Draft date: February 3, 2008

Dynamical Modeling and Control Simulation of a Large Flexible Launch Vehicle

Wei Du[1] and Bong Wie[2]
Iowa State University, Ames, IA 50011-2271

Mark Whorton[3]
NASA Marshall Space Flight Center, Huntsville, AL 35812

This paper presents dynamical models of a large flexible launch vehicle. A complete set of coupled dynamical models of propulsion, aerodynamics, guidance and control, structural dynamics, fuel sloshing, and thrust vector control dynamics are described. Such dynamical models are used to validate NASA's SAVANT Simulink-based program which is being used for the preliminary flight control systems analysis and design of NASA's Ares-1 Crew Launch Vehicle. SAVANT simulation results for validating the performance and stability of an ascent phase autopilot system of Ares-1 are also presented.

Nomenclature

R_E = Earth's equatorial radius = 20925646.325459 ft

R_P = Earth's polar radius = 20855486.595144 ft

J_2 = Earth's second order zonal coefficient = 1.082631×10^{-3}

J_3 = Earth's third order zonal coefficient= -2.55×10^{-6}

J_4 = Earth's fourth order zonal coefficient = -1.61×10^{-6}

U = Earth's gravitational potential

μ = Earth's gravitational parameter = $1.407644176\times10^{16}\ ft^3/s^2$

ϕ = Earth's geocentric latitude

$\left(g_x, g_y, g_z\right)$ = (x,y,z) components of the gravitational acceleration

$\vec{r}$ = vehicle's position vector

r = magnitude of vehicle's position vector

(x, y, z) = (x,y,z) components of vehicle's position vector in an inertial reference frame

$\left(a_x, a_y, a_z\right)$ = (x,y,z) components of vehicle's absolute acceleration in an inertial reference frame

$\vec{V}$ = vehicle's absolute velocity vector

[1] Ph.D. Student, Space Systems and Control Laboratory, Department of Aerospace Engineering, weid@iastate.edu, Student Member AIAA.
[2] Vance Coffman Endowed Chair Professor, Space Systems and Control Laboratory, Department of Aerospace Engineering, 2271 Howe Hall, Room 2355, (515) 294-3124, bongwie@iastate.edu, Associate Fellow AIAA.
[3] Chief, Guidance, Navigation, and Mission Analysis Branch, mark.whorton@nasa.gov, Associate Fellow AIAA.

(u, v, w) = (x,y,z) components of vehicle's absolute velocity vector in an inertial reference frame

$\vec{V}_{rel}$ = relative velocity vector of the vehicle (measured in a body-fixed reference frame)

$\vec{\Omega}_e$ = angular velocity vector of the Earth
ω_e = z component of the Earth's angular velocity vector = $7.2921152087 \times 10^{-5}$ rad/s
$\vec{\omega}$ = angular velocity vector of the vehicle
(p,q,r) = (x,y,z) components of vehicle's angular velocity vector in a body-fixed reference frame

$\vec{V}_w$ = velocity of the wind
$\vec{V}_m$ = air-stream velocity vector
V_m = magnitude of air-stream velocity
$(V_{m.xb}, V_{m.yb}, V_{m.zb})$ = (x,y,z) components of air-stream velocity in an inertial reference frame
M = mach number
c = speed of sound
Q = dynamic pressure
ρ = density of the air
α = angle of attack
β = angle of sideslip
F_{base} = base force
ARP = aerodynamics reference point = 275.6 ft from pin point
D = drag (axial) force
C = lateral (side) force
N = lift (normal) force
S = reference area = $116.2\ ft^2$
b_{ref} = reference length = 12.16 ft
c_{ref} = reference chord = 12.16 ft
ζ = damping ratio of the actuator dynamics = 1
ω_n = undamped natural frequency of the actuator dynamics = 32.6726 rad/sec
δ_p = pitch gimbal angle
δ_{pc} = pitch gimbal angle command
$\ddot{\delta}_p$ = pitch gimbal angle acceleration
δ_y = yaw gimbal angle
δ_{yc} = yaw gimbal angle command
$\ddot{\delta}_y$ = yaw gimbal angle acceleration of the actuator dynamics
T = total thrust inside the atmosphere
T_0 = total vacuum thrust
A_e = nozzle exit area = $122.137\ ft^2$
p_0 = local atmospheric pressure

$(F_{aero.xb}, F_{aero.yb}, F_{aero.zb})$ = (x,y,z) components of aerodynamic force in Body Frame
$(F_{rkt.xb}, F_{rkt.yb}, F_{rkt.zb})$ = (x,y,z) components of rocket engine force in Body Frame
$(F_{rcs.xb}, F_{rcs.yb}, F_{rcs.zb})$ = (x,y,z) components of reaction control force in Body Frame
$(F_{slosh.xb}, F_{slosh.yb}, F_{slosh.zb})$ = (x,y,z) components of slosh force in Body Frame
$(F_{total.xb}, F_{total.yb}, F_{total.zb})$ = (x,y,z) components of total force in Body Frame
$(F_{total.xi}, F_{total.yi}, F_{total.zi})$ = (x,y,z) components of total force in Inertial Frame
$(T_{aero.xb}, T_{aero.yb}, T_{aero.zb})$ = (x,y,z) components of aerodynamic torque in Body Frame
$(T_{rkt.xb}, T_{rkt.yb}, T_{rkt.zb})$ = (x,y,z) components of rocket engine torque in Body Frame
$(T_{rcs.xb}, T_{rcs.yb}, T_{rcs.zb})$ = (x,y,z) components of reaction control torque in Body Frame
$(T_{slosh.xb}, T_{slosh.yb}, T_{slosh.zb})$ = (x,y,z) components of slosh torque in Body Frame
T_{TWDp} = pitch torque on the vehicle due to the TWD effect
T_{TWDy} = yaw torque on the vehicle due to the TWD effect
I_e = nozzle inertia in plane of movement = $19102.0833\ lbf \cdot ft \cdot s^2$
M_e = nozzle mass = 694.4 lb
l_{cg} = distance from vehicle's center of gravity to nozzle pivot point
l_e = distance from nozzle pivot point to nozzle center of gravity = 1.2775 ft
$\vec{r}_s$ = vector from vehicle's center of mass to slosh fuel center of mass in Body Frame
$\vec{l}_{cg}$ = center of mass position vector of vehicle in Body Frame
$\vec{l}_{\text{tank}}$ = tank location vector in Body Frame
$\vec{l}_s$ = slosh moment arm
M_s = slosh mass
ζ_s = damping ratio of the slosh fuel dynamics
ω_s = undamped natural frequency of the slosh fuel dynamics
η = flex mode state
ζ_{flex} = damping ratio of flex modes
ω_{flex} = undamped natural frequency of flex modes
m = vehicle mass
(c_x, c_y, c_z) = (x,y,z) components of center of mass
T_s = sampling period = 0.02s
K_p = proportional gain
K_i = integral gain
K_d = derivative gain

I. Introduction

NOTE to Session Organizer/Reviewers: This draft manuscript summarizes the preliminary results obtained during an early phase of a new project for the dynamical modeling and flight control design of large flexible launch vehicles as applied to Ares-I Crew Launch Vehicle. During the next several months, a more detailed, rigorous study will be conducted in the areas of coupled dynamical modeling of propulsion, aerodynamics, guidance and control, and vehicle structure. A companion paper on flight control systems analysis and design for large flexible launch vehicles is also being submitted to the Space Exploration and Transportation GNC session.

II. Definition of Coordinate Frame

A. Geocentric Equatorial Inertial Frame or Inertial Frame

Geocentric equatorial inertial frame or simply inertial frame (Fig.1). Origin is at Earth Center. Axis z_i is normal to equatorial plane, pointing to North Pole; Axes x_i and y_i are in equatorial plane, axis x_i is along direction of vernal equinox, which is the direction of intersection of Earth equatorial plane and Sun ecliptic plane.

B. Geocentric Equatorial Rotating Frame or Central Earth Frame or Earth Frame

Geocentric equatorial rotating frame is fixed to the Earth, also called central Earth Frame (Fig.2). Origin is at Earth center. Axis z_e is normal to equatorial plane, pointing to North Pole, hence coincides with z_i . Axes x_e and y_e are in equatorial plane, with axis x_e in Greenwich meridian. This frame has angular velocity of Earth.

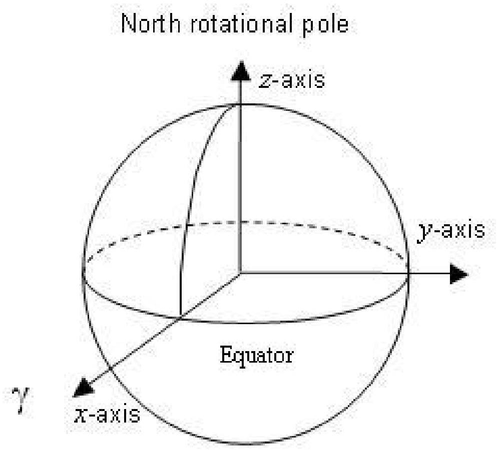

Figure 1. Inertial Frame

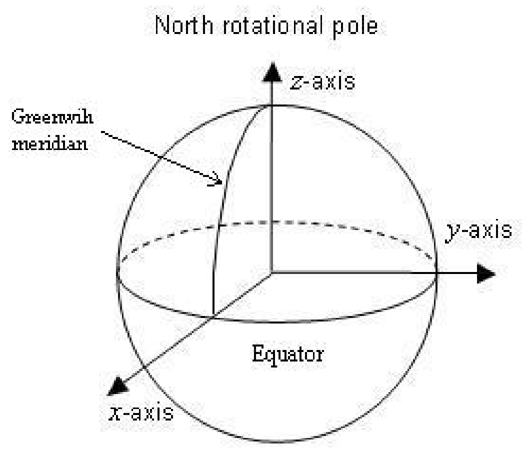

Figure 2. Earth Frame

C. Body Frame or Body-fixed Frame

Body-fixed frame, simply body frame (Fig.3), is rigidly fixed to the vehicle body. Origin is at empty vehicle center of mass; Axis x_b is along structural longitudinal axis, pointing forward; normal axis z_b is in plane of symmetry, perpendicular to x_b and pointing downward; axis y_b is perpendicular to plane of symmetry and pointing rightward.

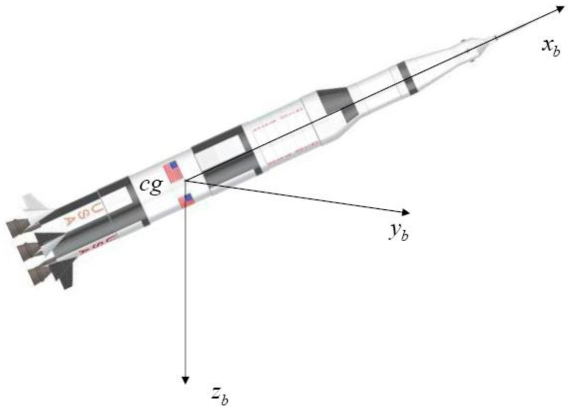

Figure 3. Body Frame

III. Aerodynamics Forces and Moments

From the state variables, we can determine the position and velocity of the vehicle in the inertial frame. And then, the geographical latitude, the height over the Earth surface, angle of attack, angle of sideslip and Mach number can be calculated. By looking up the data table, aerodynamic coefficients and base force can be found. The equations of aerodynamics forces and moments can be written as follows:

$$\vec{V}_m = \vec{V}_{rel} - \vec{V}_w = \vec{V} - \vec{\Omega}_e \times \vec{r}_{rel} - \vec{V}_w$$

$$M = \frac{V_m}{c}$$

$$Q = \frac{1}{2}\rho V_m^2$$

$$\alpha = \arctan\frac{V_{m.zb}}{V_{m.xb}}$$

$$\beta = \arctan\frac{V_{m.yb}}{\sqrt{V_{m.xb}^2 + V_{m.zb}^2}}$$

$$\begin{cases} D = C_A QS - F_{base} \\ C = C_{Y\beta}\beta QS \\ N = (C_{N0} + C_{N\alpha}\alpha)QS \end{cases}$$

Where, C_A is axial force coefficient; $C_{Y\beta}$ is side force curve slop; C_{N0} is normal force coefficient at zero angle of attack; $C_{N\alpha}$ is normal force curve slop.

$$\begin{cases} F_{aero.xb} = -D \\ F_{aero.yb} = C \\ F_{aero.zb} = -N \end{cases}$$

$$\begin{cases} T_{aero.xb} = c_y(C_{N0} + C_{N\alpha}\alpha)QS + c_z C_{Y\beta}\beta QS + C_{Mr\beta}QSb_{ref} \\ T_{aero.yb} = -c_z(F_{base} - C_A QS) + (-ARP - c_x)(C_{N0} + C_{N\alpha}\alpha)QS + (C_{Mp0} + C_{Mp\alpha}\alpha)QSc_{ref} \\ T_{aero.zb} = (-ARP - c_x)C_{Y\beta}\beta QS + c_y(F_{base} - C_\alpha QS) + C_{my\beta}\beta QSb_{ref} \end{cases}$$

Where, $C_{Mr\beta}$ is rolling moment coefficient; C_{Mp0} is pitching moment coefficient at zero angle of attack; $C_{Mp\alpha}$ is pitching moment curve slop; $C_{my\beta}$ is yawing moment curve slop.

In the SAVANT, $C_{N0} = C_{Mp0} = C_{Mr\beta} = 0$.

IV. TWD Model

Tail Wag Dog model

$$T_{TWDp} = (I_e + M_e l_{cg} l_e)\ddot{\delta}_p$$

$$T_{TWDy} = (I_e + M_e l_{cg} l_e)\ddot{\delta}_y$$

V. Flex Model

Flex state dynamics

$$\ddot{\boldsymbol{\eta}} + 2\zeta_{flex}\boldsymbol{\omega}_{\mathbf{flex}}\dot{\boldsymbol{\eta}} + \boldsymbol{\omega}^{\mathbf{2}}_{\mathbf{flex}}\boldsymbol{\eta} = \mathbf{F}^{\mathbf{T}}_{\mathbf{rkt}}\boldsymbol{\varphi}_{\mathbf{rkt}}$$

Where, $\boldsymbol{\eta}$ is the flex mode state column vector. $\mathbf{F}^{\mathbf{T}}_{\mathbf{rkt}}$ is the transpose of the rocket engine force column vector, and $\boldsymbol{\varphi}_{\mathbf{rkt}}$ is a 3 by 6 flex mode parameter matrix.

$$\mathbf{F}^{\mathbf{T}}_{\mathbf{rkt}} = (F_{rkt.xb}, F_{rkt.yb}, F_{rkt.zb})$$

Sensor error

$$\mathbf{e}_{\mathbf{angle\_flex}} = \boldsymbol{\psi}_{\mathbf{nav1}}\boldsymbol{\eta}$$

$$\dot{\mathbf{e}}_{\mathbf{rate\_flex}} = \begin{pmatrix} \boldsymbol{\psi}_{\mathbf{nav2}}\dot{\boldsymbol{\eta}} \\ \boldsymbol{\psi}_{\mathbf{nav1}}\dot{\boldsymbol{\eta}} \\ \boldsymbol{\psi}_{\mathbf{nav3}}\dot{\boldsymbol{\eta}} \end{pmatrix}$$

LOX

$$\boldsymbol{\varphi}_{\mathbf{lox\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm1}}\boldsymbol{\eta}$$
$$\dot{\boldsymbol{\varphi}}_{\mathbf{lox\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm1}}\dot{\boldsymbol{\eta}}$$
$$\ddot{\boldsymbol{\varphi}}_{\mathbf{lox\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm1}}\ddot{\boldsymbol{\eta}}$$

LH2

$$\boldsymbol{\varphi}_{\mathbf{lh2\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm2}}\boldsymbol{\eta}$$
$$\dot{\boldsymbol{\varphi}}_{\mathbf{lh2\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm2}}\dot{\boldsymbol{\eta}}$$
$$\ddot{\boldsymbol{\varphi}}_{\mathbf{lh2\_flex}} = \boldsymbol{\varphi}_{\mathbf{slm2}}\ddot{\boldsymbol{\eta}}$$

Gimbal compliance

$$\boldsymbol{\psi}_{\mathbf{rkt\_flex}} = \boldsymbol{\psi}_{\mathbf{rkt}}\boldsymbol{\eta}$$

Where, $\boldsymbol{\psi}_{\mathbf{nav1}}$, $\boldsymbol{\psi}_{\mathbf{nav2}}$, $\boldsymbol{\psi}_{\mathbf{nav3}}$, $\boldsymbol{\varphi}_{\mathbf{slm1}}$, $\boldsymbol{\varphi}_{\mathbf{slm2}}$, $\boldsymbol{\psi}_{\mathbf{rkt}}$ are 3 by 6 parameter matrix respectively.

VI. Slosh Model

The sloshing will be modeled as a spring-mass-damper system in y-z plane; we do not consider the x component. For this program, it does not include the flex model effect to the slosh model.

$$\ddot{\vec{r}}_s = -2\zeta_s\omega_s\dot{\vec{r}}_s - \omega_s^2\vec{r}_s - \{\ddot{\vec{r}}_{rel} + \dot{\vec{\omega}}\times(\vec{r}_s - \vec{l}_{cg} - \vec{l}_{\text{tank}}) + 2\vec{\omega}\times\dot{\vec{r}}_s + \vec{\omega}\times[\vec{\omega}\times(\vec{r}_s - \vec{l}_{cg} - \vec{l}_{\text{tank}})]\}$$

Matrix Form in Body Frame:

$$\begin{pmatrix} \ddot{x}_s \\ \ddot{y}_s \\ \ddot{z}_s \end{pmatrix} = -2\zeta_s\omega_s\begin{pmatrix} 0 \\ \dot{y}_s \\ \dot{z}_s \end{pmatrix} - \omega_s^2\begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} - \begin{pmatrix} a_x \\ a_y \\ a_y \end{pmatrix} - \begin{pmatrix} 0 & -\dot{r} & \dot{q} \\ \dot{r} & 0 & -\dot{p} \\ -\dot{q} & \dot{p} & 0 \end{pmatrix}\begin{pmatrix} x_{loc} - c_x - l_{\text{tank}} \\ y_s - c_y \\ z_s - c_z \end{pmatrix}$$
$$-2\begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \dot{y}_s \\ \dot{z}_s \end{pmatrix} - \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}\begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}\begin{pmatrix} x_{loc} - c_x - l_{\text{tank}} \\ y_s - c_y \\ z_s - c_z \end{pmatrix}$$

$$\begin{pmatrix} F_{slosh.xb} \\ F_{slosh.yb} \\ F_{slosh.zb} \end{pmatrix} = \begin{pmatrix} m_s\ddot{x}_s \\ m_s\ddot{y}_s \\ m_s\ddot{z}_s \end{pmatrix}$$

$$\begin{pmatrix} T_{slosh.xb} \\ T_{slosh.yb} \\ T_{slosh.zb} \end{pmatrix} = \begin{pmatrix} 0 & c_z & -c_y \\ -c_z & 0 & x_{loc} - c_x - l_{\text{tank}} \\ c_y & -x_{loc} + c_x + l_{\text{tank}} & 0 \end{pmatrix}\begin{pmatrix} F_{slosh.xb} \\ F_{slosh.yb} \\ F_{slosh.zb} \end{pmatrix}$$

VII. Rocket Model

Rocket Propulsion:

$$T = T_0 - p_0 A_e$$

The x, y and z components of the thrust in the Body Frame:

$$\begin{pmatrix} F_{rkt.xb} \\ F_{rkt.yb} \\ F_{rkt.zb} \end{pmatrix} = \begin{pmatrix} \cos\delta_p & 0 & -\sin\delta_p \\ 0 & 1 & 0 \\ \sin\delta_p & 0 & \cos\delta_p \end{pmatrix} \begin{pmatrix} \cos\delta_y & \sin\delta_y & 0 \\ -\sin\delta_y & \cos\delta_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T \\ 0 \\ 0 \end{pmatrix}$$

The x, y and z components of the torque due to the thrust in the Body Frame:

$$\begin{pmatrix} T_{rkt.xb} \\ T_{rkt.yb} \\ T_{rkt.zb} \end{pmatrix} = \begin{pmatrix} 0 & c_z & -c_y \\ -c_z & 0 & c_x - l_{rkt} \\ c_y & l_{rkt} - c_x & 0 \end{pmatrix} \begin{pmatrix} F_{rkt.xb} \\ F_{rkt.yb} \\ F_{rkt.zb} \end{pmatrix}$$

VIII. Gravity model

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\sin\phi = \frac{z}{r}$$

$$g_x = \frac{\mu x}{r^3} \left\{ \begin{aligned} &\left[-1 + \frac{3J_2 R_E^2}{2r^2}\left(3\sin^2\phi - 1\right) + \frac{4J_3 R_E^3}{2r^3}\left(5\sin^3\phi - 3\sin\phi\right) + \frac{5J_4 R_E^4}{8r^4}\left(35\sin^4\phi - 30\sin^2\phi + 3\right) \right] \\ &+ \sin\phi \left[\frac{3J_2 R_E^2}{r^2}\sin\phi + \frac{J_3 R_E^3}{2r^3}\left(15\sin^2\phi - 3\right) + \frac{J_4 R_E^4}{2r^4}\left(35\sin^3\phi - 15\sin\phi\right) \right] \end{aligned} \right\}$$

$$g_y = \frac{\mu y}{r^3} \left\{ \begin{aligned} &\left[-1 + \frac{3J_2 R_E^2}{2r^2}\left(3\sin^2\phi - 1\right) + \frac{4J_3 R_E^3}{2r^3}\left(5\sin^3\phi - 3\sin\phi\right) + \frac{5J_4 R_E^4}{8r^4}\left(35\sin^4\phi - 30\sin^2\phi + 3\right) \right] \\ &+ \sin\phi \left[\frac{3J_2 R_E^2}{r^2}\sin\phi + \frac{J_3 R_E^3}{2r^3}\left(15\sin^2\phi - 3\right) + \frac{J_4 R_E^4}{2r^4}\left(35\sin^3\phi - 15\sin\phi\right) \right] \end{aligned} \right\}$$

$$g_z = \frac{\mu z}{r^3} \left\{ \begin{aligned} &\sin\phi \left[-1 + \frac{3J_2 R_E^2}{2r^2}\left(3\sin^2\phi - 1\right) + \frac{4J_3 R_E^3}{2r^3}\left(5\sin^3\phi - 3\sin\phi\right) + \frac{5J_4 R_E^4}{8r^4}\left(35\sin^4\phi - 30\sin^2\phi + 3\right) \right] \\ &+ \cos^2\phi \left[\frac{3J_2 R_E^2}{r^2}\sin\phi + \frac{J_3 R_E^3}{2r^3}\left(15\sin^2\phi - 3\right) + \frac{J_4 R_E^4}{2r^4}\left(35\sin^3\phi - 15\sin\phi\right) \right] \end{aligned} \right\}$$

IX. Actuator Model

The actuator model is a second order system.

$$\ddot{\delta}_p + 2\zeta\omega_n\dot{\delta}_p + \omega_n^2\delta_p = \omega_n^2\delta_{pc}$$

$$\begin{pmatrix} \dot{\delta}_p \\ \ddot{\delta}_p \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n\dot{\delta}_p \end{pmatrix} \begin{pmatrix} \delta_p \\ \dot{\delta}_p \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_n^2 \end{pmatrix} \delta_{pc}$$

$$\ddot{\delta}_y + 2\zeta\omega_n\dot{\delta}_y + \omega_n^2\delta_y = \omega_n^2\delta_{yc}$$

$$\begin{pmatrix} \dot{\delta}_y \\ \ddot{\delta}_y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n\dot{\delta}_y \end{pmatrix} \begin{pmatrix} \delta_y \\ \dot{\delta}_y \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_n^2 \end{pmatrix} \delta_{yc}$$

X. Force and Momentum

In Body Frame:

$$\begin{pmatrix} F_{total.xb} \\ F_{total.yb} \\ F_{total.zb} \end{pmatrix} = \begin{pmatrix} F_{aero.xb} \\ F_{aero.yb} \\ F_{aerot.zb} \end{pmatrix} + \begin{pmatrix} F_{rkt.xb} \\ F_{rkt.yb} \\ F_{rkt.zb} \end{pmatrix} + \begin{pmatrix} F_{rcs.xb} \\ F_{rcs.yb} \\ F_{rcs.zb} \end{pmatrix} + \begin{pmatrix} F_{slosh.xb} \\ F_{slosh.yb} \\ F_{slosh.zb} \end{pmatrix}$$

To transfer the (x,y,z) components of total force from Body Frame to Inertial Frame ,by the quaternion $\begin{pmatrix} q_1 & q_2 & q_3 & q_4 \end{pmatrix}^T$.

$$\begin{pmatrix} F_{total.xi} \\ F_{total.yi} \\ F_{total.zi} \end{pmatrix} = \left[\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} + \begin{pmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \end{pmatrix}^2 \right] \begin{pmatrix} F_{total.xb} \\ F_{total.yb} \\ F_{total.zb} \end{pmatrix}$$

In Inertial Frame:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \frac{1}{m} \begin{pmatrix} F_{total.xi} \\ F_{total.yi} \\ F_{total.zi} \end{pmatrix} + \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}$$

Kinematical Equation of Rotation:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \frac{1}{\cos\theta} \begin{pmatrix} 1 & \sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Angular acceleration:

$$\begin{pmatrix} I_x & I_{xy} & I_{xz} \\ I_{xy} & I_y & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{pmatrix}\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = -\begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}\begin{pmatrix} I_x & I_{xy} & I_{xz} \\ I_{xy} & I_y & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$+\begin{pmatrix} T_{aero.xb} \\ T_{aero.yb} \\ T_{aerot.zb} \end{pmatrix} + \begin{pmatrix} T_{rkt.xb} \\ T_{rkt.yb} \\ T_{rkt.zb} \end{pmatrix} + \begin{pmatrix} T_{rcs.xb} \\ T_{rcs.yb} \\ T_{rcs.zb} \end{pmatrix} + \begin{pmatrix} T_{slosh.xb} \\ T_{slosh.yb} \\ T_{slosh.zb} \end{pmatrix} + \begin{pmatrix} 0 \\ T_{TWDp} \\ T_{TWDy} \end{pmatrix}$$

XI. Quaternion

To renormalize quaternion:

$$\begin{pmatrix} q_{1n} \\ q_{2n} \\ q_{3n} \\ q_{4n} \end{pmatrix} = (1.5 - 0.5\sqrt{q_1^2 + q_2^2 + q_3^2 + q_4^2})\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$

Where, $\begin{pmatrix} q_1 & q_2 & q_3 & q_4 \end{pmatrix}^T$ is the quaternion column vector and $\begin{pmatrix} q_{1n} & q_{2n} & q_{3n} & q_{4n} \end{pmatrix}^T$ is the quaternion column vector after renormalization.

Quaternion derivatives:

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & r & -q & p \\ -r & 0 & p & q \\ q & -p & 0 & r \\ -p & -q & -r & 0 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$

Quaternion conjugate:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}^* = \begin{pmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{pmatrix}$$

XII. Guidance Commands

XIII. Control System

Discrete PID controller for roll, pitch and yaw channels

$$u(t) = K_p e(t) + K_i Y_i(t) + K_d \dot{e}(t)$$

Discrete integration

$$Y_i(t) = Y_i(t-1) + \frac{T_s}{2}[e(t) + e(t-1)]$$

XIV. System Analysis

1. Center of Pressure

$$M_{LE} = -c_x L + M_{cg} = -x_{cp} L$$

$$x_{cp} = c_x - \frac{M_{cg}}{L}$$

Where, M_{LE} is the pitching moment relative to the pin point of the vehicle; M_{cg} is the pitching moment with respect to the center of mass, x_{cp} is the location of center of pressure.

According to the definition (force-and-moment system) of center of pressure, the plot of center of pressure can be seen in the following figure.

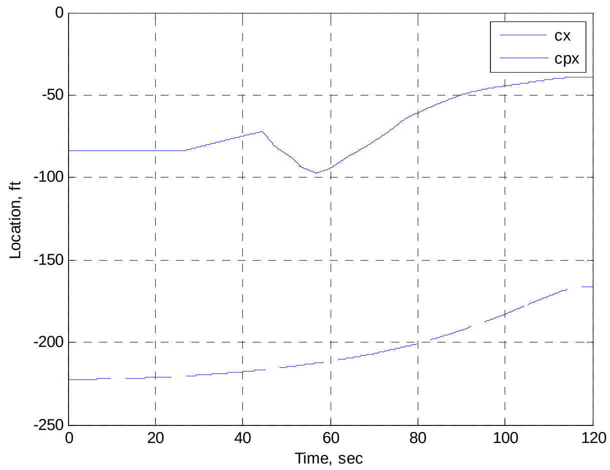

Figure 4. Location of the center of pressure center of mass

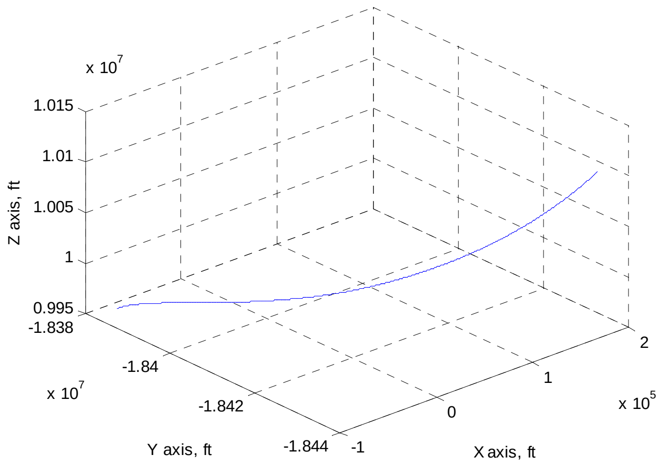

Figure 5. Vehicle position in Inertia Frame

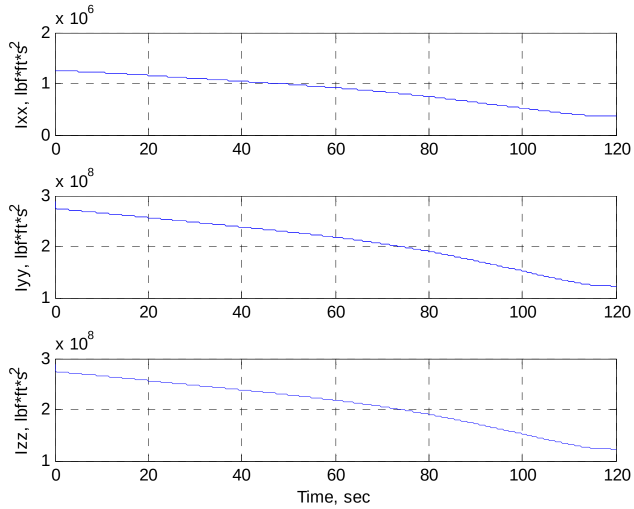

Figure 6. Moments of inertia

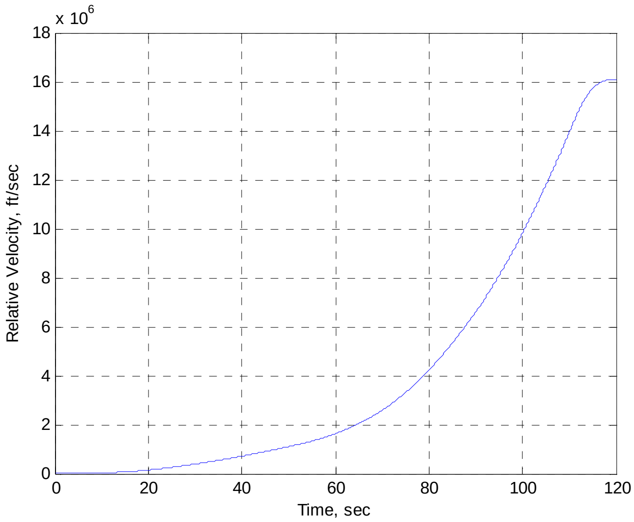

Figure 7. Relative velocity

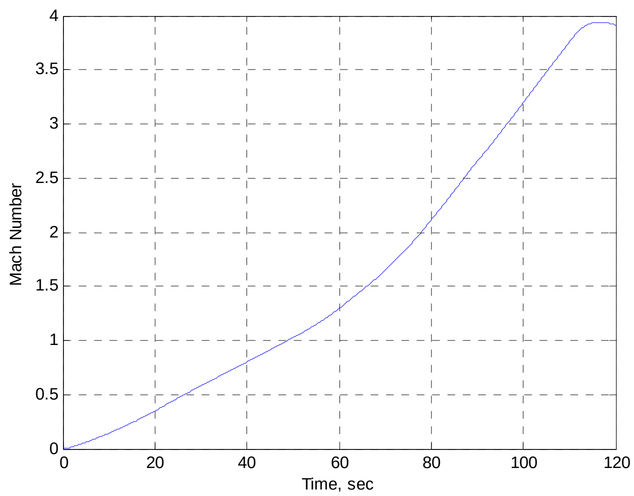

Figure 8. Mach number

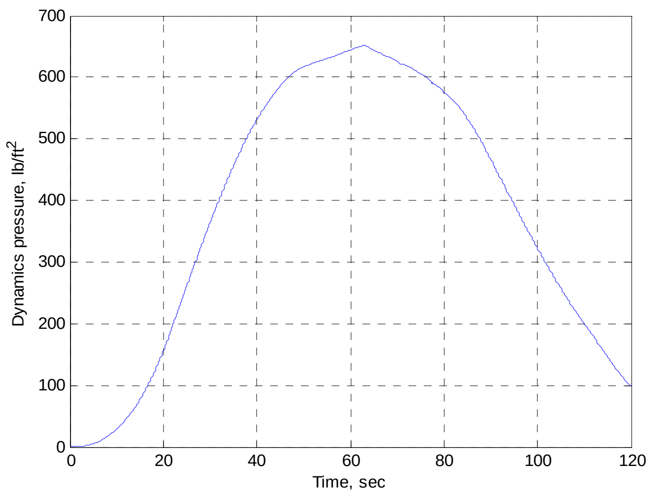

Figure 9. Dynamic pressure

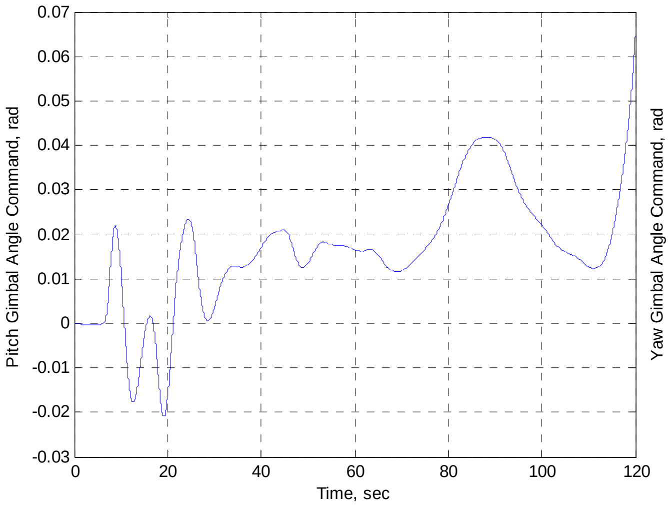

Figure 10. **Pitch gimbal angle command**

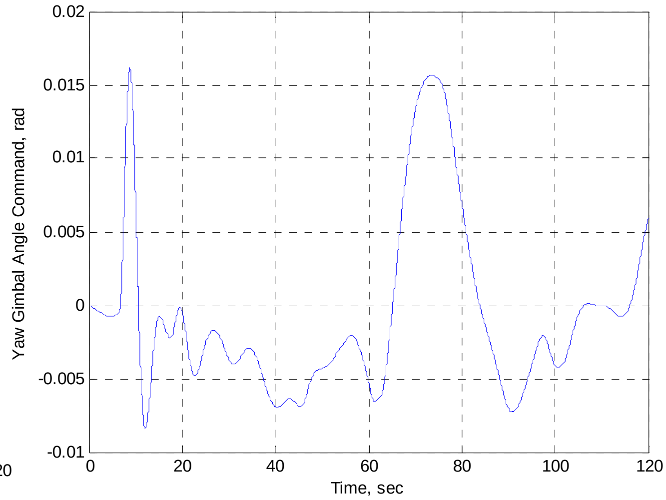

Figure 11. **Yaw gimbal angle command**

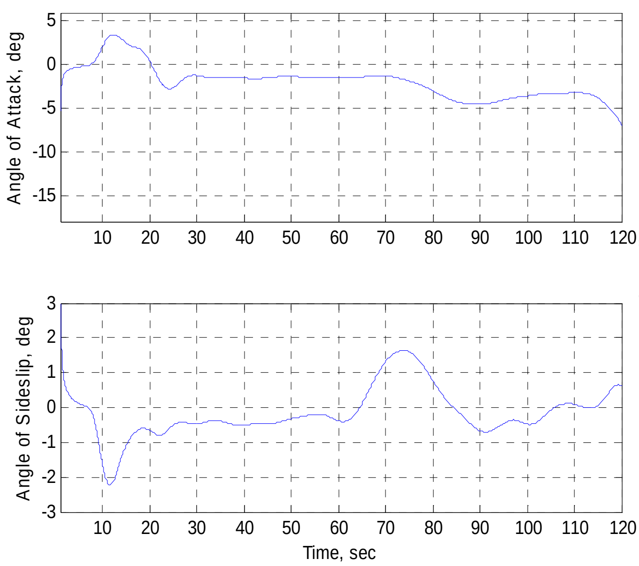

Figure 12. **Angle of attack and sideslip angle**

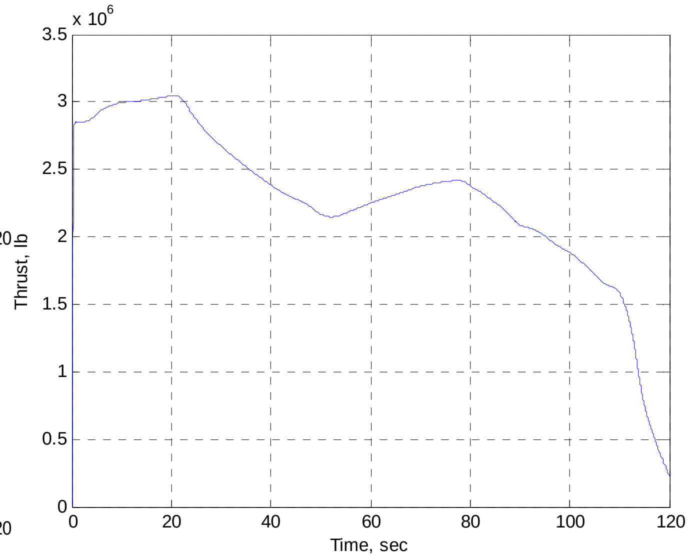

Figure 13. **Total thrust**

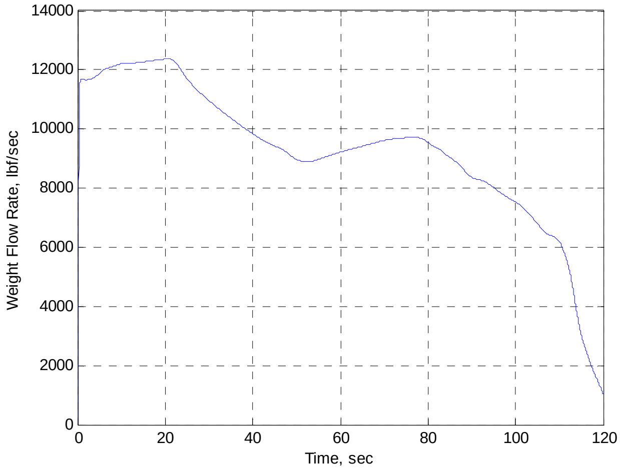

Figure. 14 Solid rocket booster weight flow rate

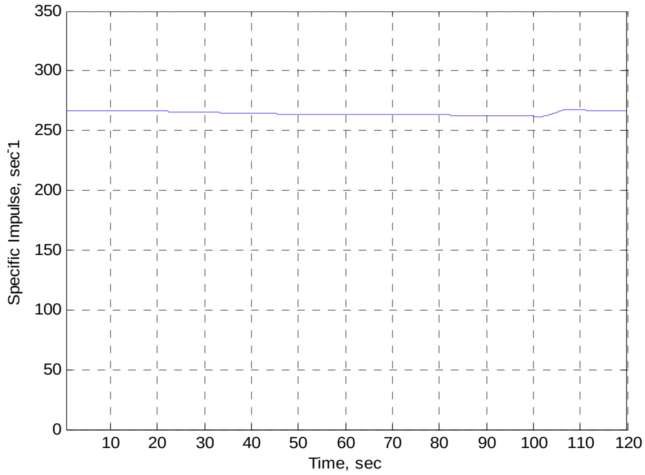

Figure 15. Solid rocket boost specific impulse

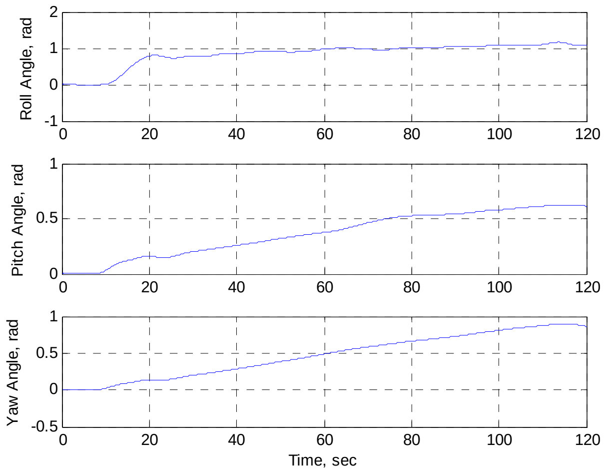

Figure 16. Euler angle

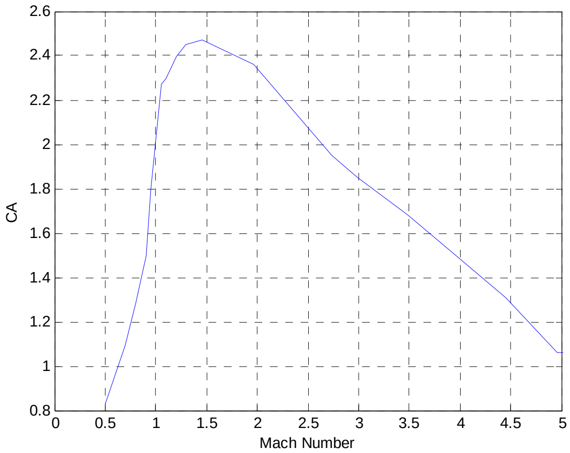

Figure 17. Axial force coefficient

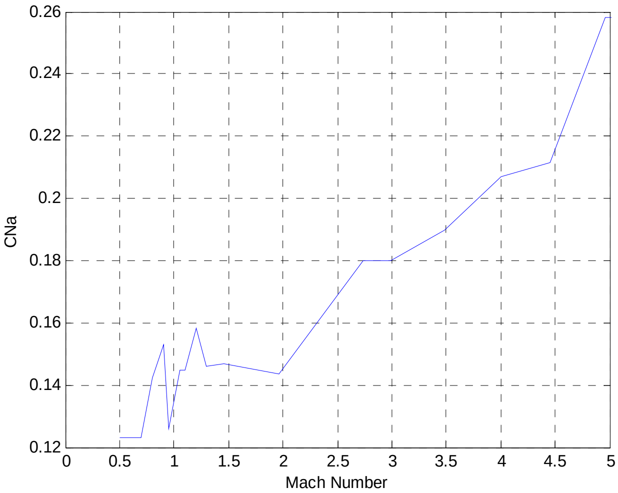

Figure 18. Normal force curve slop

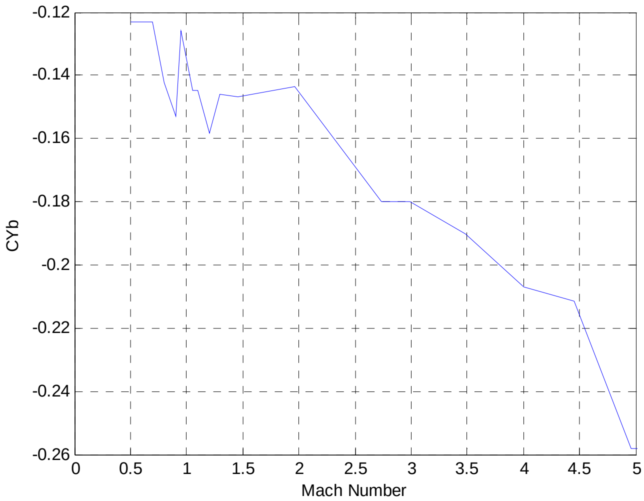

Figure 19. Side force curve slop

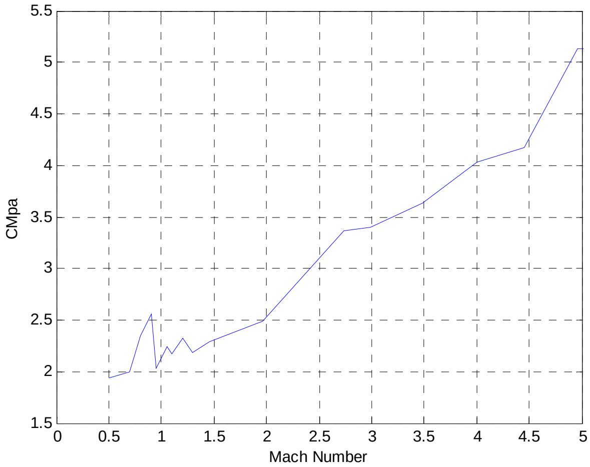

Figure 20. Pitching moment curve slop

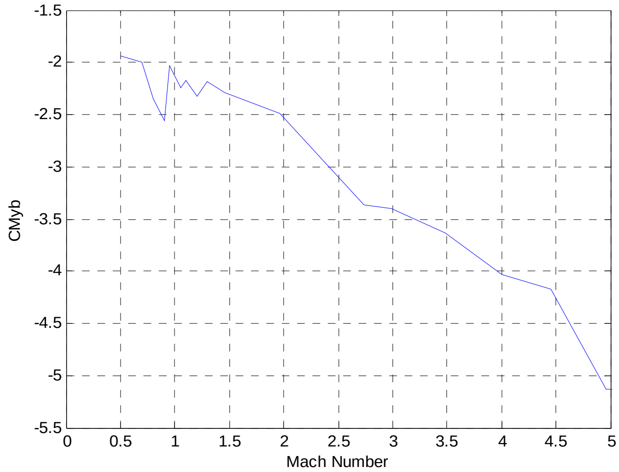

Figure 21. Yawing moment curve slop

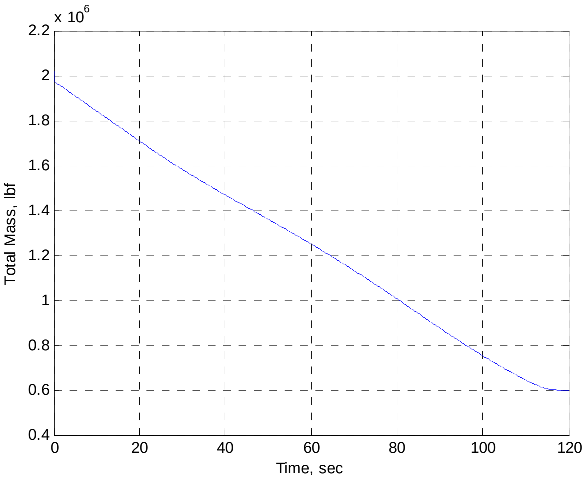

Figure 22. Total Mass

2. Stability Analysis

XV. Sample Case

$$
\boldsymbol{\varphi}_{\mathbf{rkt}} = \begin{pmatrix} 0.000000272367963 & 0.000000174392026 & -0.000000347086527 & -0.000000266173427 & 0.000000329288262 & -0.000000369058169 \\ -0.000364943105155 & 0.006281028219530 & 0.000491932740239 & -0.006259406451949 & -0.000542750533582 & -0.007673360355205 \\ 0.006281175443849 & 0.000364891432306 & -0.006260333099131 & -0.000491798506301 & 0.007676195145027 & -0.000542218216634 \end{pmatrix}
$$

Table 1. Initial States' Values

Parameter	Initial Value (unit)
U	1340.649569297657 (ft/sec)
V	6.542058392976782 (ft/sec)
W	0.244274988189246 (ft/sec)
P	0.000034916172322 (rad/sec)
Q	0.000064018398388 (rad/sec)
R	1.01643953670516e-20 (rad/sec)
X	87898.84842619013 (ft)
Y	18384832.09303243 (ft)
Z	9960462.310921878 (ft)
q1	0.359443352340896
q2	0.608859075914155
q3	0.362476166996897
q4	0.60720847366793

XVI. Conclusion

Appendix

Acknowledgments

References

[1] Betts, K. M., Rutherford R.C., MaDuffie, J., Johnson, M. D., Jackson, K., and Hall, C., "Time Domain Simulation of the NASA Crew Launch Vehicle," AIAA 2007-6621, August 2007.

[2] Betts, K. M., Rutherford R.C., MaDuffie, J., Johnson, M. D., Jackson, K., and Hall, C., "Stability Analysis of the NASA ARES I Crew Launch Vehicle Control System," AIAA 2007-6776, August 2007.

[3] Wie, B., *Space Vehicle Dynamics and Control*, AIAA Education Series, 1998.

[4] Zipfel, P. H., *Modeling and Simulation of Aerospace Vehicle Dynamics*, AIAA Education Series, 1998.

[5] Bryson, A. E., *Control of Spacecraft and Aircraft*, Princeton University Press, 1994.

[6] James, R. L., "A Three-Dimensional Trajectory Simulation Using Six Degrees of Freedom with Arbitrary Wind," NASA TN D-641, 1961.

[7] Harris, R. J., "Trajectory Simulation Applicable to Stability and Control Studies of Large Multi-Engine Vehicles," NASA TN D-1838, 1963.

[8] Fuller, D. E., and Fournier R. H., "Aerodynamic Characteristics of Several Proposed Versions of the Saturn V Launch Vehicle at Macn Numbers 1.57 to 4.65," NASA TN D-2352, 1964.

[9] Leadbetter, S. A., "Application of Analysis and Models to Structural Dynamic Problems Related to the Apollo-Saturn V Launch Vehicle," NASA TN D-5831, 1970.

[10] Curtis, H. D., *Orbital Mechanics for Engineering Students*, 1st ed., Elsevier Ltd., 2005.

[11] Ogata, K., *Modern Control Engineering*, 4th ed., Person Education, 2002.