. After initial noise (due to an initial burst of junk radiation, which is not circularly polarised), the different phase estimates agree very well, up to a common "elbow", where the binary merges. The result is a picture of BHB as a slowly changing rigid rotator – an "implicit rotating source"– generating gravitational waves all through inspiral, merger & ringdown such that each mode is tied to the corresponding multipole moment of the radiating source [16].

5. Full Late-Merger Frequency Model

One common feature of the different modes is that each (ℓ, m) mode's frequency ω increases monotonically until it plateaus after the peak amplitude, associated with the merger. The characteristic shape of the frequency-time curve is very similar for the strongest modes. It can be well modelled through merger by a smooth step-like function:

$$\Omega(t) = \Omega_i + (\Omega_f - \Omega_i)\left(\frac{1 + \tanh[\ln\sqrt{\kappa} + (t - t_0)/b]}{2}\right)^{\kappa}, \tag{3}$$

where t_0 is the time at which the rotational frequency growth rate (chirp rate) is maximum: $\dot{\Omega}_0$.

The left panel of Fig. 3 shows the leading modes in the 4:1 mass ratio case, from $60M$ before merger to $40M$ after merger, with the frequency fit (3) overlaid. This fitting works well over all mass ratios examined, and appears to be valid for more extreme mass ratios as well. The right panel of Fig. 3 shows the rotational frequency Ω_{22} as derived from the dominant (2,2) modes for the 1:1, 2:1, 4:1 & 6:1 numerical evolutions, as well as for the extreme 100:1 mass ratio[1]. The dashed horizontal line indicates the QNM ringdown frequency for an end-state Schwarzschild black hole. As might be expected, as the mass ratio increases, the final hole's spin decreases toward the Schwarzschild limit.

[1] 100:1 data supplied by T. Damour and A. Nagar

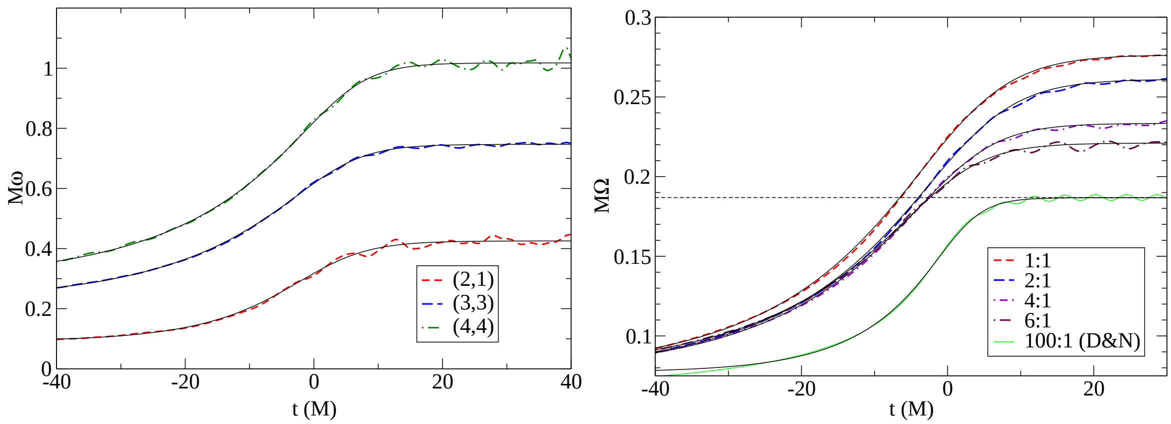

Figure 3. Left panel: fitting $\Omega_{\ell m}$ for leading modes in 4:1 mass ratio to functional form (3). Right panel: fitting Ω_{22} for all mass ratios. [Reprinted from [6].]

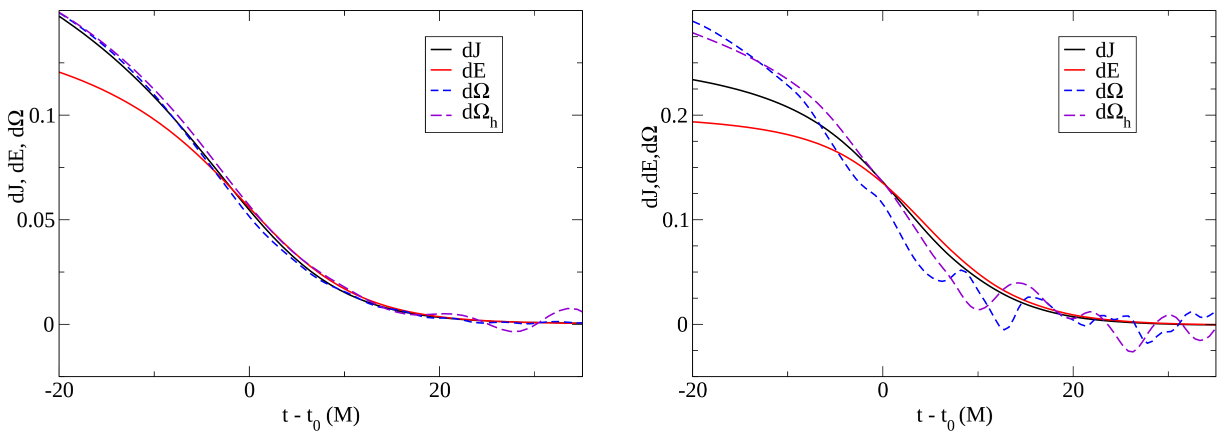

Figure 4. Decay of $E_{\ell m}$, $J_{\ell m}$, & $\Omega_{\ell m}$ for 1:1 (2,2) mode (left), 4:1 (2,1) mode (right). Both time axes are shifted relative to the peak chirp time t_0. [Reprinted from [6].]

6. Late-Merger $E(\Omega)$ & $J(\Omega)$ Relations

A consequence of the IRS picture is that both the energy and angular momentum flux assume simple forms near merger:

$$\frac{dE_{\ell m}}{dt} \propto \Omega \frac{d\Omega}{dt}, \quad \frac{dJ_{\ell m}}{dt} \propto \frac{d\Omega}{dt}. \tag{4}$$

This can be seen in Fig. 4, which shows the decay of the modal contributions to radiated E and J^z towards their final values for the equal-mass (left panel) and 4:1 (right panel) cases, as well as the decay of two measures of the rotational frequency, Ω from the strain-rate, and Ω_h from the strain (these should differ only at 2.5 PN order).

As a consistency check of this picture, if we assume that $dJ/d\Omega \sim$ constant near merger, we can use the frequency fit (3) to derive the approximate delay between peak chirp (rate of increase of Ω) and peak radiation amplitude:

$$t_{peak} - t_0 \approx -\frac{\dot{\Omega}_0^2}{\Omega_0 \dddot{\Omega}_0} > 0. \tag{5}$$

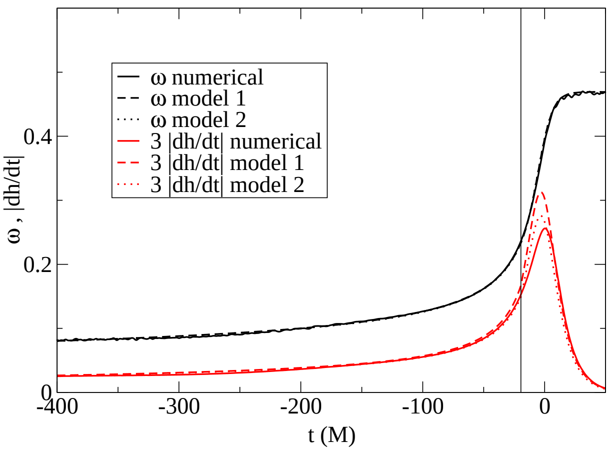

Figure 5. Frequency and strain-rate amplitude for the IRS modelled (2,2) waveform (dotted and dashed lines) compared with the numerical waveform (solid lines), for the 4:1 merger. $t = 0$ is the time of peak amplitude. [Reprinted from [6].]

Inserting values of Ω_0 and its derivatives from the frequency fit yield values of this difference of between $\sim 2 - 3M$, in good agreement with the fitted values for t_0 itself.

7. Relevance to LISA: New BHB Waveform Templates

New insights into gravitational waveforms from mergers can lead to better templates for data analysis. Although pure numerical waveforms are rarely long enough to cover the full visible lifetime of a LISA observation, they can be combined with post-Newtonian pre-merger waveforms for a hybrid template.

The simplest way to achieve this is to "stitch" the two segments together in such a way as to maintain an everywhere smooth waveform. The point or interval over which the waveforms can be stitched should be one where *both* post-Newtonian and numerical results can be trusted. An early example of this for the equal-mass case was supplied by Baker et al. [17].

The drawback with this direct approach is that each point in parameter space (mass ratio here) requires its own numerical waveform of limited duration to be attached to the post-Newtonian pre-merger waveform. As it is impractical to carry out a full evolution for every mass ratio of interest, we must develop instead a better analytic insight.

Several semi-analytical methods have been proposed involving the combination of effective-one-body (EOB) theory [18] and multiple quasinormal ringdown modes (QNMs) [19, 15], with flexibility parameters tuned using numerical merger data. The current work, with the empirical functional frequency (3), has allowed an alternative to QNMs, with a late-time strain-rate amplitude given in terms of the rotational frequency:

$$A_{\ell m} \approx \sqrt{16\pi\xi_{\ell m}\Omega_{\ell m}\dot{\Omega}_{\ell m}}, \tag{6}$$

where the constant ξ can be fixed by demanding continuity of amplitude at a matching frequency. Fig. 7 shows the modelled frequency and strain-rate amplitude (dotted lines) compared with the numerical results (solid lines) for the 4:1 evolution. We see that this new model encodes both frequency and amplitude well.

8. Summary

At the time of writing, fully numerical evolution of black-hole-binary spacetimes is possible to high accuracy. Nevertheless, LISA data analysis demands waveforms of much greater duration than is numerically feasible; additionally an understanding is required of waveforms from systems over a seven-parameter configuration space (more is eccentricity is considered crucial). To fill this gap in computational practicality, we are drawn to the development of analytical models of the gravitational-wave signals.

In this paper, we have introduced a set of numerical evolutions probing one dimension of parameter space – mass ratio. We have noted common characteristics of the dominant waveform modes, and suggested a useful heuristic picture of the radiation as being produced by an "implicit rotating source" through inspiral, merger, and ringdown. We have shown how this picture can be used to develop an alternative model to a sum of quasinormal modes for the transition through merger and ringdown. This model can then be combined with effective-one-body theory to produce analytic waveform templates that seem to capture well the important characteristics of the merger. A fuller discussion of these issues can be found in [6].

Work is underway to assess the utility of these templates in data analysis, and to extending our analysis to other parts of parameter space, involving spinning black holes.

Acknowledgments

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